MATROID THEORY, Second edition

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Errata and Update on Conjectures, Problems, and References
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The reader is encouraged to send the author <oxley@math.lsu.edu> corrections that do not appear in the table below. The errors listed below occur in the first printing of this edition. Most have been corrected in the second printing. In addition, some other changes have made in this second printing and they are also described below. The changes to the references provide updates on publication information for some papers. Again these changes have been incorporated into the second printing. Such changes also sometimes mean that a paper that had been listed as, for example, Bonin (2009a) when in preprint form has become Bonin (2010a) when it actually appeared. This necessitated corresponding changes in the text. Since the original text had no known errors with the references, these changes within the text have not been included below.

Page	**Line**	**Change**
40	1	In Fig. 1.17, the line $\{5, 6, 8\}$ should pass in front of the line $\{3, 4, 10\}$.
50	6	This should say "The Hasse diagram in Figure 1.25" (instead of 1.24).
69	7	Replace "0.1" by "$0, 1$".

Page	Line	Change
84	-15	Beginning here, the proof of this proposition has been simplified as the original proof seemed too convoluted. Another short paragraph has been added to the text following the completion of the proof. The actual change is as follows:

$$\text{“}\dim(W(1,4)\cap W(2,3)) = \dim W(1,4) + \dim W(2,3) - \dim(W(1,4)+W(2,3)) = 2+2-3 = 1.$$

Thus $W(1,4)\cap W(2,3)$ is $\langle \underline{v}\rangle$, the subspace of $V(4,\mathbb{F})$ generated by some non-zero vector $\underline{v}$. Also

$$\dim(W(1,4,5,6)\cap W(2,3,5,6)) = \dim\ W(1,4,5,6) + \dim\ W(2,3,5,6) - \dim(W(1,4,5,6)+W(2,3,5,6)) = 3+3-4 = 2.$$

But $W(5,6)$ is a 2-dimensional subspace of $W(1,4,5,6)\cap W(2,3,5,6)$, so

$$W(5,6) = W(1,4,5,6)\cap W(2,3,5,6) \supseteq W(1,4)\cap W(2,3) = \langle \underline{v}\rangle.$$

Geometrically, if V_8 is $\mathbb{F}$-representable, then we can add a point p to the diagram so that its relationship to $\{1,2,3,4,5,6\}$ is as shown in Figure 2.7(a). Of course, p corresponds to the subspace $\langle \underline{v}\rangle$ of $V(4,\mathbb{F})$. By symmetry, as $W(5,6)\supseteq\langle \underline{v}\rangle$, we deduce that $W(7,8)\supseteq\langle \underline{v}\rangle$. Thus $\langle \underline{v}\rangle\subseteq W(5,6)\cap W(7,8)$. Geometrically, this says that p is on both the line containing 5 and 6 and the line containing 7 and 8, so $\{5,6,7,8\}$ is dependent (see Figure 2.7(b)); or, in terms of subspaces:

$$\dim\ W(5,6,7,8) = \dim(W(5,6)+W(7,8)) = 2+2-\dim(W(5,6)\cap W(7,8)) \le 3.$$

This is a contradiction as $r(\{5,6,7,8\}) = 4$, so V_8 is not $\mathbb{F}$-representable. $\square$

In V_8, each of the sets $\{1,2,5,7\}$, $\{3,4,6,8\}$, $\{1,3,5,8\}$, $\{1,3,6,7\}$, $\{2,4,6,7\}$, $\{2,4,5,8\}$, $\{3,4,5,7\}$, and $\{1,2,6,8\}$ is independent. By Proposition 2.1.24, we can produce new paving matroids by making any collection of these sets into circuit-hyperplanes. While not all of these new matroids are non-isomorphic, the argument just given shows that none of them is representable.”

