QUIZ 4 FOR MATH 304-08, 2/27/2023 YOUR NAME ______________________________

Show all necessary reasoning and work for full credit.

(1) (2+6+6+6 points) Consider the set $S = \{ \begin{bmatrix}1\\4\\3\end{bmatrix}, \begin{bmatrix}5\\3\\2\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix} \}$ in the vector space $\mathbb{R}^3$.

(a) What is the dimension of the vector space? (No work needed.)
Answer: 3.

(b) Is the set linearly dependent?
Answer: Yes. There are several possible reasons.
Reason 1. The zero vector is in the set.
Reason 2. There is a linear combination of the vectors that $= \mathbf{0}$, but with coefficients that are not all 0. For example:

$$0\begin{bmatrix}1\\4\\3\end{bmatrix} + 0\begin{bmatrix}5\\3\\2\end{bmatrix} + 5\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

(c) Does the set span the vector space?
Answer: No. There are several possible reasons.
Reason 1. S has 3 vectors, and $\dim \mathbb{R}^3 = 3$. If S spans $\mathbb{R}^3$ and has 3 vectors, it is a basis (by a theorem) and therefore is linearly independent (by a theorem), but we found it is not linearly independent. Therefore, it can't span the vector space.

Reason 2. The vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$ is not in the span of S. To prove this we solve

$$c_1\begin{bmatrix}1\\4\\3\end{bmatrix} + c_2\begin{bmatrix}5\\3\\2\end{bmatrix} + c_3\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix},$$

as follows. Set up the augmented matrix and reduce (I skip steps to save space):

$$\begin{bmatrix}1&5&0&1\\4&3&0&1\\3&2&0&1\end{bmatrix} \to \begin{bmatrix}1&5&0&1\\0&-17&0&-3\\0&-13&0&-2\end{bmatrix} \to \begin{bmatrix}1&5&0&1\\0&1&0&3/17\\0&1&0&2/13\end{bmatrix} \to \begin{bmatrix}1&5&0&1\\0&1&0&3/17\\0&0&0&2/13-3/17\end{bmatrix}.$$

The last row has a pivot position in the constants column, so the system is inconsistent. Therefore, there do not exist values of c_1, c_2, c_3 that give $\begin{bmatrix}1\\1\\1\end{bmatrix}$ as a linear combination of the vectors in S.

Reason 3. Find the set of vectors that are in the span of S, by finding all vectors $\mathbf{x} = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ that are linear combinations of S. You will find that x_1, x_2, x_3 must satisfy a linear relation, so not every vector in $\mathbb{R}^3$ is a linear combination of S. I omit the details!!

(d) Is the set a basis for the vector space?
Answer: No. It is linearly dependent so it can't be a basis for a vector space.