

**ERRATA FOR  
BECK AND ROBINS  
CHAPTERS 1–6**

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**N.B. Not proofread.**

1. CONTENTS AND PREFACE

**Contents Part II title.** It shouldn't appear at or near the bottom of a page. Maybe your publisher can help with this, but watch it.

**Contents 9.1.** Delete quotation marks and use text style.

**P. 2.** I suggest the word “scaffold” is what you mean, not scaffolding. but they might both work.

**P. 2.** “refined hindsight”: I think you mean “refined insight”; how it could be hindsight is a mystery!

**P. 3.** ‘a given number of’ coin denominations: Shorten to “given”

**P. 4.** “pausing at a fixed dilation”: I think you mean “pausing at fixed dilations” since you would use more than one.

**P. 4, ¶2.** I don't understand what you're trying to say, especially when I put myself in the shoes of the reader who hasn't already read the book. You might explain more detail, without being technical. First, what are “Ehrhart polynomials in all dimensions”? Second, change “, and these” to “. These”. Third, whenever you put something in quotation marks that isn't a direct quote, it probably means you're avoiding doing the work of saying what you mean, so instead of “ ‘error terms’ ”, just explain what you meant. But do make sure this will be meaningful to someone who hasn't begun the book yet.

**P. 5, ¶1.** “steps that are given by”: you mean “implied by”.

**P. 5, ¶2.** “tool that allows us” (singular subject).

**P. 5.** In the middle of the page you need some transition to indicate you're starting a whole new topic; namely, after an overview of the subject, you're beginning a survey of the book. You could simply put a row of three \*, or you could have a centered title. (In the former case there should be a transitional sentence opening the new topic.) The space you leave is not enough of a marker for me.

**P. 5.** “explicate upon” is (a) not grammatical, (b) not what you mean. What do you mean, anyhow? You might mean “expatiate upon” (which will sound stuffy) or “explicate” (which will sound a bit fancy but maybe not too much so), or find another way to say it.

**P. 6.** “we are now ready to learn”: delete “now”, you just said it a couple of lines ago and it's repetitious.

**P. 7, top.** Don't repeat “of a polytope” so soon.

“studies” should be “studied”.

What are those quotation marks really doing? Can you say what you mean instead of (hint, hint)ing via quotation marks?

**P. 7, after the bulleted list.** It seems that the space is intended to mark a transition of topic, as did the last one, and I have the same objection: it will not help the reader. Use the \*\*\* or (my preference) a centered title.

**P. 7, first ¶ after space.** I don't see what this paragraph is doing here. It seems to belong earlier, in the first or second part of the intro. Think about where it fits best.

“(triangulated” should be “(in triangulated”); and “(computer” should be “(in computer”); don't squeeze out those little words that express the relationships between things you mention.

**P. 7.** List of recommended books: What the h— is this for? If you think each book has some value to the reader of this book, either as supplementary reading or as follow-up advanced learning or whatever, then tell the poor, naive, innocent reader why s/he might want to read it and/or why you heartily recommend it. Be specific. For instance, if a book provides a different viewpoint but is elementary, then say that (and say what the different viewpoint is).

You can tell that I find unexplicated lists of references very annoying!

**P. 7, transition.** I think there's another major transition of topic after that list. Your intro seems to have 4 parts: the intro, the survey of contents, the list of other readings, the acknowledgements. Do make that clear.

## 2. CHAPTER 1

Many places: “partial fractions expansion” should be “partial-fraction expansion” (add hyphen, remove plural). But you are “expanding in partial fractions”.

**P. 13, lines -10, -7, and more.** You are inconsistent in use of “change” vs. “exchange”. I believe you mean “change” as when you change money of certain denominations for money of other denominations. “Exchange” doesn't have that meaning.

**P. 7, line -10.** “ $n$ -th dilation” should be “ $n$ -th dilate” (and maybe elsewhere). Strictly speaking, “dilation” is a process, “dilate” is the result of the process. We often say “dilation” for the result, but I think here especially, in the definition, that's not a good idea.

**P. 16, display with  $\xi_a$ .** The comma should be semicolon.

**P. 17 and many, many places in the book.** All displayed formulas should be punctuated as if they were in text. Please check every one. (You're not consistent.) Look at (1.4): it has several parts, which if you were writing them in line, you would separate by commas. So, separate them by commas. You'll have to go through the entire book checking every display. It will be a nuisance. But otherwise I won't be happy. Don't you want to make me happy? (Don't answer.)

**P. 17, line 1.** “partitions, the” seems to me to be better as “partitions: The” because you're reintroducing not the one sentence but several paragraphs.

