Quiz 2 (not 1) for Math 304-08 2/6/2023 (not 2/3) Solutions

Remember that all answers must be fully justified by your work.

(1) (1 points) Circle the word or phrase that you think is closest in meaning to "unique".

Different Distinct Distinctive

One of a kind (Another good answer could be "Unlike all others.")

(2) (12=4+4+4 points) Here are three matrix multiplication problems. Calculate the product if possible. If it's not possible, say why. Show work on this page.

(a) $$\begin{bmatrix} 3 & 4 & -8 & -2 \\ 0 & 2 & -3 & 4 \\ 2 & 2 & 5 & 5 \\ -2 & -2 & -5 & -5 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 & 7 \\ 2 & 1 & 3 & 1 \\ 4 & -1 & -3 & -10 \end{bmatrix}$$

The product is of a 4×4 times a 3×4 matrix. Impossible. The middle numbers disagree.

(b) $$\begin{bmatrix} 3 & 4 & -2 \\ 0 & -3 & 4 \\ 2 & 2 & 5 \\ -2 & -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 & 7 \\ 2 & 1 & 3 & 1 \\ 4 & -1 & -3 & -10 \end{bmatrix}$$

The product is of a 4×3 times a 3×4 matrix. The middle numbers agree. The product exists. (I am not showing the product here because it is routine arithmetic.)

(c) $$\begin{bmatrix} 3 & 4 & -2 \\ 0 & -3 & 4 \\ 2 & 2 & 5 \\ -2 & -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 & 7 \\ 2 & 1 & 3 & 1 \\ 4 & -1 & -3 & -10 \end{bmatrix}^T$$

The product is of a 4×3 times a 4×3 matrix (after transposing). Impossible. The middle numbers disagree.

TURN OVER FOR MORE QUESTION(S)

(3) (10+5 points) Consider the line $\overline{PQ}$ in $\mathbb{R}^3$ that passes through the points $P(1,1,1)$ and $Q(1,2,3)$.

(a) Find a parametric equation of the line $\overline{PQ}$.

(b) Is the point $U(4,3,2)$ on the line $\overline{PQ}$?

(a) Solution. (This is as in the beginning of Section 3.4.) We need a direction vector for the line. This is $\overrightarrow{PQ}$. To get it we note that the position vector $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$ so $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$. The position vector of a point gives the position:

$$\overrightarrow{OP} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \overrightarrow{OQ} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{so} \quad \overrightarrow{PQ} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

We combine the direction vector with a vector in the line to get an arbitrary point $R(x_1, x_2, x_3)$ in the line. The equation is $\overrightarrow{OR} = \overrightarrow{OP} + t\overrightarrow{OQ}$ for some scalar $t \in \mathbb{R}$. (Different values of t give different points on the line.) Thus, the equation is

$$\overrightarrow{OR} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

(b) Solution. For U to be on the line, there must be a value of t that gives the vector $\overrightarrow{OU} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$. That means we must solve

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

This is three linear equations:

$$\begin{aligned} 1 + 0t &= 4, \\ 1 + 1t &= 3, \\ 1 + 1t &= 2. \end{aligned}$$

The first equation, which simplifies to $1 = 4$, already shows that there is no solution; therefore, U is not on the line.