MATH 330-05 Midterm Test Your name ____________________________________

- There are 2 pages with 10 questions.
- The exam is open book but closed notes. Small notes in your book are allowed; extensive notes in your book are not allowed.
- Show all answers and work in the blue books. Show all the work necessary to solve the problem.
- Each problem goes with a section (or more than one). You must cite every result you use (for that problem) in those sections (but not in other sections).
- Hand in blue books and this paper.

(1) (10 points) (Sect. 1.1–2) Prove that, if $i|m$ and $i|n$, then $i|(m-n)$.

(2) (5 points) (Sect. 1.1–2) For which integers m is it true that $0|m$? (Prove.)

(3) (10 points) (Sect. 2.2) Let $x, y \in \mathbb{Z}$. Prove that $x, y > 1 \implies xy > 1$.

(4) (5 points) (Sect. 4.2) Assume there are numbers a_i for all $i \in \mathbb{Z}$. Write a correct definition of $\sum_{i=m}^{n} a_i$ and state for which pairs (m, n) $(m, n \in \mathbb{Z})$ your definition is valid. Make this as many pairs as possible.

(5) (10 points) (Sect. 3.1–3) Negate this logical statement; simplify the negation as much as possible:

All apples are sweet if and only if some bananas are green.

(6) (15 points) (Sect. 4.2 and 2.3) Prove by induction that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.

Your proof should meet the standards for an induction proof as presented in class.

(7) (20 = 3+7+6+4 points) (Sect. 3.1–3, 5.1–2) A and B are subsets of the universe X.

(a) Formulate in logical form, using the definitions of set equality and containment, with any necessary quantifiers ($\forall$, $\exists$) and logical terms (and, or, not, $\implies$, $\iff$), the following statement: $(A^c)^c = A$. (Don't prove it.)

(b) Prove that $A \subseteq B \implies B^c \subseteq A^c$.

(c) State the (A) converse, (B) contrapositive, and (C) inverse of the statement in part (7b).

(d) Use the facts stated in parts (7a) and (7b) to prove the converse of (7b), without using any set-theory reasons.

TURN OVER FOR MORE PROBLEMS!

(8) (5 points) (Sect. 6.3) In $\mathbb{Z}_9$:

(a) Calculate $[4] + [13]$ and $[4] \cdot [7]$. Express your answers in the form $[s]$ where $0 \leq s \leq 8$.

(b) Does $[4]$ have a multiplicative inverse? If so, what is it?

(9) (5 points) (Sect. 6.3) Find all integers x that satisfy the equation $3x + 2 \equiv 0 \pmod{12}$.

(10) (15 points) (Sect. 6.1) Define the following relation on $A =$ the set of integers ≥ 0: $m \sim n$ if $m = n$ or $mn = 6$.

(a) Prove this is an equivalence relation.

(b) Find the equivalence class of every integer $n \in A$.