**P. 17, line 6.** End with a colon. (Again, punctuate displayed formulas exactly as if they were not displayed.)

**P. 19, top.** In the discussion of likelihood and probability, it makes no sense as stated. Your “probability” is an integer. It can't possibly approach  $\infty$ . This seems to be more of an expectation, but I don't see how to formulate it; rather, I suggest you define the probability as  $p_{a,b}(n)/n$  since  $n$  is the length of the interval. This seems to make sense, but please verify that it really is  $\leq 1$ .

**P. 20, (1.9).** I think this can be simplified. I believe I wrote this to you in an e-mail, but here's the summary: Make (1.10) into Lemma A.  $p_{a,b}(ab - a - b) = 1 - p(a + b)$  by Lemma A.  $p(a + b) = 1$  since one can only have a representation  $1a + 1b$  (actually, this proof without Popoviciu is a good exercise) (this part of the proof is due to student Garry Bowlin). QED

**P. 20, line -3.** Not by (1.10). That applies only if  $a \nmid n, b \nmid n$ . This could be an exercise (proof for all  $n < ab$ ).

**P. 20, line -1.** What is "above corollary"? No idea!

**P. 22, Ex. 1.7.** Replace "Here are" by "The table shows" and make these monster formulas into a table. Reason: you'll get better spacing. All that white space is no good—it means poor typesetting. (You might number the table. Your publisher might fix this, or might not.)

**P. 23, Note 1.** You might replace "go a little deeper" by "dig ...".

**Note 4.** The paper of P. is not a reference to this result, it is the result. More accurately, you should replace "reference to" by "appearance of". And in the next line, "is in a paper".

**Note 5.** "given for" is not English. "given of"

**P. 25, Exercises.** Should we be assuming that in every problem,  $(a, b) = 1$  (if both are integers) unless otherwise stated? You do have to do something to clear this up. The students were rather confused.

**1.4(e).** "often" sounds silly. Nowadays, it simply is, so replace by "which is".

**1.5.** "Rewrite in terms ... function as many ... as you .... (Reads much better; the syntax was confusing.)

**1.7.** Another sterling example of missing punctuation.

**P. 26, line -1.** Insert at end: "then". (I had to analyze this to decide what you meant. I generally recommend using "then" more, rather than less.)

**1.14.** "the lattice points"

**1.15.** (I think I wrote you about this already.) I think it was also Garry Bowlin who pointed out that you don't get  $-1$ . He suggested you might have meant  $n \in \mathbb{P}$ . Anyway, you'd better check this.

Also, what does "two such sequences" mean? I'm sure you mean "Such a pair of sequences" since it's the pair that makes Beatty sequences, not one sequence.

**1.16.** Your  $\binom{a}{b}$ , etc., look exactly like binomial coefficients. Please do something to avoid this problem. I can think of several possibilities, one of which is simply to write them as ordered pairs, horizontally, as we usually do.

Also, note that the parentheses around  $\binom{e}{f}$  are too big.

**1.22.** Replace "conclude" by "deduce". (More specific.)

**1.25 and anywhere else it might come up.** " $k$ -representable" has the wrong dash. You need the hyphen, not en dash. This is standard typography; just like  $k$ -dimensional,  $k$ -element set.

**1.25(e).** Run the bracketed expression into the line. (The line break is probably a typo; I'm sure it doesn't belong.)

**1.28.** Again, punctuate: you need a comma after the first line of the displayed formula, where you have cases. (This falls under the usual rule of punctuation: treat it as if it were continuous text.)

**1.33.** The first statement suggests that you consider *all*  $a, b > 0$ . But here you must assume relative primality.

**1.35.** "where the  $c_{k,j}$ "

In ¶3, line 1: make the subject “set” and verb “form” agree.  
 Step 2, hint: “Long”  
 Step 3: punctuate the displayed formula. (The usual rule.)  
**1.37.** “There are” (you have several bound mentioned).

### 3. CHAPTER 2

**P. 35, line 3.** The  $t$ th dilate has one number, its lattice point count. At least, pluralize “dilate”; I’d suggest something like “ $t$ th dilates,  $t > 0$ , of”.

**(2.1).** I advise against using  $x$  as the variable here because you typically use it as the g.f. indeterminate. Maybe  $z$  (since it can be complex)?

You also need the definition for  $n = 0$ .

