1. For matrices A, B, C, $\quad A + B = A + C \quad$ if and only if $\quad A = B$.
□ TRUE ■ **FALSE**

2. Every linear system with the same number of equations as unknowns has a unique solution.
□ TRUE ■ **FALSE**

3. Every linear system with the same number of equations as unknowns has at least one solution.
□ TRUE ■ **FALSE**

4. Every linear system with more equations than unknowns may have an infinite number of solutions.
■ **TRUE** □ FALSE

5. Every linear system with fewer equations than unknowns may have no solutions.
■ **TRUE** □ FALSE

6. Every matrix is row equivalent to a unique matrix in row echelon form.
□ TRUE ■ **FALSE**

7. If $[A \mid \mathbf{b}]$ and $[C \mid \mathbf{d}]$ are row equivalent augmented matrices, the matrix equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have the same solution set.
■ **TRUE** □ FALSE

8. A linear system with a square coefficient matrix A has a unique solution if and only if A is row equivalent to the identity matrix.
■ **TRUE** □ FALSE

9. A linear system with coefficient matrix A has an infinite number of solutions if and only if A can be row reduced to an echelon form that includes some column containing no pivot.
□ TRUE ■ **FALSE**

10. A consistent linear system with coefficient matrix A has an infinite number of solutions if and only if A can be row reduced to an echelon form that includes some column containing no pivot.
■ **TRUE** □ FALSE

11. If a square linear system $A\mathbf{x} = \mathbf{b}$ has a solution for every choice of column vector $\mathbf{b}$, then the solution is unique for each $\mathbf{b}$.
■ **TRUE** □ FALSE

12. If a square linear system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for every (appropriately sized) column vector $\mathbf{b}$.
■ **TRUE** □ FALSE

13. Multiplication of a non-zero vector by a non-zero scalar never yields the zero vector
■ **TRUE** □ FALSE

14. No vector is its own additive inverse.
□ TRUE ■ **FALSE**

15. Every vector space has at least two vectors.
□ TRUE ■ **FALSE**

16. Every vector space has at least two distinct subspaces.
□ TRUE ■ **FALSE**

17. If $\mathbf{u} + \mathbf{v}$ is in a subspace W of a vector space V, then both $\mathbf{u}$ and $\mathbf{v}$ are elements of W.
□ TRUE ■ **FALSE**

18. Two subspaces of a vector space V may have an empty intersection.
□ TRUE ■ **FALSE**

19. The rank of $A + B$ is less than or equal to the ranks of both A and B.
☐ TRUE ■ **FALSE**

20. Let A and B be 2×2 matrices such that $AB = 0$. Either $A = 0$ or $B = 0$, or both.
☐ TRUE ■ **FALSE**

21. Let A and B be 2×2 matrices such that $AB = 0$. If B is invertible, then $A = 0$.
■ **TRUE** ☐ FALSE

22. Let A and B be 2×2 matrices such that $AB = 0$. $BA = 0$.
☐ TRUE ■ **FALSE**

23. Let A and B be 2×2 matrices such that $AB = 0$. There is a vector $\mathbf{x} \neq 0$ such that $BA\mathbf{x} = \mathbf{0}$.
■ **TRUE** ☐ FALSE

24. If B is an $n \times n$ matrix and $B^2 = B$, then B is not invertible.
☐ TRUE ■ **FALSE**

25. The span of any two non-zero vectors in $\mathbb{R}^2$ is all of $\mathbb{R}^2$.
☐ TRUE ■ **FALSE**

26. If $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k$ are vectors in $\mathbb{R}^2$ such that $\text{span}(\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k) = \mathbb{R}^2$, then $k = 2$.
☐ TRUE ■ **FALSE**

27. A subset of $\mathbb{R}^n$ containing two non-zero distinct parallel vectors is dependent.
■ **TRUE** ☐ FALSE

28. If a set of non-zero vectors in $\mathbb{R}^n$ is dependent, then any two vectors in the set are parallel.
☐ TRUE ■ **FALSE**

29. Every subset of three vectors in $\mathbb{R}^2$ is dependent.
■ **TRUE** ☐ FALSE

30. Every subset of two vectors in $\mathbb{R}^2$ is independent.
☐ TRUE ■ **FALSE**

31. If a subset of two vectors in $\mathbb{R}^2$ spans $\mathbb{R}^2$, then the subset is independent.
■ **TRUE** ☐ FALSE

32. If S is independent, then each vector in V can be expressed uniquely as a linear combination of the vectors in S.
☐ TRUE ■ **FALSE**

33. If S is independent and spans V, then each vector in V can be expressed uniquely as a linear combination of the vectors in S.
■ **TRUE** ☐ FALSE

