ERRATA AND ADDENDA

to

An Introduction to Hyperplane Arrangements

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Geometric Combinatorics (2004)

by

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These errata are for the version available at

`http://www.cis.upenn.edu/~cis610/sp06stanley.pdf`.

Most of them were found by Steven Sam and Darij Grinberg. The page numbers refer to the printed page numbers of the pdf file at the above website, not the pdf page numbers.

- p. 2, various places. It is implicitly assumed that the dot product $\alpha \cdot v$ is defined via an isomorphism $V \cong K^n$.
- p. 3, lines 1–7. This discussion has some inaccuracies. What is true (and not too hard to prove) is that over $\mathbb{R}$ we can take $W = X$. Over an arbitrary field K, there exists a linear subspace W whose dimension equals $\mathrm{rank}(\mathcal{A})$, such that $\mathrm{codim}(H \cap W) = 1$ for all $H \in \mathcal{A}$. We then define $\mathcal{A}_W := \{H \cap W\ :\ H \in \mathcal{A}\}$.
- p. 4, lines 13–14. Naturally it should be assumed that $H_1, \dots, H_p$ are distinct.
- p. 4, Example 1.2, line 2. Change "L line" to "line L".
- p. 4, Example 1.2, line 2. Change $\mathcal{A}_K$ to $\mathcal{A}_k$.
- p. 7, second figure. The line $y = 0$ is missing.
- p. 8, line 12–. Though "maximal" is used in a standard way, for the sake of clarity one can define a *maximal chain* to be a chain that is contained in no larger chain. Thus in a finite poset every maximal chain is saturated, but not conversely.
- p. 8, line 8–. Replace $x < y$ by $x \leq y$ (since later the notation $\mathrm{rk}(x, x)$ is used).
- p. 10, line 10. Add at the end of the line: "The following result is known as the *Möbius inversion formula.*"
- p. 10, end of proof. Should be $\zeta g = f \iff g = \mu f$

- p. 11. line 3. Change $r(y)$ to $\mathrm{rk}(y)$.
- p. 12, Exercise 7. The term "face" is defined in Definition 2.4 on page 19.
- p. 12, Exercise 7(e). Change the difficulty rating to [3].
- p. 14, line 9–. Change $H \in \mathcal{R}(\mathcal{A}')$ to $H \in \mathcal{A}'$.
- p. 14, Lemma 2.2, lines 1–2. Replace "real arrangements" with "arrangements over a field K".
- p. 16, line 5. Change "Cross-Cut" to "Crosscut".
- p. 16, Theorem 2.2. Perhaps it should be remarked that $N_0 = 0$ unless $\#L = 1$ (i.e., $\hat{0} = \hat{1}$) and $X = \emptyset$.
- p. 18, line 6– to 1–. Replace these six lines by the following.

  In the latter case, let $\mathcal{B} \subseteq \mathcal{A}$ be central and $H_0 \in \mathcal{B}$. Set $\hat{\mathcal{B}} = (\mathcal{B} - \{H_0\})^{H_0}$, a subarrangement of $\mathcal{A}^{H_0} = \mathcal{A}''$. Suppose that $\hat{\mathcal{B}} = \{H_1, \ldots, H_k\}$. Let

  $$c_i = \#\{H \in \mathcal{A} : H \cap H_0 = H_i\}.$$

  Consider the contribution to the sum $\sum_{\substack{H_0 \in \mathcal{B} \subseteq \mathcal{A} \\ \mathcal{B}\,\text{central}}} (-1)^{\#\mathcal{B}} t^{n-\mathrm{rank}(\mathcal{B})}$ from all subarrangements $\mathcal{C} \subseteq \mathcal{A}$ satisfying $\hat{\mathcal{C}} = \hat{\mathcal{B}}$. Note that $\mathrm{rank}(\mathcal{C}) = 1 + \mathrm{rank}(\hat{\mathcal{C}}) = 1 + \mathrm{rank}(\hat{\mathcal{B}})$. The sum is given by

  $$\begin{aligned} \sum_{\substack{H_0 \in \mathcal{C} \subseteq \mathcal{A} \\ \hat{\mathcal{C}} = \hat{\mathcal{B}}}} (-1)^{\#\mathcal{C}} t^{n-\mathrm{rank}(\mathcal{C})} &= t^{n-(1+\mathrm{rank}(\hat{\mathcal{B}}))} \sum_{i_1=1}^{c_1} \cdots \sum_{i_k=1}^{c_k} (-1)^{i_1+\cdots+i_k+1} \binom{c_1}{i_1} \cdots \binom{c_k}{i_k} \\ &= -t^{n-1-\mathrm{rank}(\hat{\mathcal{B}})} (-1)^k \\ &= -t^{n-1-\mathrm{rank}(\hat{\mathcal{B}})} (-1)^{\#\hat{\mathcal{B}}}. \end{aligned}$$

