

HW Sec 7.1 5 insights :

#1 a) $3x+1 \geq 2+x$

$$x \geq \frac{1}{2}$$

$$[\frac{1}{2}, \infty)$$

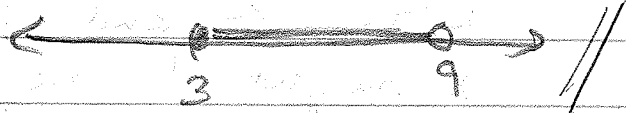


b) $-1 < 2 - \frac{x}{3} \leq 1$

$$-3 < -\frac{x}{3} \leq -1$$

(multiply by -3) $9 > x \geq 3$

$$[3, 9)$$



c) $x^2 - x - 6 > 0$

$$(x-3)(x+2) > 0$$

Both factors need to be (+) or both (-). It's most efficient to check the intervals around zeros.



Check $x = -3, 0, 4$

by noting signs of factors when value is substituted.

$x = -3$: $(-)(-) = (+) > 0$ good

$x = 0$: $(-)(+) = (-) < 0$ nope

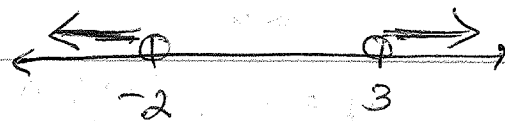
$x = 4$: $(+)(+) = (+) > 0$ good

d) $(x+3)(x-2)^2(x-1)^4 < 0$

Notice that we need all factors negative or just one. Since only $(x+3)$ can be negative, solve just $x+3 < 0$

$$x < -3$$

$$(-\infty, -3)$$



$$(-\infty, -2) \cup (3, \infty)$$

#(e) $\frac{2x}{x-2} > 0 \quad (x \neq 2)$

Again, use the zeros of numerator + denominator to partition the number line, inspect signs.

Zeros: $2x = 0 \rightarrow x = 0$ $x - 2 = 0 \rightarrow x = 2$



Check

$x = -1: (-)/(-) = (+) > 0$ good

$x = 1: (+)/(-) = (-) < 0$ nope

$x = 3: (+)/(+) = (+) > 0$ good

#(f)

$\frac{2}{x} \leq \frac{x}{2} \Rightarrow$

If you try to cross multiply, you'll have to consider both $\leq$ and $\geq$, since x might be negative,

So set all = zero

$\frac{2}{x} - \frac{x}{2} \leq 0$

$\Leftarrow$ which changes direction of $\leq$

$\frac{4 - x^2}{2x} \leq 0$

Zeros: $x = 2, -2, 0$

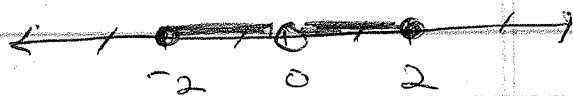
Check: $x = -3, -1, 1, 3$

$x = -3: (-)/(-) = (+)$ ~~no~~

$x = -1: (+)/(-) = (-)$ good

$x = 1: (+)/(+) = (+)$ no

$x = 3: (-)/(+) = (-)$ good



$[-2, 0) \cup (0, 2]$

See 7.1 HW con'd

#2

a) $-1 < \frac{3-x}{2} \leq 1 \rightarrow -2 < 3-x \leq 2$

$$-5 < -x \leq -1$$

$$5 > x \geq 1$$

Notice that the

soln. is automatic

since x is bounded in the algebra.

$$[1, 5) //$$

b) $x^3 + 2x^2 - 4x - 8 \leq 0$

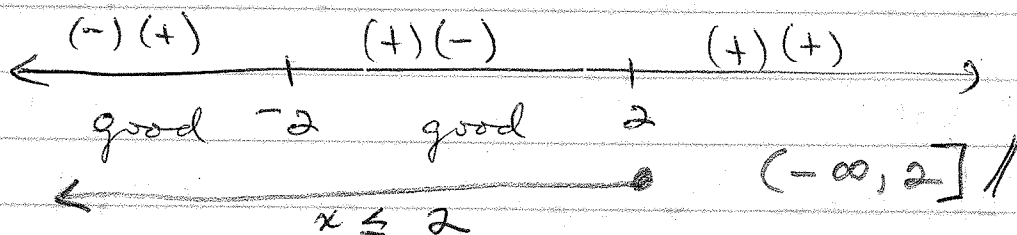
$$x^2(x+2) - 4(x+2) \leq 0$$

factor by grouping

$$(x+2)(x^2-4) \leq 0$$

zeros: $x = -2, 2$

Check intervals:



c) $\frac{1}{x} \geq \frac{1}{x+3}$

Again, "cross multiplying" will require we inspect $>$ and $\leq$. Better to set ≥ 0

