Math 304 **Midterm 2** **Spring 2019**

No books, no notes, no calculators. You must show work, unless the question is a true/false, multiple choice, or fill-in-the-blank question.

Name: ______________________________ Section: ____________________

Section Number	Instructor	Meeting Time
1	David Biddle	8:00
2	Daniel Rossi	8:00
3	David Biddle	9:40
4	Thomas Kilcoyne	11:20
5	Dikran Karagueuzian	11:20
6	Thomas Kilcoyne	1:10
7	Matt Evans	2:50
8	Joshua Carey	4:40

Page:	2	3	4	5	6	7	8	Total
Points:	24	8	8	8	9	9	9	75
Score:								

1. Fill in the blanks in the following definitions and statements of results from the textbook.

   (a) (4 points) If $F\colon V \to V$ is a linear transformation, we say that a vector w is an *eigenvector* of F associated to the *eigenvalue* α if two conditions are met:

      1. ____

      2. ____

   (b) (6 points) A list of vectors $(w_1, w_2, \ldots, w_n)$ in a vector space is said to be *linearly dependent* if there are ________________________ $a_1, a_2, \ldots, a_n$ such that the linear combination

   $$\underline{\hspace{6cm}} = \underline{\hspace{2cm}}$$

   and ____________________________________.

   (c) (4 points) A *basis* of a vector space V is a list $(v_1, v_2, \ldots, v_n)$ of vectors in V with two properties:

      1. ____

      2. ____

   (d) (10 points) Fill in the blanks in the following statments of properties of the determinant. Throughout, you may assume that A, B are $n \times n$ matrices of real numbers.

      (a) If A is the identity matrix, the determinant of A is ________.

      (b) If B is obtained by exchanging two rows of A, then $\det B =$ ______________.

      (c) The determinant is a __________________ function of each row separately.

(d) If two rows of A are equal, then $\det A$ is __________.

(e) If B is obtained by subtracting a multiple of one row of A from another row of A, then $\det B =$ ________________________.

(f) If A is a matrix with a row of zeroes, then $\det A =$ __________.

(g) If A is not invertible, then $\det A$ ________________________.

(h) How is $\det(AB)$ related to $\det(A)$ and $\det(B)$? $\det(AB) =$ ________________________.

(i) How is $\det(A^T)$ related to $\det(A)$? $\det(A^T) =$ ________________________.

(e) (5 points) Fill in the blanks in the following statement of the "Subspace Theorem" (Theorem 9.1.1):

Let U be a non-empty subset of a vector space V. Then U is a subspace if and only if, for any two vectors v and w in ______________, and any two ____________________ a and b, we have

$$\underline{\hspace{4cm}} \in \underline{\hspace{4cm}}$$

(f) (3 points) The *characteristic polynomial* of an $n \times n$ matrix A is, by definition:

$$p_A(\lambda) = \underline{\hspace{6cm}}$$

2. (8 points) Find the characteristic polynomial of the matrix A below:

$$A = \begin{bmatrix} 1 & 2 & 2 \\ -3 & 6 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

3. (8 points) After some computations, you find that the characteristic polynomial of the matrix

$$B = \begin{bmatrix} 7 & 6 & -2 \\ -4 & -3 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

is $p_B(\lambda) = \lambda^3 - 3\lambda^2 + 4$. Find all eigenvalues of B.

4. (9 points) After some computation, you determine that 2 is an eigenvalue of the matrix

$$C = \begin{bmatrix} 1 & 6 & 5 \\ 0 & -4 & -3 \\ 1 & 4 & 2 \end{bmatrix}$$

Find a basis of the eigenspace of C associated to the eigenvalue 2.

5. (9 points) Let $\mathbb{P}_2$ be the vector space of polynomials with real coefficients with degree less than or equal to 2. Let

$$B = (1, x, x^2) \qquad \text{and} \qquad C = (1, 1 + x, 1 + x + \frac{1}{2}x^2)$$

be two ordered bases of $\mathbb{P}_2$. (You may assume that B and C are bases and need not verify this.) Find the change of basis matrix P that helps us convert from the old basis B to the new basis C.

6. (9 points) The matrix of the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ with respect to the standard basis E is

$$_E T_E = A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & 3 \end{bmatrix}$$

After some computing, you find that the vectors

$$u = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \qquad v = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad w = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

are eigenvectors of A associated to the eigenvalues 1, 2, and 2 respectively. Let $C = (u, v, w)$ be the new basis of $\mathbb{R}^3$ formed by these three vectors, in this order.

Find $_C T_C$, the matrix of T with respect to the basis C (used as both the input basis and the output basis). *You need not verify statements made in the problem. In particular you may assume that u, v, and w are eigenvectors and that C is a basis.*