

Homework 12, due on Wednesday, May 6

Study carefully Chapter 7 of the book. Solve problems 18, 20 e), 29, 36, 39b), c), 42 in Chapter 7.

Also solve the following problems.

Problem 1. Let $d > 1$ be an integer which is not a square (so \sqrt{d} is irrational). Suppose that positive integers m, n satisfy $m^2 - n^2d = \pm 1$. Prove that m/n is a convergent of \sqrt{d} .

Hint Show that $|\sqrt{d} - m/n| < 1/2n^2$.

Problem 2. Let x be an irrational number, let the simple continued fraction expression for x be $x = [k_0, k_1, \dots]$, and let p_i/q_i , $i = 0, 1, \dots$ be the convergents of x .

a) Prove that if $n > 0$ then

$$\frac{1}{(k_{n+1} + 2)q_n^2} < \left| x - \frac{p_n}{q_n} \right| < \frac{1}{k_{n+1}q_n^2}.$$

b) Prove that there is $c > 0$ such that $|x - \frac{a}{b}| > \frac{1}{cb^2}$ for every fraction a/b if and only if the sequence k_0, k_1, \dots is bounded. Numbers x with this property are called **poorly approximable**.