  Thus

  $$\begin{aligned} \sum_{\substack{H_0 \in \mathcal{B} \subseteq \mathcal{A} \\ \mathcal{B}\,\text{central}}} (-1)^{\#\mathcal{B}} t^{n-\mathrm{rank}(\mathcal{B})} &= - \sum_{\substack{\hat{\mathcal{B}} \subseteq \mathcal{A}'' \\ \hat{\mathcal{B}}\,\text{central}}} t^{n-1-\mathrm{rank}(\hat{\mathcal{B}})} (-1)^{\#\hat{\mathcal{B}}} \\ &= -\chi_{\mathcal{A}''}(t), \end{aligned}$$

  and the proof follows. $\square$
- p. 22, line 3. $(1)^k$ should be $(-1)^k$
- p. 24, lines 3– to 2–. Change "by first choosing the size $i = \#\kappa([n])$ of its image in $\binom{q}{i}$ ways" with "by first choosing the size $i = \#\kappa([n])$ of its image, then choosing its image $\kappa([n])$ itself in $\binom{q}{i}$ ways".

- p. 25, line 7–. Change the second $=$ to $-$.
- p. 27, line 4. Change $L(\mathcal{A})$ to $L(\mathcal{A}_G)$.
- p. 27, line 9. Insert after "sublattice" the following parenthetical statement.

  (i.e., it is not true that if $\sigma, \tau \in L_G$ then $\sigma \wedge \tau \in L_G$ and $\sigma \vee \tau \in L_G$, where $\wedge$ and $\vee$ are computed in Π_n)

- p. 27, line 10. Insert after "[why?]." the following sentence:

  In other words, if $\sigma, \tau \in L_G$ then $\sigma \vee \tau \in L_G$, where $\vee$ is computed in Π_n.

- p. 28, line 9–. Change $\mathfrak{o}$ to $\bar{\mathfrak{o}}$.
- p. 30, Exercise 4, line 3. Chang $r(\mathcal{A})$ to $r(\mathcal{A}_G)$.
- p. 30, Exercise 7, line 1. Change "the the" to "the".
- p. 30, Exercise 9. Change the difficulty level to [4–]. In fact, the stronger result $c_i^2 \geq c_{i-1}c_{i+1}$ is now known to be true.
- p. 32, line 5–. Change "diagam" to "diagram".
- p. 35, proof of Proposition 3.6, last line. Change $\overline{B}'$ to $\overline{B'}$.
- p. 36, Definition 3.9. In order for condition (1) to make sense, it should be assumed that L is graded. Let us point out, moreover, that a finite lattice satisfying condition (2) is automatically graded.
- p. 36, line 14–. Change $\vee I$ to $\bigvee I$.
- p. 36, line 6–. Delete "$y \in S$ but".
- p. 37, line 5. Change second $S \cup T$ to $S \cap T$.
- p. 37, line 8. Change $L_{\mathcal{A}}$ to $L(\mathcal{A})$.
- p. 38, line equation (26). Change χ_M to $\chi_{M_{\mathcal{A}}}$.
- p. 41, lines 7– to 6–. Change "of the affine matroid M of Figure 1" to "of a certain affine matroid M".
- p. 42, line 6. Change $i-1$ to $i+1$.
- p. 42, Lemma 4.4. It should be assumed that $\hat{0} < \hat{1}$. Otherwise we need to add a term c_0 to the formula for $\mu(\hat{0}, \hat{1})$.

