



**AUFMANN &
LOCKWOOD**

**INTERMEDIATE
ALGEBRA**
AN APPLIED APPROACH

8E **AIM** **AUFMANN
INTERACTIVE
METHOD**

Aufmann Interactive Method **AIM**

The Aufmann Interactive Method (AIM) is a proven learning system that has helped thousands of students master concepts and achieve results.

To follow the **AIM**, step through the **HOW TO** examples that are provided and then work through the matched **EXAMPLE / YOU TRY IT** pairs.

Aufmann

HOW TO • 1

Factor: $4x^2 - 81y^2$

Write the binomial as the difference of two perfect squares.

The factors are the sum and difference of the square roots of the perfect squares.

$$4x^2 - 81y^2 = (2x)^2 - (9y)^2$$

$$= (2x + 9y)(2x - 9y)$$

Interactive

EXAMPLE • 1

Factor: $25x^2 - 1$

Solution

$$\begin{aligned} 25x^2 - 1 &= (5x)^2 - (1)^2 \\ &= (5x + 1)(5x - 1) \end{aligned}$$

• Difference of two squares

YOU TRY IT • 1

Factor: $x^2 - 36y^4$

Your solution

For extra support, you can find the complete solutions to the **YOU TRY IT** problems in the back of the text.

Method

SOLUTIONS TO CHAPTER 5 “YOU TRY IT”

You Try It 1

$$\begin{aligned} x^2 - 36y^4 &= x^2 - (6y^2)^2 \\ &= (x + 6y^2)(x - 6y^2) \end{aligned}$$

• Difference of two squares

Dick Aufmann



Joanne Lockwood



We have taught math for many years. During that time, we have had students ask us a number of questions about mathematics and this course. Here you find some of the questions we have been asked most often, starting with the big one.

Why do I have to take this course? You may have heard that “*Math is everywhere.*” That is probably a slight exaggeration but math does find its way into many disciplines. There are obvious places like engineering, science, and medicine. There are other disciplines such as business, social science, and political science where math may be less obvious but still essential. If you are going to be an artist, writer, or musician, the direct connection to math may be even less obvious. Even so, as art historians who have studied the Mona Lisa have shown, there is a connection to math. But, suppose you find these reasons not all that compelling. **There is still a reason to learn basic math skills: You will be a better consumer and able to make better financial choices for you and your family.** For instance, is it better to buy a car or lease a car? Math can provide an answer.

I find math difficult. Why is that? It is true that some people, even very smart people, find math difficult. Some of this can be traced to previous math experiences. If your basic skills are lacking, it is more difficult to understand the math in a new math course. Some of the difficulty can be attributed to the ideas and concepts in math. They can be quite challenging to learn. Nonetheless, most of us can learn and understand the ideas in the math courses that are required for graduation. **If you want math to be less difficult, practice. When you have finished practicing, practice some more.** Ask an athlete, actor, singer, dancer, artist, doctor, skateboarder, or (name a profession) what it takes to become successful and the one common characteristic they all share is that they practiced—a lot.

Why is math important? As we mentioned earlier, math is found in many fields of study. There are, however, other reasons to take a math course. Primary among these reasons is to become a better problem solver. Math can help you learn critical thinking skills. It can help you develop a logical plan to solve a problem. Math can help you see relationships between ideas and to identify patterns. **When employers are asked what they look for in a new employee, being a problem solver is one of the highest ranked criteria.**

What do I need to do to pass this course? The most important thing you must do is to know and understand the requirements outlined by your instructor. These requirements are usually given to you in a syllabus. Once you know what is required, you can chart a course of action. Set time aside to study and do homework. If possible, choose your classes so that you have a free hour after your math class. Use this time to review your lecture notes, rework examples given by the instructor, and to begin your homework. All of us eventually need help, so know where you can get assistance with this class. This means knowing your instructor’s office hours, know the hours of the math help center, and how to access available online resources. And finally, do not get behind. **Try to do some math EVERY day, even if it is for only 20 minutes.**

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Intermediate Algebra

An Applied Approach

EIGHTH EDITION

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Palomar College

Joanne S. Lockwood

Nashua Community College

**Intermediate Algebra: An Applied Approach,
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Preface

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The goal in any textbook revision is to improve upon the previous edition, taking advantage of new information and new technologies, where applicable, in order to make the book more current and appealing to students and instructors. While change goes hand-in-hand with revision, a revision must be handled carefully, without compromise to valued features and pedagogy. In the eighth edition of *Intermediate Algebra: An Applied Approach*, we endeavored to meet these goals.

As in previous editions, the focus remains on the **Aufmann Interactive Method (AIM)**. Students are encouraged to be active participants in the classroom and in their own studies as they work through the How To examples and the paired Examples and You Try It problems. The role of “active participant” is crucial to success. Providing students with worked examples, and then affording them the opportunity to immediately work similar problems, helps them build their confidence and eventually master the concepts.

To this point, simplicity plays a key factor in the organization of this edition, as in all other editions. All lessons, exercise sets, tests, and supplements are organized around a carefully constructed hierarchy of objectives. This “objective-based” approach not only serves the needs of students, in terms of helping them to clearly organize their thoughts around the content, but instructors as well, as they work to design syllabi, lesson plans, and other administrative documents.

In order to enhance the AIM and the organization of the text around objectives, we have introduced a new design. We believe students and instructors will find the page even easier to follow. Along with this change, we have introduced several new features and modifications that we believe will increase student interest and renew the appeal of presenting the content to students in the classroom, be it live or virtual.

Changes to the Eighth Edition

With the eighth edition, previous users will recognize many of the features that they have come to trust. Yet, they will notice some new additions and changes:

- Enhanced WebAssign® now accompanies the text
- Revised exercise sets with new applications
- New **In the News** applications
- New **Think About It** exercises
- Revised Chapter Review Exercises and Chapter Tests
- End-of-chapter materials now include Concept Reviews
- Revised Chapter Openers, now with Prep Tests

Take AIM and Succeed!

Intermediate Algebra: An Applied Approach is organized around a carefully constructed hierarchy of **OBJECTIVES**. This “objective-based” approach provides an integrated learning environment that allows students and professors to find resources such as assessment (both within the text and online), videos, tutorials, and additional exercises.

Chapter Openers are set up to help you organize your study plan for the chapter. Each opener includes Objectives, Are You Ready? and a Prep Test.

Each Chapter Opener outlines the **OBJECTIVES** that appear in each section. The list of objectives serves as a resource to guide you in your study and review of the topics.

ARE YOU READY? outlines what you need to know to be successful in the coming chapter.

Complete each **PREP TEST** to determine which topics you may need to study more carefully, versus those you may only need to skim over to review.

CHAPTER 5

Polynomials

OBJECTIVES

SECTION 5.1

- A To multiply monomials
- B To divide monomials and simplify expressions with negative exponents
- C To write a number using scientific notation
- D To solve application problems

SECTION 5.2

- A To evaluate polynomial functions
- B To add or subtract polynomials

SECTION 5.3

- A To multiply a polynomial by a monomial
- B To multiply polynomials
- C To multiply polynomials that have special products
- D To solve application problems

SECTION 5.4

- A To divide a polynomial by a monomial
- B To divide polynomials
- C To divide polynomials by using synthetic division
- D To evaluate a polynomial function using synthetic division

SECTION 5.5

- A To factor a monomial from a polynomial
- B To factor by grouping
- C To factor a trinomial of the form $x^2 + bx + c$
- D To factor $ax^2 + bx + c$

SECTION 5.6

- A To factor the difference of two perfect squares or a perfect-square trinomial
- B To factor the sum or the difference of two perfect cubes
- C To factor a trinomial that is quadratic in form
- D To factor completely

SECTION 5.7

- A To solve an equation by factoring
- B To solve application problems

ARE YOU READY?

Take the Chapter 5 Prep Test to find out if you are ready to learn to:

- Add, subtract, multiply, and divide polynomials
- Simplify expressions with negative exponents
- Write a number in scientific notation
- Factor a polynomial completely
- Solve an equation by factoring

PREP TEST

Do these exercises to prepare for Chapter 5.

For Exercises 1 to 5, simplify.

1. $-4(3y)$
2. $(-2)^3$
3. $-4a - 8b + 7a$
4. $3x - 2[y - 4(x + 1) + 5]$
5. $-(x - y)$
6. Write 40 as a product of prime numbers.
7. Find the GCF of 16, 20, and 24.
8. Evaluate $x^3 - 2x^2 + x + 5$ for $x = -2$.
9. Solve: $3x + 1 = 0$

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OBJECTIVE A To multiply a polynomial by a monomial

To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions.

HOW TO 1 Multiply: $-3x^2(2x^2 - 5x + 3)$

$$\begin{aligned} & -3x^2(2x^2 - 5x + 3) \\ &= -3x^2(2x^2) - (-3x^2)(5x) + (-3x^2)(3) \\ &= -6x^4 + 15x^3 - 9x^2 \end{aligned}$$

- Use the Distributive Property.
- Use the Rule for Multiplying Exponential Expressions.

HOW TO 2 Simplify: $5x(3x - 6) + 3(4x - 2)$

$$\begin{aligned} & 5x(3x - 6) + 3(4x - 2) \\ &= 5x(3x) - 5x(6) + 3(4x) - 3(2) \\ &= 15x^2 - 30x + 12x - 6 \\ &= 15x^2 - 18x - 6 \end{aligned}$$

- Use the Distributive Property.
- Simplify.

HOW TO 3 Simplify: $2x^2 - 3x[2 - x(4x + 1) + 2]$

$$\begin{aligned} & 2x^2 - 3x[2 - x(4x + 1) + 2] \\ &= 2x^2 - 3x[2 - 4x^2 - x + 2] \\ &= 2x^2 - 3x[-4x^2 - x + 4] \\ &= 2x^2 + 12x^3 + 3x^2 - 12x \\ &= 12x^3 + 5x^2 - 12x \end{aligned}$$

- Use the Distributive Property to remove the parentheses.
- Simplify.
- Use the Distributive Property to remove the brackets.
- Simplify.

EXAMPLE 1

Multiply: $(3a^2 - 2a + 4)(-3a)$

Solution

$$\begin{aligned} & (3a^2 - 2a + 4)(-3a) \\ &= 3a^2(-3a) - 2a(-3a) + 4(-3a) \\ &= -9a^3 + 6a^2 - 12a \end{aligned}$$

- Use the Distributive Property.

YOU TRY IT 1

Multiply: $(2b^2 - 7b - 8)(-5b)$

Your solution

In each section, **OBJECTIVE STATEMENTS** introduce each new topic of discussion.

In each section, the **HOW TO'S** provide detailed explanations of problems related to the corresponding objectives.

The **EXAMPLE/YOU TRY IT** matched pairs are designed to actively involve you in learning the techniques presented. The You Try Its are based on the Examples. They appear side-by-side so you can easily refer to the steps in the Examples as you work through the You Try Its.

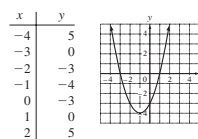
Complete, **WORKED-OUT SOLUTIONS** to the

You Try It problems are found in an appendix at the back of the text. Compare your solutions to the solutions in the appendix to obtain immediate feedback and reinforcement of the concept(s) you are studying.

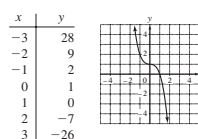
You Try It 2 The leading coefficient is -3 , the constant term is -12 , and the degree is 4.

- You Try It 3**
- Yes, this is a polynomial function.
 - No, this is not a polynomial function. A polynomial function does not have a variable expression raised to a negative power.
 - No, this is not a polynomial function. A polynomial function does not have a variable expression within a radical.

You Try It 4



You Try It 5



SECTION 5.3

You Try It 1

$$\begin{aligned} & (2b^2 - 7b - 8)(-5b) \\ &= 2b^2(-5b) - 7b(-5b) - 8(-5b) \\ &= -10b^3 + 35b^2 + 40b \end{aligned}$$

- Use the Distributive Property.

You Try It 2 $x^2 - 2x[x - x(4x - 5) + x^2]$

$$\begin{aligned} &= x^2 - 2x[x - 4x^2 + 5x + x^2] \\ &= x^2 - 2x[6x - 3x^2] \\ &= x^2 - 12x^2 + 6x^3 \\ &= 6x^3 - 11x^2 \end{aligned}$$

You Try It 3

$$\begin{aligned} & -2b^2 + 5b - 4 \\ & \quad - 3b + 2 \\ \hline & -2b^2 + 8b - 2 \\ & 6b^3 - 15b^2 + 12b \\ \hline & 6b^3 - 19b^2 + 22b - 8 \end{aligned}$$

- $2(-2b^2 + 5b - 4)$
- $-3b(-2b^2 + 5b - 4)$

You Try It 4

$$\begin{aligned} & (3x - 4)(2x - 3) = 6x^2 - 9x - 8x + 12 \\ &= 6x^2 - 17x + 12 \end{aligned}$$

- FOIL

You Try It 5

$$\begin{aligned} & (3ab + 4)(5ab - 3) = 15a^2b^2 - 9ab + 20ab - 12 \\ &= 15a^2b^2 + 11ab - 12 \end{aligned}$$

Intermediate Algebra: An Applied Approach contains **A WIDE VARIETY OF EXERCISES** that promote skill building, skill maintenance, concept development, critical thinking, and problem solving.

THINK ABOUT IT exercises promote conceptual understanding. Completing these exercises will deepen your understanding of the concepts being addressed.

New

Working through the application exercises that contain **REAL DATA** will help prepare you to answer questions and/or solve problems based on your own experiences, using facts or information you gather.

Completing the **WRITING** exercises will help you to improve your communication skills, while increasing your understanding of mathematical concepts.

5.4 EXERCISES

OBJECTIVE A To divide a polynomial by a monomial

For Exercises 1 to 12, divide and check.

1. $\frac{3x^2 - 6x}{3x}$
2. $\frac{10y^2 - 6y}{2y}$
3. $\frac{5x^2 - 10x}{-5x}$
4. $\frac{3y^2 - 27y}{-3y}$
5. $\frac{5x^2y^2 + 10xy}{5xy}$
6. $\frac{8x^2y^2 - 24xy}{8xy}$

13. If $\frac{P(x)}{3x} = 2x^2 + 7x - 5$, what is $P(x)$?

14. If $\frac{6x^3 + 15x^2 - 24x}{ax} = 2x^2 + 5x - 8$, what is the value of a ?

35. **Meteorology** Draw a scatter diagram for the data in the article.

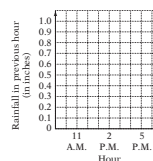
In the News

Tropical Storm Fay Lashes Coast

Tropical storm Fay hit the Florida coast today, with heavy rain and high winds. Here's a look at the amount of rainfall over the course of the afternoon.

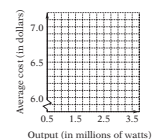
Hour	11 A.M.	12 P.M.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.
Inches of rain in preceding hour	0.25	0.69	0.85	1.05	0.70	0	0.08

Source: www.weather.gov



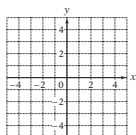
36. **Utilities** A power company suggests that a larger power plant can produce energy more efficiently and therefore at lower cost to consumers. The table below shows the output and average cost for power plants of various sizes. Draw a scatter diagram for these data.

Output (in millions of watts)	0.7	2.2	2.6	3.2	2.8	3.5
Average Cost (in dollars)	6.9	6.5	6.3	6.4	6.5	6.1

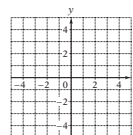


Applying the Concepts

37. Graph the ordered pairs (x, x^2) , where $x \in \{-2, -1, 0, 1, 2\}$.



38. Graph the ordered pairs $(x, \frac{1}{x})$, where $x \in \{-2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 1, 2\}$.



39. Describe the graph of all the ordered pairs (x, y) that are 5 units from the origin.
40. Consider two distinct fixed points in the plane. Describe the graph of all the points (x, y) that are equidistant from these fixed points.
41. Draw a line passing through every point whose abscissa equals its ordinate.
42. Draw a line passing through every point whose ordinate is the additive inverse of its abscissa.

In the News

A Mars Landing for Phoenix

At 7:53 P.M., a safe landing on the surface of Mars brought an end to the *Phoenix* spacecraft's 296-day, 422-million-mile journey to the Red Planet.

Source: The Los Angeles Times



In the News

Forest Fires Spread Seeds

Forest fires may be feared by humans, but not by the lodgepole pine, a tree that uses the intense heat of a fire to release its seeds from their cones. After a blaze that burned 12,000,000 acres of Yellowstone National Park, scientists counted 2 million lodgepole pine seeds on a single acre of the park. One year later, they returned to find 12,000 lodgepole pine seedlings growing.

Source: National Public Radio

125. **Astronomy** It took 11 min for the commands from a computer on Earth to travel to the *Phoenix* Mars Lander, a distance of 119 million miles. How fast did the signals from Earth to Mars travel?
126. **Forestry** Use the information in the article at the right. If every burned acre of Yellowstone Park had 12,000 lodgepole pine seedlings growing on it 1 year after the fire, how many new seedlings would be growing?
127. **Forestry** Use the information in the article at the right. Find the number of seeds released by the lodgepole pine trees for each surviving seedling.
128. One light-year is approximately 5.9×10^{12} mi and is defined as the distance light can travel in a vacuum in 1 year. *Voyager 1* is approximately 15 light-hours away from Earth and took about 30 years to travel that distance. One light-hour is approximately 6.7×10^8 mi. True or false: $\frac{5.9 \times 10^{12}}{6.7 \times 10^8} \approx$ number of hours in 1 year.

