QUIZ: (1) Define what it means that two functions f, g are homotopic.

(2) Define a contractible space.

(3) Suppose that X is contractible, Y any topological space. Prove that any continuous function $f: X \to Y$ is homotopic to a constant function.

Solution: (1) f, g are homotopic if $f, g: X \to Y$ are continuous functions and there is a continuous function $H: X \times I \to Y$ such that $f(x) = H(x,0)$, $g(x) = H(x,1)$ for all $x \in I$.

(2) X is contractible if the identity function $id: X \to X$ is homotopic to a constant function $X \to X$, i.e. if there is $H: X \times I \to X$ continuous and such that $H(x,0) = x$ and $H(x,1) = a$ for all $x \in X$ and some $a \in X$.

(3) Since X is contractible, we have a continuous function $H: X \times I \to X$ such that $H(x,0) = x$, $H(x,1) = a$, for all $x \in X$ and some $a \in X$. The composition $H_1 = f \circ H: X \times I \to Y$ is a continuous function such that $H_1(x,0) = f \circ H(x,0) = f(x)$ and $H_1(x,1) = f \circ H(x,1) = f(a)$ so H_1 is a homotopy between f and a constant function.