- p. 43, line 9. Change second x_1 to x_2.
- p. 43, proof of Theorem 4.11. The proof assumes that $n > 0$, i.e., $\hat{0} < \hat{1}$. Of course the case $n = 0$ is trivial.
- p. 44, equation (27). Change n to $n - 1$.
- p. 44, line 12. Change $\cdots$ to $\vdots$.
- p. 44, line 12–. Change $\lambda(x_i) > \lambda(x_{i+1})$ to $\lambda(x_{i-1}, x_i) > \lambda(x_i, x_{i+1})$.
- p. 45, line 12. Insert after "with" the phrase "the edge ordering $\mathcal{O}$ (in large numbers) and".
- p. 46, line 2. Replace $>$ with $\geq$.
- p. 46, line 5. Change "increasing" to "strictly increasing".
- p. 46,line 7. Change $\lambda(C)$ to $\tilde{\lambda}(C)$.
- p. 46, line 22. Change $\hat{0} := y_0$ to $\hat{0} = y_0$.
- p. 47, Example 4.9(c), line 3. Change "and" to "with".
- p. 47, Example 4.9(e), line 4. Change $\mathbb{F}_n(q)$ to $\mathbb{F}_q^n$.
- p. 48, line 1. Change L to $B_n(q)$ (twice).
- p. 49, line 11. change $B_2 - b, \ldots, B_3$ to $B_2 - b, B_3$.
- p. 49, line 12. Change B_l to B_k.
- Theorem 4.13, line 1. Although the meaning should be clear, to avoid any ambiguity the first sentence should be changed to "Let L be a geometric lattice of rank n, and let z be a modular element of L."
- Theorem 4.13. The characteristic polynomial of a (finite) graded poset P with $\hat{0}$ needs to be defined. Suppose that P has rank n, so that every maximal chain of P has length n. Define
$$\chi_P(t) = \sum_{x \in P} \mu(\hat{0}, x) t^{n - \mathrm{rk}(x)}.$$
- p. 50, line 4. Change $x^n - ax^{n-1} + \cdots$ with $t^n - at^{n-1} + \cdots$.
- p. 50, equation (33). Change $\sum_{y \wedge z = \hat{0}}$ to $\sum_{y \,:\, y \wedge z = \hat{0}}$.
- p. 52, line 3–. Change $x\mathcal{A}$ to $c\mathcal{A}$.

- p. 53, lines 5–6. Should be displayed so = signs are aligned.
- p. 53, Definition 4.13, line 3. Change $L_{\mathcal{A}}$ to $L(\mathcal{A})$.
- p. 54, line 8–. Change $B_1 \subset B_2 \cdots$ to $B_1 \subset B_2 \subset \cdots$.
- p. 54, line 7–. Change "atoms covered by π_i" to "atoms less than or equal to π_i".
- p. 54, line 1–. Change $\mathcal{B}_n(t)$ to $\mathcal{B}_n$.
- p. 55, line 10. Change $p_i(H)$ to $(p_1(H), \ldots, p_n(H))$.
- p. 55, line 16–. Change $L_{\mathcal{A}}$ to $L(\mathcal{A})$.
- p. 59, (22)(b). Change "internal activity 0" to "internal activity 1".
- p. 61, line 4–. Change $v_i, a_i \in \mathbb{Z}^n$ to "$v_i \in \mathbb{Z}^n$ and $a_i \in \mathbb{Z}$".
- p. 62, second line of proof. Change F_q to $\mathbb{F}_q$.
- p. 63, line 4. Change $\mathbb{F}_1^n$ to $\mathbb{F}_q^n$.
- p. 72, line 10. Change "intervals" to "interval".
- p. 78, Lemma 5.6, line 2. Change $\sigma(x) = \sigma(y)$ to $\sigma(x) = y$.
- p. 84, Exercise 19(a). Change $\sum_{k=1}^n$ to $\sum_{k=0}^n$.
- p. 86, Exercise 27(b). Change the rating to [3–]. A solution was found by Seunghyun Seo.
- p. 90, line 7. Change $(c_1, \cdots, c_n)$ to $(c_1, \ldots, c_n)$.
- p. 90, line 12–. Change "is easy" to "it is easy".
- p. 90, line 3–. Change $\mathrm{sep}(R_0, u)$ to $\mathrm{sep}(R_0, R_u)$.
- p. 92, Definition 6.15, line 2. Change "rearrangment" to "rearrangement".
- p. 94, line 1–. Change "parking function" to "parking functions".
- p. 95, line 17–. Change "connect it the roots" to "connect it to the roots".
- p. 98, line 5. Change "(bbb])" to "(bbb)])".
- p. 101, line 5–. Change x_{d+1} to x_{d-1}.
- p. 104, entry (6,2) of V. Change $a_a a_2 a_3$ to $a_1 a_2 a_3$.
- p. 104, entry (7,2) of V. Change $a_1 a_3$ to $a_2 a_3$.

- p. 106, Exercise 5. For a solution, see S. Sivasubramanian, Interpreting the two variable distance enumerator of the Shi hyperplane arrangement, `arXiv:math/0610780`.