MATROID THEORY, Second edition

James Oxley

Oxford University Press, New York, 2011

Errata and Update on Conjectures, Problems, and References
Latest update: March 20, 2020

The reader is encouraged to send the author <oxley@math.lsu.edu> corrections that do not appear in the table below. Some of the errors listed below were corrected in the second printing of this book. The changes to the references provide updates on publication information for some papers. Such changes also sometimes mean that a paper that had been listed as, for example, Bonin (2009a) when in preprint form has become Bonin (2010a) when it actually appeared. This necessitated corresponding changes in the text. Since the original text had no known errors with the references, these changes within the text have not been included below.

I thank the following people for pointing out errors: Nick Brettell, Farid Bouya, Cameron Crenshaw, F.M. Dong, James Dylan Douthitt, Jonathan Farley, Tara Fife, Zachary Gershkoff, Kevin Grace, Stefan Kratsch, Criel Merino, Peter Nelson, Jakayla Robbins, Jagdeep Singh, Avery St. Dizier, Ben Warren, and Michael Welsh.

Page	Line	Change
12	-15	N. Bansal, R.A. Pendavingh, and J.G. van der Pol (On the number of matroids, *Combinatorica* **35** (2015), 253–277) have sharpened Piff's upper bound on the number $f(n)$ of non-isomorphic matroids on an n-element set by proving that $$\log_2 \log_2 f(n) \le n - \tfrac{3}{2}\log_2 n + \tfrac{1}{2}\log_2 \tfrac{2}{\pi} + 1 + o(1).$$
22	-15	Replace "(R2)" by "(R1)".
38	-1	This should say "38 non-isomorphic matroids on a 5-set".
40	1	In Fig. 1.17, the line $\{5,6,8\}$ should pass in front of the line $\{3,4,10\}$.
41	17	Part (ii) of Proposition 1.5.18 can be simplified to "the union of any $r-1$ of $L_1, L_2, \dots, L_r$ has rank r". Moreover, part (iii) should be deleted.
50	6	This should say "The Hasse diagram in Figure 1.25" (instead of 1.24).
65	-7	Replace "Proposition 1.4.10(iii)" with "Proposition 1.4.10(iv)".
69	7	Replace "0.1" by "$0,1$".
73	8	Replace "Proposition 1.4.10(iii)" with "Proposition 1.4.10(iv)".

Page	Line	Change
84	-15	Beginning here, the proof of this proposition has been simplified as the original proof seemed too convoluted. Another short paragraph has been added to the text following the completion of the proof. The actual change is as follows:

$$\text{“}\dim(W(1,4)\cap W(2,3)) = \dim W(1,4)+\dim W(2,3) - \dim(W(1,4)+W(2,3)) = 2+2-3=1.$$

Thus $W(1,4)\cap W(2,3)$ is $\langle \underline{v}\rangle$, the subspace of $V(4,\mathbb{F})$ generated by some non-zero vector $\underline{v}$. Also

$$\dim(W(1,4,5,6)\cap W(2,3,5,6)) = \dim W(1,4,5,6)+\dim W(2,3,5,6) - \dim(W(1,4,5,6)+W(2,3,5,6)) = 3+3-4=2.$$

But $W(5,6)$ is a 2-dimensional subspace of $W(1,4,5,6)\cap W(2,3,5,6)$, so

$$W(5,6)=W(1,4,5,6)\cap W(2,3,5,6)\supseteq W(1,4)\cap W(2,3)=\langle \underline{v}\rangle.$$

Geometrically, if V_8 is $\mathbb{F}$-representable, then we can add a point p to the diagram so that its relationship to $\{1,2,3,4,5,6\}$ is as shown in Figure 2.7(a). Of course, p corresponds to the subspace $\langle \underline{v}\rangle$ of $V(4,\mathbb{F})$. By symmetry, as $W(5,6)\supseteq\langle \underline{v}\rangle$, we deduce that $W(7,8)\supseteq\langle \underline{v}\rangle$. Thus $\langle \underline{v}\rangle\subseteq W(5,6)\cap W(7,8)$. Geometrically, this says that p is on both the line containing 5 and 6 and the line containing 7 and 8, so $\{5,6,7,8\}$ is dependent (see Figure 2.7(b)); or, in terms of subspaces:

$$\dim W(5,6,7,8) = \dim(W(5,6)+W(7,8)) = 2+2-\dim(W(5,6)\cap W(7,8))\leq 3.$$

This is a contradiction as $r(\{5,6,7,8\})=4$, so V_8 is not $\mathbb{F}$-representable. □

In V_8, each of the sets $\{1,2,5,7\},\{3,4,6,8\},\{1,3,5,8\},\{1,3,6,7\},\{2,4,6,7\}$, $\{2,4,5,8\},\{3,4,5,7\}$, and $\{1,2,6,8\}$ is independent. By Proposition 2.1.24, we can produce new paving matroids by making any collection of these sets into circuit-hyperplanes. While not all of these new matroids are non-isomorphic, the argument just given shows that none of them is representable.”