Applying the Concepts

129. Correct the error in each of the following expressions. Explain which rule or property was used incorrectly.
- a. $x^0 = 0$ b. $(x^4)^5 = x^9$ c. $x^2 \cdot x^3 = x^6$

New

IN THE NEWS application exercises help you master the utility of mathematics in our everyday world. They are based on information found in popular media sources, including newspapers and magazines, and the Web.

APPLYING THE CONCEPTS

exercises may involve further exploration of topics, or they may involve analysis. They may also integrate concepts introduced earlier in the text. **Optional** calculator exercises are included, denoted by .

Applying the Concepts

111. For what values of the variable is the equation true? Write the solution set in set-builder notation.
- a. $|x + 3| = x + 3$ b. $|a - 4| = 4 - a$
112. Write an absolute value inequality to represent all real numbers within 5 units of 2.
113. Replace the question mark with $\leq$, $\geq$, or $=$.
- a. $|x + y| ? |x| + |y|$ b. $|x - y| ? |x| - |y|$

PROJECTS AND GROUP ACTIVITIES

Astronomical Distances and Scientific Notation



Astronomers have units of measurement that are useful for measuring vast distances in space. Two of these units are the astronomical unit and the light-year. An **astronomical unit** is the average distance between Earth and the sun. A **light-year** is the distance a ray of light travels in 1 year.

- Light travels at a speed of 1.86×10^5 mi/s. Find the measure of 1 light-year in miles. Use a 365-day year.
- The distance between Earth and the star Alpha Centauri is approximately 25 trillion miles. Find the distance between Earth and Alpha Centauri in light-years. Round to the nearest hundredth.
- The Coma cluster of galaxies is approximately 2.8×10^8 light-years from Earth. Find the distance, in miles, from the Coma cluster to Earth. Write the answer in scientific notation.
- One astronomical unit (A.U.) is 9.3×10^7 mi. The star Pollux in the constellation Gemini is 1.8228×10^{12} mi from Earth. Find the distance from Pollux to Earth in astronomical units.



PROJECTS AND GROUP ACTIVITIES appear at the end of each chapter. Your instructor may assign these to you individually, or you may be asked to work through the activity in groups.

Intermediate Algebra: An Applied Approach addresses students' broad range of study styles by offering **A WIDE VARIETY OF TOOLS FOR REVIEW.**

At the end of each chapter you will find a **SUMMARY** with **KEY WORDS** and **ESSENTIAL RULES AND PROCEDURES**. Each entry includes an example of the summarized concept, an objective reference, and a page reference to show where each concept was introduced.

CHAPTER 5	
SUMMARY	
KEY WORDS	EXAMPLES
A <i>monomial</i> is a number, a variable, or a product of numbers and variables. [5.1A, p. 260]	5 is a number; y is a variable. $8a^2b^2$ is a product of a number and variables. 5, y , and $8a^2b^2$ are monomials.
The <i>degree of a monomial</i> is the sum of the exponents on the variables. [5.1A, p. 260]	The degree of $8x^4y^2z$ is 10.
A <i>polynomial</i> is a variable expression in which the terms are monomials. [5.2A, p. 272]	$x^4 - 2xy - 32x + 8$ is a polynomial. The terms are x^4 , $-2xy$, $-32x$, and 8.

CHAPTER 5	
CONCEPT REVIEW	
Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.	
1. How do you determine the degree of a monomial with several variables?	
2. How do you write a very small number in scientific notation?	
3. How do you multiply two binomials?	

CONCEPT REVIEWS actively engage you as you study and review the contents of a chapter. The **ANSWERS** to the questions are found in an appendix at the back of the text. After each answer, look for an objective reference that indicates where the concept was introduced.

By completing the chapter **REVIEW EXERCISES**, you can practice working problems that appear in an order that is different from the order they were presented in the chapter. The **ANSWERS** to these exercises include references to the section objectives upon which they are based. This will help you to quickly identify where to go to review the concepts if needed.

CHAPTER 5	
REVIEW EXERCISES	
1. Factor: $18a^2b^2 - 12a^3b^3 + 30a^2b$	2. Divide: $\frac{15x^2 + 2x - 2}{3x - 2}$
3. Multiply: $(2x^{-1}y^2z^3)(-3x^3yz^{-3})^2$	4. Factor: $2ax + 4bx - 3ay - 6by$
5. Factor: $12 + x - x^2$	6. Use the Remainder Theorem to evaluate $P(x) = x^3 - 2x^2 + 3x - 5$ when $x = 2$.
7. Subtract: $(5x^2 - 8xy + 2y^2) - (x^2 - 3y^2)$	8. Factor: $24x^2 + 38x + 15$
9. Factor: $4x^2 + 12xy + 9y^2$	10. Multiply: $(-2a^2b^4)(3ab^2)$
11. Factor: $64a^3 - 27b^3$	12. Divide: $\frac{4x^3 + 27x^2 + 10x + 2}{x + 6}$

Each chapter **TEST** is designed to simulate a possible test of the concepts covered in the chapter.

The **ANSWERS** include references to section objectives.

References to How Tos, worked Examples, and You Try Its, that provide solutions to similar problems, are also included.

CHAPTER 5

TEST

- Factor: $16r^2 + 24r + 9$
- Multiply: $-6rs^2(3r - 2s - 3)$
- Given $P(x) = 3x^2 - 8x + 1$, evaluate $P(2)$.
- Factor: $27x^3 - 8$
- Factor: $16x^2 - 25$
- Multiply: $(3t^3 - 4t^2 + 1)(2t^2 - 5)$

CUMULATIVE REVIEW EXERCISES

- Simplify: $8 - 2[-3 - (-1)]^2 + 4$
- Evaluate $\frac{2a-b}{b-c}$ when $a = 4$, $b = -2$, and $c = 6$.
- Identify the property that justifies the statement $2x + (-2x) = 0$.
- Simplify: $2x - 4[x - 2(3 - 2x) + 4]$
- Solve: $\frac{2}{3} - y = \frac{5}{6}$
- Solve: $8x - 3 - x = -6 + 3x - 8$
- Divide: $\frac{x^3 - 3}{x - 3}$
- Solve: $3 - |2 - 3x| = -2$
- Given $P(x) = 3x^2 - 2x + 2$, evaluate $P(-2)$.
- What values of x are excluded from the domain of the function $f(x) = \frac{x+1}{x+2}$?

CUMULATIVE REVIEW EXERCISES

which appear at the end of each chapter (beginning with Chapter 2), help you maintain skills you previously learned. The **ANSWERS** include references to the section objectives upon which the exercises are based.

FINAL EXAM

- Simplify: $12 - 8[3 - (-2)]^2 + 5 - 3$
- Evaluate $\frac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.
- Given: $f(x) = 3x - 7$ and $t(x) = x^2 - 4x$, find $(f \circ t)(3)$.
- Solve: $\frac{3}{4}x - 2 = 4$
- Solve: $\frac{2-4x}{3} - \frac{x-6}{12} = \frac{5x-2}{6}$
- Solve: $8 - |5 - 3x| = 1$

A **FINAL EXAM** appears after the last chapter in the text. It is designed to simulate a possible examination of all the concepts covered in the text. The **ANSWERS** to the exam questions are provided in the answer appendix at the back of the text and include references to the section objectives upon which the questions are based.

Other Key Features

MARGINS Within the margins, students can find the following.



Take Note boxes alert students to concepts that require special attention.



Point of Interest boxes, which may be historical in nature or be of general interest, relate to topics under discussion.



Integrating Technology boxes, which are offered as optional instruction in the proper use of the scientific calculator, appear for selected topics under discussion.



Tips for Success boxes outline good study habits.

HOW TO 1 Write $\log_3 81 = 4$ in exponential form.

$\log_3 81 = 4$ is equivalent to $3^4 = 81$.

HOW TO 2 Write $10^{-2} = 0.01$ in logarithmic form.

$10^{-2} = 0.01$ is equivalent to $\log_{10}(0.01) = -2$.

It is important to note that the exponential function is a 1-1 function and thus has an inverse function. The inverse function of the exponential function is called a logarithm.

IMPORTANT POINTS Passages of text are now highlighted to help students recognize what is most important and to help them study more effectively.

PROBLEM-SOLVING STRATEGIES

The text features a carefully developed approach to problem solving that encourages students to develop a Strategy for a problem and then to create a Solution based on the Strategy.

EXAMPLE 2

The length of a rectangle is 8 in. more than the width. The area of the rectangle is 240 in^2 . Find the width of the rectangle.

Strategy

Draw a diagram. Then use the formula for the area of a rectangle.



Solution

$$\begin{aligned} A &= LW \\ 240 &= (x + 8)x \\ 240 &= x^2 + 8x \\ 0 &= x^2 + 8x - 240 \\ 0 &= (x + 20)(x - 12) \end{aligned}$$

$$\begin{array}{ll} x + 20 = 0 & x - 12 = 0 \\ x = -20 & x = 12 \end{array}$$

The width cannot be negative.
The width is 12 in.

YOU TRY IT 2

The height of a triangle is 3 cm more than the length of the base of the triangle. The area of the triangle is 54 cm^2 . Find the height of the triangle and the length of the base.

Your strategy

Your solution

Solution on p. S18

FOCUS ON PROBLEM SOLVING

At the end of each chapter, the Focus on Problem Solving fosters further discovery of new problem-solving strategies, such as applying solutions to other problems, working backwards, inductive reasoning, and trial and error.

FOCUS ON PROBLEM SOLVING

Find a Counterexample When you are faced with an assertion, it may be that the assertion is false. For instance, consider the statement "Every prime number is an odd number." This assertion is false because the prime number 2 is an even number.

Finding an example that illustrates that an assertion is false is called finding a *counterexample*. The number 2 is a counterexample to the assertion that every prime number is an odd number.

If you are given an unfamiliar problem, one strategy to consider as a means of solving the problem is to try to find a counterexample. For each of the following problems, answer *true* if the assertion is always true. If the assertion is not true, answer *false* and give a counterexample. If there are terms used that you do not understand, consult a reference to find the meaning of the term.

1. If x is a real number, then x^2 is always positive.
2. The product of an odd integer and an even integer is an even integer.
3. If m is a positive integer, then $2m + 1$ is always a positive odd integer.

General Revisions

- Chapter Openers now include Prep Tests for students to test their knowledge of prerequisite skills for the new chapter.
- Each exercise set has been thoroughly reviewed to ensure that the pace and scope of the exercises adequately cover the concepts introduced in the section.
- The variety of word problems has increased. This will appeal to instructors who teach to a range of student abilities and want to address different learning styles.
- Think About It exercises, which are conceptual in nature, have been added. They are meant to assess and strengthen a student's understanding of the material presented in an objective.
- In the News exercises have been added and are based on a media source such as a newspaper, a magazine, or the Web. The exercises demonstrate the pervasiveness and utility of mathematics in a contemporary setting.
- Concept Reviews now appear in the end-of-chapter materials to help students more actively study and review the contents of the chapter.
- The Chapter Review Exercises and Chapter Tests have been adjusted to ensure that there are questions that assess the key ideas in the chapter.
- The design has been significantly modified to make the text even easier for students to follow.

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Instructor Resources

Print Ancillaries

Complete Solutions Manual (0-538-49393-3)

Ellena Reda, *Dutchess Community College*

The Complete Solutions Manual provides worked-out solutions to all of the problems in the text.

Instructor's Resource Binder (0-538-49776-9)

Maria H. Andersen, *Muskegon Community College*

New

The Instructor's Resource Binder contains uniquely designed Teaching Guides, which include instruction tips, examples, activities, worksheets, overheads, and assessments, with answers to accompany them.

Appendix to accompany Instructor's Resource Binder (0-538-49776-9)

Richard N. Aufmann, *Palomar College*

Joanne S. Lockwood, *Nashua Community College*

New! The Appendix to accompany the Instructor's Resource Binder contains teacher resources that are tied directly to *Intermediate Algebra: An Applied Approach*, 8e. Organized by objective, the Appendix contains additional questions and short, in-class activities. The Appendix also includes answers to Writing Exercises, Focus on Problem Solving, and Projects and Group Activities found in the text.

Electronic Ancillaries

Enhanced WebAssign ENHANCED WebAssign

New

Used by over one million students at more than 1,100 institutions, WebAssign allows you to assign, collect, grade, and record homework assignments via the Web. This proven and reliable homework system includes thousands of algorithmically generated homework problems, links to relevant textbook sections, video examples, problem-specific tutorials, and more.

Solution Builder (0-840-04555-7)

This online solutions manual allows instructors to create customizable solutions that they can print out to distribute or post as needed. This is a convenient and expedient way to deliver solutions to specific homework sets.

PowerLecture with Diploma®

(0-538-45122-X)

New

This CD-ROM provides the instructor with dynamic media tools for teaching. Create, deliver, and customize tests (both print and online) in minutes with Diploma's Computerized Testing featuring algorithmic equations. Easily build solution sets for homework or exams using Solution Builder's online solutions manual. Quickly and easily update your syllabus with the new Syllabus Creator, which was created by the authors and contains the new edition's table of contents. Practice Sheets, First Day of Class PowerPoint® lecture slides, art and figures from the book, and a test bank in electronic format are also included on this CD-ROM.

Text Specific DVDs (0-538-79792-4)

Hosted by Dana Mosely and captioned for the hearing-impaired, these DVDs cover all sections in the text. Ideal for promoting individual study and review, these comprehensive DVDs also support students in online courses or those who may have missed a lecture.

Student Resources

Print Ancillaries

Student Solutions Manual (0-538-49392-5)

Ellena Reda, *Dutchess Community College*

The Student Solutions Manual provides worked-out solutions to the odd-numbered problems in the textbook.

Student Workbook (0-538-49583-9)

Maria H. Andersen, *Muskegon Community College*

New

Get a head-start! The Student Workbook contains assessments, activities, and worksheets from the Instructor's Resource Binder. Use them for additional practice to help you master the content.

Electronic Ancillaries

Enhanced WebAssign ENHANCED WebAssign

New

If you are looking for extra practice or additional support, Enhanced WebAssign offers practice problems, videos, and tutorials that are tied directly to the problems found in the textbook.

Text Specific DVDs (0-538-79792-4)

Hosted by Dana Mosley, an experienced mathematics instructor, the DVDs will help you to get a better handle on topics found in the textbook. A comprehensive set of DVDs for the entire course is available to order.

→ AIM for Success: Getting Started

Welcome to *Intermediate Algebra: An Applied Approach*! Students come to this course with varied backgrounds and different experiences in learning math. We are committed to your success in learning mathematics and have developed many tools and resources to support you along the way. Want to excel in this course? Read on to learn the skills you'll need and how best to use this book to get the results you want.

Motivate Yourself



Take Note

Motivation alone won't lead to success. For example, suppose a person who cannot swim is rowed out to the middle of a lake and thrown overboard. That person has a lot of motivation to swim, but will most likely drown without some help. You'll need motivation and learning in order to succeed.

You'll find many real-life problems in this book, relating to sports, money, cars, music, and more. We hope that these topics will help you understand how you will use mathematics in your real life. However, to learn all of the necessary skills and how you can apply them to your life outside this course, you need to stay motivated.

**THINK ABOUT WHY YOU WANT TO SUCCEED IN THIS COURSE.
LIST THE REASONS HERE (NOT IN YOUR HEAD . . . ON THE PAPER!):**

We also know that this course may be a requirement for you to graduate or complete your major. That's OK. If you have a goal for the future, such as becoming a nurse or a teacher, you will need to succeed in mathematics first. Picture yourself where you want to be, and use this image to stay on track.

Make the Commitment

Stay committed to success! With practice, you will improve your math skills. Skeptical? Think about when you first learned to ride a bike or drive a car. You probably felt self-conscious and worried that you might fail. But with time and practice, it became second nature to you.

You will also need to put in the time and practice to do well in mathematics. Think of us as your "driving" instructors. We'll lead you along the path to success, but we need you to stay focused and energized along the way.



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**LIST A SITUATION IN WHICH YOU ACCOMPLISHED YOUR GOAL
BY SPENDING TIME PRACTICING AND PERFECTING YOUR
SKILLS (SUCH AS LEARNING TO PLAY THE PIANO OR
PLAYING BASKETBALL):**

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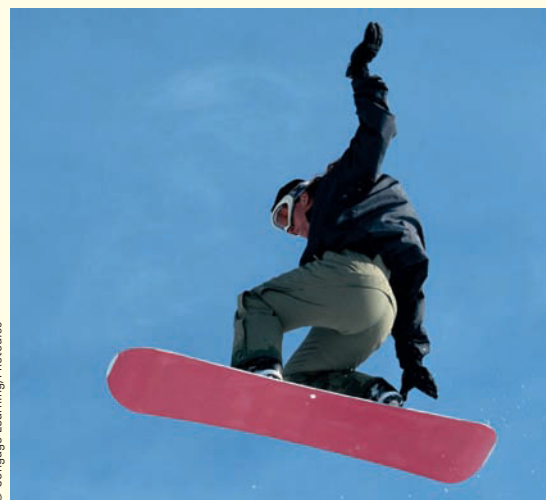


If you spend time learning and practicing the skills in this book, you will also succeed in math.

**Think You Can't
Do Math?
Think Again!**

You can do math! When you first learned the skills you just listed, you may have not done them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

It is difficult for us to emphasize how important it is to overcome the “I Can’t Do Math Syndrome.” If you listen to interviews of very successful athletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a “Can Do” attitude. Develop this attitude toward math and you will succeed.