**P. 35, line -8.** Delete “incidentally”. Too many words.

**P. 35, line -3.** New ¶ at “In our case” since you’re beginning a new thought, i.e., about □.

**P. 35.** Notation for Eulerian numbers: I strongly request  $A(d, k)$  as in Stanley, Comtet (a classic of old-style enumerative combinatorics), many others. The giant angle brackets are very ugly (opinion!) and are not so very common. I also think that the few kinds of brackets should be reserved for very special uses, as in binomial coefficients  $\binom{\phantom{x}}{\phantom{x}}$  and Gaussian numbers  $\lfloor \phantom{x} \rfloor$ , which are analogous (to each other) and exceptionally special.

**(2.2).** Definition is invalid for  $d = 0$ . The ranges of summation should be  $j \geq 0$  and  $k \geq 0$ .

**Next line.** “In other words” is appropriate for something that’s a triviality. This is far from it. Omit the words. Maybe replace by “One can prove that”.

**Next display.** You have too many  $d$ ’s. Suggestion: All  $d/dx$  should be replaced by  $d/dx$  (roman  $d$ ). I’ve seen this in physics books, for instance. If so, do it consistently in the book.

**Next line.** Don’t begin a paragraph. Do restrict  $0 \leq k$  and insert a line in the list:  $d = 0 : 1$  and a column under it of all 0’s for  $k = 0$ .

**Th. 2.1(c).** Several comments. First, display the formula.

Second, let  $k \geq 0$ .

I have doubts about the exponent  $x^{k-1}$ : should it be  $x^k$ ?

Add another expression on the right:

$$= \left( x \frac{d}{dx} \right)^d \left( \frac{1}{1-x} \right).$$

**P. 36 etc..** VERY minor suggestion: instead of the Greek letter  $\Delta$  for a simplex, use  $\triangle$  (i.e., `\triangle`). Of course you’d have to go through the whole book and it would be unnoticed by most readers. But you’d be strictly parallel to □. On the other hand, I don’t really care for □ that much, so forget it.

**P. 35, line -5.** Delete “defined as”

**P. 35, line -1.** Delete comma.

**P. 40, line 2.** Remove extra  $()$  from numerator.

**P. 43, etc..** “Cross polytope”: I looked this up (I think). I believe it’s quite standard to hyphenate this name: “cross-polytope”. I recommend that.

**P. 44, line -9.** You cannot “use that”. You must “use the fact that” (for correct English).

**P. 46, line -2.** “any convex polygon”.

**P. 47, line 4.** Delete first comma. As written, it means that an “integral rectangle” necessarily has sides parallel to the axes. Even if true, it’s confusing to make it part of the definition.

**P. 48, 2.7, ¶2, line -2.** Replace , by ; in “regions,”.

**P. 48, 2.7, ¶3.** What do you mean by “right-angled rectangle”? I know! You mean a rectangle whose angles are all right angles! Right? (Sick.) Seriously, you’ll lose your readers. Laughing, maybe. I think you mean “rectangle with sides parallel to the axes”. I don’t know any way to say that except to say it. Every time.

**P. 48, 2.7, ¶3.** What do you mean by “right-angled rational triangle”? You mean “right triangle” (not the same as “right-angled triangle”) with rational vertices and sides parallel to the axes. Say this! E.g., “rational right triangle whose sides are parallel to the axes”. (There must be a better word than “sides”: could it be “legs”? I’m sure it exists. I may be able to look it up if you remind me.)

**Next sentence, “We now adjust”.** This is pretty confusing. E.g., what does “introduced earlier” mean? Be specific! Rewrite the whole sentence.

**Next sentence.** Delete “given as”.

**P. 48, line -5.** “about  $x$ - or  $y$ -axis” should be “about the  $x$ - or  $y$ -axis” (italics).

**P. 48, fn. 1.** Delete “However”. No need for it.

**P. 49, line -6.** What is a “Taylor coefficient”? Why not just “coefficient”?

**P. 49, line -5.** “of” should be “in”. (Too many “of”s.)

**(2.16).** Do something to make all the big () of the same size.

**P. 50, line after (2.18).** “So we can” should be “Then we”.

**P. 51, line -2.** Not a quadratic polynomial.  $u$  and  $v$  are step functions of  $t$ . The pure polynomial part includes the terms with  $-u - v$  as well as the Dedekind sums.

**P. 52, line 10.** “structure” should be “structural”

**line 14.** Remove “maximum”. There might be a minimum period, but I think you defined period in such a way that the word minimum is not needed. (You might just verify that?)

**P. 52, Proof.** More of those pesky right-angled rectangles and triangles!

**(2.21).** The superscript  $d$  on  $\mathbb{R}$  is not the original  $d$  in which  $P$  lives. This is confusing. At least, mention it.

