Multiple Choice Questions

There is no penalty for guessing. Four points per question, so a total of 64 points for this section.

1. What is the determinant of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$?

   (a) 2

   (b) 4

   (c) 6

   (d) None of the above

2. What is the determinant of the matrix $\begin{bmatrix} \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \end{bmatrix}$?

   (a) 0

   (b) λ

   (c) λ^3

   (d) None of the above

3. Suppose the determinant of a 2×2 matrix A is equal to 5. What is the determinant of $2A$?

   (a) 5

   (b) 10

   (c) 20

   (d) 25

   (e) There is insufficient information to answer the question.

4. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

   (a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

   (b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

   (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

   (e) There is insufficient information to answer the question

5. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$?

   (a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (d) None of the above

6. $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue?

   (a) 4/3

   (b) 5

   (c) -2

   (d) None of the above

7. Choose the correct completion of the following statement: If A and B are square matrices such that $AB = I$, then zero is an eigenvalue of

   (a) A but not of B

   (b) B but not of A

   (c) Both A and B

   (d) Neither A nor B

8. How many subspaces does $\mathbb{R}^2$ have?

   (a) two: 0 and $\mathbb{R}^2$

   (b) four: 0, $\mathbb{R} \times 0$, $0 \times \mathbb{R}$ (the "axes"), and $\mathbb{R}^2$ itself

   (c) infinitely many

   (d) None of the above answers is correct

9. To determine whether a set of n vectors from $\mathbb{R}^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

   (a) A row of all zeros

   (b) A row that has all zeros except in the last position

   (c) A column of all zeros

   (d) An identity matrix

10. Which of the following sets of vectors forms a basis for $\mathbb{R}^3$? (Hint: at least one of these sets is a basis!)

i.) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \right\}$

ii.) $\left\{ \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

iii.) $\left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix} \right\}$

iv.) $\left\{ \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} \right\}$

(a) ii, iii, and iv only

(b) ii and iii only

(c) i, ii, and iii only

(d) iii and iv only

(e) ii only

11. Which of the following sets is linearly independent? (Hint: only one set is linearly independent!)

(a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -12 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

12. Let V be a vector space, and let S be a subset of V. What does it mean when we say that S spans V?

(a) The elements of S are all distinct from each other

(b) Every vector in V has exactly one representation as a linear combination of vectors in S

(c) S has at least as many elements as the dimension V

(d) S is a basis for V

(e) Every vector in V can be expressed as a linear combination of vectors in S

13. Let V be a five-dimensional vector space, and let S be a subset of V consisting of three vectors. Then S

    (a) May or may not be linearly independent, and may or may not span V
    (b) Must be linearly dependent, but may or may not span$V$
    (c) Must be linearly independent, but cannot span V
    (d) Can span V, but only if it is linearly independent, and vice versa
    (e) Cannot span V, but can be linearly independent or dependent

14. Let V be a three-dimensional vector space, and let S be a subset of V consisting of five vectors. Then S

    (a) Can span V,but only if it is linearly independent, and vice versa
    (b) Must be linearly independent, but cannot span V
    (c) Must be linearly dependent, and must span V
    (d) Must be linearly independent, but may or may not span V
    (e) Must be linearly dependent, but may or may not span V

15. Let A and B be the 2×2 matrices

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

    Which of the following statements is most accurate?

    (a) Neither A nor B is diagonalizable.
    (b) Both A and B are diagonalizable.
    (c) A is diagonalizable but B is not.
    (d) B is diagonalizable but A is not.

16. Let A and B be the 2×2 matrices

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

    Which of the following statements is most accurate?

    (a) Neither A nor B is diagonalizable.
    (b) Both A and B are diagonalizable.
    (c) A is diagonalizable but B is not.
    (d) B is diagonalizable but A is not.