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**Skills for
Success**

GET THE BIG PICTURE If this were an English class, we wouldn’t encourage you to look ahead in the book. But this is mathematics—go right ahead! Take a few minutes to read the table of contents. Then, look through the entire book. Move quickly: scan titles, look at pictures, notice diagrams.

Getting this big picture view will help you see where this course is going. To reach your goal, it’s important to get an idea of the steps you will need to take along the way.

As you look through the book, find topics that interest you. What’s your preference? Horse racing? Sailing? TV? Amusement parks? Find the Index of Applications at the back of the book and pull out three subjects that interest you. Then, flip to the pages in the book where the topics are featured and read the exercises or problems where they appear.



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WRITE THE TOPIC HERE:

**WRITE THE CORRESPONDING
EXERCISE/PROBLEM HERE:**

You’ll find it’s easier to work at learning the material if you are interested in how it can be used in your everyday life.

Use the following activities to think about more ways you might use mathematics in your daily life. Flip open your book to the following exercises to answer the questions.

- (see p. 95, #109) I’m thinking of getting a new checking account. I need to use algebra to . . .

- (see p. 367, #33) I'm considering walking to work as part of a new diet. I need algebra to . . .

- (see p. 174, #82) I just had an hour-long phone conversation. I need algebra to . . .

You know that the activities you just completed are from daily life, but do you notice anything else they have in common? That's right—they are **word problems**. Try not to be intimidated by word problems. You just need a strategy. It's true that word problems can be challenging because we need to use multiple steps to solve them:

- Read the problem.
- Determine the quantity we must find.
- Think of a method to find it.
- Solve the problem.
- Check the answer.

In short, we must come up with a **strategy** and then use that strategy to find the **solution**.

We'll teach you about strategies for tackling word problems that will make you feel more confident in branching out to these problems from daily life. After all, even though no one will ever come up to you on the street and ask you to solve a multiplication problem, you will need to use math every day to balance your checkbook, evaluate credit card offers, etc.

Take a look at the following example. You'll see that solving a word problem includes finding a *strategy* and using that strategy to find a *solution*. If you find yourself struggling with a word problem, try writing down the information you know about the problem. Be as specific as you can. Write out a phrase or a sentence that states what you are trying to find. Ask yourself whether there is a formula that expresses the known and unknown quantities. Then, try again!



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EXAMPLE • 12

The radius of a circle is $(3x - 2)$ cm.
Find the area of the circle in terms of the variable x .
Use 3.14 for π .

Strategy

To find the area, replace the variable r in the equation $A = \pi r^2$ by the given value, and solve for A .

Solution

$$\begin{aligned} A &= \pi r^2 \\ A &\approx 3.14(3x - 2)^2 \\ &= 3.14(9x^2 - 12x + 4) \\ &= 28.26x^2 - 37.68x + 12.56 \end{aligned}$$

The area is $(28.26x^2 - 37.68x + 12.56)$ cm².

YOU TRY IT • 12

The radius of a circle is $(2x + 3)$ cm.
Find the area of the circle in terms of the variable x .
Use 3.14 for π .

Your strategy

Your solution

Solutions on p. S16

Take Note

Take a look at your syllabus to see if your instructor has an **attendance policy** that is part of your overall grade in the course.

The attendance policy will tell you:

- How many classes you can miss without a penalty
- What to do if you miss an exam or quiz
- If you can get the lecture notes from the professor if you miss a class

Take Note

When planning your schedule, give some thought to how much time you realistically have available each week. For example, if you work 40 hours a week, take 15 units, spend the recommended study time given at the right, and sleep 8 hours a day, you will use over 80% of the available hours in a week. That leaves less than 20% of the hours in a week for family, friends, eating, recreation, and other activities.

Visit http://college.cengage.com/masterstudent/shared/content/time_chart/chart.html and use the Interactive Time Chart to see how you're spending your time—you may be surprised.

GET THE BASICS On the first day of class, your instructor will hand out a **syllabus** listing the requirements of your course. Think of this syllabus as your personal roadmap to success. It shows you the destinations (topics you need to learn) and the dates you need to arrive at those destinations (by when you need to learn the topics). Learning mathematics is a journey. But, to get the most out of this course, you'll need to know what the important stops are and what skills you'll need to learn for your arrival at those stops.

You've quickly scanned the table of contents, but now we want you to take a closer look. Flip open to the table of contents and look at it next to your syllabus. Identify when your major exams are and what material you'll need to learn by those dates. For example, if you know you have an exam in the second month of the semester, how many chapters of this text will you need to learn by then? What homework do you have to do during this time? Managing this important information will help keep you on track for success.

MANAGE YOUR TIME We know how busy you are outside of school. Do you have a full-time or a part-time job? Do you have children? Visit your family often? Play basketball or write for the school newspaper? It can be stressful to balance all of the important activities and responsibilities in your life. Making a **time management plan** will help you create a schedule that gives you enough time for everything you need to do.

Let's get started! Create a weekly schedule.

First, list all of your responsibilities that take up certain set hours during the week. Be sure to include:

- each class you are taking
- time you spend at work
- any other commitments (child care, tutoring, volunteering, etc.)

Then, list all of your responsibilities that are more flexible. Remember to make time for:

- **STUDYING** You'll need to study to succeed, but luckily you get to choose what times work best for you. Keep in mind:
 - Most instructors ask students to spend twice as much time studying as they do in class (3 hours of class = 6 hours of study).
 - Try studying in chunks. We've found it works better to study an hour each day, rather than studying for 6 hours on one day.
 - Studying can be even more helpful if you're able to do it right after your class meets, when the material is fresh in your mind.
- **MEALS** Eating well gives you energy and stamina for attending classes and studying.
- **ENTERTAINMENT** It's impossible to stay focused on your responsibilities 100% of the time. Giving yourself a break for entertainment will reduce your stress and help keep you on track.
- **EXERCISE** Exercise contributes to overall health. You'll find you're at your most productive when you have both a healthy mind and a healthy body.



Here is a sample of what part of your schedule might look like:

	8–9	9–10	10–11	11–12	12–1	1–2	2–3	3–4	4–5	5–6
Monday	History class Jenkins Hall 8–9:15 →	Eat 9:15–10 →	Study/Homework for History 10–12 →		Lunch and Nap! 12–1:30 →		Work 2–6 →			
Tuesday	Breakfast	Math Class Douglas Hall 9–9:45 →	Study/Homework for Math 10–12 →		Eat 12–1 →	English Class Scott Hall 1–1:45 →	Study/Homework for English 2–4 →		Hang out with Alli and Mike 4–6 →	

Features for Success in This Text

ORGANIZATION Let’s look again at the table of contents. There are 12 chapters in this book. You’ll see that every chapter is divided into **sections**, and each section contains a number of **learning objectives**. Each learning objective is labeled with a letter from A to D. Knowing how this book is organized will help you locate important topics and concepts as you’re studying.

PREPARATION Ready to start a new chapter? Take a few minutes to be sure you’re ready, using some of the tools in this book.

- **CUMULATIVE REVIEW EXERCISES:** You’ll find these exercises after every chapter, starting with Chapter 2. The questions in the Cumulative Review Exercises are taken from the previous chapters. For example, the Cumulative Review for Chapter 3 will test all of the skills you have learned in Chapters 1, 2, and 3. Use this to refresh yourself before moving on to the next chapter, or to test what you know before a big exam.

Here’s an example of how to use the Cumulative Review:

- Turn to page 199 and look at the questions for the Chapter 3 Cumulative Review, which are taken from the current chapter and the previous chapters.
- We have the answers to all of the Cumulative Review Exercises in the back of the book. Flip to page A14 to see the answers for this chapter.
- Got the answer wrong? We can tell you where to go in the book for help! For example, scroll down page A14 to find the answer for exercise #9, which is –4.5. You’ll see that after this answer, there is an **objective reference** [1.3B]. This means that the question was taken from Chapter 1, Section 3, Objective B. Go here to restudy the objective.
- **PREP TESTS:** These tests are found at the beginning of every chapter and will help you see if you’ve mastered all of the skills needed for the new chapter.

Here’s an example of how to use the Prep Test:

- Turn to page 201 and look at the Prep Test for Chapter 4.
- All of the answers to the Prep Tests are in the back of the book. You’ll find them in the first set of answers in each answer section for a chapter. Turn to page A14 to see the answers for this Prep Test.
- Restudy the objectives if you need some extra help.



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- Before you start a new section, take a few minutes to read the **Objective Statement** for that section. Then, browse through the objective material. Especially note the words or phrases in bold type—these are important concepts that you'll need as you're moving along in the course.
- As you start moving through the chapter, pay special attention to the **rule boxes**. These rules give you the reasons certain types of problems are solved the way they are. When you see a rule, try to rewrite the rule in your own words.

Determinant of a 2×2 Matrix

The **determinant** of a 2×2 matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is written $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$. The value of this determinant is given by the formula

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Page 226

Knowing what to pay attention to as you move through a chapter will help you study and prepare.

INTERACTION We want you to be actively involved in learning mathematics and have given you many ways to get hands-on with this book.

- **HOW TO EXAMPLES** Take a look at page 206 shown here. See the HOW TO example? This contains an explanation by each step of the solution to a sample problem.

HOW TO 4 Solve by the substitution method: (1) $6x + 2y = 8$
(2) $3x + y = 2$

(3) $3x + y = 2$
 $y = -3x + 2$

(1) $6x + 2y = 8$
 $6x + 2(-3x + 2) = 8$
 $6x - 6x + 4 = 8$
 $0x + 4 = 8$
 $4 = 8$

- We will solve Equation (2) for y .
- This is Equation (3).
- This is Equation (1).
- Equation (3) states that $y = -3x + 2$.
- Substitute $-3x + 2$ for y in Equation (1).
- Solve for x .

Page 206

Grab a paper and pencil and work along as you're reading through each example. When you're done, get a clean sheet of paper. Write down the problem and try to complete the solution without looking at your notes or at the book. When you're done, check your answer. If you got it right, you're ready to move on.

- **EXAMPLE/YOU TRY IT PAIRS** You'll need hands-on practice to succeed in mathematics. When we show you an example, work it out beside our solution. Use the Example/You Try It pairs to get the practice you need.

Take a look at page 206, Example 4 and You Try It 4 shown here:

EXAMPLE 4	YOU TRY IT 4
Solve by substitution: (1) $3x - 2y = 4$ (2) $-x + 4y = -3$ Solution Solve Equation (2) for x. $-x + 4y = -3$ $-x = -4y - 3$	Solve by substitution: $3x - y = 3$ $6x + 3y = -4$ Your solution

Page 206

You'll see that each Example is fully worked-out. Study this Example carefully by working through each step. Then, try your hand at it by completing the You Try It. If you get stuck, the solutions to the You Try Its are provided in the back of the book. There is a page number following the You Try It, which shows you where you can find the completely worked-out solution. Use the solution to get a hint for the step on which you are stuck. Then, try again!

When you've finished the solution, check your work against the solution in the back of the book. Turn to page S11 to see the solution for You Try It 4.

Remember that sometimes there can be more than one way to solve a problem. But, your answer should always match the answers we've given in the back of the book. If you have any questions about whether your method will always work, check with your instructor.

REVIEW We have provided many opportunities for you to practice and review the skills you have learned in each chapter.

- **SECTION EXERCISES** After you're done studying a section, flip to the end of the section and complete the exercises. If you immediately practice what you've learned, you'll find it easier to master the core skills. Want to know if you answered the questions correctly? The answers to the odd-numbered exercises are given in the back of the book.
- **CHAPTER SUMMARY** Once you've completed a chapter, look at the Chapter Summary. This is divided into two sections: *Key Words* and *Essential Rules and Procedures*. Flip to page 249 to see the Chapter Summary for Chapter 4. This summary shows all of the important topics covered in the chapter. See the reference following each topic? This shows you the objective reference and the page in the text where you can find more information on the concept.
- **CONCEPT REVIEW** Following the Chapter Summary for each chapter is the Concept Review. Flip to page 252 to see the Concept Review for Chapter 4. When you read each question, jot down a reminder note on the right about whatever you feel will be most helpful to remember if you need to apply that concept during an exam. You can also use the space on the right to mark what concepts your instructor expects you to know for the next test. If you are unsure of the answer to a concept review question, flip to the answers appendix at the back of the book.



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- **CHAPTER REVIEW EXERCISES** You'll find the Chapter Review Exercises after the Concept Review. Flip to page 438 to see the Chapter Review Exercises for Chapter 7. When you do the review exercises, you're giving yourself an important opportunity to test your understanding of the chapter. The answer to each review exercise is given at the back of the book, along with the objective the question relates to. When you're done with the Chapter Review Exercises, check your answers. If you had trouble with any of the questions, you can restudy the objectives and retry some of the exercises in those objectives for extra help.

- **CHAPTER TESTS** The Chapter Tests can be found after the Chapter Review Exercises and can be used to prepare for your exams. The answer to each test question is given at the back of the book, along with a reference to a How To, Example, or You Try It that the question relates to. Think of these tests as “practice runs” for your in-class tests. Take the test in a quiet place and try to work through it in the same amount of time you will be allowed for your exam.

Here are some strategies for success when you’re taking your exams:

- Scan the entire test to get a feel for the questions (get the big picture).
- Read the directions carefully.
- Work the problems that are easiest for you first.
- Stay calm, and remember that you will have lots of opportunities for success in this class!

EXCEL Visit www.cengage.com/math/aufmann to learn about additional study tools!

- **Enhanced WebAssign®** online practice exercises and homework problems match the textbook exercises.
- **DVDs** Hosted by Dana Mosley, an experienced mathematics instructor, the DVDs will help you to get a better handle on topics that may be giving you trouble. A comprehensive set of DVDs for the entire course is available to order.

Get Involved Have a question? Ask! Your professor and your classmates are there to help. Here are some tips to help you jump in to the action:



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- Raise your hand in class.
- If your instructor prefers, email or call your instructor with your question. If your professor has a website where you can post your question, also look there for answers to previous questions from other students. Take advantage of these ways to get your questions answered.
- Visit a **math center**. Ask your instructor for more information about the math center services available on your campus.
- Your instructor will have **office hours** where he or she will be available to help you. Take note of where and when your instructor holds office hours. Use this time for one-on-one help, if you need it.
- Form a **study group** with students from your class. This is a great way to prepare for tests, catch up on topics you may have missed, or get extra help on problems you’re struggling with. Here are a few suggestions to make the most of your study group:
 - **Test each other by asking questions.** Have each person bring a few sample questions when you get together.



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- **Practice teaching each other.** We've found that you can learn a lot about what you know when you have to explain it to someone else.
- **Compare class notes.** Couldn't understand the last five minutes of class? Missed class because you were sick? Chances are someone in your group has the notes for the topics you missed.
- **Brainstorm test questions.**
- **Make a plan for your meeting.** Agree on what topics you'll talk about and how long you'll be meeting. When you make a plan, you'll be sure that you make the most of your meeting.



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Ready, Set, Succeed! It takes hard work and commitment to succeed, but we know you can do it! Doing well in mathematics is just one step you'll take along the path to success.

I succeeded in Intermediate Algebra!

We are confident that if you follow our suggestions, you will succeed. Good luck!



Rubberball

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Review of Real Numbers

OBJECTIVES

SECTION 1.1

- A** To use inequality and absolute value symbols with real numbers
- B** To write and graph sets
- C** To find the union and intersection of sets

SECTION 1.2

- A** To add, subtract, multiply, and divide integers
- B** To add, subtract, multiply, and divide rational numbers
- C** To evaluate exponential expressions
- D** To use the Order of Operations Agreement

SECTION 1.3

- A** To use and identify the properties of the real numbers
- B** To evaluate a variable expression
- C** To simplify a variable expression

SECTION 1.4

- A** To translate a verbal expression into a variable expression
- B** To solve application problems

ARE YOU READY?

Take the Chapter 1 Prep Test to find out if you are ready to learn to:

- Write sets, graph sets, and find the union and intersection of sets
- Add, subtract, multiply, and divide integers and rational numbers
- Evaluate numerical expressions
- Evaluate variable expressions
- Simplify variable expressions
- Translate a verbal expression into a variable expression

PREP TEST

Do these exercises to prepare for Chapter 1.

For Exercises 1 to 8, add, subtract, multiply, or divide.

1. $\frac{5}{12} + \frac{7}{30}$

2. $\frac{8}{15} - \frac{7}{20}$

3. $\frac{5}{6} \cdot \frac{4}{15}$

4. $\frac{4}{15} \div \frac{2}{5}$

5. $8 + 29.34 + 7.065$

6. $92 - 18.37$

7. $2.19(3.4)$

8. $32.436 \div 0.6$

9. Which of the following numbers are greater than -8 ?

- a. -6 b. -10 c. 0 d. 8

10. Match the fraction with its decimal equivalent.

a. $\frac{1}{2}$ A. 0.75

b. $\frac{7}{10}$ B. 0.89

c. $\frac{3}{4}$ C. 0.5

d. $\frac{89}{100}$ D. 0.7

SECTION

1.1

Introduction to Real Numbers

OBJECTIVE A

To use inequality and absolute value symbols with real numbers



Point of Interest

The Big Dipper, known to the Greeks as *Ursa Major*, the great bear, is a constellation that can be seen from northern latitudes. The stars of the Big Dipper are Alkaid, Mizar, Alioth, Megrez, Phecda, Merak, and Dubhe. The star at the bend of the handle, Mizar, is actually two stars, Mizar and Alcor. An imaginary line from Merak through Dubhe passes through Polaris, the north star.

It seems to be a human characteristic to put similar items in the same place. For instance, an astronomer places stars in *constellations*, and a geologist divides the history of Earth into *eras*.

