**Show full justification of every answer.**

(1) (10+5 points) Here is a matrix $A = \begin{bmatrix} 0 & 1 & 2 & 4 \\ 3 & 0 & 7 & 8 \\ -3 & 0 & -7 & 0 \\ -4 & 0 & 1 & 1 \end{bmatrix}$.

(a) Find the determinant $|A|$. Hint: Look for the easiest way to do it.
**Solution.** Expand in column 2:

$$|A| = (-1)^{1+2} 1 \begin{vmatrix} 3 & 7 & 8 \\ -3 & -7 & 0 \\ -4 & 1 & 1 \end{vmatrix},$$

then add row 2 to row 1 (which does not change the determinant) and expand in row 1:

$$= - \begin{vmatrix} 0 & 0 & 8 \\ -3 & -7 & 0 \\ -4 & 1 & 1 \end{vmatrix} = -(-1)^{0+2} 8 \begin{vmatrix} -3 & -7 \\ -4 & 1 \end{vmatrix} = -8[(-3)(1) - (-7)(-4)] = 8 \cdot 41 = 324.$$

(b) Use your answer to part (a) to decide whether 0 is an eigenvalue of $A$.
**Solution.** Since $|A| \neq 0$, 0 can't be an eigenvalue.

(2) (15 points) Here is a matrix $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$. Find the eigenvalues of $B$ and their eigenspaces.

**Solution.** $\begin{vmatrix} 2-\lambda & 0 \\ 3 & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda)$ so the eigenvalues are 1 and 2.

For eigenvalue 1 we want the null space of $B - I = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$. The reduced echelon form is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, which means an eigenvector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ must satisfy $x_1 + 0x_2 = 0$, i.e., $x_1 = 0$; that is, the eigenspace is $\{\begin{bmatrix} 0 \\ x_2 \end{bmatrix} : x_2 \in \mathbb{R}\} = \text{Span}\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$.

For eigenvalue 2 we want the null space of $B - 2I = \begin{bmatrix} 0 & 0 \\ 3 & -1 \end{bmatrix}$. The reduced echelon form is $\begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$, which means an eigenvector $\mathbf{x}$ must satisfy $3x_1 - x_2 = 0$, i.e., $x_2 = 3x_1$; that is, the eigenspace is $\{\begin{bmatrix} x_1 \\ 3x_1 \end{bmatrix} : x_1 \in \mathbb{R}\} = \text{Span}\{\begin{bmatrix} 3 \\ 1 \end{bmatrix}\}$.