# Signed Graphs Homework Problems[1]

**Problem 1** (Due 6 February 2017)**.** Let $\Sigma$ be a signed graph. Show the following:

1. Every negation set of $\Sigma$ is a deletion set.
2. Deletion sets of $\Sigma$ need not be negation sets.
3. Every minimal deletion set of $\Sigma$ is a minimal negation set.[2]
4. A negation set of $\Sigma$ need not be a minimal deletion set.

Is every minimal negation set a minimal deletion set?

**Problem 2** (Due 10 February 2017)**.** Show that the maximal frustration index of $K_n$ is equal to the frustration index of $-K_n$ and compute the value thereof.[3] In symbols, show $\ell_{\max}(K_n) = \ell(-K_n)$ and compute.

**Problem 3** (Due 10 February 2017)**.** Show that the frustration index of $\Sigma$ satisfies:

$$\ell(\Sigma) = \min\left\{\#E^-(\Sigma^S) \;\middle|\; S \subseteq V(\Sigma)\right\}$$

**Problem 4** (Open)**.** Characterize the graphs $\Gamma$ for which $\ell_{\max}(\Gamma) = \ell(-\Gamma)$.

**Problem 5** (Open)**.** Let integers $3 \leq r \leq s$ be given, and let $q(r,s)$ denote the number of switching isomorphism classes of $K_{r,s}$. Compute $q(r,s)$.[4]

**Problem 6** (Due 13 February 2017)**.** Let integer $3 \leq s$ be given and consider $K_{3,s}$.

1. Compute $q(3,s)$ for all $s \geq 3$.
2. Describe the switching isomorphism classes of $K_{3,3}$.

   Repeat for $K_{3,s}$ if it is feasable to do so.
3. Compute the frustration index of each switching isomorphism class of $K_{3,3}$.

   Repeat for $K_{3,s}$ if it is feasable to do so.
4. Compute a minimal reresentative of each switching isomorphism class of $K_{3,3}$.[5]

   Repeat for $K_{3,s}$ if it is feasable to do so.

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[1] These are assigned problems from lectures in Signed Graphs given by Tom Zaslavsky during the spring of 2017 as Bingamton University. This list was LaTeXed by Chris Eppolito (email eppolito-at-math-dot-binghamton-dot-edu with errors), and this version was most recently compiled March 29, 2017.

[2] "Minimal" means "minimal with respect to containment."

[3] This result is attributed to Petersdorf.

[4] Various results are known which could be used to relate $q(r,s)$ to other invariants.

[5] "Minimal" here means "minimal with respect to the number of negative edges among all elements of the class."