Mathematicians likewise place objects with similar properties in *sets*. A **set** is a collection of objects. The objects are called **elements** of the set. Sets are denoted by placing braces around the elements in the set.

The numbers that we use to count things, such as the number of books in a library or the number of CDs sold by a record store, have similar characteristics. These numbers are called the *natural numbers*.

$$\text{Natural numbers} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots\}$$

Each natural number greater than 1 is a *prime* number or a *composite* number. A **prime number** is a natural number greater than 1 that is divisible (evenly) only by itself and 1. For example, 2, 3, 5, 7, 11, and 13 are the first six prime numbers. A natural number that is not a prime number is a **composite number**. The numbers 4, 6, 8, and 9 are the first four composite numbers.



Point of Interest

The concept of zero developed very gradually over many centuries. It has been variously denoted by leaving a blank space, by a dot, and finally as 0. Negative numbers, although evident in Chinese manuscripts dating from 200 B.C., were not fully integrated into mathematics until late in the 14th century.

The natural numbers do not have a symbol to denote the concept of none—for instance, the number of trees taller than 1000 feet. The *whole numbers* include zero and the natural numbers.

$$\text{Whole numbers} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, \dots\}$$

The whole numbers alone do not provide all the numbers that are useful in applications. For instance, a meteorologist needs numbers below zero and above zero.

$$\text{Integers} = \{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$$

The integers $\dots, -5, -4, -3, -2, -1$ are **negative integers**. The integers 1, 2, 3, 4, 5, $\dots$ are **positive integers**. Note that the natural numbers and the positive integers are the same set of numbers. The integer zero is neither a positive nor a negative integer.

Still other numbers are necessary to solve the variety of application problems that exist. For instance, a landscape architect may need to purchase irrigation pipe that has a diameter of $\frac{5}{8}$ in. The numbers that include fractions are called *rational numbers*.

$$\text{Rational numbers} = \left\{ \frac{p}{q}, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0 \right\}$$

The numbers $\frac{2}{3}$, $-\frac{9}{2}$, and $\frac{5}{1}$ are examples of rational numbers. Note that $\frac{5}{1} = 5$, so all integers are rational numbers. The number $\frac{4}{\pi}$ is not a rational number because π is not an integer.

A rational number written as a fraction can be written in decimal notation by dividing the numerator by the denominator.

HOW TO 1 Write $\frac{3}{8}$ as a decimal.

Divide 3 by 8.

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

← This is a terminating decimal.

← The remainder is zero.

$$\frac{3}{8} = 0.375$$

HOW TO 2 Write $\frac{2}{15}$ as a decimal.

Divide 2 by 15.

$$\begin{array}{r} 0.13\overline{3} \\ 15 \overline{)2.000} \\ \underline{-15} \\ 50 \\ \underline{-45} \\ 50 \\ \underline{-45} \\ 5 \end{array}$$

← This is a repeating decimal.

← The remainder is never zero.

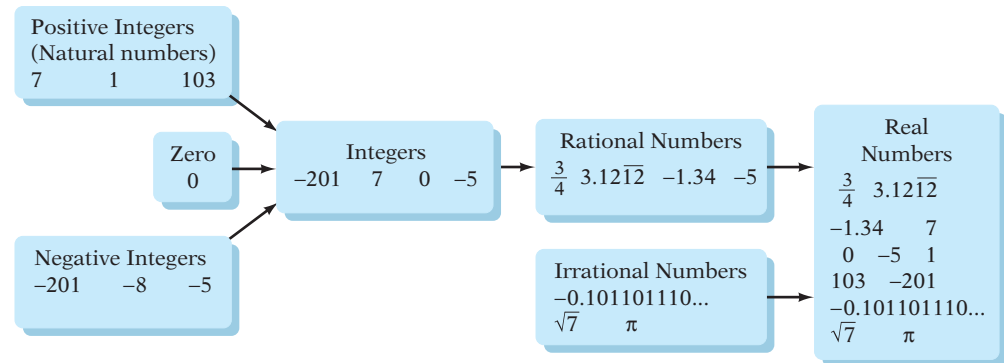
• The bar over 3 indicates that this digit repeats.

$$\frac{2}{15} = 0.1\overline{3}$$

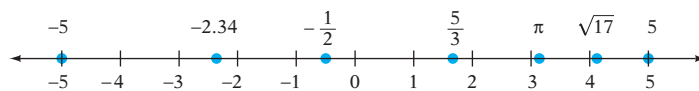
Some numbers cannot be written as terminating or repeating decimals—for example, $0.01001000100001\dots$, $\sqrt{7} \approx 2.6457513$, and $\pi \approx 3.1415927$. These numbers have decimal representations that neither terminate nor repeat. They are called **irrational numbers**. The rational numbers and the irrational numbers taken together are the **real numbers**.

Take Note

The real numbers are the rational numbers and the irrational numbers. The relationships among sets of numbers are shown in the figure at the right, along with examples of elements in each set.



The **graph of a real number** is made by placing a heavy dot directly above the number on a number line. The graphs of some real numbers follow.



Consider the following sentences:

- A restaurant's chef prepared a dinner and served *it* to the customers.
- A maple tree was planted and *it* grew two feet in one year.

In the first sentence, “*it*” means dinner; in the second sentence, “*it*” means tree. In language, the word *it* can stand for many different objects. Similarly, in mathematics, a letter of the alphabet can be used to stand for some number. A letter used in this way is called a **variable**.

It is convenient to use a variable to represent or stand for any one of the elements of a set. For instance, the statement “ x is an *element of* the set $\{0, 2, 4, 6\}$ ” means that x can be replaced by 0, 2, 4, or 6. The set $\{0, 2, 4, 6\}$ is called the **domain** of the variable.

The symbol for “is an element of” is $\in$; the symbol for “is not an element of” is $\notin$. For example,

$$2 \in \{0, 2, 4, 6\} \quad 6 \in \{0, 2, 4, 6\} \quad 7 \notin \{0, 2, 4, 6\}$$

Variables are used in the next definition.

Definition of Inequality Symbols

If a and b are two real numbers and a is to the left of b on the number line, then a is **less than** b . This is written $a < b$.

If a and b are two real numbers and a is to the right of b on the number line, then a is **greater than** b . This is written $a > b$.

Here are some examples.

$$5 < 9 \quad -4 > -10 \quad \pi < \sqrt{17} \quad 0 > -\frac{2}{3}$$

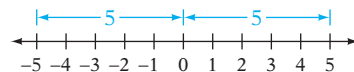
The inequality symbols $\leq$ (is less than or equal to) and $\geq$ (is greater than or equal to) are also important. Note the examples below.

$$4 \leq 5 \text{ is a true statement because } 4 < 5.$$

$$5 \leq 5 \text{ is a true statement because } 5 = 5.$$

The numbers 5 and -5 are the same distance from zero on the number line but on opposite sides of zero. The numbers 5 and -5 are called **additive inverses** or **opposites** of each other.

The additive inverse (or opposite) of 5 is -5 . The additive inverse of -5 is 5. The symbol for additive inverse is $-$.



$-(2)$ means the additive inverse of *positive* 2.

$$-(2) = -2$$

$-(-5)$ means the additive inverse of *negative* 5.

$$-(-5) = 5$$

The **absolute value** of a number is its distance from zero on the number line. The symbol for absolute value is $| \quad |$.

Note from the figure above that the distance from 0 to 5 is 5. Therefore, $|5| = 5$. That figure also shows that the distance from 0 to -5 is 5. Therefore, $|-5| = 5$.

Absolute Value

The absolute value of a positive number is the number itself.

The absolute value of a negative number is the opposite of the negative number.

The absolute value of zero is zero.



Integrating Technology

See the Keystroke Guide: *Math* for instructions on using a graphing calculator to evaluate absolute value expressions.

HOW TO 3

Evaluate: $-|-12|$

$$-|-12| = -12$$

- The absolute value symbol does not affect the negative sign in front of the absolute value symbol.

EXAMPLE • 1

Let $y \in \{-7, 0, 6\}$. For which values of y is the inequality $y < 4$ a true statement?

Solution

Replace y by each of the elements of the set and determine whether the inequality is true.

$$\begin{aligned} y &< 4 \\ -7 &< 4 \quad \text{True} \\ 0 &< 4 \quad \text{True} \\ 6 &< 4 \quad \text{False} \end{aligned}$$

The inequality is true for -7 and 0 .

YOU TRY IT • 1

Let $z \in \{-10, -5, 6\}$. For which values of z is the inequality $z > -5$ a true statement?

Your solution**EXAMPLE • 2**

Let $y \in \{-12, 0, 4\}$.

- Determine $-y$, the additive inverse of y , for each element of the set.
- Evaluate $|y|$ for each element of the set.

Solution

- Replace y in $-y$ by each element of the set and determine the value of the expression.

$$\begin{aligned} -y \\ -(-12) &= 12 \\ -(0) &= 0 \end{aligned} \quad \bullet \text{ 0 is neither positive nor negative.}$$

$$-(4) = -4$$

- Replace y in $|y|$ by each element of the set and determine the value of the expression.

$$\begin{aligned} |y| \\ |-12| &= 12 \\ |0| &= 0 \\ |4| &= 4 \end{aligned}$$

YOU TRY IT • 2

Let $d \in \{-11, 0, 8\}$.

- Determine $-d$, the additive inverse of d , for each element of the set.
- Evaluate $|d|$ for each element of the set.

Your solution

Solutions on p. S1

OBJECTIVE B**To write and graph sets**

The **roster method** of writing a set encloses a list of the elements of the set in braces. The set of even natural numbers less than 10 is written $\{2, 4, 6, 8\}$. This is an example of a **finite set**; all the elements of the set can be listed. The set of whole numbers, written $\{0, 1, 2, 3, 4, \dots\}$, and the set of natural numbers, written $\{1, 2, 3, 4, \dots\}$, are **infinite sets**. The pattern of numbers continues without end. It is impossible to list all the elements of an infinite set.

The set that contains no elements is called the **empty set**, or **null set**, and is symbolized by $\emptyset$ or $\{ \}$.

The set of trees over 1000 feet tall is the empty set.

HOW TO • 4

Use the roster method to write the set of whole numbers less than 5.

$\{0, 1, 2, 3, 4\}$

• Recall that the whole numbers include 0.

A second method of representing a set is **set-builder notation**. Set-builder notation can be used to describe almost any set, but it is especially useful when writing infinite sets.

In set-builder notation, the set of integers greater than -3 is written

$$\{x | x > -3, x \in \text{integers}\}$$

The set of all x
such that
 $x > -3$ and x is an element of the integers.

This is an infinite set. It is impossible to list all the elements of the set, but the set can be described using set-builder notation.

The set of real numbers less than 5 is written

$$\{x | x < 5, x \in \text{real numbers}\}$$

and is read “the set of all x such that x is less than 5 and x is an element of the real numbers.”

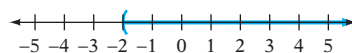
HOW TO • 5

Use set-builder notation to write the set of integers greater than 20.

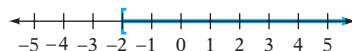
$\{x | x > 20, x \in \text{integers}\}$

Set-builder notation and the inequality symbols $>$, $<$, $\geq$, and $\leq$ are used to describe infinite sets of real numbers. These sets can also be graphed on the real number line.

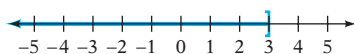
The graph of $\{x | x > -2, x \in \text{real numbers}\}$ is shown below. The set is the real numbers greater than -2 . The parenthesis on the graph indicates that -2 is not included in the set.



The graph of $\{x|x \geq -2, x \in \text{real numbers}\}$ is shown below. The set is the real numbers greater than or equal to -2 . The bracket at -2 indicates that -2 is included in the set.

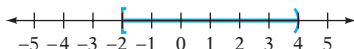


In many cases, we will assume that real numbers are being used and omit “ $x \in \text{real numbers}$ ” from set-builder notation. For instance, the above set is written $\{x|x \geq -2\}$.

HOW TO 6Graph $\{x|x \leq 3\}$.

- Draw a bracket at 3 to indicate that 3 is in the set. Draw a solid line to the left of 3.

$\{x|-2 \leq x < 4\}$ is read “the set of all x such that x is greater than or equal to -2 and less than 4.”

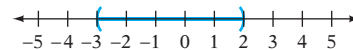
HOW TO 7Graph $\{x|-2 \leq x < 4\}$.

- This is the set of real numbers between -2 and 4, including -2 but not including 4. Draw a bracket at -2 and a parenthesis at 4.

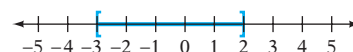
Some sets can also be expressed using **interval notation**. For example, the interval notation $(-3, 2]$ indicates the interval of all real numbers greater than -3 and less than or equal to 2. As on the graph of a set, the left parenthesis indicates that -3 is not included in the set. The right bracket indicates that 2 is included in the set.

An interval is said to be a **closed interval** if it includes both endpoints; it is an **open interval** if it does not include either endpoint. An interval is a **half-open interval** if one endpoint is included and the other is not. In each example given below, -3 and 2 are the **endpoints of the interval**. In each case, the set notation, the interval notation, and the graph of the set are shown.

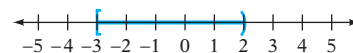
$\{x|-3 < x < 2\}$ $(-3, 2)$
Open interval



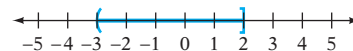
$\{x|-3 \leq x \leq 2\}$ $[-3, 2]$
Closed interval



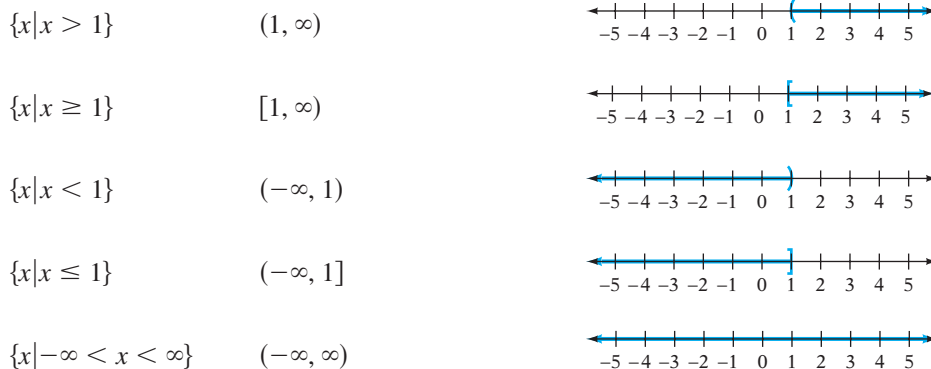
$\{x|-3 \leq x < 2\}$ $[-3, 2)$
Half-open interval



$\{x|-3 < x \leq 2\}$ $(-3, 2]$
Half-open interval



To indicate an interval that extends forever in one or both directions using interval notation, we use the **infinity symbol** ∞ or the **negative infinity symbol** $-\infty$. The infinity symbol is not a number; it is simply a notation to indicate that the interval is unlimited. In interval notation, a parenthesis is always used to the right of an infinity symbol or to the left of a negative infinity symbol, as shown in the following examples.

**EXAMPLE • 3**

Use the roster method to write the set of positive integers less than or equal to 7.

Solution

$\{1, 2, 3, 4, 5, 6, 7\}$

YOU TRY IT • 3

Use the roster method to write the set of negative integers greater than -6 .

Your solution**EXAMPLE • 4**

Use set-builder notation to write the set of integers less than -9 .

Solution

$\{x|x < -9, x \in \text{integers}\}$

YOU TRY IT • 4

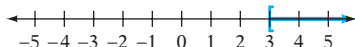
Use set-builder notation to write the set of whole numbers greater than or equal to 15.

Your solution**EXAMPLE • 5**

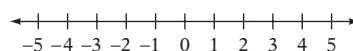
Graph $\{x|x \geq 3\}$.

Solution

Draw a bracket at 3 to indicate that 3 is in the set.
Draw a solid line to the right of 3.

**YOU TRY IT • 5**

Graph $\{x|x < 0\}$.

Your solution

EXAMPLE • 6

Write each set in interval notation.

- a. $\{x|x > 3\}$
 b. $\{x|-2 < x \leq 4\}$

Solution

- a. The set $\{x|x > 3\}$ is the numbers greater than 3. In interval notation, this is written $(3, \infty)$.
 b. The set $\{x|-2 < x \leq 4\}$ is the numbers greater than -2 and less than or equal to 4. In interval notation, this is written $(-2, 4]$.

YOU TRY IT • 6

Write each set in interval notation.

- a. $\{x|x < -1\}$
 b. $\{x|-2 \leq x < 4\}$

Your solution**EXAMPLE • 7**

Write each set in set-builder notation.

- a. $(-\infty, 4]$
 b. $[-3, 0]$

Solution

- a. $(-\infty, 4]$ is the numbers less than or equal to 4. In set-builder notation, this is written $\{x|x \leq 4\}$.
 b. $[-3, 0]$ is the numbers greater than or equal to -3 and less than or equal to 0.
 In set-builder notation, this is written $\{x|-3 \leq x \leq 0\}$.

YOU TRY IT • 7

Write each set in set-builder notation.

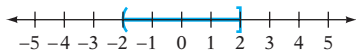
- a. $(3, \infty)$
 b. $(-4, 1]$

Your solution**EXAMPLE • 8**

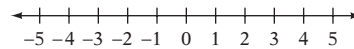
Graph $(-2, 2]$.

Solution

Draw a parenthesis at -2 to show that it is not in the set. Draw a bracket at 2 to show that it is in the set. Draw a solid line between -2 and 2.