34. Every independent subset of V is a subset of some basis for V.
■ **TRUE** ☐ FALSE

35. If H is a row echelon form of a matrix A, then the non-zero column vectors in H form a basis for the column space of A.
☐ TRUE ■ **FALSE**

36. For all positive integers m and n, the nullity (dimension of the null space) of an $m \times n$ matrix might be any number from 0 to n.
☐ TRUE ■ **FALSE**

37. For all positive integers m and n, the nullity of an $m \times n$ matrix might be any number from 0 to m.
☐ TRUE ■ **FALSE**

38. For all positive integers $m \geq n$, the nullity of an $m \times n$ matrix might be any number from 0 to n.
■ **TRUE** □ FALSE

39. There is a unique coordinate vector associated with each vector $\mathbf{v} \in V$.
□ TRUE ■ **FALSE**

40. There is a unique coordinate vector associated with each vector $\mathbf{v} \in V$ relative to a basis for V.
□ TRUE ■ **FALSE**

41. There is a unique coordinate vector associated with each vector $\mathbf{v} \in V$ relative to an ordered basis of V.
■ **TRUE** □ FALSE

42. Distinct vectors in V have distinct coordinate vectors relative to the same ordered basis $\mathcal{B}$ of V.
■ **TRUE** □ FALSE

43. The same vector in V cannot have the same coordinate vector relative to different ordered bases $\mathcal{B}$ and C of V.
□ TRUE ■ **FALSE**

44. Every change-of-coordinates matrix is square.
■ **TRUE** □ FALSE

45. There are six possible ordered bases for $\mathbb{R}^3$.
□ TRUE ■ **FALSE**

46. There are six possible ordered bases for $\mathbb{R}^3$ consisting of the standard basis vectors.
■ **TRUE** □ FALSE

47. If A is invertible, the row space of A^{-1} must be the same as the row space of A.
■ **TRUE** □ FALSE

48. If A is a 6×8 matrix then the dimension of Null(A) is at least two.
■ **TRUE** □ FALSE

49. If A is a 6×8 matrix and the dimension of the null space of A is three, then the dimension of Col(A^T) is five.
■ **TRUE** □ FALSE

50. If A is a 3×2 matrix with independent columns, then there exists a 2×3 matrix B such that $AB = I_3$.
□ TRUE ■ **FALSE**

51. Let A be an $n \times n$ matrix. If $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_n\}$ is a basis for the row space of A (these $\mathbf{b}_i's$ are written in row vector shape), then the columns of the matrix $\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_n \end{bmatrix}$ form a basis for the column space of A.
■ **TRUE** □ FALSE

52. Let V and W be linear spaces (vector spaces) and let $T : V \to W$ be a linear transformation. Let $v_1, v_2 ..., v_n$ be vectors in V. If $T(v_1), T(v_2), ..., T(v_n)$ are linearly independent, then $v_1, v_2 ..., v_n$ are linearly independent.
■ **TRUE** □ FALSE

53. The determinant of a 2×2 matrix is a vector.
□ TRUE ■ **FALSE**

54. If two rows of 3×3 matrix are interchanged, the sign of the determinant is changed.
■ **TRUE** □ FALSE

55. The determinant of a 3×3 matrix is zero if two rows of the matrix are parallel vectors in $\mathbb{R}^3$
■ **TRUE** □ FALSE

56. In order for the determinant of a 3×3 matrix to be zero, two rows of the matrix must be parallel vectors in $\mathbb{R}^3$
□ TRUE ■ **FALSE**

57. The determinant of A, $\det(A)$, is defined for any matrix A.
□ TRUE ■ **FALSE**

58. $\det(A)$ is defined for each square matrix A.
■ **TRUE** □ FALSE

59. $\det(A)$ is a scalar.
■ **TRUE** □ FALSE

60. If a matrix A is multiplied by the scalar k, then the determinant of the resulting matrix is $k \cdot \det(A)$.
□ TRUE ■ **FALSE**

61. If an $n \times n$ matrix A is multiplied by the scalar k, then the determinant of the resulting matrix is $|k| \cdot \det(A)$.
□ TRUE ■ **FALSE**

62. If an $n \times n$ matrix A is multiplied by the scalar k, then the determinant of the resulting matrix is $k^n \cdot \det(A)$.
■ **TRUE** □ FALSE

