NAME (Printed): ________________________________ Section: ______

Math 304-5,6 Linear Algebra Fall 2018 Quiz 3 Feingold

INSTRUCTIONS: Show all calculations needed to justify your answers.

Let $L_A : \mathbf{R}^2 \to \mathbf{R}^3$ and $L_B : \mathbf{R}^2 \to \mathbf{R}^2$ be the functions

$$L_A(X) = L_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ -x_1 + 2x_2 \\ 2x_1 - x_2 \end{bmatrix} \qquad \text{and} \qquad L_B(Y) = L_B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix}.$$

(1) (2 points) Find the matrices A and B.

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(2) (4 points) Use the formulas for L_A and L_B to get the **formula** for the composition $(L_A \circ L_B)(Y)$.

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(3) (2 points) Use the formula you got for $(L_A \circ L_B)(Y)$ in part 2 to find the matrix C such that $L_C = L_A \circ L_B$.

Math 304-5,6 Linear Algebra Fall 2018 Quiz 3 Solutions Feingold

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$$L_A(X) = L_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ -x_1 + 2x_2 \\ 2x_1 - x_2 \end{bmatrix} \quad \text{and} \quad L_B(Y) = L_B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix}.$$

(1) **Solution:** (2 points) $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ since

$$\begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ -x_1 + 2x_2 \\ 2x_1 - x_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix}.$$

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(2) Use the formulas for L_A and L_B to get the **formula** for the composition $(L_A \circ L_B)(Y)$.
Solution: (4 points)

$$(L_A \circ L_B) \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = L_A \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix} = \begin{bmatrix} (2y_1 - y_2) + (y_1 + 2y_2) \\ -(2y_1 - y_2) + 2(y_1 + 2y_2) \\ 2(2y_1 - y_2) - (y_1 + 2y_2) \end{bmatrix} = \begin{bmatrix} 3y_1 + y_2 \\ 5y_2 \\ 3y_1 - 4y_2 \end{bmatrix}$$

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(3) **Solution:** (2 points) $C = \begin{bmatrix} 3 & 1 \\ 0 & 5 \\ 3 & -4 \end{bmatrix}$ since $\begin{bmatrix} 3 & 1 \\ 0 & 5 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3y_1 + y_2 \\ 5y_2 \\ 3y_1 - 4y_2 \end{bmatrix}.$