**YOU TRY IT • 8**

Graph $[-2, \infty)$.

Your solution

OBJECTIVE C**To find the union and intersection of sets**

Just as operations such as addition and multiplication are performed on real numbers, operations are performed on sets. Two operations performed on sets are *union* and *intersection*.

Union of Two Sets

The **union of two sets**, written $A \cup B$, is the set of all elements that belong to either set A or set B . In set-builder notation, this is written

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

Given $A = \{2, 3, 4\}$ and $B = \{0, 1, 2, 3\}$, the union of A and B contains all the elements that belong to either A or B . Any elements that belong to both sets are listed only once.

$$A \cup B = \{0, 1, 2, 3, 4\}$$

Intersection of Two Sets

The **intersection of two sets**, written $A \cap B$, is the set of all elements that are common to both set A and set B . In set-builder notation, this is written

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

Given $A = \{2, 3, 4\}$ and $B = \{0, 1, 2, 3\}$, the intersection of A and B contains all the elements that are common to both A and B .

$$A \cap B = \{2, 3\}$$

**Point of Interest**

The symbols $\in$, $\cup$, and $\cap$ were first used by Giuseppe Peano in *Arithmetices Principia, Nova Exposita* (The Principle of Mathematics, a New Method of Exposition), published in 1889. The purpose of this book was to deduce the principles of mathematics from pure logic.

HOW TO 8

Given $A = \{2, 3, 5, 7\}$ and $B = \{0, 1, 2, 3, 4\}$, find $A \cup B$ and $A \cap B$.

$$A \cup B = \{0, 1, 2, 3, 4, 5, 7\}$$

- List the elements of each set. The elements that belong to both sets are listed only once.

$$A \cap B = \{2, 3\}$$

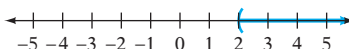
- List the elements that are common to both A and B .

The union of two sets is the set of all elements belonging to either one or the other of the two sets. The set $\{x|x \leq -1\} \cup \{x|x > 3\}$ is the set of real numbers that are either less than or equal to -1 or greater than 3 .

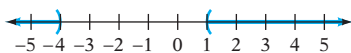


The set is written $\{x|x \leq -1 \text{ or } x > 3\}$.

The set $\{x|x > 2\} \cup \{x|x > 4\}$ is the set of real numbers that are either greater than 2 or greater than 4 . Because any number greater than 4 is also greater than 2 , this is the set $\{x|x > 2\}$.

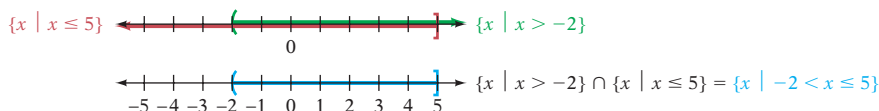


HOW TO 9 Graph $\{x|x > 1\} \cup \{x|x < -4\}$.



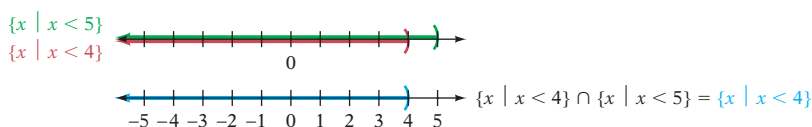
- The graph includes all the numbers that are either greater than 1 or less than -4 .

The intersection of two sets is the set that contains the elements common to both sets. The set $\{x|x > -2\} \cap \{x|x \leq 5\}$ is the set of real numbers that are greater than -2 and less than or equal to 5 . This is shown graphically below.

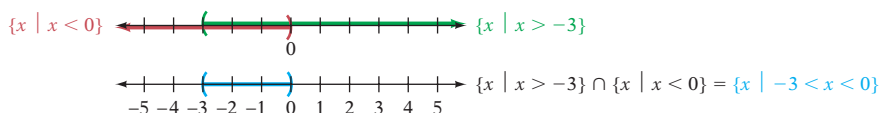


Note that although -2 is an element of $\{x|x \leq 5\}$, -2 is not an element of $\{x|x > -2\}$ and therefore -2 is not an element of the intersection of the two sets. Indicate this with a parenthesis at -2 . However, 5 is an element of $\{x|x \leq 5\}$ and 5 is an element of $\{x|x > -2\}$. Therefore, 5 is an element of the intersection of the two sets. Indicate this with a bracket at 5 .

The set $\{x|x < 4\} \cap \{x|x < 5\}$ is the set of real numbers that are less than 4 and less than 5 . This is the set of real numbers that are less than 4 , as shown in the graphs below.



HOW TO 10 Graph $\{x|x > -3\} \cap \{x|x < 0\}$.



EXAMPLE • 9

Given $A = \{0, 2, 4, 6, 8, 10\}$ and $B = \{0, 3, 6, 9\}$, find $A \cup B$.

Solution

$$A \cup B = \{0, 2, 3, 4, 6, 8, 9, 10\}$$

YOU TRY IT • 9

Given $C = \{1, 5, 9, 13, 17\}$ and $D = \{3, 5, 7, 9, 11\}$, find $C \cup D$.

Your solution**EXAMPLE • 10**

Given $A = \{x | x \in \text{natural numbers}\}$ and $B = \{x | x \in \text{negative integers}\}$, find $A \cap B$.

Solution

$$A \cap B = \emptyset$$

- There are no natural numbers that are also negative integers.

YOU TRY IT • 10

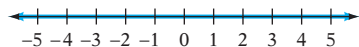
Given $E = \{x | x \in \text{odd integers}\}$ and $F = \{x | x \in \text{even integers}\}$, find $E \cap F$.

Your solution**EXAMPLE • 11**

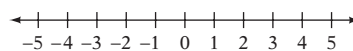
Graph $\{x | x > -1\} \cup \{x | x < 2\}$.

Solution

This is the set of real numbers greater than -1 or less than 2 . Any real number satisfies this condition. The graph is the entire real number line.

**YOU TRY IT • 11**

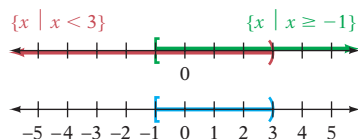
Graph $\{x | x < -2\} \cup \{x | x > -1\}$.

Your solution**EXAMPLE • 12**

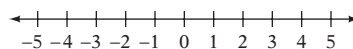
Graph $\{x | x < 3\} \cap \{x | x \geq -1\}$.

Solution

The graph is the set of real numbers that are common to the two intervals.

**YOU TRY IT • 12**

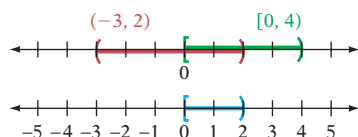
Graph $\{x | x < 1\} \cap \{x | x > -3\}$.

Your solution**EXAMPLE • 13**

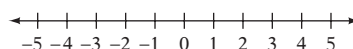
Graph $(-3, 2) \cap [0, 4)$.

Solution

The graph is the set of real numbers that are common to the two intervals.

**YOU TRY IT • 13**

Graph $[-4, 0) \cup [1, 3]$.

Your solution

1.1 EXERCISES

OBJECTIVE A

To use inequality and absolute value symbols with real numbers

For Exercises 1 and 2, determine which of the numbers are **a.** integers, **b.** rational numbers, **c.** irrational numbers, **d.** real numbers. List all that apply.

1. $-\frac{15}{2}, 0, -3, \pi, 2.\overline{33}, 4.232232223\dots, \frac{\sqrt{5}}{4}, \sqrt{7}$

2. $-17, 0.3412, \frac{3}{\pi}, -1.010010001\dots, \frac{27}{91}, 6.\overline{12}$

For Exercises 3 to 12, find the additive inverse of the number.

3. 27

4. -3

5. $\frac{3}{4}$

6. $\sqrt{17}$

7. 0

8. $-\pi$

9. $-\sqrt{33}$

10. -1.23

11. -91

12. $-\frac{2}{3}$

For Exercises 13 to 20, solve.

13. Let $y \in \{-6, -4, 7\}$. For which values of y is $y > -4$ true?

14. Let $x \in \{-6, -3, 3\}$. For which values of x is $x < -3$ true?

15. Let $w \in \{-2, -1, 0, 1\}$. For which values of w is $w \leq -1$ true?

16. Let $p \in \{-10, -5, 0, 5\}$. For which values of p is $p \geq 0$ true?

17. Let $b \in \{-9, 0, 9\}$. Evaluate $-b$ for each element of the set.

18. Let $a \in \{-3, -2, 0\}$. Evaluate $-a$ for each element of the set.

19. Let $c \in \{-4, 0, 4\}$. Evaluate $|c|$ for each element of the set.

20. Let $q \in \{-3, 0, 7\}$. Evaluate $|q|$ for each element of the set.



21. Are there any real numbers x for which $-x > 0$? If so, describe them.



22. Are there any real numbers y for which $-|y| > 0$? If so, describe them.

OBJECTIVE B To write and graph sets

For Exercises 23 to 28, use the roster method to write the set.

23. the integers between -3 and 5	24. the integers between -4 and 0
25. the even natural numbers less than 14	26. the odd natural numbers less than 14
27. the positive-integer multiples of 3 that are less than or equal to 30	28. the negative-integer multiples of 4 that are greater than or equal to -20

For Exercises 29 to 36, use set-builder notation to write the set.

29. the integers greater than 4	30. the integers less than -2
31. the real numbers greater than or equal to -2	32. the real numbers less than or equal to 2
33. the real numbers between 0 and 1	34. the real numbers between -2 and 5
35. the real numbers between 1 and 4 , inclusive	36. the real numbers between 0 and 2 , inclusive

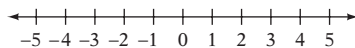


For Exercises 37 to 42, let $A = \{x | x \geq -3, x \in \text{integers}\}$. State whether the given number is an element of A .

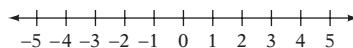
37. -3	38. 3.5	39. $\frac{1}{2}$	40. 1	41. -5	42. 5

For Exercises 43 to 50, graph.

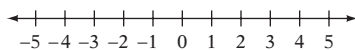
43. $\{x | x < 2\}$



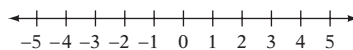
44. $\{x | x < -1\}$



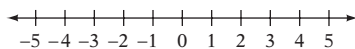
45. $\{x | x \geq 1\}$



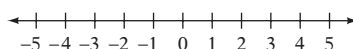
46. $\{x | x \leq -2\}$



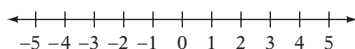
47. $\{x | -1 < x < 5\}$



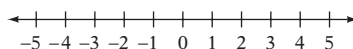
48. $\{x | 1 < x < 3\}$



49. $\{x | 0 \leq x \leq 3\}$



50. $\{x | -1 \leq x \leq 1\}$



For Exercises 51 to 58, write each set of real numbers in interval notation.

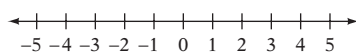
51. $\{x \mid -2 < x < 4\}$ 52. $\{x \mid 0 < x < 3\}$ 53. $\{x \mid -1 \leq x \leq 5\}$ 54. $\{x \mid 0 \leq x \leq 3\}$
 55. $\{x \mid x < 1\}$ 56. $\{x \mid x \leq 6\}$ 57. $\{x \mid x \geq -2\}$ 58. $\{x \mid x \geq 3\}$

For Exercises 59 to 68, write each interval in set-builder notation.

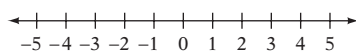
59. $(0, 8)$ 60. $(-2, 4)$ 61. $[-5, 7]$ 62. $[3, 4]$ 63. $[-3, 6)$
 64. $(4, 5]$ 65. $(-\infty, 4]$ 66. $(-\infty, -2)$ 67. $(5, \infty)$ 68. $[-2, \infty)$

For Exercises 69 to 76, graph.

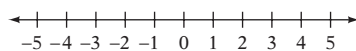
69. $(-2, 5)$



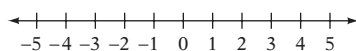
71. $[-1, 2]$



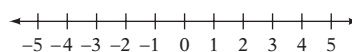
73. $(-\infty, 3]$



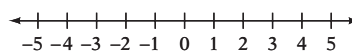
75. $[3, \infty)$



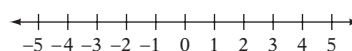
70. $(0, 3)$



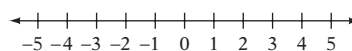
72. $[-3, 2]$



74. $(-\infty, -1)$



76. $[-2, \infty)$



OBJECTIVE C

To find the union and intersection of sets

For Exercises 77 to 84, find $A \cup B$.

77. $A = \{1, 4, 9\}, B = \{2, 4, 6\}$ 78. $A = \{-1, 0, 1\}, B = \{0, 1, 2\}$
 79. $A = \{2, 3, 5, 8\}, B = \{9, 10\}$ 80. $A = \{1, 3, 5, 7\}, B = \{2, 4, 6, 8\}$
 81. $A = \{-4, -2, 0, 2, 4\}, B = \{0, 4, 8\}$ 82. $A = \{-3, -2, -1\}, B = \{-2, -1, 0, 1\}$
 83. $A = \{1, 2, 3, 4, 5\}, B = \{3, 4, 5\}$ 84. $A = \{2, 4\}, B = \{0, 1, 2, 3, 4, 5\}$

For Exercises 85 to 92, find $A \cap B$.

85. $A = \{6, 12, 18\}, B = \{3, 6, 9\}$ 86. $A = \{-4, 0, 4\}, B = \{-2, 0, 2\}$
 87. $A = \{1, 5, 10, 20\}, B = \{5, 10, 15, 20\}$ 88. $A = \{1, 3, 5, 7, 9\}, B = \{1, 9\}$

89. $A = \{1, 2, 4, 8\}, B = \{3, 5, 6, 7\}$

90. $A = \{-3, -2, -1, 0\}, B = \{1, 2, 3, 4\}$

91. $A = \{2, 4, 6, 8, 10\}, B = \{4, 6\}$

92. $A = \{-9, -5, 0, 7\}, B = \{-7, -5, 0, 5, 7\}$



93. Which set is the empty set?

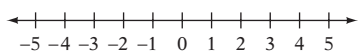
- (i) $\{x|x \in \text{integers}\} \cap \{x|x \in \text{rational numbers}\}$
 (ii) $\{-4, -2, 0, 2, 4\} \cup \{-3, -1, 1, 3\}$
 (iii) $[5, \infty) \cap (0, 5)$

94. Which set is *not* equivalent to the interval $[-1, 6)$?

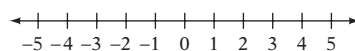
- (i) $\{x|-1 \leq x < 6\}$
 (ii) $\{x|x \geq -1\} \cup \{x|x < 6\}$
 (iii) $\{x|x < 6\} \cap \{x|x \geq -1\}$

For Exercises 95 to 106, graph.

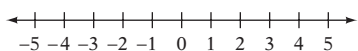
95. $\{x|x > 1\} \cup \{x|x < -1\}$



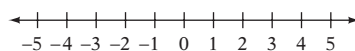
96. $\{x|x \leq 2\} \cup \{x|x > 4\}$



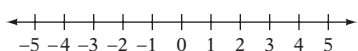
97. $\{x|x \leq 2\} \cap \{x|x \geq 0\}$



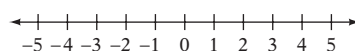
98. $\{x|x > -1\} \cap \{x|x \leq 4\}$



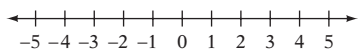
99. $\{x|x > 1\} \cap \{x|x \geq -2\}$



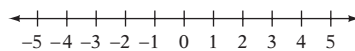
100. $\{x|x < 4\} \cup \{x|x \leq 0\}$



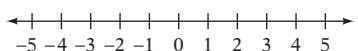
101. $\{x|x > -3\} \cup \{x|x < 1\}$



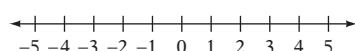
102. $\{x|x > -2\} \cap \{x|x < -4\}$



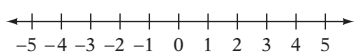
103. $[-5, 0) \cup (1, 4]$



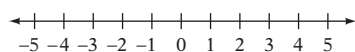
104. $(-\infty, -2] \cup [3, \infty)$



105. $[-3, 3] \cap [0, 5]$



106. $(-\infty, 1) \cap (-4, \infty)$



Applying the Concepts

Let $R = \{\text{real numbers}\}$, $A = \{x|-1 \leq x \leq 1\}$, $B = \{x|0 \leq x \leq 1\}$, $C = \{x|-1 \leq x \leq 0\}$, and $\emptyset$ be the empty set. Answer Exercises 107 to 116 using R , A , B , C , or $\emptyset$.

107. $A \cup B$

108. $A \cup A$

109. $B \cap B$

110. $A \cup C$

111. $A \cap R$

112. $C \cap R$

113. $B \cup R$

114. $A \cup R$

115. $R \cup R$

116. $R \cap \emptyset$

117. The set $B \cap C$ cannot be expressed using R , A , B , C , or $\emptyset$. What real number is represented by $B \cap C$?



118. A student wrote $-3 > x > 5$ as the inequality that represents the real numbers less than -3 or greater than 5 . Explain why this is incorrect.

SECTION

1.2

Operations on Rational Numbers

OBJECTIVE A

To add, subtract, multiply, and divide integers 

An understanding of the operations on integers is necessary to succeed in algebra. Let's review those properties, beginning with the sign rules for addition.



