NAME (Printed): ________________________________ Section: ______

Math 304-5,6 Linear Algebra Fall 2018 Quiz 6 Feingold

INSTRUCTIONS: Show all calculations and reasons needed to justify your answers.

(1) (3 Pts) Let $V = \mathbf{R}^2_2$. Find a **basis** for subspace $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{R}^2_2 \mid a - b + 2c - 3d = 0 \right\}$.

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(2) (2 Pts) Suppose $S = \{u_1, u_2, \cdots, u_m\}$ is an **independent** subset of a vector space U and $u \in \langle S \rangle$. Then $T = \{u_1, u_2, \cdots, u_m, u\}$ **must** be ________________ since

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(3) (2 Pts) Suppose $S = \{v_1, v_2, \cdots, v_n\}$ spans V. What does this imply about $dim(V)$?

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(1) (3 Pts) Let $V = \mathbf{R}^2_2$. Find a **basis** for subspace $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{R}^2_2 \mid a - b + 2c - 3d = 0 \right\}$.

Solution: Since $a = b - 2c + 3d$ we can write any vector in W as

$$\begin{bmatrix} b-2c+3d & b \\ c & d \end{bmatrix} = b \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = bw_1 + cw_2 + dw_3$$

so $W = \langle w_1, w_2, w_3 \rangle$. Also, $\{w_1, w_2, w_3\}$ is independent since $bw_1 + cw_2 + dw_3 = 0^2_2$ means $\begin{bmatrix} b-2c+3d & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ so $b = c = d = 0$. Then $\{w_1, w_2, w_3\}$ is a basis of W.

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(2) (2 Pts) Suppose $S = \{u_1, u_2, \cdots, u_m\}$ is an **independent** subset of a vector space U and $u \in \langle S \rangle$. Then $T = \{u_1, u_2, \cdots, u_m, u\}$ **must** be __________________ since

Solution: T must be **dependent** since $u \in \langle S \rangle$ means $u = \sum_{i=1}^m a_i u_i$ for some $a_i \in \mathbf{R}$, we have the non-trivial dependence relation $1u - \sum_{i=1}^m a_i u_i = \theta$ among the vectors in T.

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(3) (2 Pts) Suppose $S = \{v_1, v_2, \cdots, v_n\}$ spans V. What does this imply about $dim(V)$?

Solution: If S is independent, then S is a basis for V so $dim(V) = n$. But if S is dependent, there is at least one redundant vector which can be removed, leaving a subset of $n-1$ vectors that span V. Continuing to remove redundant vectors from that subset, eventually we can reduce S to an independent set that spans V, a basis for V. So in general, the most we can say is that $dim(V) \leq n$.
