MATH 304-08 QUIZ 1 1/25/2023 YOUR NAME ______________________________

(1) (10=1+2+5+2 points) Here is a linear system:

$$\begin{aligned} 2x_1 + x_3 &= 0 \\ x_1 + x_3 &= 1 \\ x_2 + x_3 &= 0 \end{aligned}$$

(a) How many variables are there in the system? <u>3 (or more!)</u>

(b) Write the augmented matrix of this system.

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(c) Solve the system using elementary row operations. Use the space below.

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

Solution: $x_1 = -1$, $x_2 = -2$, $x_3 = 2$, or

$(x_1, x_2, x_3) = (-1, -2, 2)$, or $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$.

(d) How many solutions does this linear system have? Circle the right answer. Explain why your answer is correct.

Zero <u>One</u> Two Many (i.e., more than two)

Reason: The solution is shown, there is no inconsistency, and it has no free variables.

TURN OVER FOR ANOTHER QUESTION

(2) (10=1+3+4+2 points) Here is a linear system:

$$\begin{aligned} 2x_1 + x_4 &= 0 \\ x_1 + x_3 &= 0 \end{aligned}$$

(a) How many variables are there in the system? <u>4 (or more)</u>

(b) Write the augmented matrix of this system.

$$\begin{bmatrix} 2 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(c) Solve the system using elementary row operations. Use the space below.

$$\begin{bmatrix} 2 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}.$$

x_1, x_3 are basic variables, x_2, x_4 are free variables. The new equations are

$$\begin{aligned} x_1 + \frac{1}{2}x_4 &= 0, \\ x_3 - \frac{1}{2}x_4 &= 0. \end{aligned}$$

Express $x_2 = s$, $x_4 = t$ as parameters s, t (this is optional), and solve for $x_1 = -\frac{1}{2}t$, $x_3 = \frac{1}{2}t$. The solution is
$x_1 = -\frac{1}{2}t$, $x_2 = s$, $x_3 = \frac{1}{2}t$, $x_4 = t$, with parameters s, t. Or you can write

$(x_1, x_2, x_3, x_4) = (-\frac{1}{2}t, s, \frac{1}{2}t, t)$, or $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}t \\ s \\ \frac{1}{2}t \\ t \end{bmatrix}$.

(d) How many solutions does this linear system have? Circle the right answer. Explain why your answer is correct.

Zero One Two <u>Many (i.e., more than two)</u>

Reason: There are free variables and there is no inconsistency, so there are infinitely many solutions.