Class Problem for Math 304-08, 3/13/2023

(1) In $\mathbb{R}^2$ I give you two bases: the standard basis $\mathcal{E}_2$ and another basis, $\mathcal{B} = \{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$.

(a) What is the basis-change ("transition") matrix from $\mathcal{E}_2$ to $\mathcal{B}$?

(b) What is the transition matrix from $\mathcal{B}$ to $\mathcal{E}_2$?

PLEASE TURN OVER FOR PROBLEM 2

(2) In $\mathcal{P}_2(x)$ I give you an ordered basis $\mathcal{A} = \{x+1, x-1, x^2\}$. In $\mathbb{R}^2$ I give you the basis $\mathcal{B} = \{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$ from Problem 1. I also give you a linear transformation $T : \mathcal{P}_2(x) \to \mathbb{R}^2$ by the formula $T(p(x)) = \begin{bmatrix}p(1)\\p(2)\end{bmatrix}$.

Problem: Find the matrix $[T]_{\mathcal{A}}^{\mathcal{B}}$ that represents T with respect to those two bases.