ERRATA AND SUPPLEMENTA FOR *COMBINATORICS OF FINITE SETS* BY IAN ANDERSON (OXFORD, 1987)

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Page 2:
A *chain* of sets is a class of sets in which for each pair of sets, one contains the other. (A definition appears only on p. 14 (says Zaslavsky).)
A definition of *interval* should appear somewhere (says Zaslavsky). The interval $[x, y]$ in a poset is the set $\{z : x \leq z \leq y\}$.

Page 3:
It should be pointed out (says Zaslavsky) that $\max_k \binom{n}{k}$ occurs when $k = \lfloor \frac{n}{2} \rfloor$ and $\lceil \frac{n}{2} \rceil$.

Page 3, line 10:
Replace $p_k = 1$ by $p_k = \binom{n}{k}$.

Page 5, line -2:
$A_k \cap B_j \neq \emptyset$.

Page 7, Exerc. 1.7:
"Inside" means "in the interior of".

Page 7, Exerc. 1.7, hint:
First explain why we may assume that each $x_i > 0$. Then associate with each sum $\sum(2\varepsilon_i - 1)x_i$ the corresponding set of indices i for which $(2\varepsilon_i - 1)x_i > 0$.

Page 8, Exerc. 1.9:
In the binomial coefficient the bottom entry [*binominator*?] should be b_i, not a_i.

Page 11, line -7:
Replace the first i_0 by p_{i_0}.

Page 14:
A good notation for "y covers x" is $y \gtrdot x$. (This has been used by several people. It is available in AMSTex as `\gtrdot` $\gtrdot$ and `\lessdot` $\lessdot$, but you have to load the `amssymb` package.)
The definition of a maximal chain is not optimal (says Zaslavsky). A better definition is: a chain that is not a proper subset of another chain. (This is better for two reasons. First, it applies to all posets. Second, it conforms with the usual meaning of the word "maximal.")
Given the more general definition, here is a suggested additional exercise (says Zaslavsky): Prove that in a ranked poset, any maximal chain (in the new definition) meets every rank set exactly once.
For this the necessary definitions are: *Ranked poset*: a poset in which rank is well defined (see the middle of the page). A *rank set* is the set of all elements having a fixed rank.

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Page 17, Theorem 2.3.4:
It is incorrect as stated. Either replace $n+i$ by $n-i$, or else replace the floor by the ceiling function.

Page 22, top:
Note (says Zaslavsky) that a partition into $k(n)$ blocks will cover any partition in $\mathcal{C}$ to which it is related. (This is only implicit in the text.)

Page 24–25, Exerc. 2.16:
The correct rank function is $r(\pi) = n -$ the number of blocks (says Zaslavsky). All the statements should be revised accordingly.

Page 28, line 13:
Example 3.2 should be Example 3.1.2.

Page 30:
See suggested Exercise $3.1\frac{1}{2}$ (p. 53).

Page 31, line 18:
"a(i)" should be: New paragraph; "(i)"

Page 36, lines 4–6 up:
There's nothing dubious about the antichain $\{\emptyset\}$ (according to Zaslavsky). It meets all the criteria for an antichain, exactly as much as does $\{[n]\}$.

Page 38, line 19:
In the exponent of this displayed formula, the 0 (zero) should be O (Oh); thus, $0(\log n/n)$ should be $O(\log n/n)$.

Page 39, line -1, to 40, line 2:
It appears that the subscripts e, f should be italicized.

Page 40, lines 4—8:
The $(k+1)$'s should be k's.

Page 43, bottom line:
It should read $i = \lceil \frac{1}{2}n \rceil$.

Page 44, Def., line 2:
... saturated and either the maximum rank of an element in each chain is the same, or the minimum rank in each is the same, or the chain containing ...

Page 48, line 8:
In the set definition of $T(n)$ there should also be the proviso that $1 \leq a$ and $c \leq n-1$.

Page 50, Eq. (3.5):
The indexing range $j = J$ should be $j \in J$.

Page 51, line 1:
Anderson 1967b

Page 51, display line after "so that":
Missing left parenthesis inside the last pair of vertical bars.

