QUIZ 6 FOR MATH 304-08, 3/10/2023 YOUR NAME _____

Show full justification of every answer.

(1) (15 points) A is an $m \times n$ matrix whose nullity is 55 and whose rank is 20.
   (a) (5 points) Is it possible to find the value of n? If yes, what is n?
   (b) (5 points) Is it possible to find the value of m? If yes, what is m?
   (c) (5 points) If I reveal that the linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ defined by $T(\mathbf{x}) = A\mathbf{x}$ is surjective, can you find the value of m? If yes, what is m?

Extra space for problem 2 work:

TURN OVER FOR QUESTION 2

(2) (25 points) For the linear transformation $T : \mathcal{P}_2(x) \to \mathbb{R}^4$ defined by $T(p(x)) = \begin{bmatrix} p(0) \\ p(1) \\ p(2) \\ p(3) \end{bmatrix}$:

(a) (5 points) What is $T(2 - x - x^2)$?
(b) (10 points) Find the kernel ("null space") of T.
(c) (3 points) Is T injective (one-to-one)?
(d) (7 points) Are the values of T on the standard basis of the domain—that is, the values $T(1)$, $T(x)$, $T(x^2)$—linearly dependent or independent?