Page	Line	Change
91	-7	Replace “connected planar graph G_1” by “connected plane graph G_1”.
92	-8	In Exercise 8, replace “$G[X]$” by “$G/(E(G)-X)$.”
105	7	Add “and $M_1\backslash e = M_2\backslash e$” at the end of the first sentence of this proposition, and omit “ $M_1\backslash e = M_2\backslash e$ and” from (i).
130	12	Omit “that” after “with”.
130	-3	Add “as $\{e_2,e_3\}$ is a circuit of M/e_1.”
132	1	Replace “4.3.12” by “4.3.15” in the caption for Figure 4.7.

Page	Line	Change
132	-15	Replace "The cycle matroid of the graph in Figure 4.7" by "Modify the graph G in Figure 4.7 by splitting one of the degree-4 vertices of G adding a new edge e so that $H/e = G$, each end of e has degree 3 in H, and the four 4-cycles of G remain 4-cycles in H. The cycle matroid of H"
134	7	Replace "simple non-empty connected matroid" by "simple connected matroid with at least two elements".
140	-12	Change this sentence to "Suppose now that G and G' are arbitrary graphs for which such a bijection exists." We want an arbitrary graph to be allowed to have an abstract dual although it will follow from what is shown in this section that a graph has an abstract dual if and only if it is planar.
142	-9	Change the end of this sentence and the beginning of the next sentence to produce a sentence that ends with "we eventually obtain, in case (ii), a 2-connected non-planar subgraph G_m of G such that $M(G_m)$ is a component of $M(G)$."
157	-23	In Exercise 3, add "one-edge" before "chordal paths".
161	8	Note that the isomorphism being referred to here is isomorphism of projective geometries as defined on the previous page. It is not isomorphism of matroids. To see the distinction, consider the projective geometry $PG(3,2)$. Viewed as a matroid, this has rank 4. Truncating this matroid gives a rank-3 matroid $T(PG(3,2))$. The sets of points and lines and the incidence relations between them are the same in $PG(3,2)$ and $T(PG(3,2))$, so these quite distinct matroids give rise to the same projective geometry. From 6.1.3 onwards, whenever projective geometries are considered, it is their corresponding matroid structure that is the focus of attention.
194	18	Replace "3.1.16" by "3.1.17".
210	-3	Replace "Exercise 8" by "Exercise 9".
238	-23	Change to "the *frame matroid* or *bias matroid*". After consultation with Tom Zaslavsky, I have changed most occurrences of "bias matroid" to "frame matroid" throughout this section.
240	16	Replace this sentence by: "We refer the reader seeking a more detailed discussion of frame matroids to the sequence of papers on this topic by Zaslavsky (1982a, 1987b, 1989, 1990, 1991, 1994) and to his extensive bibliography of papers in this and related areas (Zaslavsky 1998a) noting that frame matroids were originally called bias matroids."
257	-17	For the first part of (ii), replace the exception by "unless p is a coloop of M_2 but is not a coloop of M_1"; for the second part of (ii), replace the exception by "unless p is a loop of M_2 but is not a loop of M_1".
263	-11	In Exercise 1(a), replace "$A_1 \subseteq E(M_1)$ and $A_2 \subseteq E(M_2)$" by "$A_1 \subseteq E(M_1) - p$ and $A_2 \subseteq E(M_2) - p$".
272	5	Replace "$k \geq 3$" by "$k \geq 4$".
281	-18	In Exercise 3, replace "f is in $E(F_7) - e$" by "f is in $E(M) - e$".
322	2	Replace the equals sign on this line by a greater-than-or-equal-to sign.
348	-22	Exercise 2 should be replaced by: "Prove that a matroid M is binary if and only if, for every two distinct hyperplanes H_1 and H_2, there is a hyperplane H_3 that contains $H_1 \cap H_2$ such that $H_1 \cup H_2 \cup H_3 = E(M)$."

