

Homework 3

due on Friday, February 13

Read sections 2.1, 2.2 and 2.3 in the book. Solve problems 13, 21, 26, 28 e,f; 29 e,f; in Chapter 2.

Also solve the following problems.

Problem 1. Let a, b, c be positive integers such that $\gcd(a, b) = 1$. We showed in class that if $c > ab$ then the equation $ax + by = c$ is solvable in positive integers x, y and that for $c = ab$ it does not have solutions in positive integers. Suppose now that $1 \leq c < ab$.

a) Show that if neither a nor b divides c then exactly one of the equations:

$$ax + by = c, \quad ax + by = ab - c$$

has solutions in positive integers x, y .

b) If $c = am$ for some m then the only solution to $ax + by = c$ in non-negative integers is $x = m, y = 0$.

Problem 2. John needs to pay \$2.05. He has only dimes and quarters. In how many ways he can pay?

Problem 3. Let m, n be positive integers. How many multiples of n are in the sequence $m, 2m, 3m, \dots, nm$?

Problem 4. Find a positive integer such that half of it is a square, a third of it is a cube, and a fifth of it is a fifth power. Hint: Look for a number of the form $2^a 3^b 5^c$.