**P. 52, fn. 2.** Too many “Here”s here and about.

**P. 53, middle.** In the displayed formulas, when you have two lines one above the other you still need that comma!

Worse, there are two versions of  $P$  given here: one in the plane and one in 4-space. They cannot be “=”. Rewrite to make clear there are two  $P$ ’s, that are (we know) equivalent (for our purposes).

**P. 53, line -6.** Comma should be ; in “series; we”

**P. 53, line -5.** Remove “now” (one is inclined to use too many nows).

**P. 54, 3 lines above (2.23).** Break into two sentences: “(2.21). Denote”

**P. 55, Note 1.** Change “collection” to the more precise “range”.

**Note 2.** I can’t imagine what you mean by “The analogy of ...”. Rewrite.

**Note 3.** Line 2: Change “on” to “in”.

Line 2: Change “(they” to “(which”.

Lines 2–4: The grammar is bad and the sentence is pointless unless the reader is *very* well educated mathematically. Either explain a lot better (in more than one sentence; this one is crammed), or say less.

**Note 5.** Line 1: delete “certain”.

Line 3: “they” is bad grammar. You probably meant the “enumerators”? That works.

**Note 6.** Delete “incidentally”.

**Note 7.** Do you mean *non-self-intersecting* curves (that need not be convex)?

**Note 10.** “The potential of” is inadequate. You need a noun like “The potential value/usefulness of”.

**Exerc. 2.3.** “the face” should be “a face”.

**2.7.** You need , at the end of the first line.

**2.8.** Do you mean  $d \geq 0$ ? (If you’re going to restate (2.5), do it completely.)

**2.9.** The first line needs a comma and can be improved: “For  $t, k \in \mathbb{Z}$  and  $d \in \mathbb{Z}_{>0}$ ”

**2.10.** “kind<sub>1</sub>  $s(n, m)$ ” (N.B. If anything should be written with giant angle brackets it’s Stirling numbers. But I don’t like that, personally. The  $S$  and  $s$  for them are pretty standard nowadays, I think.)

**2.13.** “Prove that”

and in line 3, “that” should be “which”.

**2.17.** “represent any polynomial as a sum”.

**2.19.** The summand is hard to interpret. Is it a product,  $B_n$  times  $(x + \frac{k}{m})$ ? Or is it a Bernoulli polynomial evaluated at  $x + \frac{k}{m}$ ? If the former, reverse the order. If the latter, shrink the (). Or do something!

**2.21.** Take out the (): “polynomials<sub>1</sub> as ... (2.7):”

(b)  $nB_{n-1}(x)$

(c)  $= 0$

(I believe Garry Bowlin again was the finder of these points.)

**2.22.** (□)

**2.23.** Add commas at the ends of the first three lines (including the display).

**2.31.** The typography, with vertical fractions in exponents, is hard to read. Suggest: Write them as  $bm/a$  and  $an/b$ .

“only track” should be “track only”.

**2.35, Hint.** Capitalize “You”.

**2.38, last line.** “relationship between”. “Relationship of” implies that  $v_1$  and  $v_2$  have a relationship with something else.

**2.40(a).** Plural or singular: you have one of each (subject, verb).

**2.42.** Delete “: they”.

#### 4. CHAPTER 3

**P. 63, first line of intro.** “the polynomial” should be “polynomial”

**P. 63, first bulleted item.** Use text style.

**P. 64, line 6.** “such that” should be “, which is such that”

**P. 64, line 9.** Remove , before “by”.

**P. 64, line 11.** “Just like ... have” is bad English. Correct English is “Just as ... have”. (You say “Just like” when comparing nouns: A is just like B. You say “Just as” when comparing verbs.—A poor explanation, but mine own.)

**P. 64, line 12.** Clarify: replace “pointed ... rational” by “so do pointed cones: a rational”

**Line 15.** “such that the intersection is  $d$ -dimensional and the corresponding hyperplanes” (Delete “of the form”.)

**Lines -7, -6.** Delete “based on”.

**Line -5.** Delete , after  $\mathbb{R}^{d+1}$ .

**Line -4.** Insert , after “So”.

**P. 65, Line 2.** Replace “set” by “translate” or “translated set”.

**P. 65, line 6.** I recommend that you display the two alternatives for  $\mathcal{K}$  in one line. Also, that you move the condition  $\mathcal{K} \cap H = \emptyset$  to after this display, to make clear that it is not part of the definition of  $\mathcal{K}$  lying on one side of  $H$ .

**line -5.** Insert , after “So”.

**P. 66, line 7.** The hyperplane  $x_1 = 1$  might not be the right hyperplane, for general  $\mathcal{K}$ . Either take a more general hyperplane (a translate of a hyperplane whose intersection with  $\mathcal{K}$  is the origin only), or restrict the pointed cone (obviously, not the most desirable thing).