Point of Interest

Rules for operating with positive and negative numbers have existed for a long time. Although there are older records of these rules (from the 3rd century A.D.), one of the most complete records is contained in *The Correct Astronomical System of Brahma*, written by the Indian mathematician Brahmagupta around A.D. 600.

Rules for Addition of Real Numbers

To add numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

To add numbers with different signs, find the absolute value of each number. Subtract the smaller of these two numbers from the larger. Then attach the sign of the number with the larger absolute value.

HOW TO 1

Add: a. $-65 + (-48)$ b. $27 + (-53)$

$$\text{a. } -65 + (-48) = -113$$

$$\begin{aligned} \text{b. } 27 + (-53) \\ |27| = 27 \quad |-53| = 53 \\ 53 - 27 = 26 \end{aligned}$$

$$27 + (-53) = -26$$

• The signs are the same. Add the absolute values of the numbers. Then attach the sign of the addends.

• The signs are different. Find the absolute value of each number.

• Subtract the smaller number from the larger.

• Because $|-53| > |27|$, attach the sign of -53 .

Subtraction is defined as addition of the additive inverse.

Rule for Subtraction of Real Numbers

If a and b are real numbers, then $a - b = a + (-b)$. In words, a minus b equals a plus the opposite of b .

HOW TO 2

Subtract: a. $48 - (-22)$ b. $-31 - 18$

$$\begin{array}{c} \text{Change to } + \\ \downarrow \\ \text{a. } 48 - (-22) = 48 + 22 = 70 \\ \uparrow \\ \text{Opposite of } -22 \end{array}$$

$$\begin{array}{c} \text{Change to } + \\ \downarrow \\ \text{b. } -31 - 18 = -31 + (-18) = -49 \\ \uparrow \\ \text{Opposite of } 18 \end{array}$$

HOW TO 3

Simplify: $-3 - (-16) + (-12)$

$$\begin{aligned} -3 - (-16) + (-12) &= -3 + 16 + (-12) \\ &= 13 + (-12) = 1 \end{aligned}$$

• Write subtraction as addition of the opposite.

• Add from left to right.

The sign rules for multiplying real numbers are given below.

Rules for Multiplication of Real Numbers

The product of two numbers with the same sign is positive.

The product of two numbers with different signs is negative.

HOW TO 4

Multiply: **a.** $-4(-9)$ **b.** $84(-4)$

a. $-4(-9) = 36$

• The product of two numbers with the same sign is positive.

b. $84(-4) = -336$

• The product of two numbers with different signs is negative.

The **multiplicative inverse** of a nonzero real number a is $\frac{1}{a}$. This number is also called the **reciprocal** of a . For instance, the reciprocal of 2 is $\frac{1}{2}$ and the reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$. Division of real numbers is defined in terms of multiplication by the multiplicative inverse.

Rule for Division of Real Numbers

If a and b are real numbers and $b \neq 0$, then $a \div b = a \cdot \frac{1}{b}$.

Because division is defined in terms of multiplication, the sign rules for dividing real numbers are the same as the sign rules for multiplying.

HOW TO 5

Divide: **a.** $\frac{-54}{9}$ **b.** $(-21) \div (-7)$ **c.** $-\frac{63}{-9}$

a. $\frac{-54}{9} = -6$

• The quotient of two numbers with different signs is negative.

b. $(-21) \div (-7) = 3$

• The quotient of two numbers with the same sign is positive.

c. $-\frac{63}{-9} = 7$

Note that $\frac{-12}{3} = -4$, $\frac{12}{-3} = -4$, and $-\frac{12}{3} = -4$. This suggests the following result: If a and b are real numbers and $b \neq 0$, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

Properties of Zero and One in Division

• Zero divided by any number other than zero is zero.

$$\frac{0}{a} = 0, a \neq 0$$

**Take Note**

Suppose $\frac{4}{0} = n$. The

related multiplication problem is $n \cdot 0 = 4$. But $n \cdot 0 = 4$ is impossible because any number times 0 is 0. Therefore, division by zero is not defined.

- Division by zero is not defined.

$$\frac{a}{0} \text{ is undefined.}$$

- Any number other than zero divided by itself is 1.

$$\frac{a}{a} = 1, a \neq 0$$

- Any number divided by 1 is the number.

$$\frac{a}{1} = a$$

EXAMPLE • 1

Simplify: $-3 - (-5) - 9$

Solution

$$\begin{aligned} -3 - (-5) - 9 \\ &= -3 + 5 + (-9) \\ &= 2 + (-9) \\ &= -7 \end{aligned}$$

- Rewrite subtraction as addition of the opposite. Then add.

YOU TRY IT • 1

Simplify: $6 - (-8) - 10$

Your solution**EXAMPLE • 2**

Simplify: $-6|-5|(-15)$

Solution

$$\begin{aligned} -6|-5|(-15) \\ &= -6(5)(-15) \\ &= -30(-15) \\ &= 450 \end{aligned}$$

- Find the absolute value of -5 . Then multiply.

YOU TRY IT • 2

Simplify: $12(-3)|-6|$

Your solution**EXAMPLE • 3**

Simplify: $-\frac{36}{-3}$

Solution

$$-\frac{36}{-3} = -(-12) = 12$$

YOU TRY IT • 3

Simplify: $-\frac{28}{|-14|}$

Your solution

Solutions on p. S2

OBJECTIVE B**To add, subtract, multiply, and divide rational numbers**

Recall that a rational number is one that can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Examples of rational numbers are $-\frac{5}{9}$ and $\frac{12}{5}$.

HOW TO • 6 Add: $-12.34 + 9.059$

$$12.340 - 9.059 = 3.281$$

$$-12.34 + 9.059 = -3.281$$

- The signs are different. Subtract the absolute values of the numbers.

- Attach the sign of the number with the larger absolute value.

HOW TO • 7 Multiply: $(-0.23)(0.04)$

$$(-0.23)(0.04) = -0.0092$$

- The signs are different. The product is negative.

HOW TO • 8 Divide: $(-4.0764) \div (-1.72)$

$$(-4.0764) \div (-1.72) = 2.37$$

- The signs are the same. The quotient is positive.

To add or subtract rational numbers written as fractions, first rewrite the fractions as equivalent fractions with a common denominator. For the common denominator, we will use the **least common multiple (LCM) of the denominators**.

HOW TO • 9 Add: $\frac{5}{6} + \left(-\frac{7}{8}\right)$

$$\begin{aligned}\frac{5}{6} + \left(-\frac{7}{8}\right) &= \frac{5}{6} \cdot \frac{4}{4} + \left(\frac{-7}{8} \cdot \frac{3}{3}\right) \\ &= \frac{20}{24} + \left(\frac{-21}{24}\right) = \frac{20 + (-21)}{24} \\ &= \frac{-1}{24} = -\frac{1}{24}\end{aligned}$$

- The common denominator is 24. Write each fraction in terms of the common denominator.
- Add the numerators. Place the sum over the common denominator.

 Take Note

Although the sum could have been left as $\frac{-1}{24}$,

all answers in this text that are negative fractions are written with the negative sign in front of the fraction.

HOW TO • 10 Subtract: $-\frac{7}{15} - \left(-\frac{1}{6}\right)$

$$\begin{aligned}-\frac{7}{15} - \left(-\frac{1}{6}\right) &= -\frac{7}{15} + \frac{1}{6} \\ &= \frac{-7}{15} \cdot \frac{2}{2} + \frac{1}{6} \cdot \frac{5}{5} = \frac{-14}{30} + \frac{5}{30} \\ &= \frac{-9}{30} = -\frac{3}{10}\end{aligned}$$

- Write subtraction as addition of the opposite.
- Write each fraction in terms of the common denominator, 30.
- Add the numerators. Write the answer in simplest form.

The product of two fractions is the product of the numerators over the product of the denominators.

HOW TO • 11 Multiply: $-\frac{5}{12} \cdot \frac{8}{15}$

$$\begin{aligned}-\frac{5}{12} \cdot \frac{8}{15} &= -\frac{5 \cdot 8}{12 \cdot 15} = -\frac{40}{180} \\ &= -\frac{20 \cdot 2}{20 \cdot 9} = -\frac{2}{9}\end{aligned}$$

- The signs are different. The product is negative. Multiply the numerators and multiply the denominators.
- Write the answer in simplest form.

In the last problem, the fraction was written in simplest form by dividing the numerator and denominator by 20, which is the largest integer that divides evenly into both 40 and 180. The number 20 is called the **greatest common factor (GCF)** of 40 and 180. To write a fraction in simplest form, divide the numerator and denominator by the GCF. If you have difficulty finding the GCF, try finding the prime factorization of the numerator and the denominator and then divide by the common prime factors. For instance,

$$-\frac{5}{12} \cdot \frac{8}{15} = -\frac{\overset{1}{\cancel{5}} \cdot (\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2)}{(\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3) \cdot (3 \cdot \overset{1}{\cancel{5}})} = -\frac{2}{9}$$

Division of Fractions

To divide two fractions, multiply by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

HOW TO • 12 Divide: $\frac{3}{8} \div \frac{9}{16}$

$$\frac{3}{8} \div \frac{9}{16} = \frac{3}{8} \cdot \frac{16}{9} = \frac{48}{72} = \frac{2}{3}$$

- Multiply by the reciprocal of the divisor. Write the answer in simplest form.

EXAMPLE • 4

Simplify: $-\frac{3}{8} - \frac{5}{12} - \left(-\frac{9}{16}\right)$

Solution

$$\begin{aligned} -\frac{3}{8} - \frac{5}{12} - \left(-\frac{9}{16}\right) &= \frac{-3}{8} + \frac{-5}{12} + \frac{9}{16} \\ &= \frac{-3}{8} \cdot \frac{6}{6} + \frac{-5}{12} \cdot \frac{4}{4} + \frac{9}{16} \cdot \frac{3}{3} \\ &= \frac{-18 + (-20) + 27}{48} \\ &= \frac{-11}{48} = -\frac{11}{48} \end{aligned}$$

YOU TRY IT • 4

Simplify: $\frac{5}{6} - \frac{3}{8} - \frac{7}{9}$

Your solution**EXAMPLE • 5**

Simplify: $\left(-\frac{5}{12}\right)\left(\frac{4}{25}\right)$

Solution

$$\left(-\frac{5}{12}\right)\left(\frac{4}{25}\right) = -\frac{5 \cdot 4}{12 \cdot 25} = -\frac{1}{15}$$

YOU TRY IT • 5

Simplify: $\frac{5}{8} \div \left(-\frac{15}{40}\right)$

Your solution

Solutions on p. S2

OBJECTIVE C**To evaluate exponential expressions****Integrating Technology**

The caret key $\wedge$ on a graphing calculator is used to enter an exponent. For example, to evaluate the expression at the right, press 2 $\wedge$ 6 ENTER . The display reads 64.

Repeated multiplication of the same factor can be written using an exponent.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 \leftarrow \begin{array}{l} \text{Exponent} \\ \uparrow \\ \text{Base} \end{array}$$

$$b \cdot b \cdot b \cdot b \cdot b = b^5 \leftarrow \begin{array}{l} \text{Exponent} \\ \uparrow \\ \text{Base} \end{array}$$

The **exponent** indicates how many times the factor, called the **base**, occurs in the multiplication. The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in **factored form**. The exponential expression 2^6 is in **exponential form**. The exponent is also said to indicate the **power** of the base.

2^1 is read “the first power of two” or just “two.”
 2^2 is read “the second power of two” or “two squared.”
 2^3 is read “the third power of two” or “two cubed.”
 2^4 is read “the fourth power of two.”
 2^5 is read “the fifth power of two.”
 b^5 is read “the fifth power of b .”

- Usually the exponent 1 is not written.

n*th Power of *a

If a is a real number and n is a positive integer, the n th power of a is the product of n factors of a .

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{a \text{ as a factor } n \text{ times}}$$

 Take Note

Examine the results of $(-3)^4$ and -3^4 very carefully. As another example, $(-2)^6 = 64$ but $-2^6 = -64$.

$$5^3 = 5 \cdot 5 \cdot 5 = 125$$

$$(-3)^4 = (-3)(-3)(-3)(-3) = 81$$

$$-3^4 = -(3)^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$$

Note the difference between $(-3)^4$ and -3^4 . The placement of the parentheses is very important.

EXAMPLE 6

Evaluate $(-2)^5$ and -5^2 .

Solution

$$\begin{aligned} (-2)^5 &= (-2)(-2)(-2)(-2)(-2) = -32 \\ -5^2 &= -(5 \cdot 5) = -25 \end{aligned}$$

YOU TRY IT 6

Evaluate -2^5 and $(-5)^2$.

Your solution**EXAMPLE 7**

Evaluate $\left(-\frac{3}{4}\right)^3$.

Solution

$$\begin{aligned} \left(-\frac{3}{4}\right)^3 &= \left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) \\ &= -\frac{3 \cdot 3 \cdot 3}{4 \cdot 4 \cdot 4} = -\frac{27}{64} \end{aligned}$$

YOU TRY IT 7

Evaluate $\left(\frac{2}{3}\right)^4$.

Your solution

Solutions on p. S2

OBJECTIVE D**To use the Order of Operations Agreement**

Suppose we wish to evaluate $16 + 4 \cdot 2$. There are two operations, addition and multiplication. The operations could be performed in different orders.

Multiply first. $16 + \underbrace{4 \cdot 2}_{24}$

Then add. $\underbrace{16 + 8}_{24}$

Add first. $\underbrace{16 + 4 \cdot 2}_{20}$

Then multiply. $\underbrace{20 \cdot 2}_{40}$

Note that the answers are different. To avoid possibly getting more than one answer to the same problem, an *Order of Operations Agreement* is followed.

 Integrating Technology

A graphing calculator uses the Order of Operations Agreement. Press 16 $\div$ 4

$\times$ 2 $\div$ ENTER. The display reads 24.

Order of Operations Agreement

- Step 1.** Perform operations inside grouping symbols. **Grouping symbols** include parentheses (), brackets [], braces { }, the absolute value symbol, and the fraction bar.
- Step 2.** Simplify exponential expressions.
- Step 3.** Do multiplication and division as they occur from left to right.
- Step 4.** Do addition and subtraction as they occur from left to right.

**Tips for Success**

The HOW TO feature indicates an example with explanatory remarks. Using paper and pencil, you should work through the example. See *AIM for Success* in the Preface.

HOW TO • 13Simplify: $8 - \frac{2 - 22}{4 + 1} \cdot 2^2$

$$\begin{aligned} 8 - \frac{2 - 22}{4 + 1} \cdot 2^2 &= 8 - \frac{-20}{5} \cdot 2^2 \\ &= 8 - \frac{-20}{5} \cdot 4 \\ &= 8 - (-4) \cdot 4 \\ &= 8 - (-16) \\ &= 24 \end{aligned}$$

- The fraction bar is a grouping symbol. Perform the operations above and below the fraction bar.
- Simplify exponential expressions.
- Do multiplication and division as they occur from left to right.
- Do addition and subtraction as they occur from left to right.

One or more of the steps in the Order of Operations Agreement may not be needed. In that case, just proceed to the next step.

HOW TO • 14Simplify: $14 - [(25 - 9) \div 2]^2$

$$\begin{aligned} 14 - [(25 - 9) \div 2]^2 &= 14 - [16 \div 2]^2 \\ &= 14 - [8]^2 \\ &= 14 - 64 \\ &= -50 \end{aligned}$$

- Perform the operations inside grouping symbols.
- Simplify exponential expressions.
- Do addition and subtraction as they occur from left to right.

A **complex fraction** is a fraction in which the numerator or denominator contains one or more fractions. The fraction bar that is placed between the numerator and denominator of a complex fraction is called the **main fraction bar**. Examples of complex fractions are shown at the right.

$$\frac{\frac{2}{3}}{-\frac{5}{2}}, \frac{\frac{1}{7} + 5}{\frac{3}{4} - \frac{7}{8}} \leftarrow \text{Main fraction bar}$$

When simplifying complex fractions, recall that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

HOW TO • 15Simplify: $\frac{\frac{3}{4} - \frac{1}{3}}{\frac{1}{5} - 2}$

$$\begin{aligned} \frac{\frac{3}{4} - \frac{1}{3}}{\frac{1}{5} - 2} &= \frac{\frac{9}{12} - \frac{4}{12}}{\frac{1}{5} - \frac{10}{5}} = \frac{\frac{5}{12}}{-\frac{9}{5}} \\ &= \frac{5}{12} \left(-\frac{5}{9} \right) \\ &= -\frac{25}{108} \end{aligned}$$

- Perform the operations above and below the main fraction bar.
- Multiply the numerator of the complex fraction by the reciprocal of the denominator of the complex fraction.