Page 52, line -8:
The case of odd n ...

Page 53:

Exercise $3.1\frac{1}{2}$ (suggested by Zaslavsky): Use symmetric chain decompositions to prove that the only antichains of maximum size in the power set of an n-set are the one or two middle ranks.

Page 54, Exerc. 3.9 and 3.10:

Theorem 3.4.2.

Page 59, Thm. 4.13:

"rank numbers for multisets" means "rank numbers of the lattice of multisubsets of a multiset", equivalently of the lattice of divisors of an integer.

Page 61, lines 8–16:

The subscripts i are not the rank; they should be j to avoid confusion.

Page 62, lines -6 to -4:

The previous erratum "$L(m)$ should be $\sum k_i$." is wrong. Instead, $\sum k_i$ should be $L(m)$ (twice). Also, in line -6, there should be set braces around $0, 1, \ldots, L(m)$.

Page 63, line 4:

a_i should be d_i.

Page 69, Exerc. 4.7:

This might work better without following the suggested solution outline (says Griggs).

Page 69, Exerc. 4.7(ii):

$m \geq p$ should be $m > p$.

A trick for simplifying calculations (Griggs): define $f_i = g_i = 0$ for $i < 0$, then carry out sums over all $m, p \in \mathbb{Z}$.

Page 72, Lemma 5.1.3:

The proof is wrong. It should be replaced from near the end of line 3 by:
...other sets are the $k-1$ sets A_i beginning with x_i (for $2 \leq i \leq k$) and the sets B_i ending with x_i (for $1 \leq i \leq k-1$). But at most one of A_{i+1}, B_i can be in the collection (for $1 \leq i \leq k-1$), and, since $A_1 = B_k$, the collection can contain at most $1 + (k-1) = k$ of these sets.

Page 73, Lemma 5.2.1:

Adapt the proof similarly.

Page 73, Lemma 5.1.5:

This requires proof.

Page 73, Theorem 5.22:

It should be Theorem 5.2.2.

Page 77, proof of Theorem 5.3.1:

The proof fails at p. 77, line -2, if $|A_i| = n$ since there are only $(n-1)!$ α's. Thus one should check the theorem separately for the case $\mathcal{A} = \{[n]\}$.

Page 79, line 9:

$|A|$ should be $|\mathcal{A}|$.

Page 80, line -9:

$\geq$ should be $=$. Reason: For $k = n/2$, $M_k - \binom{n-1}{\frac{1}{2}n-1} = 0$.

Page 82, Theorem 5.5.3:
Omit the (inside the set braces in the first line.

Page 89, line -2:
$\mathcal{A} = \mathfrak{U} \cap \mathfrak{D}$ should be $\mathcal{A} \subseteq \mathfrak{U} \cap \mathfrak{D}$.

Page 90, Theorem 6.1.5:
The disjointness is obviously unnecessary; the theorem without this assumption follows immediately from that with it.

Page 91, (6.6):
A should be $\mathcal{A}$.

Page 103, line -12:
$k \leq \frac{1}{2}n$

Page 120, Lemma 7.3.2:
Let $\mathcal{A}$ be a collection of k-subsets ...

Page 165, line -4:
In the right-hand summations, $i \in I$, $i \in J$ should be $1 \in I$, $1 \in J$.

Page 186, Exerc. 11.7(i):
greater than $\pi/3$ and less than $2\pi/3$.

Page 214, Hint 1.2:
subsets of size $\geq \frac{1}{2}(n+1)$.

Page 214, Hint 1.5:
A simpler solution than the hint implies is this (says Griggs): Each $x \in S$ belongs to 0, 1, or 2 of the sets in a pair $X \subseteq Y$, and not every x belongs to 0 or 2 of them, since $X \neq Y$. So we get $3^n - 2^n$ such pairs.

Page 214, Hint 1.7:
The hard part (says Griggs) is to observe that it suffices to consider $x_i > 0$ since replacing any $x_i < 0$ by $-x_i$ does not change the relative position of the 2^n sums–it only translates them–providing one makes the correspondence of sums with (respectively, without) $i \in I$ before and sums without (respectively, with) $i \in I$ after the switch.