**P. 66, line 12.** Change “by all” to “by the” (or “by the set of all”, but that’s unnecessarily long).

**P. 66, line 14.** Italicize “*a fortiori*”. What does it mean, anyway? I don’t see how it can be the right phrase here.

**line 10.** Somehow “our old friend:” doesn’t sound right. I suggest either “an old friend” with : or else just delete the : .

**P. 67, line -3.** Do you mean *simple  $d$ -cones*?

**P. 68, line 7.** Change “every” to “each” since you’re referring only to one point in your description.

**P. 68, line -9.** Use hyphen, not en dash.

**line -6.** “multiply everything out” (this is the English equivalent of a German separable prefix).

**line -3.** I don’t know what this sentence is supposed to mean. Are you translating  $v + \mathcal{K}$  with  $v + \Pi$  and tiling all other translates of  $v + \Pi$  by nonnegative integral combinations? That’s what you wrote. I suggest putting in clarification using more words. Note that  $v + \Pi$  is a translate of  $v + \Pi$  by a nonnegative combination, so you don’t need two cases. I think therefore you might just delete “the parallelepiped ... and” and replace it by “the”.

Does the reader know what you mean by “*tile*”?

**P. 69, line 5.** “Chapter 4” should be “Section 3.4”.

**P. 69, Proof of 3.6.** I don’t like “goes over almost verbatim” here. I like better: Note that  $\sigma_{v+\Pi^\circ} = \sigma_{v+\Pi}$  and apply Theorem 3.5. (Here I mean  $\Pi$  to be half open in the appropriate way.)

**Next line.** “A general” should be “Since a general”, and delete “So that”.

**Next line.** Use hyphen in “inclusion-exclusion”. This is not a sequence of names.

**Cor. 3.7, line 1.** Change “the” to “any”.

**Line -8.** Add , after “Later”.

**P. 71, line 3.** “More generally,”

**P. 72, Lemma 3.10.**  $(1, 1, \dots, 1, z)$

**P. 72, line 14.** I believe you need something stronger than  $g(1) \neq 0$ . Note that you have  $g(1) \geq 1 > 0$  because  $g(1) = \sigma_\Pi(1, 1, \dots, 1, 1)$  and  $\sigma_\Pi(1, \dots, 1, z) = 1$  (for  $m = 0$ ) +

additional terms in  $z$  with positive coefficients. (Because each term of  $\sigma_{\Pi}$  has coefficient 1. I hope I'm remembering this correctly.)

You're missing some technicalities when you simply assert that it's sufficient to do the proof for simplicial cones. Let's look at the details. You decompose  $P$  into integral simplices  $S_1, S_2, \dots, S_k$  of full dimension and  $S_{k+1}, \dots, S_l$  of lower dimensions. Assume you've done that; then you sum the series  $g_i(x)/(1-x)^{d+1}$ , where  $g_i$  is the numerator for  $S_i$  normalized so the denominators are all  $(1-x)^{d+1}$ . Thus,  $g_i(1) = 0$  if  $i > k$ . By my previous remark,  $g_i(1) > 0$  if  $i \leq k$ . This shows there is no cancellation in evaluating  $g_P(1)$  so it is nonzero (in fact, positive). If you didn't prove positivity you couldn't be sure of that.

**P. 73, 3.11, last line.** "first variable" should be "last coordinate".

**P. 74, proof of 3.13.** I believe your discussion doesn't describe what you've really done. What you're really proving is that  $L_P(0) = 1$  is dictated by the cone interpretation of  $\text{Ehr}_P(t)$ , by counting the origin. Thus, by Lemma 3.9, if you want  $L_P$  to be a polynomial, you can only choose the value 1 dictated by the cone.

**Note.** I used in class the notation  $K_v(P)$  for the cone with apex  $v$  generated by  $P$ , thus  $K_0(P)$  for your  $\text{cone}(P)$ . I like this better than  $\text{cone}(P)$  because it's easier to read in text and because it's more flexible and mostly because (I think) I'm used to it from other places. Just opinion, but if you like this, you could change to it. I'm pretty sure I didn't make it up myself.

**P. 75, 3.15.** This is really just my argument above, re 3.13. You might combine 3.13 and 3.15 since they are different aspects of the same fact due to the same reason.

