MATH 478: TEST 1

1. Test 1

Problem 1 (25 points)**.** Prove using only the definition of limit that

$$\lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n+1}-1} = 0.$$

As part of your solution you should state the definition of limit, i.e. say what it means for a sequence $\{a_n\}$ to converge to a limit L.

Problem 2 (25 points)**.** Let $\{a_n\}$ be a monotone increasing sequence of real numbers which is bounded above and has least upper bound L. Show that

$$\lim_{n\to\infty} a_n = L.$$

Problem 3 (25 points)**.** State the definition of a contractive sequence and prove that the sequence defined by $a_1 = 1$ and $a_{n+1} = \sqrt{9-2a_n}$ is contractive. (Hint: First show that $0 \le a_n \le 3$ for all n.)

Problem 4 (25 points)**.** Indicate in your bluebook whether each of the following statements is true or false. No reasoning or proof is required for this question. However, you will lose 2.5 points for each incorrect answer.

(1) Every subsequence of a convergent sequence converges.
(2) There exists a convergent sequence which is unbounded.
(3) Every sequence of real numbers has a monotone subseqence.
(4) Every Cauchy sequence is a contractive sequence.
(5) If a sequence of real numbers $\{a_n\}$ has the property that $a_{n+1} > a_n$ for all n (note the strict inequality), then $\{a_n\}$ diverges to $+\infty$.
(6) A monotone decreasing sequence of real numbers is bounded above.
(7) If $\{a_n\}$ is a sequence of positive real numbers and $a_n \to 0$, then there exist real numbers C, r with $C > 0$ and $0 < r < 1$ such that $a_n \le Cr^n$ for all sufficiently large n.
(8) If a is a positive real number and $a < 1$, then for any positive integer k, $lim_{n\to\infty} n^k a^n = 0$.
(9) Every bounded sequence has a convergent subsequence.
(10) There exists a convergent sequence which is not a Cauchy seqence.

Mathematics Department, Binghamton Univeristy, P. O. Box 6000, Binghamton, New York, 13902-6000
E-mail address: dikran@math.binghamton.edu

Date: October 7, 2005.