QUIZ 5 FOR MATH 304-08, 3/1/2023 YOUR NAME _____

Show all necessary reasoning and work for full credit.

(1) (6 points) Find the coordinate vector, $[p(t)]_{\mathcal{B}}$, of $p(t) = 5t^0 + 2t - 7t^2$ with respect to the standard basis $\mathcal{B} = \{t^0, t_1, t^2\}$ of $\mathcal{P}_2(t)$.

(2) (4+6 points) A linear transformation $T : \mathcal{P}_2(t) \to \mathbb{R}^3$ is given by the rule

$$T(a_0t^0 + a_1t + a_2t^2) = \begin{bmatrix} a_0 - a_2 \\ 2a_2 \\ a_1 + a_0 \end{bmatrix}.$$

(a) Evaluate $T(3t - 5t^2)$.

(b) Find the coordinate vector, $[T(3t - 5t^2)]_{\mathcal{E}}$, [OMIT: of $p(t) = 5t^0 + 2t - 7t^2$] with respect to the standard basis $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ of $\mathbb{R}^3$.

TURN OVER FOR MORE QUESTION(S)

(3) (8+8+4 points) For the same linear transformation $T(a_0t^0+a_1t+a_2t^2) = \begin{bmatrix} a_0 - a_2 \\ 2a_2 \\ a_1 + a_0 \end{bmatrix}$ as in the previous problem:

(a) Is T one-to-one (injective)?

(b) What is the range of T?

(c) Is T surjective?