**P. 77, line 1.** Remove (); you are referring to the proof, not the statement, of the lemma.

**P. 77, displays.** Where you have

$$\left(\frac{1}{t}\mathbb{Z}\right)^d$$

I think it would look better (less garish because smaller parentheses) if you had text style

$$\left(\frac{1}{t}\mathbb{Z}\right)^d$$

**P. 77, 3.18.** This strikes me as a lemma, not a theorem, in terms of your topic and development.

**P. 78, line 1.** "(since  $X$  could"

**P. 78, 3.19.** This is the theorem, not 3.18. It's a main result! A main result shouldn't be called a corollary.

**P. 78, line 17.** "as such is"

**P. 78, line 20.** Remove () and change "interpolate" to "interpolating".

**P. 78, lines -2, -1.** "vol  $P$  but even the discrete volume  $L_P$  from finite data."

**P. 79, just after (3.5).** Delete "Incidentally": it's not incidental, it's definition of the terminology.

**P. 80, line 6.** "structure" to "structural".

**P. 80, 3.22.** Capitalize the whole title of the theorem: "Rational Polytopes".

**P. 81, middle of page.** The displayed formula " $w_1 = \dots$ " should end with semicolon.

**P. 81, line -9.** "as does"

**line -5.** I don't understand what "less developed" means in this context. You might mean "less well known"? I'm puzzled.



**line -2.** Remove comma at end.

**P. 82, line 3.** *General comment* about “lattice-point enumerators”, “lattice-point enumeration functions”: When you (Matt) talk about enumerators I thought you generally mean the generating functions  $\sigma_P$  that have a monomial for each point. The functions  $L_P$  are *counting functions*. Do you care to make a distinction in this book? I don’t think it’s necessary but it occurred to me to raise the point since you do employ both and you might want to have distinctive terminology. However, you would have to go through the whole book checking every use of “enumerata” with a text editor. Probably not worth it!

**P. 82, Note 2.** “foundation for”: there is no such thing as “foundation to”.

**P. 83, Exercise 3.1(b).** Change “another one” to “any other”.

**Exerc. 3.4.** Doesn’t this work if the apex is rational? Then you don’t need to say anything, since a rational cone has rational apex.

**Exerc. 3.6.** “with apex at the origin”

**Exerc. 3.9.** Put a word space in the definition of  $\varphi_d$  after the comma. (Space is too small.)

**Exerc. 3.11.** “the convex hull”

**Exerc. 3.13, 3.14.** “translated so that”

**Exerc. 3.15.** Actually, the transpose is the Vandermonde matrix.

## 5. CHAPTER 4

**P. 89, line 2 after (4.1).** Comma after “Namely”.

**P. 89, line -5.** Word space after “*interior*”. I don’t know what the problem is: either you forgot to type a space, or you need an italic correction symbol  $\backslash/$ , or you need a word space symbol.

**P. 89, line -2.** Comma before “algebraically”.

**P. 90, Proof, line 1.** “similar”: Similar to what? I can’t tell what you’re referring to.

**P. 90, Proof, line 2.** Change “result” to “formula” (you’re citing formulas, not results).

**P. 91, bottom.** I assume your publisher will fix this and other giant white spaces, but make sure it’s done.

**P. 92, line 8.** Change “a very similar manner to our” (poor English) to “a manner very similar to that of our”.

**P. 92, Proof, line 2.** “ensures”

**P. 92, line -1.** And (4.5’). That is the formula

$$\partial(v + K_j) \cap \mathbb{Z}^d = \emptyset, \quad \forall j$$

which you don’t have. You need to add this formula to have a correct proof. (I made this into a homework problem together with (4.4) and (4.5).) The proof I have in my notes is:

$$\begin{aligned} \sigma_K(1/z) &\stackrel{(4.6)}{=} \sigma_{-v+K}(1/z) \stackrel{(4.5')}{=} \sum_{j=1}^m \sigma_{-v+K_j}(1/z) \stackrel{\text{Theorem 4.2}}{=} \sum_{j=1}^m (-1)^d \sigma_{v+K_j}(z) \\ &\stackrel{(4.5)}{=} (-1)^d \sigma_{v+K}(z) \stackrel{(4.4)}{=} (-1)^d \sigma_{K^\circ}(z). \end{aligned}$$

**P. 93, line 3.** “preparation for” (correct English).

**P. 93, line 6.** “convention of beginning” (correct English).

**P. 93, line 8.** What does “Much in sync” mean? It’s extremely slangy and I only get a hazy idea of the meaning.

**P. 93, line -6.** Replace “the identity of Lemma 3.10” by “the Ehrhart series of  $P$ ”. I don’t believe 3.10 is involved here at all.

**P. 94, §4.4, line 4.** Replace “an integral polytope” by “a polytope”. Add at end of line ‘if it is integral and’. I believe you mean that for a polytope to be reflexive, by definition it must be integral. If I’m wrong, then your statement was correct but you might put in a word to explain that.

