Homework 1

due on Wednesday, January 28

Read carefully Appendix A and sections 1.1, 1.2, 1.3, 1.4 in the book. Read the section about induction in the link on the course web page. Solve the following problems.

Problem 1. Prove that every natural number is a sum of disctinct powers of 2 (e.g. $1 = 2^0$; $2 = 2^1$, $3 = 2^0 + 2^1$, etc.). Hint: Assume not and consider the smallest counterexample. Consider the case when it is even and the case when it is odd. Alternatively, prove it by induction.

Extra credit: prove that such expression is unique. Hint: Observe that $1 + 2 + 4 + \ldots + 2^n < 2^{n+1}$

Problem 2. We defined in class $v(n)$ to be the number of positive divisors of n. Charcterize positive integrs n such that $v(n) = 3$.

Problem 3. Use the Euclidean algorithm to find the greatest common divisor d of 441 and 1155. Then find integers k, l such that $d = 441k + 1155l$. Verify your answer. **You should not use a calculator in this problem.**

Problem 4. Prove that if a, b are relatively prime integers such that $a|c$ and $b|c$ then $ab|c$. Hint: Write $ua + wb = 1$ for some integers u, w and use this to show that $b|c_1$, where $c = ac_1$.