EXAMPLE • 8Simplify: $-3 \cdot 4^2 + [2 - 5(8 \div 2)]$ **Solution**

$$\begin{aligned}
 & -3 \cdot 4^2 + [2 - 5(8 \div 2)] \\
 & = -3 \cdot 4^2 + [2 - 5(4)] && \bullet \text{ Inside parentheses} \\
 & = -3 \cdot 4^2 + [2 - 20] && \bullet \text{ Inside brackets} \\
 & = -3 \cdot 4^2 + [-18] \\
 & = -3 \cdot 16 + [-18] && \bullet \text{ Exponents} \\
 & = -48 + [-18] && \bullet \text{ Multiplication} \\
 & = -66 && \bullet \text{ Addition}
 \end{aligned}$$

YOU TRY IT • 8Simplify: $-2 \left[3 - 3^3 + \frac{16 + 2}{1 - 10} \right]$ **Your solution****EXAMPLE • 9**Simplify: $\left(\frac{1}{2}\right)^3 - \left[\left(\frac{2}{3} + \frac{1}{4}\right) \div \frac{5}{6}\right]$ **Solution**

$$\begin{aligned}
 & \left(\frac{1}{2}\right)^3 - \left[\left(\frac{2}{3} + \frac{1}{4}\right) \div \frac{5}{6}\right] \\
 & = \left(\frac{1}{2}\right)^3 - \left[\frac{11}{12} \div \frac{5}{6}\right] && \bullet \text{ Inside parentheses} \\
 & = \left(\frac{1}{2}\right)^3 - \left[\frac{11}{12} \cdot \frac{6}{5}\right] && \bullet \text{ Inside brackets} \\
 & = \left(\frac{1}{2}\right)^3 - \frac{11}{10} \\
 & = \frac{1}{8} - \frac{11}{10} && \bullet \text{ Exponents} \\
 & = -\frac{39}{40} && \bullet \text{ Subtraction}
 \end{aligned}$$

YOU TRY IT • 9Simplify: $\frac{1}{3} + \frac{5}{8} \div \frac{15}{16} - \frac{7}{12}$ **Your solution****EXAMPLE • 10**Simplify: $9 \cdot \frac{\frac{5}{6} - 2}{\frac{3}{8}} \div \frac{7}{6}$ **Solution**

$$\begin{aligned}
 9 \cdot \frac{\frac{5}{6} - 2}{\frac{3}{8}} \div \frac{7}{6} &= 9 \cdot \frac{-\frac{7}{6}}{\frac{3}{8}} \div \frac{7}{6} \\
 &= 9 \cdot \left(-\frac{7}{6} \cdot \frac{8}{3}\right) \div \frac{7}{6} \\
 &= 9 \cdot \left(-\frac{28}{9}\right) \div \frac{7}{6} \\
 &= -28 \div \frac{7}{6} && \bullet \text{ Multiplication} \\
 &= -28 \cdot \frac{6}{7} = -24 && \bullet \text{ Division}
 \end{aligned}$$

YOU TRY IT • 10Simplify: $\frac{11}{12} - \frac{\frac{5}{4}}{2 - \frac{7}{2}} \cdot \frac{3}{4}$ **Your solution**

1.2 EXERCISES

OBJECTIVE A

To add, subtract, multiply, and divide integers



1. a. Explain how to add two integers with the same sign. b. Explain how to add two integers with different signs.



2. Explain how to rewrite $8 - (-12)$ as addition of the opposite.

For Exercises 3 to 38, simplify.

3. $-18 + (-12)$

4. $-18 - 7$

5. $5 - 22$

6. $16(-60)$

7. $3 \cdot 4 \cdot (-8)$

8. $18 \cdot 0 \cdot (-7)$

9. $18 \div (-3)$

10. $25 \div (-5)$

11. $-60 \div (-12)$

12. $(-9)(-2)(-3)(10)$

13. $-20(35)(-16)$

14. $54(19)(-82)$

15. $8 - (-12)$

16. $-6 - (-3)$

17. $|12(-8)|$

18. $|7 - 18|$

19. $|15 - (-8)|$

20. $|-16 - (-20)|$

21. $|-56 \div 8|$

22. $|81 \div (-9)|$

23. $|-153 \div (-9)|$

24. $|-4| - |-2|$

25. $-|-8| + |-4|$

26. $-|-16| - |24|$

27. $-30 + (-16) - 14 - 2$

28. $3 - (-2) + (-8) - 11$

29. $-2 + (-19) - 16 + 12$

30. $-6 + (-9) - 18 + 32$

31. $13 - |6 - 12|$

32. $-9 - |-7 - (-15)|$

33. $738 - 46 + (-105) - 219$



34. $-871 - (-387) - 132 - 46$



35. $-442 \div (-17)$



36. $621 \div (-23)$



37. $-4897 \div 59$

38. $-17|-5|$



39. What is the sign of the product of an odd number of negative factors?



40. What is the sign of the product of an even number of negative factors?

OBJECTIVE B**To add, subtract, multiply, and divide rational numbers**

- 41. a.** Describe the least common multiple of two numbers. **b.** Describe the greatest common factor of two numbers.



- 42.** Explain how to divide two fractions.

For Exercises 43 to 70, simplify.

43. $\frac{7}{12} - \left(-\frac{5}{16}\right)$

44. $\frac{3}{8} - \frac{5}{12}$

45. $-\frac{5}{9} - \frac{14}{15}$

46. $\frac{1}{2} + \frac{1}{7} - \frac{5}{8}$

47. $-\frac{1}{3} + \frac{5}{9} - \frac{7}{12}$

48. $\frac{1}{3} + \frac{19}{24} - \frac{7}{8}$

49. $\frac{2}{3} - \frac{5}{12} + \frac{5}{24}$

50. $-\frac{7}{10} + \frac{4}{5} + \frac{5}{6}$

51. $\frac{5}{8} - \frac{7}{12} + \frac{1}{2}$

52. $-\frac{1}{3} \cdot \frac{5}{8}$

53. $\left(\frac{6}{35}\right)\left(-\frac{5}{16}\right)$

54. $\frac{2}{3}\left(-\frac{9}{20}\right) \cdot \frac{5}{12}$

55. $-\frac{8}{15} \div \frac{4}{5}$

56. $-\frac{2}{3} \div \left(-\frac{6}{7}\right)$

57. $-\frac{11}{24} \div \frac{7}{12}$

58. $\frac{7}{9} \div \left(-\frac{14}{27}\right)$

59. $\left(-\frac{5}{12}\right)\left(\frac{4}{35}\right)\left(\frac{7}{8}\right)$

60. $\frac{6}{35}\left(-\frac{7}{40}\right)\left(-\frac{8}{21}\right)$

61. $-14.27 + 1.296$

62. $-0.4355 + 172.5$

63. $1.832 - 7.84$

64. $-3.52 - (-4.7)$

65. $(0.03)(10.5)(6.1)$

66. $(1.2)(3.1)(-6.4)$

67. $5.418 \div (-0.9)$

68. $-0.2645 \div (-0.023)$

69. $-0.4355 \div 0.065$

70. $-6.58 - 3.97 + 0.875$



- 71.** If the product of six numbers is negative, how many of those numbers could be negative?



- 72.** If the product of seven numbers is positive, how many of those numbers could be negative?

For Exercises 73 to 76, simplify. Round to the nearest hundredth.



73. $38.241 \div [-(-6.027)] - 7.453$



74. $-9.0508 - (-3.177) + 24.77$



75. $-287.3069 \div 0.1415$



76. $6472.3018 \div (-3.59)$

OBJECTIVE C**To evaluate exponential expressions**

For Exercises 77 to 96, simplify.

77. 5^3

78. 3^4

79. -2^3

80. -4^3

81. $(-5)^3$

82. $(-8)^2$

83. $2^2 \cdot 3^4$

84. $4^2 \cdot 3^3$

85. $-2^2 \cdot 3^2$

86. $-3^2 \cdot 5^3$

87. $(-2)^3(-3)^2$

88. $(-4)^3(-2)^3$

89. $2^3 \cdot 3^3$

90. $(-3)^2(4^2)$

91. $-2^2(-2)^2$

92. $(-2)^2(-5)^3$

93. $\left(-\frac{2}{3}\right)^2 \cdot 3^3$

94. $-\left(\frac{2}{5}\right)^3 \cdot 5^2$



95. $2^5(-3)^4 \cdot 4^5$



96. $4^4(-3)^5(6)^2$



For Exercises 97 to 100, without finding the product, state whether the given expression simplifies to a positive or a negative number.

97. $(-9)^7$

98. -8^6

99. $(-9^{10})(-5^4)$

100. $-(3^4)(-2^5)$

OBJECTIVE D**To use the Order of Operations Agreement**

101. Why do we need an Order of Operations Agreement?



102. Describe each step in the Order of Operations Agreement.

For Exercises 103 to 126, simplify.

103. $5 - 3(8 \div 4)^2$

104. $4^2 - (5 - 2)^2 \cdot 3$

105. $16 - \frac{2^2 - 5}{3^2 + 2}$

106. $\frac{4(5 - 2)}{4^2 - 2^2} \div 4$

107. $\frac{3 + \frac{2}{3}}{\frac{11}{16}}$

108. $\frac{\frac{11}{14}}{4 - \frac{6}{7}}$

109. $5[(2 - 4) \cdot 3 - 2]$

110. $2[(16 \div 8) - (-2)] + 4$

111. $16 - 4\left(\frac{8 - 2}{3 - 6}\right) \div \frac{1}{2}$

112. $25 \div 5\left(\frac{16 + 8}{-2^2 + 8}\right) - 5$

113. $6[3 - (-4 + 2) \div 2]$

114. $12 - 4[2 - (-3 + 5) - 8]$

$$115. \frac{1}{2} - \left(\frac{2}{3} \div \frac{5}{9} \right) + \frac{5}{6} \qquad 116. \left(-\frac{3}{5} \right)^2 - \frac{3}{5} \cdot \frac{5}{9} + \frac{7}{10} \qquad 117. \frac{1}{2} - \frac{\frac{17}{25}}{4 - \frac{3}{5}} \div \frac{1}{5}$$

$$118. \frac{3}{4} + \frac{3 - \frac{7}{9}}{\frac{5}{6}} \cdot \frac{2}{3} \qquad 119. \frac{2}{3} - \left(\frac{3}{8} + \frac{5}{6} \right) \div \frac{3}{5} \qquad 120. \frac{3}{4} \div \left(\frac{5}{8} - \frac{5}{12} \right) + 2$$



$$121. 0.4(1.2 - 2.3)^2 + 5.8$$



$$122. 5.4 - (0.3)^2 \div 0.09$$



$$123. 1.75 \div 0.25 - (1.25)^2$$



$$124. (3.5 - 4.2)^2 - 3.50 \div 2.5$$



$$125. 27.2322 \div (6.96 - 3.27)^2$$



$$126. (3.09 - 4.77)^3 - 4.07 \cdot 3.66$$



127. Which expression is equivalent to $8^2 - 2^2(5 - 3)^3$?
 (i) $6^2(2)^3$ (ii) $64 - 4(8)$ (iii) $60(2)^3$ (iv) $64 - 8^3$



128. Which expression is equivalent to $32 + 32 \div 4 - 2^3$?
 (i) $64 \div 4 - 8$ (ii) $32 + 32 \div (-4)$ (iii) $32 + 8 - 8$ (iv) $32 + 32 \div 8$

Applying the Concepts

129. A number that is its own additive inverse is _____.

130. Which two numbers are their own multiplicative inverse?

131. Do all real numbers have a multiplicative inverse? If not, which ones do not have a multiplicative inverse?

132. What is the tens digit of 11^{22} ?

133. What is the ones digit of 7^{18} ?

134. What are the last two digits of 5^{33} ?

135. What are the last three digits of 5^{234} ?

136. **a.** Does $(2^3)^4 = 2^{(3^4)}$? **b.** If not, which expression is larger?

137. What is the Order of Operations Agreement for a^{b^c} ? *Note:* Even calculators that normally follow the Order of Operations Agreement may not do so for this expression.

SECTION

1.3

Variable Expressions

OBJECTIVE A

To use and identify the properties of the real numbers



The properties of the real numbers describe the way operations on numbers can be performed. Following is a list of some of the real-number properties and an example of each property.

Properties of the Real Numbers**The Commutative Property of Addition**

$$a + b = b + a$$

$$\begin{aligned} 3 + 2 &= 2 + 3 \\ 5 &= 5 \end{aligned}$$

The Commutative Property of Multiplication

$$a \cdot b = b \cdot a$$

$$\begin{aligned} (3)(-2) &= (-2)(3) \\ -6 &= -6 \end{aligned}$$

The Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

$$\begin{aligned} (3 + 4) + 5 &= 3 + (4 + 5) \\ 7 + 5 &= 3 + 9 \\ 12 &= 12 \end{aligned}$$

The Associative Property of Multiplication

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

$$\begin{aligned} (3 \cdot 4) \cdot 5 &= 3 \cdot (4 \cdot 5) \\ 12 \cdot 5 &= 3 \cdot 20 \\ 60 &= 60 \end{aligned}$$

The Addition Property of Zero

$$a + 0 = 0 + a = a$$

$$3 + 0 = 0 + 3 = 3$$

The Multiplication Property of Zero

$$a \cdot 0 = 0 \cdot a = 0$$

$$3 \cdot 0 = 0 \cdot 3 = 0$$

The Multiplication Property of One

$$a \cdot 1 = 1 \cdot a = a$$

$$5 \cdot 1 = 1 \cdot 5 = 5$$

The Inverse Property of Addition

$$a + (-a) = (-a) + a = 0$$

$$4 + (-4) = (-4) + 4 = 0$$

$-a$ is called the *additive inverse* of a . a is the additive inverse of $-a$. The sum of a number and its additive inverse is 0.

The Inverse Property of Multiplication

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1, \quad a \neq 0$$

$$(4)\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)(4) = 1$$

$\frac{1}{a}$ is called the *multiplicative inverse* of a . $\frac{1}{a}$ is also called the *reciprocal* of a . The product of a number and its multiplicative inverse is 1.

The Distributive Property

$$a(b + c) = ab + ac$$

$$\begin{aligned} 3(4 + 5) &= 3 \cdot 4 + 3 \cdot 5 \\ 3 \cdot 9 &= 12 + 15 \\ 27 &= 27 \end{aligned}$$

EXAMPLE • 1

Complete the statement by using the Commutative Property of Multiplication.

$$(x)\left(\frac{1}{4}\right) = (?) (x)$$

Solution

$$(x)\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)(x)$$

YOU TRY IT • 1

Complete the statement by using the Inverse Property of Addition.

$$3x + ? = 0$$

Your solution**EXAMPLE • 2**

Identify the property that justifies the statement:
 $3(x + 4) = 3x + 12$

Solution

The Distributive Property

YOU TRY IT • 2

Identify the property that justifies the statement:
 $(a + 3b) + c = a + (3b + c)$

Your solution

OBJECTIVE B**To evaluate a variable expression**

An expression that contains one or more variables is a **variable expression**. The variable expression $6x^2y - 7x - z + 2$ contains four **terms**: $6x^2y$, $-7x$, $-z$, and 2 . The first three terms are **variable terms**. The 2 is a **constant term**.

Each variable term is composed of a **numerical coefficient** and a **variable part**.

Variable Term	Numerical Coefficient	Variable Part
$6x^2y$	6	x^2y
$-7x$	-7	x
$-z$	-1	z

- When the coefficient is 1 or -1, the 1 is usually not written.

Replacing the variables in a variable expression by a numerical value and then simplifying the resulting expression is called **evaluating the variable expression**.

**Integrating Technology**

See the Keystroke Guide: *Evaluating Variable Expressions* for instructions on using a graphing calculator to evaluate variable expressions.

HOW TO • 1

Evaluate $a^2 - (a - b^2c)$ when $a = -2$, $b = 3$, and $c = -4$.

$$a^2 - (a - b^2c)$$

$$\begin{aligned} (-2)^2 - [(-2) - 3^2(-4)] &= (-2)^2 - [(-2) - 9(-4)] \\ &= (-2)^2 - [(-2) - (-36)] \\ &= (-2)^2 - [34] \\ &= 4 - 34 \\ &= -30 \end{aligned}$$

- Replace each variable with its value: $a = -2$, $b = 3$, $c = -4$. Use the Order of Operations Agreement to simplify the resulting numerical expression.

EXAMPLE • 3

Evaluate $2x^3 - 4(2y - 3z)$ when $x = 2$, $y = 3$, and $z = -2$.

Solution

$$\begin{aligned} 2x^3 - 4(2y - 3z) &= 2(2)^3 - 4[2(3) - 3(-2)] \\ &= 2(2)^3 - 4[6 - (-6)] \\ &= 2(8) - 4[12] \\ &= 16 - 48 = -32 \end{aligned}$$

YOU TRY IT • 3

Evaluate $-2x^2 + 3(4xy - z)$ when $x = -3$, $y = -1$, and $z = 2$.

Your solution**EXAMPLE • 4**

Evaluate $3 - 2|3x - 2y^2|$ when $x = -1$ and $y = -2$.

Solution

$$\begin{aligned} 3 - 2|3x - 2y^2| &= 3 - 2|3(-1) - 2(-2)^2| \\ &= 3 - 2|3(-1) - 2(4)| \\ &= 3 - 2|(-3) - 8| \\ &= 3 - 2|-11| = 3 - 2(11) \\ &= 3 - 22 = -19 \end{aligned}$$

YOU TRY IT • 4

Evaluate $2x - y|4x^2 - y^2|$ when $x = -2$ and $y = -6$.

Your solution

OBJECTIVE C**To simplify a variable expression**

Like terms of a variable expression are terms with the same variable part.

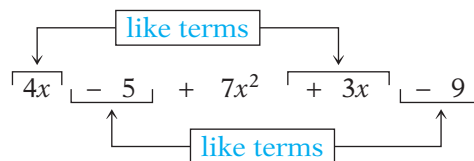
Constant terms are like terms.

To simplify a variable expression, combine the like terms by using the Distributive Property. For instance,

$$7x + 4x = (7 + 4)x = 11x$$

Adding the coefficients of like terms is called **combining like terms**.

The Distributive Property is also used to remove parentheses from a variable expression so that like terms can be combined.

**Tips for Success**

One of the key instructional features of this text is the Example/You Try It pairs. Each Example is completely worked. You are to solve the You Try It problems. When you are ready, check your solution against the one given in the Solution section. The solutions for You Try Its 5 and 6 below are given on page S2 (see the reference at the bottom right of the You Try It box). See *AIM for Success* in the Preface.