**P. (4.10).** I don’t feel quite happy with this. I believe what you really mean is:

$$L_{P^\circ}(t+1) = L_P(t), \forall t > 0 \text{ and } L_{P^\circ}(1) = 1.$$

Your statement of (4.10) is equivalent to this, but my version is more intuitively meaningful, therefore I suggest you begin with one version (possibly mine) and then point out that it’s equivalent to the other version.

**P. 95, lines -9, -8, -7.** Rewrite: “By condition . . . only if

$$\text{Ehr}_{P^\circ}(x) = x + \sum_{t \geq 2} = \dots.”$$

Also, in the summations, there’s the wrong kind of  $t$  under the summation signs (twice): it’s boldface but should be italic.

**P. 96, Note 3, line 3.** Replace ; by period.

**Note 3, line -3.** There’s something wrong here, maybe, that I can’t quite make out from my notes. I think a student (probably Garry Bowlin again!) noticed that it’s not the sum of the sums of the edge lengths but the sum of the products in  $P$  and  $P^*$ , or the product of the sums. And my notes say this can be an exercise.

**note 4, line 2.** Delete “Here”.

Also, note that the so-called “dual” is generally known as the *polar polytope* (I think; you can look this up).

**Note 4, line 3.** “to  $P$ ” should be “of  $P$ ”. ( $P^*$  is dual to  $P$  but  $P^*$  is the dual of  $P$ .)

**Note 4, line 4.** “concept of polar duality”, to be specific, since there are other kinds of duality.

**Note 4, line 5.** Suddenly you use the word “polarity” without explanation. By inserting polar terminology as I suggested you will have made clear what you mean.

## 6. CHAPTER 5

**P. 99, line -12, -11.** Overly detailed. Revise: “with Ehrhart–Macdonald reciprocity.”

**P. 99, line -9.** Change “define” to “denote”: you aren’t defining.

**line -8.** “about the”

**line -4.**  $0 \leq k \leq d$  (You have a tendency to omit indications of the range of variables. It would be beneficial to put in more detail about that.)

**P. 100, line 3.** Delete “can”.

**P. 100, line 6.** “open faces that contain them.”

**P. 100, line -1.** “relatively open face” should be “relative interior”. (Make sure this term is in the index.)

**P. 101, line 2.** Again,  $0 \leq k \leq d$ . (You think it’s obvious that  $k \geq 0$ . The reader may wonder whether  $k \geq 1$ .)

**P. 101, line 9.** The sum does not “represent”, it *is*.

Also, again the wrong dash in “inclusion-exclusion”.

I note that you are making a claim here for which you don't have a proof or an explanation. In fact, I suggest you don't have *the* inclusion-exclusion formula but rather *an* inclusion-exclusion formula, deducible from the usual one with sufficient knowledge of face posets (that you don't discuss at all) but not identical to it. Thus, it would be better to say the sum is "an" or "a kind of", or some such. I don't find this satisfactory, unless you make it clear this is a remark that can't be understood properly without going outside the scope of the book. And, as a consequence, I don't see how you can justify saying "we recover ... [reciprocity]"; you don't, although someone who knows face poset Möbius functions could do it.

**P. 101, line 12.** Again, too formal: just "E–M reciprocity".

**line 16.** Same: over-formal, and repetitious.

**P. 103.** Several wrong dashes: "right-hand" like "inclusion-exclusion" uses a hyphen. Please search your ms. for all such examples.

**P. 103, line -12.** Delete "We note that": no need.

**P. 103, line -5.** "coefficients"

**P. 104, line 2.** "(since *X*)" (You have a tendency to leave out these valuable connecting words, that tell the reader how your thoughts are related to each other. Don't.)

**P. 105, line 4 of text.** Too many "that"s. "found in Cor. 5.5 that the second"

**P. 105, line -1.** The range of summation should have text "a facet of"; probably you typed it this way and it was italicized for being in a declaration, but you might check it.

**P. 106, Note 1.** "Duncan": Sommerville was consistently known by his initials (three of them, which I don't remember just now). It's disrespectful to change his name, even if his friends called him "Duncan" (as they may have done). I suggest you apply this principle to all authors, e.g., "Ian" Macdonald has chosen to be known as "I.G." in his professional publications—although in his case, he's alive and you can ask him, or maybe he does have "Ian" publications. I know you both mean well, but when I read something like this I think the authors are showing off their obscure learning. (I am possibly guilty of this myself. That doesn't make it correct. I would hate for some pseudo-scholar to dig out my middle name and refer to me by it. I've consistently chosen not to use it because I don't think of myself by that name.)