63. $\det(AA^T) = \det(A^T A) = [\det(A)]^2$
■ **TRUE** □ FALSE

64. The determinant of an elementary matrix is nonzero.
■ **TRUE** □ FALSE

65. If $\det(A) = 2$ and $\det(B) = 2$, then $\det(A + B) = 4$.
□ TRUE ■ **FALSE**

66. If $\det(A) = 2$ and $\det(B) = 2$, then $\det(AB) = 4$.
□ TRUE ■ **FALSE**

67. If A and B are $n \times n$ matrices and $\det(A) = 2$ and $\det(B) = 3$, then $\det(AB) = 6$.
■ **TRUE** □ FALSE

68. The determinant of a square matrix is the product of the entries on its main diagonal.
□ TRUE ■ **FALSE**

69. The determinant of an upper triangular matrix is the product of the entries on its main diagonal.
■ **TRUE** □ FALSE

70. The determinant of a lower triangular matrix is the product of the entries on its main diagonal.
■ **TRUE** □ FALSE

71. A square matrix is invertible if and only if its determinant is positive.
□ TRUE ■ **FALSE**

72. The column vectors of an $n \times n$ matrix are independent if and only if the determinant of the matrix is nonzero.
■ **TRUE** □ FALSE

73. A homogeneous square linear system has a nontrivial solution if and only if the determinant of its coefficient matrix is zero.
■ **TRUE** □ FALSE

74. Every $n \times n$ matrix has real eigenvalues.
□ TRUE ■ **FALSE**

75. There can only be one eigenvalue associated with an eigenvector.
■ **TRUE** □ FALSE

76. There can only be one eigenvector associated with a distinct eigenvalue.
□ TRUE ■ **FALSE**

77. If **v** is an eigenvector for A then **v** is an eigenvector for $A - kI_n$, for all scalars k.
■ **TRUE** □ FALSE

78. If ł is an eigenvalue for A then ł is an eigenvalue for $A - kI_n$, for all scalars k.
□ TRUE ■ **FALSE**

79. If **v** is an eigenvector for A^{-1} then k**v** is an eigenvector for A, for all nonzero scalars k.
■ **TRUE** □ FALSE

80. Every vector in $\mathbb{R}^n$ is an eigenvector for I_n.
□ TRUE ■ **FALSE**

81. If an $n \times n$ matrix has n distinct real eigenvalues it is diagonalizable.
■ **TRUE** □ FALSE

82. An $n \times n$ matrix is diagonalizable if and only if it has n distinct eigenvalues.
□ TRUE ■ **FALSE**

83. Let A and B be 3×3 matrices. If $\det(A) = 1$ and $\det(B) = 0$, then rank $A >$ rank B.
■ **TRUE** □ FALSE

84. An $n \times n$ matrix is diagonalizable if and only if the algebraic multiplicity of each of its eigenvalues equals the geometric multiplicity.
■ **TRUE** □ FALSE

85. Every invertible matrix is diagonalizable.
□ TRUE ■ **FALSE**

86. Every triangular matrix is diagonalizable.
□ TRUE ■ **FALSE**

87. If A and B are similar and A is diagonalizable, then B is also diagonalizable.
■ **TRUE** □ FALSE

88. If A and B are similar, then $\det(A) = \det(B)$.
■ **TRUE** □ FALSE

89. If A is diagonalizable, there is a unique diagonal matrix D that is similar to A.
□ TRUE ■ **FALSE**

90. If A and P are $n \times n$ matrices with P invertible, then $\det(PAP^{-1}) = \det(A)$.
■ **TRUE** □ FALSE

91. Similar matrices have the same eigenvalues and eigenvectors.
□ TRUE ■ **FALSE**

92. Similar matrices have the same eigenvalues with the algebraic and geometric multiplicities.
■ **TRUE** □ FALSE

93. If A and B are $n \times n$ matrices with B invertible and **v** an eigenvector of A, then B**v** is an eigenvector of BAB^{-1}.
■ **TRUE** □ FALSE

94. If A and B are $n \times n$ matrices with B invertible and **v** an eigenvector of A, then B^{-1}**v** is an eigenvector of $B^{-1}AB$.
■ **TRUE** □ FALSE

95. Any two $n \times n$ diagonalizable matrices having the same eigenvalues of the same algebraic multiplicities are similar.
■ **TRUE** □ FALSE

96. Any two $n \times n$ diagonalizable matrices having the same eigenvectors are similar.
□ TRUE ■ **FALSE**

97. Any two $n \times n$ diagonal matrices are similar.
□ TRUE ■ **FALSE**

98. Every non-zero vector in $\mathbb{R}^n$ has non-zero magnitude.
■ **TRUE** □ FALSE

99. Every vector of non-zero magnitude in $\mathbb{R}^n$ is non-zero.
■ **TRUE** □ FALSE

100. There are exactly two unit vectors parallel to any given non-zero vector in $\mathbb{R}^n$.
■ **TRUE** □ FALSE

101. There are exactly two unit vectors orthogonal to any given non-zero vector in $\mathbb{R}^n$.
□ TRUE ■ **FALSE**

102. The dot product of a vector with itself yields the magnitude of the vector.
□ TRUE ■ **FALSE**

103. For a vector $\mathbf{v} \in \mathbb{R}^n$ and r a scalar, the magnitude of r times $\mathbf{v}$ is r times the magnitude of $\mathbf{v}$.
□ TRUE ■ **FALSE**