Page	Line	Change
130	-3	Add "as $\{e_2, e_3\}$ is a circuit of M/e_1."
188	1	In Figure 6.14(b), the columns should be labelled "1 4 7" instead of "5 6 7".
238	-23	Change to "the *frame matroid* or *bias matroid*". After consultation with Tom Zaslavsky, I have changed most occurrences of "bias matroid" to "frame matroid" throughout this section.
240	16	Replace this sentence by: "We refer the reader seeking a more detailed discussion of frame matroids to the sequence of papers on this topic by Zaslavsky (1982a, 1987b, 1989, 1990, 1991, 1994) and to his extensive bibliography of papers in this and related areas (Zaslavsky 1998a) noting that frame matroids were originally called bias matroids."
263	-11	Replace "$A_1 \subseteq E(M_1)$ and $A_2 \subseteq E(M_2)$" by "$A_1 \subseteq E(M_1) - p$ and $A_2 \subseteq E(M_2) - p$".
272	9	Exercise 8 should be marked with an asterisk (*) indicating it is a hard problem.
282	-11	Add to the hypothesis of this exercise that M_2 is a weak-map image of M_1 and replace (b) by: "For all pairs $\{I, J\}$ of independent sets in M_1, if $\mathrm{cl}_1(I) = \mathrm{cl}_1(J)$, then $r_2(I) = r_2(J)$."
434	-3	Replace the last sentence of this exercise by: "Assume that G is connected. Prove that D has a Hamiltonian path, a directed path using all the edges of D, if and only if $M(G)$ has a basis that is independent in both $M[\mathcal{T}]$ and $M[\mathcal{H}]$."
455	1	In Figure 11.22, the third column should be labelled x_3 instead of y_3.
561	12	In this sentence, omit "when q is prime" and insert "projectively" before "inequivalent".
561	15	Insert "projectively" before "inequivalent" in this sentence (twice).
586	5	Recent important progress that is reported in this section necessitated the removal of the sentence "This section has very few updates to the corresponding section in the first edition of this book, reflecting how little progress has been made on these conjectures."
586	16	This paragraph has been altered to incorporate the fact that part (i) of Conjecture 15.2.2 has been proved for representable matroids by Matthias Lenz (The f-vector of a realizable matroid complex is strictly log-concave, 2011, http://arxiv.org/abs/1106.2944).
595	-7	The text has been updated here to reflect the fact that what had been Conjecture 15.5.5 has now been proved. The paragraph that had ended on l.-7 has been extended and the text from there to the end of the page has been replaced with the following. "Very recently, Oxley, Semple, Warshauer, and Welsh (On properties of almost all matroids, 2011) proved the following result thereby verifying a strengthening of one of these conjectures, namely that almost all matroids are connected. **Proposition 15.5.5** *Asymptotically, almost every labelled matroid is* 3-*connected.* □ This result leaves open the following."
609	5	Update reference to "*J. Eur. Math. Soc.*, to appear."
610	-21	Update reference to "Bonin, J. E. (2010a). Lattice path matroids: the excluded minors. *J. Combin. Theory Ser. B* **100**, 585–599."
610	-18	Update reference to "*SIAM J. Discrete Math.* **24**, 1742–1752."

Page	Line	Change
613	1	Update reference to "Chun, C. and Oxley, J. (2009). Unavoidable parallel minors and series minors of regular matroids. *European J. Combin.* **32** (2011), 762–774."
616	22	Update reference to "Geelen, J. (2009). Small cocircuits in matroids. *European J. Combin.* **32** (2011), 795–801."
617	7	Update reference to "Geelen, J., Gerards, B., and Whittle, G. (2010). On inequivalent representations of matroids over non-prime fields. *J. Combin. Theory Ser. B* **100**, 740–743."
617	21	Update reference to "Geelen, J. and Nelson, P. (2010). The number of points in a matroid with no n-point line as a minor. *J. Combin. Theory Ser. B* **100**, 625–630."
618	-8	Update reference to "Hall, R., Mayhew, D., and van Zwam, S. (2009). The excluded minors for near-regular matroids. *European J. Combin.* **32** (2011), 802–830."
621	-2	Update reference to "Kahn, J. and Neiman, M. (2010). Negative correlation and log-concavity. *Random Structures Algorithms* **37**, 367–388."
626	14	Update reference to "Mayhew, D., Newman, M., Welsh, D., and Whittle, G. (2009). On the asymptotic proportion of connected matroids. *European J. Combin.* **32** (2011), 882–890."
626	19	Update reference to "Mayhew, D., Oporowski, B., Oxley, J., and Whittle, G. (2009). The excluded minors for the matroids that are either binary or ternary. *European J. Combin.* **32** (2011), 891–930."
626	-20	Update reference to "Mayhew, D., Royle, G., and Whittle, G. (2009a). The internally 4-connected binary matroids with no $M(K_{3,3})$-minor. *Mem. Amer. Math. Soc.* **208** (2010), no. 981."
626	-15	Update reference to " Mayhew, D., Whittle, G., and van Zwam, S. (2009). An obstacle to a decomposition theorem for near-regular matroids, *SIAM J. Discrete Math.* **25** (2011), 271–279."
626	-13	Update reference to "Mayhew, D., Whittle, G., and van Zwam, S. (2010). Stability, fragility, and Rota's Conjecture, *J. Combin. Theory Ser. B*, to appear."
630	22	Update reference to " Pendavingh, R. A. and van Zwam, S. H. M. (2010b). Confinement of matroid representations to subsets of partial fields *J. Combin. Theory Ser. B* **100**, 510–545."
630	22	Update reference to "Wagner, D. K. (2010). On Mighton's characterization of graphic matroids. *J. Combin. Theory Ser. B* **100**, 493–496."
656	1	In the diagram of the Desargues configuration, the highest over-under crossing in the diagram needs to be switched. This makes the middle of the three lines meeting the highest point of the diagram pass behind the first line it encounters when proceeding from the top.