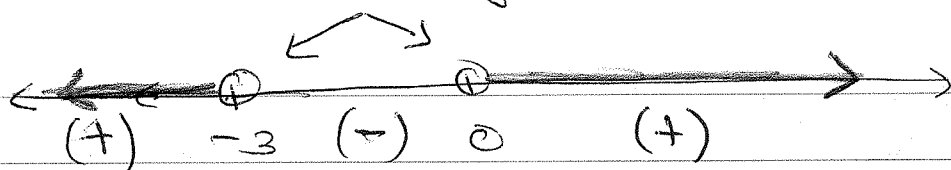
$$\frac{1}{x} - \frac{1}{x+3} \geq 0$$

LCD

$$\frac{x+3-x}{x^2+3x} \geq 0 \rightarrow \frac{3}{x^2+3x} \geq 0$$

Zeros: $x = 0$ and -3 , but note the denom. is usually positive and numerator (3) is always positive!

holes (why?) - now check $3/x^2 + 3x$ at intervals



$$(-\infty, -3) \cup (0, \infty)$$

#2 d)

$$\frac{x}{2} - \frac{8x}{3} + \frac{x}{4} > \frac{23}{6}$$

$$(LCD = 12)$$

$$6x - 32x + 3x - 46 > 0$$

$$-23x - 46 > 0 \quad (\div \text{ by } -23)$$

$$x + 2 < 0 \rightarrow x < -2 \quad (\text{only one interval})$$

$$(-\infty, -2) //$$

#2e) $x^2 - 2x + 1 \leq 0$ factors as $(x-1)^2 \leq 0$

Notice $(x-1)^2$ is either positive or zero, so there is just a one-pt. soln.

$$x = 1 //$$

#3 a)

$$-6x + 3 > x + 5$$

$$-7x - 2 > 0$$

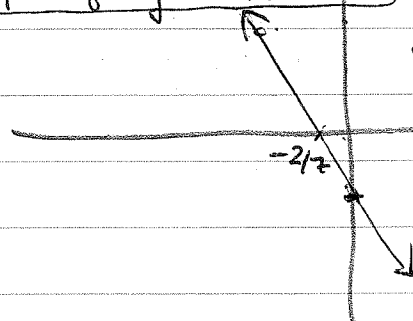
$$x < -2/7$$

b) $x^4 - 16 < 0$

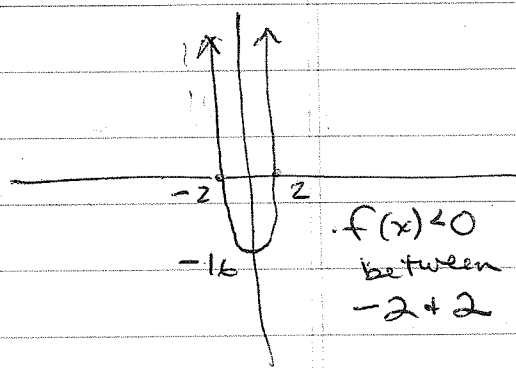
$$x^4 < 16 \text{ when}$$

$$-2 < x < 2$$

Graph of $y = -7x - 2$



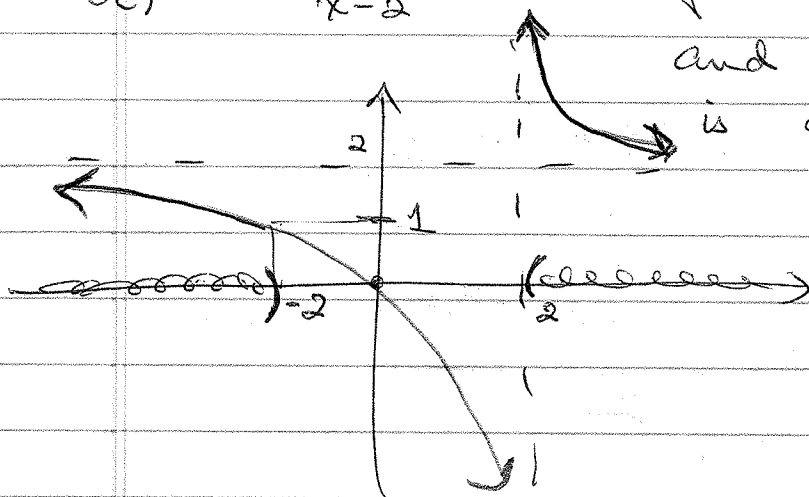
Notice the graph is negative below $-2/7$.