353	-15	In Exercise 4(c), replace "(i) and (ii)" by "(a) and (b)".
353	-11	At the end of Exercise 4(e), replace "$M\|S$" by "$(M\{N\})^*\|S$."
353	-8	At the end of Exercise 4(f), replace "$M.S$" by "$(M\{N\})^*.S$."
368	-2	Replace "C_1^* *separates* C_2^* *from* C_2^*" by "C_1^* *separates* C_2^* *from* C_3^*".
410	10	Replace "$f(X)$" by "$f(T)$" in the definition of $\mathcal{I}(\mathcal{E}, f)$.
422	8	Change the beginning of the line to "for every flat X of N that contains a circuit containing e".
432	-19	Change the sentence that begins here to "Conversely, if (11.15) holds, let B be a basis of $M_1 \vee M_2^*$. Then $B = I_1 \dot\cup I_2^*$ where $I_1 \in \mathcal{I}(M_1)$ and $I_2^* \in \mathcal{I}(M_2^*)$ and such sets are chosen so that the latter has maximal cardinality, Clearly $I_2^* \subseteq D_2^*$, a basis of M_2^*. As $I_1 \dot\cup I_2^*$ is a basis of $M_1 \vee M_2^*$, we see that $I_1 \dot\cup I_2^* = I_1 \cup D_2^*$. The choice of I_2^* implies that $I_2^* = D_2^*$. Thus $$\|I_1\| + r_2(M_2^*) = \|I_1 \dot\cup D_2^*\| = \|B\| = r(M_1 \vee M_2^*) \geq k + r_2(M_2^*).\text{"}$$
434	-3	Replace the last sentence of this exercise by: "Assume that G is connected. Prove that D has a Hamiltonian path, a directed path using all the edges of D, if and only if $M(G)$ has a basis that is independent in both $M[\mathcal{T}]$ and $M[\mathcal{H}]$."
451	-14	Change "For all $n \geq 2$" to "For all $n \geq 3$".
451	-9	Change the sentence that begins on this line to "By Lemma 11.5.5, X is a modular line of Θ_n if $n \geq 3$. Moreover, if $n = 2$, then X is a modular line of $\mathrm{si}(\Theta_n)$."
516	-17	In Exercise 2, the correct alternatives are "either $\kappa_{M\backslash e}(X, Y) < k$ and N is not a minor of M/e; or $\kappa_{M/e}(X, Y) < k$ and N is not a minor of $M\backslash e$."
552	-10	Replace "Proposition 14.6.6" by "Theorem 14.6.6".
584	15	Simeon Ball and Jan De Beule (On sets of vectors of a finite vector space in which every subset of basis size is a basis II, *Des. Codes Cryptog.* **65** (2012), 5–14) proved Conjecture 15.1.2 for $r \leq 2p - 2$ where $q = p^h$ and $h > 1$.
586	12	Part (i) of Conjecture 15.2.2 has been proved by Karim Adiprasito, June Huh and Eric Katz (Hodge theory for combinatorial geometries, *Ann. of Math. (2)* **188** (2018), 381–452). In the same paper, they proved Conjecture 15.2.7.
588	13	Update the reference for June Huh's paper to "Milnor numbers of projective hypersurfaces and the chromatic polynomial of graphs, *J. Amer. Math. Soc.* **25** (2012), 907–927." More significantly, Karim Adiprasito, June Huh and Eric Katz (Hodge theory for combinatorial geometries, *Ann. of Math. (2)* **188** (2018), 381–452) have proved Conjectures 15.2.7 and 15.2.2(i).
592	14	Problem 15.4.3 should be modified to say "Are all matroids with at least two elements minor-reconstructible?" since neither of the two one-element matroids, $U_{0,1}$ and $U_{1,1}$, is minor-reconstructible.
595	-7	The text has been updated here to reflect the fact that what had been Conjecture 15.5.5 has now been proved. The paragraph that had ended on l.-7 has been extended and the text from there to the end of the page has been replaced. This text can now be further updated as follows: Pendavingh and Van der Pol (On the number of bases of almost all matroids, *Combinatorica* **38** (2018), 955–985) have proved Conjecture 15.5.6. Indeed, they have shown that there is a positive constant c such that asymptotically almost all matroids on n elements have connectivity at least $c\sqrt{\log(n)}$.

596	-13	Peter Nelson (Almost all matroids are nonrepresentable, *Bull. Lond. Math. Soc.* **50** (2018), 245–248) has proved Conjecture 15.5.11.
598	-15	Stefan Kratsch and Magnus Wahlström (Representative sets and irrelevant vertices: new tools for kernelization, *2012 IEEE 53rd Annual Symposium on Foundations of Computer Science–FOCS 2012*, 450–459, *IEEE Computer Soc., Los Alamitos, CA*, 2012) have given an algorithm that shows that a gammoid M with n elements can be obtained as a restriction of a strict gammoid for which the associated digraph has $O(n^3)$ vertices. Indeed, if $r(M) = r$, then such a digraph can be found having $O(nr^2)$ vertices.
607	-15	Replace "G is the additive group of $GF(p^n)$ where p is a prime and n is a positive integer" by "G is an abelian group of order p^n where p is a prime and n is a positive integer".
609	5	Update this reference to "On sets of vectors of a finite vector space in which every subset of basis size is a basis, *J. Eur. Math. Soc.* **14** (2012), 733–748."
610	-21	Update reference to "Bonin, J. E. (2010a). Lattice path matroids: the excluded minors. *J. Combin. Theory Ser. B* **100**, 585–599."
610	-18	Update reference to "*SIAM J. Discrete Math.* **24**, 1742–1752."
617	7	Update reference to "Geelen, J., Gerards, B., and Whittle, G. (2010). On inequivalent representations of matroids over non-prime fields. *J. Combin. Theory Ser. B* **100**, 740–743."
617	21	Update reference to "Geelen, J. and Nelson, P. (2010). The number of points in a matroid with no n-point line as a minor. *J. Combin. Theory Ser. B* **100**, 625–630."
621	-2	Update reference to "Kahn, J. and Neiman, M. (2010). Negative correlation and log-concavity. *Random Structures Algorithms* **37**, 367–388."
636	-22	Replace "313–316" by "375–377".
656	1	In the diagram of the Desargues configuration, the highest over-under crossing in the diagram needs to be switched. This makes the middle of the three lines meeting the highest point of the diagram pass behind the first line it encounters when proceeding from the top.