

Contents

Preface	vii
1. The Basics	1
1.1 Graphs*	2
1.2 The degree of a vertex*	5
1.3 Paths and cycles*	6
1.4 Connectivity*	10
1.5 Trees and forests*	13
1.6 Bipartite graphs*	17
1.7 Contraction and minors*	18
1.8 Euler tours*	22
1.9 Some linear algebra	23
1.10 Other notions of graphs	28
Exercises	30
Notes	32
2. Matching, Covering and Packing	33
2.1 Matching in bipartite graphs*	34
2.2 Matching in general graphs ^(*)	39
2.3 Packing and covering	44
2.4 Tree-packing and arboricity	46
2.5 Path covers	49
Exercises	51
Notes	53

* Sections marked by an asterisk are recommended for a first course.
Of sections marked ^(*), the beginning is recommended for a first course.

3. Connectivity	55
3.1 2-Connected graphs and subgraphs*	55
3.2 The structure of 3-connected graphs ^(*)	57
3.3 Menger's theorem*	62
3.4 Mader's theorem	67
3.5 Linking pairs of vertices ^(*)	69
Exercises	78
Notes	80
4. Planar Graphs	83
4.1 Topological prerequisites*	84
4.2 Plane graphs*	86
4.3 Drawings	92
4.4 Planar graphs: Kuratowski's theorem*	96
4.5 Algebraic planarity criteria	101
4.6 Plane duality	103
Exercises	106
Notes	109
5. Colouring	111
5.1 Colouring maps and planar graphs*	112
5.2 Colouring vertices*	114
5.3 Colouring edges*	119
5.4 List colouring	121
5.5 Perfect graphs	126
Exercises	133
Notes	136
6. Flows	139
6.1 Circulations ^(*)	140
6.2 Flows in networks*	141
6.3 Group-valued flows	144
6.4 k -Flows for small k	149
6.5 Flow-colouring duality	152
6.6 Tutte's flow conjectures	156
Exercises	160
Notes	161

7. Extremal Graph Theory	163
7.1 Subgraphs*	164
7.2 Minors ^(*)	169
7.3 Hadwiger's conjecture*	172
7.4 Szemerédi's regularity lemma	175
7.5 Applying the regularity lemma	183
Exercises	189
Notes	192
8. Infinite Graphs	195
8.1 Basic notions, facts and techniques*	196
8.2 Paths, trees, and ends ^(*)	204
8.3 Homogeneous and universal graphs*	212
8.4 Connectivity and matching	216
8.5 The topological end space	226
Exercises	237
Notes	244
9. Ramsey Theory for Graphs	251
9.1 Ramsey's original theorems*	252
9.2 Ramsey numbers ^(*)	255
9.3 Induced Ramsey theorems	258
9.4 Ramsey properties and connectivity ^(*)	268
Exercises	271
Notes	272
10. Hamilton Cycles	275
10.1 Simple sufficient conditions*	275
10.2 Hamilton cycles and degree sequences*	278
10.3 Hamilton cycles in the square of a graph	281
Exercises	289
Notes	290

11. Random Graphs	293
11.1 The notion of a random graph*	294
11.2 The probabilistic method*	299
11.3 Properties of almost all graphs*	302
11.4 Threshold functions and second moments	306
Exercises	312
Notes	313
12. Minors, Trees and WQO	315
12.1 Well-quasi-ordering*	316
12.2 The graph minor theorem for trees*	317
12.3 Tree-decompositions	319
12.4 Tree-width and forbidden minors	327
12.5 The graph minor theorem ^(*)	341
Exercises	350
Notes	354
A. Infinite sets	357
B. Surfaces	361
Hints for all the exercises	369
Index	393
Symbol index	409