"Ironically ...": Do you really know this to be true? How?

"used much" should be "much used"

"their": only one thing was rediscovered (grammatically, since the subject is "Theorem 5.1"). (Twice.)

Say "famous" or "widely read"; both constitute overkill.

**Note 2.** What do you mean by "this case is *attributed to* Euler"? Either he did it or he didn't. If he didn't, then don't attribute it to him. If someone else did so (wrongly), that's not relevant here.

"(like numerous later"

Lines 5–8 are poorly written. E.g., I don't understand "nicely inductively". E.g., is it *you* who call it shelling?—no, it *is* called shelling; that's standard terminology. E.g., a fact is not called "shelling". And more. Rewrite at greater length, to clarify; and maybe you can leave out some of this.

**P. 107, Exerc. 5.7.** Put that punctuation in the display formula!

**Exerc. 5.13.** Delete "Cor. 3.19 and", and also "a geometric" (repetition).

## 7. CHAPTER 6

**P. 109, caption.** The name of the church is not “Sagrada Familia”, that’s a convenient and common abbreviation. Assuming your reader might not know about it, I recommend giving a fuller title or something that indicates it’s a church and not a salt cellar, for instance.

**P. 109, last 4 lines.** I don’t think this is quite the right picture of magic squares. I think the main interest in magic squares is still not that of professional mathematicians, so their reductionist ideas of what a magic square is shouldn’t be given great weight. I recommend some minor changes to reflect this without upsetting anyone:

Line -4: “(usually re-”

Line -3: “often restricted” or better yet, “usually” (I would say “usually” is clearly the fact, in real life).

Lines -3, -2: “usually required to have” (same reason; I think you should put some extra emphasis on this fact, especially since it is not what you’ll be doing; and you should point out somewhere that you’ll *not* be observing this rule—my opinion, of course).

Line -1: “number, called the *magic sum*NO.” (I think this is general terminology, though not absolutely universal; no need to be cautious about saying it’s the name.)

## 8. APPENDIX A

## 9. REFERENCES

[24]. Capitalize the proper names in the title. (Maybe also in other papers.)

**Consistent style.** This is incredibly tedious but necessary. Get a student to do it?

**Book titles.** My opinion is that book titles in English should be fully capitalized. You can enforce capitalizations on Bibtex; see the manual. German and French have their own customs.

**Dover.** Omit the “Publications, Inc.” This is customarily omitted, except when essential, as in a small math publisher whose name escapes me; but not Dover. A similar example: “Wiley” is the usual way to refer to John Wiley and Sons. This is a completely standard method of citing a publisher.

[1] **and similar ones.** Make up your mind: either it’s a Ph.D. thesis, referred to in thesis style, or it’s a technical report. (Not “in” a technical report. Unless the report contains other articles.) If both, then it’s a Ph.D. thesis first; use separate sentences. State whose technical report it is; there are thousands of places producing technical reports. “Univ. of Pennsylvania” is probably incorrect, it’s probably “Department of Mathematics, U of P” (write it in full) for the report. For the Ph.D. thesis citation, just “U of P, date” is correct. One reason for this is that the university itself maintains a file of theses but not of technical reports, which will be kept by the individual departments. All this is about making documents attainable by the reader. It’s especially important for obscure sources.

[2]. [12], **others.** *Do not italicize book series names.* It’s misleading. Here, e.g., you’re giving the impression that *Algorithms Combin.* is a journal and the article appeared in vol. 25.

**Books of articles.** It’s desirable to list the editors (before the title).

## 10. INDEX

**Overall comments.** Limited use of the index suggests it needs much improvement. It seems there is a need to check every entry (sorry!!!).

- (1) Every entry should have a definition in the book that is cited in the index.
- (2) Every entry must appear on the pages listed in the index. (Yes, I found discrepancies.)
- (3) All significant terms must be indexed; I found some missing terms. That includes variant forms; as: “arrangement of hyperplanes: see hyperplane arrangements” (or vice versa). (Do the best you can. The definition is the essential part, obviously.)
- (4) All significant appearances of each term should be indexed. (Do what you can. This is especially hard, but not as important as the others.)

**Missing entries that I noticed.**

- arrangement of hyperplanes
- fundamental domain
- integer part (of a real number)
- lattice

**Assorted errors.**

- greatest integer function *and* greatest-integer function are indexed separately. Choose a consistent spelling.
- hyperplane: no definition is indexed.
- Khovanskii–Postnikov’s Theorem: remove the ‘s.
- lattice point: no definition is indexed. (This just needs you to index lattice.)