Math 386 Quiz 4 December 5, 2022 Solutions

(1) (4 pts) What is the difference between a sequence and a generating function?

A sequence is a list. A generating function is a sum, specifically it is a power series.

(2) (4 pts) If I ask you for the sequence $h_n = c_1(-3)^n$ that satisfies $h_0 = 1$, what is your answer?

$h_0 = c_1(-3)^0 = c_1$, so $c_1 = 1$. Thus, $h_n = (-3)^n$. The sequence is

$$1, -3, 9, -27, \ldots, (-3)^n, \ldots,$$

or

$$1, -3, (-3)^2, (-3)^3, \ldots, (-3)^n, \ldots$$

(both are the same sequence; there are various ways to write it). Notice that the terms are separated by commas.

(3) If I ask you for the generating function of the same sequence $h_n = c_1(-3)^n$ with $h_0 = 1$, what is your answer?

(a) (4 pts) Before simplifying the generating function.

$$\sum_{n=0}^{\infty}(-3)^n x^n \quad \text{or} \quad \sum_{n=0}^{\infty}(-3x)^n$$

or

$$1 - 3x + 9x^2 - 27x^3 + \cdots + (-3x)^n + \cdots$$

(various ways to write it). Notice that the terms are added.

(b) (4 pts) After simplifying the generating function.

$$\frac{1}{1+3x}$$

by the geometric series or Newton's Binomial Theorem.