NAME (Printed): ____

Math 304-5, 6 Linear Algebra Fall 2018 Quiz 1 Feingold

INSTRUCTIONS: Show all necessary work for each problem.

Let $A = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 3 & 5 \\ 2 & 3 & -1 & 0 \end{bmatrix}$.

(1) (4 Points) Use row reduction to Reduced Row Echelon Form (RREF) to find all solutions $X \in \mathbf{R}^4$ to the linear system $AX = 0^4_1$ in terms of some free variables.

(2) (4 Points) Let $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \in \mathbf{R}^4$. Use row reduction to Reduced Row Echelon Form (RREF) to find the conditions on the entries of B required for the linear system $AX = B$ to be consistent.

Math 304-5, 6 Linear Algebra Fall 2018 Quiz 1 Solutions Feingold

1. (4 Points) Use row reduction to Reduced Row Echelon Form (RREF) to solve the linear system by row reducing

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ -1 & 1 & 3 & 5 & 0 \\ 2 & 3 & -1 & 0 & 0 \end{array}\right] \text{ to } \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{ so } \begin{array}{c} x_1 = 2r + 3s \\ x_2 = -r - 2s \\ x_3 = r \in \mathbf{R} \\ x_4 = s \in \mathbf{R} \end{array}.$$

The solutions in terms of free variables are $\left\{ X = \begin{bmatrix} 2r + 3s \\ -r - 2s \\ r \\ s \end{bmatrix} \in \mathbf{R}^4 \;\middle|\; r, s \in \mathbf{R} \right\}$.

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2. (4 Points) For $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \in \mathbf{R}^4$ find conditions on the entries of B for the system $AX = B$ to be consistent. Row reduce the augmented matrix and find consistency conditions from rows with all zeros on the left side:

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & b_1 \\ 1 & 2 & 0 & 1 & b_2 \\ -1 & 1 & 3 & 5 & b_3 \\ 2 & 3 & -1 & 0 & b_4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 2b_1 - b_2 \\ 0 & 1 & 1 & 2 & -b_1 + b_2 \\ 0 & 0 & 0 & 0 & 3b_1 - 2b_2 + b_3 \\ 0 & 0 & 0 & 0 & b_1 + b_2 - b_4 \end{array}\right] \begin{array}{c} \text{is consistent iff} \\ 0 = 3b_1 - 2b_2 + b_3 \\ \text{and} \\ 0 = b_1 + b_2 - b_4 \end{array}.$$