#3c)

$$\frac{2x}{x-2} > 1$$

You could graph $\frac{2x}{x-2}$
and see where the fun.
is greater than 1.



$$\text{VA: } x=2 \quad \left. \begin{array}{l} \text{HA: } y=2 \end{array} \right\} \text{why?}$$

$$\text{root: } x=0$$

$$y\text{-int: } y=0$$

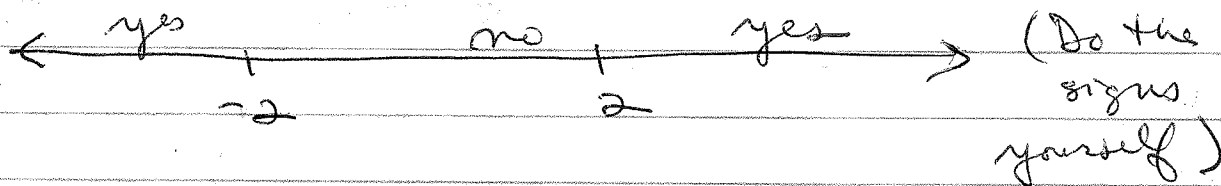
The ~~wavy~~ parts show where the
fun's graph has a value greater than 1.
Note that you're looking at the y values
on these intervals.

How do we get these values? Again,
we ^{could} set all > 0 . But first, notice
that $\frac{2x}{x-2} = 1$ at $x = -2$. Since

we have the graph we can read the soln.
off the picture!

$$\text{Algebraically: } \frac{2x}{x-2} - 1 > 0 \longrightarrow$$

$$\left(\begin{array}{l} \text{LCD} \\ = x-2 \end{array} \right) \longrightarrow \frac{2x - x + 2}{x-2} > 0 \longrightarrow \frac{x+2}{x-2} > 0$$



3d)

$$\frac{-x^2 + x}{x+2} > -x+3$$

$$\frac{-x^2 + x}{x+2} - \frac{-x+3}{1} \geq 0$$

$$\left(\begin{array}{l} \text{LCD} \\ = x+2 \end{array} \right) \frac{(-x^2 + x) - (x+2)(-x+3)}{x+2} \geq 0$$

$$\frac{-x^2 + x + x^2 - x - 6}{x+2} > 0$$

$$\frac{-6}{x+2} > 0 \quad \text{for } x+2 < 0 \quad \text{only (why?)}$$

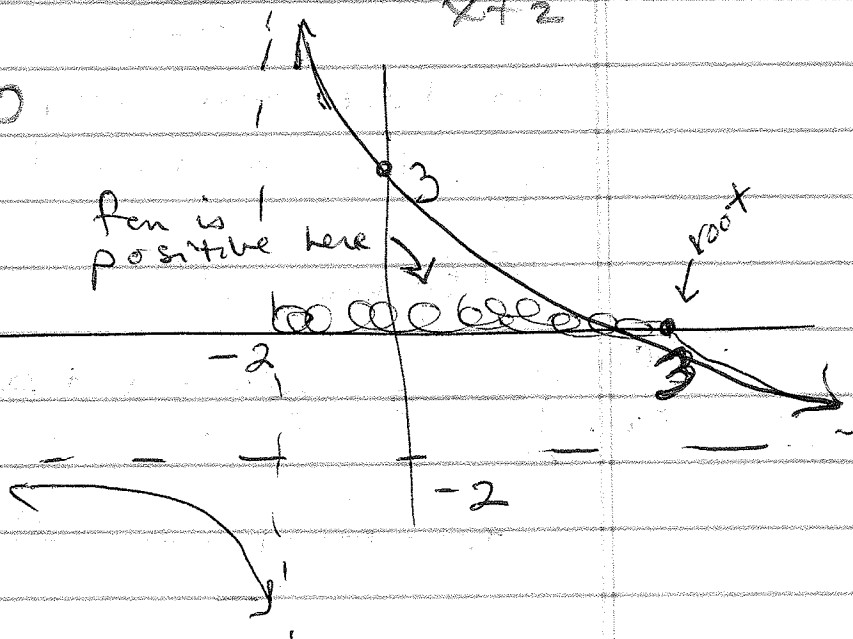
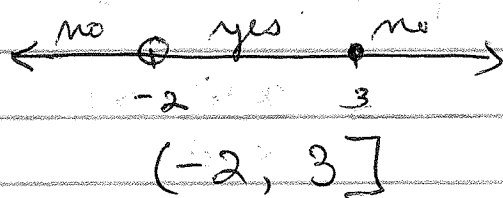
So $x < -2$ is soln. //