104. If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ of the same magnitude, then the magnitude of $\mathbf{v} - \mathbf{w}$ is 0.
□ TRUE ■ **FALSE**

105. The set of all vectors in $\mathbb{R}^n$ orthogonal to every vector $\mathbf{w}$ of a subspace W is a subspace of $\mathbb{R}^n$.
■ **TRUE** □ FALSE

106. The intersection of W and $W^\perp$ is empty.
□ TRUE ■ **FALSE**

107. If vectors $\mathbf{u}$ and $\mathbf{v}$ have the same projection onto the subspace W of V, then $\mathbf{u} = \mathbf{v}$.
□ TRUE ■ **FALSE**

108. All vectors in an orthogonal basis have length 1.
□ TRUE ■ **FALSE**

109. If A has kernel $\{\mathbf{0}\}$, then the Gram matrix, $A^T A$, is invertible.
■ **TRUE** □ FALSE

110. The parallelogram in $\mathbb{R}^2$ determined by nonzero vectors $\mathbf{x}$ and $\mathbf{y}$ is a square if and only if $\mathbf{x} \cdot \mathbf{y} = 0$.
□ TRUE ■ **FALSE**

111. The box in $\mathbb{R}^3$ determined by vectors $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ is a cube if and only if $\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot \mathbf{z} = \mathbf{y} \cdot \mathbf{z}0$, and $\mathbf{x} \cdot \mathbf{x} = \mathbf{y} \cdot \mathbf{y} = \mathbf{z} \cdot \mathbf{z}$.
■ **TRUE** □ FALSE

112. The projection of $\mathbf{x}$ onto the span of $\mathbf{y}$ is a scalar multiple of $\mathbf{x}$.
□ TRUE ■ **FALSE**

113. The projection of $\mathbf{x}$ onto the span of $\mathbf{y}$ is a scalar multiple of $\mathbf{y}$.
■ **TRUE** □ FALSE

114. The set of all vectors in $\mathbb{R}^n$ which are orthogonal to a subspace W of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$.
■ **TRUE** □ FALSE

115. If the projection of $\mathbf{x}$ onto the subspace W of $\mathbb{R}^n$ is $\mathbf{x}$ itself, then $\mathbf{x}$ is orthogonal to every vector in W.
□ TRUE ■ **FALSE**

116. If the projection of $\mathbf{x}$ onto the subspace W of $\mathbb{R}^n$ is $\mathbf{x}$ itself, then $\mathbf{x} \in W$.
■ **TRUE** □ FALSE

117. The intersection of W and $W^\perp$ is empty.
□ TRUE ■ **FALSE**

118. if $\text{proj}_W \mathbf{x} = \text{proj}_W \mathbf{y}$ then $\mathbf{x} = \mathbf{y}$.
□ TRUE ■ **FALSE**

119. All vectors in an orthogonal basis have length 1.
□ TRUE ■ **FALSE**

120. Every nontrivial subspace of R^n has an orthonormal basis.
■ **TRUE** □ FALSE

121. Every vector in $\mathbb{R}^n$ is in some orthonormal basis for R^n.
□ TRUE ■ **FALSE**

122. Every unit vector in $\mathbb{R}^n$ is in some orthonormal basis for R^n.
■ **TRUE** □ FALSE

123. A matrix is orthogonal when its column vectors are orthogonal.
□ TRUE ■ **FALSE**

124. A square matrix is orthonormal when its column vectors are orthonormal.
□ TRUE ■ **FALSE**

125. Every orthogonal matrix has a trivial nullspace.
■ **TRUE** □ FALSE

126. If A^T is orthogonal then A is orthogonal.
■ **TRUE** □ FALSE

127. If A is a symmetric orthogonal $n \times n$ matrix, then $A^2 = I_n$.
■ **TRUE** □ FALSE

128. If A is a symmetric $n \times n$ matrix with $A^2 = I_n$, then A is an orthogonal matrix.
■ **TRUE** □ FALSE

129. If A and B are both $n \times n$ orthogonal matrices, then AB is an orthogonal matrix.
■ **TRUE** □ FALSE