Multiple Choice Questions

There is no penalty for guessing. Three points per question, so a total of 48 points for this section.

1. What is the complete relationship between homogeneous linear systems of equations, and the zero solution (all unknowns equal to zero)?

   (a) If a solution to a homogeneous linear system exists at all, then the zero solution will be a solution

   (b) The zero solution is always a solution to both homogeneous and inhomogeneous linear systems

   (c) The zero solution can be a solution to both homogeneous and inhomogeneous linear systems, but only if the equations are solvable

   (d) The zero solution is never a solution to inhomogeneous linear systems, and may or may not be a solution to homogeneous linear systems

   (e) The zero solution is always a solution to homogeneous linear systems, and never a solution to inhomogeneous linear systems

   Solution: (e). This is covered in Section 2.5.

2. Let V be a vector space, and let W be a subset of V. What does it mean when we say that W is closed under scalar multiplication?

   (a) Whenever x is in W and c is a scalar, then cx is in V.

   (b) Whenever x is in V and c is a scalar, then cx is in V.

   (c) Whenever x is in V and c is a scalar, then cx is in W.

   (d) Whenever x is in W and c is a scalar, then cx is in W.

   (e) If cx is in W and c is a scalar, then x is in W.

   Solution: (d)

3. Which of the following statements is not an axiom for vector spaces?

   (a) For all $x, y \in V$ we have $x + y = y + x$

   (b) For all $x, y, z \in V$, we have $(x + y) + z = x + (y + z)$

   (c) For all $x, y, z \in V$, we have $(xy)z = x(yz)$

   (d) All of the above are axioms for vector spaces.

Solution: (c) There is no such thing as multiplication of vectors in a general vector space.

4. What is the solution to the following system of equations?

$$2x + y = 3$$
$$3x - y = 7$$

   (a) $x = 4$ and $y = -5$
   (b) $x = 2$ and $y = -1$
   (c) $x = 2$ and $y = \frac{1}{2}$
   (d) There are an infinite number of solutions to this system.
   (e) There are no solutions to this system.

Solution: (b).

5. A system of 5 linear equations in 7 variables could not have exactly ______ solutions.

   (a) 0
   (b) 1
   (c) infinite
   (d) More than one of these is impossible.
   (e) All of these are possible numbers of solutions.

Solution: (b).

6. Which augmented matrix represents the following system of equations? (The augmented matrix is constructed with order of variables x, y.)

$$x + 2y = 3$$
$$4y + 5x = 6$$

   (a) $\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$
   (b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$
   (c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 5 & 4 & 6 \end{array}\right]$

(d) $\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 5 & 4 & 6 \end{array}\right]$

Solution: (c).

7. What is the solution to the system of equations represented with this augmented matrix? (Assume that the variables are x, y, z, in that order.)

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(a) $x = 2,\ y = 3,\ z = 4$

(b) $x = -1,\ y = 1,\ z = 1$

(c) There are infinitely many solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

Solution: (c). We see that the matrix is in reduced row echelon form and that z is a free variable.

8. If A is a 3×3 matrix such that $A \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $A \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then the product $A \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$ is

(a) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}$

(e) Not uniquely determined by the information given

Solution: (b). Notice that

$$2 \cdot \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} - 1 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$$

and use the fact that matrix multiplication is a linear function.

9. Calculate the matrix product $\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 \\ 6 & -2 \end{bmatrix}$

   (d) None of the above answers is correct.

   (e) This matrix multiplication is not defined.

Solution: (a).

10. When we put a matrix A into reduced row echelon form, we get the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. This means that

    (a) The matrix A has no inverse.

    (b) The matrix we have found is the inverse of the matrix A.

    (c) Matrix A has an inverse, but this isn't it.

    (d) This tells us nothing about whether A has an inverse.

Solution: (a). A matrix has an inverse iff its reduced row echelon form is the identity.

11. Find a matrix A such that $\left(2A^T + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

    and give its first row

    (a) $(2, -1)$

    (b) $(0, 0)$

(c) $(-1/2, 1/2)$

(d) $(0, 1/2)$

(e) $(1/2, 0)$

Solution: (b). Use $(A+B)^T = A^T + B^T$ and $(A^T)^T = A$.

12. Which matrix product is defined?

(a) $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Solution: (d). This is a 1×4 matrix times a 4×1 matrix.

13. If the augmented matrix $[A|\mathbf{b}]$ of a system $A\mathbf{x} = \mathbf{b}$ is row equivalent to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

Which of the following is true?

(a) The system is inconsistent.

(b) $\mathbf{x} = \begin{bmatrix} 5 \\ -2-s \\ 1 \end{bmatrix}$ is a solution for any value of s.

(c) $\mathbf{x} = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ is the unique solution of the system.

(d) $\mathbf{x} = \begin{bmatrix} 5s \\ -2s \\ s \end{bmatrix}$ is a solution for any value of s.

(e) $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ is the unique solution to the system.

> **Solution:** (e). Notice that this matrix is *not* in reduced row echelon form.

14. If C is a $n \times 4$ matrix and $D = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then the second column of the matrix CD is

(a) The same as the second column of C

(b) The sum of the first and second columns of C

(c) The sum of the second and fourth columns of C

(d) The same as the third row of D

(e) The sum of the first and the third columns of C

> **Solution:** (a). Note that the second column of CD is Cd_2, where d_2 is the second column of D. Then note that $d_2 = e_2$, and Ce_2 is the second column of C.

15. What is the dot product of the vectors $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

(b) 5

(c) 0

(d) The dot product of this pair of vectors is not defined.

> **Solution:** (b)

16. What can we say about two vectors whose dot product is negative?

(a) The vectors are orthogonal

(b) The angle between the two vectors is less than $90°$

(c) The angle between the two vectors is greater than $90°$

(d) None of the above statements is correct.

Solution: (c). Use the formula $\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}||||\mathbf{v}|| \cos(\theta)$.