NAME (Printed): __________________________________ Section: ______

Math 304-5,6 Linear Algebra Fall 2018 Quiz 9 Feingold

INSTRUCTIONS: Show all calculations and reasons needed to justify your answers.

(1) (2 Pts) Suppose $L : V \to V$ and $v \in V$ is an eigenvector for L with eigenvalue $\lambda \in \mathbf{R}$. Show v is also an eigenvector for $L^2 = L \circ L$ with eigenvalue λ^2.

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(2) (2 Pts) Let $A, B, C \in \mathbf{R}_n^n$ with $det(A) = 3$, $det(B) = -7$ and $\det(C) = 2$. Find $det(A^T B^{-1} C^3)$.

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(3) (4 Pts) Find det $\begin{bmatrix} 18 & -20 & -20 & -20 \\ 5 & -7 & -5 & -5 \\ 5 & -5 & -7 & -5 \\ 5 & -5 & -5 & -7 \end{bmatrix}$.

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Solution: Since $L(v) = \lambda v$ we have $L^2(v) = (L \circ L)(v) = L(L(v)) = L(\lambda v) = \lambda L(v) = \lambda\lambda v = \lambda^2 v$ shows v is an eigenvector for L^2 with eigenvalue λ^2.

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(2) (2 Pts) Let $A, B, C \in \mathbf{R}^n_n$ with $det(A) = 3$, $det(B) = -7$ and $\det(C) = 2$. Find $det(A^T B^{-1} C^3)$.

Solution: $det(A^T B^{-1} C^3) = \dfrac{det(A) det(C)^3}{det(B)} = \dfrac{(3)(2^3)}{-7} = \dfrac{24}{-7}$.

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(3) (4 Pts) Find $\det \begin{bmatrix} 18 & -20 & -20 & -20 \\ 5 & -7 & -5 & -5 \\ 5 & -5 & -7 & -5 \\ 5 & -5 & -5 & -7 \end{bmatrix}$.

Solution: $\det \begin{bmatrix} 18 & -20 & -20 & -20 \\ 5 & -7 & -5 & -5 \\ 5 & -5 & -7 & -5 \\ 5 & -5 & -5 & -7 \end{bmatrix} = \det \begin{bmatrix} -2 & 0 & 0 & 8 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -2 & 2 \\ 5 & -5 & -5 & -7 \end{bmatrix} =$
$(-2)^3 \det \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 5 & -5 & -5 & -7 \end{bmatrix} = (-2)^3 \det \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix} = (-2)^3(1)(1)(1)(3) = -24.$

In the first step, three elementary adder row operations were applied:

$$-1\ Row_4 + Row_3 \to Row_3, \quad -1\ Row_4 + Row_2 \to Row_2, \quad -4\ Row_4 + Row_1 \to Row_1.$$

In the second step, a factor of -2 was brought out from each of the first three rows. In the third step, three elementary adder row operations were applied,

$$-5\ Row_1 + Row_4 \to Row_4, \quad 5\ Row_2 + Row_4 \to Row_4, \quad 5\ Row_3 + Row_4 \to Row_4.$$

They had no effect on the determinant, but made zeros in the first three columns of Row_4, and made the last column of Row_4 into $-7 + 20 - 5 - 5 = 20 - 17 = 3$. The determinant of the final upper triangular matrix was the product of the diagonal entries.
