NAME (Printed): ______________________________ Section: ______

Math 304-5,6 Linear Algebra Fall 2018 Quiz 4 Feingold

INSTRUCTIONS: Show all calculations needed to justify your answers.

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $W = \left\{ B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{R}^2_2 \mid AB = 0^2_2 \right\}$.

(1) (4 Pts) Show that W is a subspace of $\mathbf{R}^2_2$.

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(2) (3 Pts) Find exactly what vectors B of $\mathbf{R}^2_2$ are in W by solving a linear system.

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(1) (4 Pts) Show that W is a subspace of $\mathbf{R}^2_2$.

SOLUTION: First show W is closed under addition. Let $B_1, B_2 \in W$, so $AB_1 = 0^2_2$ and $AB_2 = 0^2_2$. Then $A(B_1 + B_2) = AB_1 + AB_2 = 0^2_2 + 0^2_2 = 0^2_2$ means $B_1 + B_2 \in W$.

Next show W is closed under scalar multiplication. Let $B \in W$ so $AB = 0^2_2$, and let $r \in \mathbf{R}$. Then $A(rB) = r(AB) = r0^2_2 = 0^2_2$ means $rB \in W$.

Finally, $0^2_2 \in W$ since $A0^2_2 = 0^2_2$.

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(2) (3 Pts) Find exactly what vectors B of $\mathbf{R}^2_2$ are in W by solving a linear system.

SOLUTION: To find exactly what vectors $B \in \mathbf{R}^2_2$ are in W, we write the condition $AB = 0^2_2$ as

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} (a+2c) & (b+2d) \\ (2a+4c) & (2b+4d) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

This gives us four equations in four variables which we solve by row reduction as follows:

$$\text{Row reduce } \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \\ 0 & 2 & 0 & 4 & 0 \end{array}\right] \text{ to } \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{ so } \begin{array}{c} a = -2c \\ b = -2d \\ c \in \mathbf{R} \text{ free} \\ d \in \mathbf{R} \text{ free} \end{array}$$

so $W = \left\{ B = \begin{bmatrix} -2c & -2d \\ c & d \end{bmatrix} \in \mathbf{R}^2_2 \mid c, d \in \mathbf{R} \right\}$.
