CORRECTIONS TO "DUALITY AND LINEAR APPROXIMATIONS IN HOCHSCHILD HOMOLOGY, K-THEORY, AND STRING TOPOLOGY"

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- On page 3, I claim that the theorem of Blumberg and Mandell implies that $K(DX)$ is equivalent to $\mathbb{V}(X)$. This is true if we interpret $\mathbb{V}(X)$ as the "geometric Swan theory" defined in their paper, but it is not equivalent to what is more commonly called $\mathbb{V}(X)$. Their definition uses the K-theory of modules over $\Sigma^\infty_+ \Omega X$ lying in the thick subcategory of the module $\mathbb{S}$, which is different from the modules whose *underlying* spectrum is finite. Interestingly enough, this is not the only issue that comes up in defining $\mathbb{V}(X)$, and there seem to be many different inequivalent definitions. There are also other trace methods that let us attack $\mathbb{V}(X)$ as opposed to $K(DX)$ − more on those soon!
- On page 46, Definition 2.6.4, the claims about the category of nondegenerate simplices are not quite right. One should just realize the category of all simplices and recover the thick geometric realization. This is corrected in the published version of "A tower connecting gauge groups to string topology."
- On page 69, I claim that the coassembly map for $\mathbb{V}(BG)$ is split surjective after p-completion. This does not follow from the contents of the following section. I most likely meant to write the corresponding claim for THH, as in Corollary 1.2.5.
- On page 103, the Prop 3.2.21, the given square commutes only if the chosen bijection $C_{mn} \cong C_m \times C_n$ respects the C_n-action and also preserves the choice of isomorphism between the C_n-quotient and the set C_m. This is corrected in "On the topological Hochschild homology of DX."
- On page 109, Prop 3.3.5, I only demonstrate that this map is a bijection on the underlying sets, so it might not be a homeomorphism. It doesn't matter, since we are only interested in the derived totalization, which does in fact give $\Sigma^\infty_+ LX$ back.
- On page 135, the ring structures given are not correct − they are based on a geometric argument that was not properly carried out. For DS^{2n+1}_+ the ring structure is more of a divided polynomial algebra. See "On the topological Hochschild homology of DX." For DS^1_+, the ring structure appears to be far more interesting. Nick Kuhn has done some work on describing it more explicitly.
- On page 137, Lemma 3.6.1 is not correct unless $G \cong H \times G/H$. In fact, taking X to be $E(G/H)$, it would imply that $BG \simeq BH \times B(G/H)$ for any normal subgroup $H \trianglelefteq G$, which is clearly false. The flaw in the proof is the assumption that G acts trivially on BH – it doesn't, and the leftover must be accounted for when taking the orbits.

  The correct statement is that there is a fiber sequence

$$BH \longrightarrow X_{hG} \longrightarrow X_{G/H}.$$

We prove this by forming the homotopy pullback square

$$\begin{array}{ccc} B(*, G, X) & \longrightarrow & B(*, G/H, X) \\ \downarrow & & \downarrow \\ B(*, G, *) & \longrightarrow & B(*, G/H, *) \end{array}$$

and then recalling that the fiber of the bottom map is BH. Fortunately, the published version of "The topological cyclic homology of the dual circle" gets the calculation of $TC(DS^1)$ without relying on this lemma.