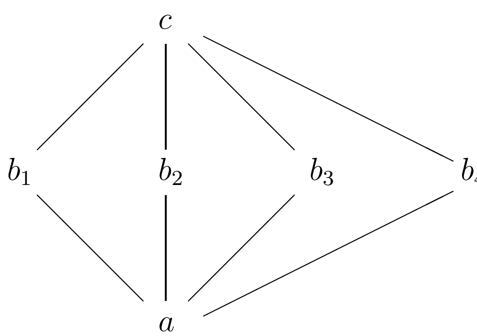
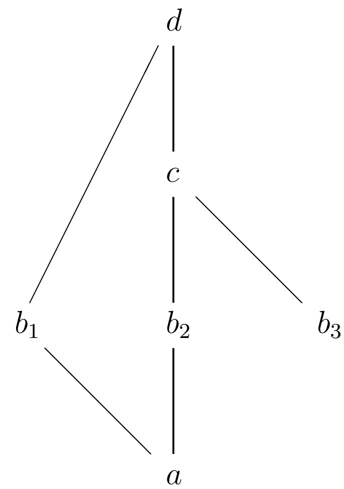
Math 386 Quiz 2 November 2, 2022 Your name ______________________________

- **Show all your work**. Show it on this paper.
- No electronics, no notes, no book.

I made the Hasse diagrams of two posets, P and Q.



P



Q

(1) (8 pts) Compute the Möbius function values $\mu(a, y)$ for all y in the poset P. Solution:

$\mu(a, a) = 1$ by the definition of μ.

$$\mu(a, b_i) = -\sum_{z:a\leq z<b_i} \mu(a, z) = -\mu(a, a) = -1 \text{ for all } b_i.$$

$$\mu(a, c)) = -\sum_{z:a\leq z<c} \mu(a, z) = -[\mu(a, a) + \mu(a, b_1) + \mu(a, b_2) + \mu(a, b_3) + \mu(a, b_4)]$$

$$= -[1 - 1 - 1 - 1 - 1] = 3.$$

(2) (8 pts) Compute the Möbius function values $\mu(a, y)$ for all y in the poset Q. Solution:

$\mu(a, a) = 1$.

$$\mu(a, b_i) = -\sum_{z:a\leq z<b_i} \mu(a, z) = -\mu(a, a) = -1 \text{ for } b_1 \text{ and } b_2.$$

$\mu(a, b_3) = 0$ because $a \not\leq b_3$.

$$\mu(a, c) = -\sum_{z:a\leq z<c} \mu(a, z) = -[\mu(a, a) + \mu(a, b_2)] = -[1 - 1] = 0.$$

$$\mu(a, d) = -\sum_{z:a\leq z<d} \mu(a, z) = -[\mu(a, a) + \mu(a, b_1) + \mu(a, b_2) + \mu(a, c)]$$

$$= -[1 - 1 - 1 + 0] = 1.$$

b_3 is not one of the z's because $b_3 \not\geq a$.