CLASS PROBLEM FOR MATH 304-08, 4/19/2023

Write a good reason for each answer.

(1) Is the set S an orthogonal set? Use the stated inner product.

(a) Vector space $\mathbb{R}^3$ with the dot product. $S = \{ \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \}$.

(b) Vector space $\mathcal{P}_2(x)$ with the integral inner product $\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)dx$.
$S = \{x, x^2 - 1\}$.

PLEASE TURN OVER FOR ANOTHER PROBLEM

(2) In an inner product space V, the vectors of the set $A = \{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}\}$ are an orthogonal set. Are they linearly independent? Circle one, and explain.

Yes, always   No, never   Yes and no are both possible   No idea!