$$e) \quad \frac{x+12}{x+2} - 3 \geq 0$$

$$\text{LCD} = x+2$$

$$\frac{x+12 - 3x - 6}{x+2} \geq 0 \rightarrow \frac{-2x+6}{x+2} \geq 0$$

$$\rightarrow \frac{-2(x-3)}{(x+2)} \geq 0$$



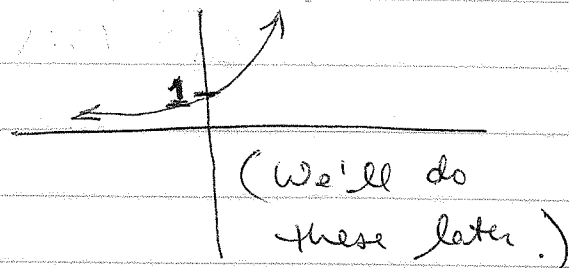
#3f)

$$2^x(x-1) < 0$$

Solve just $x-1 < 0$

$$\text{or } x < 1 //$$

The exponential fn.
 2^x is always positive
 (see graph)



#4

These are like ~~problems~~ problems that are restricted to certain functional values. Namely, anything above. But we need to name the interval.

$$a) f(x) = \sqrt{x^2 + 4x + 3}$$

$$x^2 + 4x + 3 \geq 0 \rightarrow (x+3)(x+1) \geq 0$$

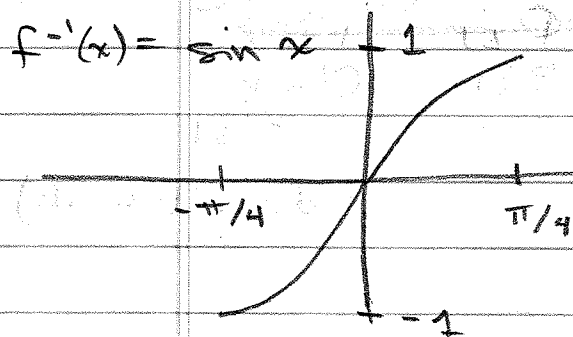
Checking intervals gives $(-\infty, -3] \cup [-1, \infty)$

Remember that a radicand can = zero.

$$b) f(x) = \sin^{-1}(1-x^2)$$

↑ This is not an angle.

Let's look at the its inverse fn.



The range of f^{-1} is the domain of f .

That is, $(-1, 1)$ is the interval $1-x^2$ lies in:

$$-1 \leq 1-x^2 \leq 1$$

← Solve this

Thus,

$$-\sqrt{2} \leq x \leq \sqrt{2}$$

← Answer

#5) The formula for the sum of the first
 n integers is $\frac{n(n+1)}{2}$.

This is what $\sum_{i=1}^n i$ is essentially asking.

$$\text{Thus, } \sum_{i=1}^n i = \frac{n(n+1)}{2} < 465$$

$$\begin{aligned}\text{Solving: } n^2 + n &< 930 \\ n^2 + n - 930 &< 0 \\ (n-30)(n+31) &< 0\end{aligned}$$

The zeros are $n=30$ + $n=-31$,
but we discard the negative answer,
since n is assumed non-negative.

Since $n-30 < 0 \Rightarrow n < 30$ where n is non-negative $\mathbb{Z}$
we choose integers from 1 to 29.

$$1 \leq n \leq 29 //$$

Graphically, the intervals would be