HOW TO • 2

Simplify: $3(x + 2y) + 2(4x - 3y)$

$$\begin{aligned} 3(x + 2y) + 2(4x - 3y) \\ &= 3x + 6y + 8x - 6y \\ &= (3x + 8x) + (6y - 6y) \\ &= 11x \end{aligned}$$

- Use the Distributive Property.
- Use the Commutative and Associative Properties of Addition to rearrange and group like terms.
- Combine like terms.

HOW TO • 3

Simplify: $2 - 4[3x - 2(5x - 3)]$

$$\begin{aligned} 2 - 4[3x - 2(5x - 3)] &= 2 - 4[3x - 10x + 6] \\ &= 2 - 4[-7x + 6] \\ &= 2 + 28x - 24 \\ &= 28x - 22 \end{aligned}$$

- Use the Distributive Property to remove the inner parentheses.
- Combine like terms.
- Use the Distributive Property to remove the brackets.
- Combine like terms.

EXAMPLE • 5

Simplify: $7 + 3(4x - 7)$

Solution

$$\begin{aligned} 7 + 3(4x - 7) \\ &= 7 + 12x - 21 \\ &= 12x - 14 \end{aligned}$$

- The Order of Operations Agreement requires multiplication before addition.

YOU TRY IT • 5

Simplify: $9 - 2(3y - 4) + 5y$

Your solution**EXAMPLE • 6**

Simplify: $5 - 2(3x + y) - (x - 4)$

Solution

$$\begin{aligned} 5 - 2(3x + y) - (x - 4) \\ &= 5 - 6x - 2y - x + 4 \\ &= -7x - 2y + 9 \end{aligned}$$

- Distributive Property
- Combine like terms.

YOU TRY IT • 6

Simplify: $6z - 3(5 - 3z) + 5(2z - 3)$

Your solution

1.3 EXERCISES**OBJECTIVE A****To use and identify the properties of the real numbers**

For Exercises 1 to 14, use the given property of the real numbers to complete the statement.

1. The Commutative Property of Multiplication $3 \cdot 4 = 4 \cdot ?$	2. The Commutative Property of Addition $7 + 15 = ? + 7$
3. The Associative Property of Addition $(3 + 4) + 5 = ? + (4 + 5)$	4. The Associative Property of Multiplication $(3 \cdot 4) \cdot 5 = 3 \cdot (? \cdot 5)$
5. The Division Property of Zero $\frac{5}{?}$ is undefined.	6. The Multiplication Property of Zero $5 \cdot ? = 0$
7. The Distributive Property $3(x + 2) = 3x + ?$	8. The Distributive Property $5(y + 4) = ? \cdot y + 20$
9. The Division Property of Zero $\frac{?}{-6} = 0$	10. The Inverse Property of Addition $(x + y) + ? = 0$
11. The Inverse Property of Multiplication $\frac{1}{mn}(mn) = ?$	12. The Multiplication Property of One $? \cdot 1 = x$
13. The Associative Property of Multiplication $2(3x) = ? \cdot x$	14. The Commutative Property of Addition $ab + bc = bc + ?$

For Exercises 15 to 26, identify the property that justifies the statement.

15. $\frac{0}{-5} = 0$	16. $-8 + 8 = 0$
17. $(-12)\left(-\frac{1}{12}\right) = 1$	18. $(3 \cdot 4) \cdot 2 = 2 \cdot (3 \cdot 4)$
19. $y + 0 = y$	20. $2x + (5y + 8) = (2x + 5y) + 8$
21. $\frac{-9}{0}$ is undefined.	22. $(x + y)z = xz + yz$
23. $6(x + y) = 6x + 6y$	24. $(-12y)(0) = 0$
25. $(ab)c = a(bc)$	26. $(x + y) + z = (y + x) + z$



27. The sum of a positive number n and its additive inverse is multiplied by the reciprocal of the number n . What is the result?



28. The product of a negative number n and its reciprocal is multiplied by the number n . What is the result?

OBJECTIVE B**To evaluate a variable expression**

For Exercises 29 to 58, evaluate the variable expression when $a = 2$, $b = 3$, $c = -1$, and $d = -4$.

29. $ab + dc$

30. $2ab - 3dc$

31. $4cd \div a^2$

32. $b^2 - (d - c)^2$

33. $(b - 2a)^2 + c$

34. $(b - d)^2 \div (b - d)$

35. $(bc + a)^2 \div (d - b)$

36. $\frac{1}{3}b^3 - \frac{1}{4}d^3$

37. $\frac{1}{4}a^4 - \frac{1}{6}bc$

38. $2b^2 \div \frac{ad}{2}$

39. $\frac{3ac}{-4} - c^2$

40. $\frac{2d - 2a}{2bc}$

41. $\frac{3b - 5c}{3a - c}$

42. $\frac{2d - a}{b - 2c}$

43. $\frac{a - d}{b + c}$

44. $|a^2 + d|$

45. $-a|a + 2d|$

46. $d|b - 2d|$

47. $\frac{2a - 4d}{3b - c}$

48. $\frac{3d - b}{b - 2c}$

49. $-3d \div \left| \frac{ab - 4c}{2b + c} \right|$

50. $-2bc + \left| \frac{bc + d}{ab - c} \right|$

51. $2(d - b) \div (3a - c)$

52. $(d - 4a)^2 \div c^3$

53. $-d^2 - c^3a$

54. $a^2c - d^3$

55. $-d^3 + 4ac$

56. b^a

57. $4^{(a^2)}$

58. a^b



For Exercises 59 and 60, determine whether the expression $\frac{abc}{b-a}$ is positive or negative for the given conditions on a , b , and c .

59. $a = -38, b = -52, c > 0$

60. $a > 20, b = 18, c < 0$

OBJECTIVE C**To simplify a variable expression**

For Exercises 61 to 94, simplify.

61. $5x + 7x$

62. $3x + 10x$

63. $3x - 5x + 9x$

64. $-2x + 5x - 7x$

65. $5b - 8a - 12b$

66. $-2a + 7b + 9a$

67. $\frac{1}{3}(3y)$

68. $12\left(\frac{1}{12}x\right)$

69. $-\frac{2}{5}\left(-\frac{5}{2}z\right)$

70. $5(x + 2)$

71. $3(a - 5)$

72. $-3(x - 2)$

73. $-5(x - 9)$

74. $-(-x - y)$

75. $-(x + y)$

76. $3(x - 2y) - 5$

77. $4x - 3(2y - 5)$

78. $3x + 8(3x + 5)$

79. $25x + 10(9 - x)$

80. $2x - 3(x - 2y)$

81. $3[x - 2(x + 2y)]$

82. $5[-2 - 6(a - 5)]$

83. $3[a - 5(5 - 3a)]$

84. $5[y - 3(y - 2x)]$

85. $-2(x - 3y) + 2(3y - 5x)$

86. $4(-a - 2b) - 2(3a - 5b)$

87. $5(3a - 2b) - 3(-6a + 5b)$

88. $-7(2a - b) + 2(-3b + a)$

89. $3x - 2[y - 2(x + 3[2x + 3y])]$

90. $2x - 4[x - 4(y - 2[5y + 3])]$

91. $4 - 2(7x - 2y) - 3(-2x + 3y)$

92. $3x + 8(x - 4) - 3(2x - y)$

93. $\frac{1}{3}[8x - 2(x - 12) + 3]$

94. $\frac{1}{4}[14x - 3(x - 8) - 7x]$



95. State whether the given coefficient or constant will be positive, negative, or zero after the expression $31a - 102b + 73 - 88a + 256b - 73$ is simplified.

- a. the coefficient of a b. the coefficient of b c. the constant term



96. State whether the given expression is equivalent to $3[5 - 2(y - 6)]$.

- a. $3[3(y - 6)]$ b. $15 - 6(y - 6)$

Applying the Concepts

Exercises 97 to 100 show some expressions that you will encounter in subsequent chapters in the text. Simplify each expression.

97. $0.052x + 0.072(x + 1000)$

98. $0.07x + 0.08(10,000 - x)$

99. $\frac{t}{20} + \frac{t}{30}$

100. $\frac{t}{4} + \frac{t}{5}$

SECTION

1.4

Verbal Expressions and Variable Expressions

OBJECTIVE A

To translate a verbal expression into a variable expression 

Point of Interest

Mathematical symbolism, as shown on this page, has advanced through various stages: rhetorical, syncopical, and modern. In the rhetorical stage, all mathematical description was through words. In the syncopical stage, there was a combination of words and symbols. For instance, x plano 4 in y meant $4xy$. The modern stage, which is used today, began in the 17th century. Modern symbolism is still changing. For example, there are advocates of a system of symbolism that would place all operations last. Using this notation, 4 plus 7 would be written $4\ 7\ +$; 6 divided by 4 would be $6\ 4\ \div$.

One of the major skills required in applied mathematics is the ability to translate a verbal expression into a mathematical expression. Doing so requires recognizing the verbal phrases that translate into mathematical operations. Following is a partial list of the verbal phrases used to indicate the different mathematical operations.

Addition	more than	8 more than w	$w + 8$
	added to	x added to 9	$9 + x$
	the sum of	the sum of z and 9	$z + 9$
	the total of	the total of r and s	$r + s$
	increased by	x increased by 7	$x + 7$
Subtraction	less than	12 less than b	$b - 12$
	the difference between	the difference between x and 1	$x - 1$
	minus	z minus 7	$z - 7$
	decreased by	17 decreased by a	$17 - a$
Multiplication	times	negative 2 times c	$-2c$
	the product of	the product of x and y	xy
	multiplied by	3 multiplied by n	$3n$
	of	three-fourths of m	$\frac{3}{4}m$
	twice	twice d	$2d$
Division	divided by	v divided by 15	$\frac{v}{15}$
	the quotient of	the quotient of y and 3	$\frac{y}{3}$
	the ratio of	the ratio of x to 7	$\frac{x}{7}$
Power	the square of or the second power of	the square of x	x^2
	the cube of or the third power of	the cube of r	r^3
	the fifth power of	the fifth power of a	a^5

Translating a phrase that contains the word *sum*, *difference*, *product*, or *quotient* can be difficult. In the examples at the right, note where the operation symbol is placed.

the <i>sum</i> of x and y	$x + y$
the <i>difference</i> between x and y	$x - y$
the <i>product</i> of x and y	$x \cdot y$
the <i>quotient</i> of x and y	$\frac{x}{y}$

HOW TO • 1

Translate “three times the sum of c and five” into a variable expression.

Identify the words that indicate the mathematical operations.

3 times the sum of c and 5

Use the identified words to write the variable expression. Note that the phrase times the sum of requires parentheses.

$3(c + 5)$

HOW TO • 2

The sum of two numbers is thirty-seven. If x represents the smaller number, translate “twice the larger number” into a variable expression.

Write an expression for the larger number by subtracting the smaller number, x , from 37.

larger number: $37 - x$

Identify the words that indicate the mathematical operations.

twice the larger number

Use the identified words to write a variable expression.

$2(37 - x)$

HOW TO • 3

Translate “five less than twice the difference between a number and seven” into a variable expression. Then simplify.

Identify the words that indicate the mathematical operations.

the unknown number: x

5 less than twice the difference between x and 7

Use the identified words to write the variable expression.

$2(x - 7) - 5$

Simplify the expression.

$2x - 14 - 5 = 2x - 19$

EXAMPLE • 1

Translate “the quotient of r and the sum of r and four” into a variable expression.

Solution

the quotient of r and the sum of r and four

$$\frac{r}{r + 4}$$

YOU TRY IT • 1

Translate “twice x divided by the difference between x and seven” into a variable expression.

Your solution**EXAMPLE • 2**

Translate “the sum of the square of y and six” into a variable expression.

Solution

the sum of the square of y and six

$$y^2 + 6$$

YOU TRY IT • 2

Translate “the product of negative three and the square of d ” into a variable expression.

Your solution**EXAMPLE • 3**

The sum of two numbers is twenty-eight. Using x to represent the smaller number, translate “the sum of three times the larger number and the smaller number” into a variable expression. Then simplify.

Solution

The smaller number is x .

The larger number is $28 - x$.

the sum of three times the larger number
and the smaller number

$$\begin{aligned} 3(28 - x) + x & \quad \bullet \text{ This is the variable expression.} \\ = 84 - 3x + x & \quad \bullet \text{ Simplify.} \\ = 84 - 2x \end{aligned}$$

YOU TRY IT • 3

The sum of two numbers is sixteen. Using x to represent the smaller number, translate “the difference between twice the smaller number and the larger number” into a variable expression. Then simplify.

Your solution**EXAMPLE • 4**

Translate “eight more than the product of four and the total of a number and twelve” into a variable expression. Then simplify.

Solution

Let the unknown number be x .

8 more than the product of 4 and the total of
 x and 12

$$\begin{aligned} 4(x + 12) + 8 & \quad \bullet \text{ This is the variable expression.} \\ = 4x + 48 + 8 & \quad \bullet \text{ Simplify.} \\ = 4x + 56 \end{aligned}$$

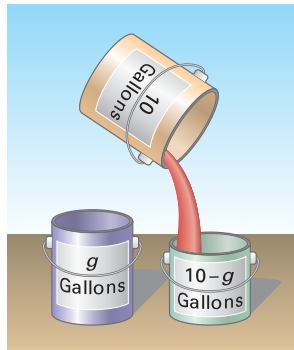
YOU TRY IT • 4

Translate “the difference between fourteen and the sum of a number and seven” into a variable expression. Then simplify.

Your solution

OBJECTIVE B**To solve application problems**

Many of the applications of mathematics require that you identify the unknown quantity, assign a variable to that quantity, and then attempt to express other unknowns in terms of that quantity.

**HOW TO • 4**

Ten gallons of paint were poured into two containers of different sizes. Express the amount of paint poured into the smaller container in terms of the amount poured into the larger container.

Assign a variable to the amount of paint poured into the larger container.

Express the amount of paint in the smaller container in terms of g .
(g gallons of paint were poured into the larger container.)

The number of gallons of paint poured into the larger container: g

The number of gallons of paint poured into the smaller container: $10 - g$

EXAMPLE • 5

A cyclist is riding at a rate that is twice the speed of a runner. Express the speed of the cyclist in terms of the speed of the runner.

Solution

The speed of the runner: r

The speed of the cyclist is twice r : $2r$

YOU TRY IT • 5

The length of the Carnival cruise ship *Destiny* is 56 ft more than the height of the Empire State Building. Express the length of the *Destiny* in terms of the height of the Empire State Building.

Your solution**EXAMPLE • 6**

The length of a rectangle is 2 ft more than three times the width. Express the length of the rectangle in terms of the width.

Solution

The width of the rectangle: w

The length is 2 more than 3 times w : $3w + 2$

YOU TRY IT • 6

The depth of the deep end of a swimming pool is 2 ft more than twice the depth of the shallow end. Express the depth of the deep end in terms of the depth of the shallow end.

Your solution**EXAMPLE • 7**

A chemist combined a 5% acid solution with a 7% acid solution to create 12 L of solution. If x represents the number of liters of the 5% solution, write an expression for the number of liters of the 7% solution.

Solution

Liters of 5% solution: x

Liters of 7% solution $12 - x$

YOU TRY IT • 7

A financial advisor suggested that a client split a \$5000 savings account between a mutual fund and a certificate of deposit. If x represents the amount the client placed in the mutual fund, write an expression for the amount placed in the certificate of deposit.

Your solution

1.4 EXERCISES**OBJECTIVE A****To translate a verbal expression into a variable expression**

For Exercises 1 to 6, translate into a variable expression.

1. eight less than a number
2. the product of negative six and a number
3. four-fifths of a number
4. the difference between a number and twenty
5. the quotient of a number and fourteen
6. a number increased by two hundred



For Exercises 7 and 8, state whether the given phrase translates into the given variable expression.

7. five subtracted from the product of the cube of eight and a number; $8n^3 - 5$
8. fifteen more than the sum of five times a number and two; $(5n + 2) + 15$

For Exercises 9 to 18, translate into a variable expression. Then simplify.

9. a number minus the sum of the number and two
10. a number decreased by the difference between five and the number
11. five times the product of eight and a number
12. a number increased by two-thirds of the number
13. the difference between seventeen times a number and twice the number
14. one-half of the total of six times a number and twenty-two
15. the difference between the square of a number and the total of twelve and the square of the number
16. eleven more than the square of a number added to the difference between the number and seventeen
17. the sum of five times a number and twelve added to the product of fifteen and the number
18. four less than twice the sum of a number and eleven
19. The sum of two numbers is fifteen. Using x to represent the smaller of the two numbers, translate “the sum of two more than the larger number and twice the smaller number” into a variable expression. Then simplify.
20. The sum of two numbers is twenty. Using x to represent the smaller of the two numbers, translate “the difference between two more than the larger number and twice the smaller number” into a variable expression. Then simplify.
21. The sum of two numbers is thirty-four. Using x to represent the larger of the two numbers, translate “the quotient of five times the smaller number and the difference between the larger number and three” into a variable expression.
22. The sum of two numbers is thirty-three. Using x to represent the larger of the two numbers, translate “the difference between six more than twice the larger number and the sum of the smaller number and three” into a variable expression. Then simplify.

OBJECTIVE B To solve application problems

-  **23. Global Warming** Use the information in the article at the right. Express the temperature of the Arctic Ocean in 2007 in terms of **a.** the historical average temperature and **b.** the historical maximum temperature.
-  **24. Online Advertising** eMarketer, a website that conducts market research about online business, predicts that in 2011 the dollars spent for online advertising will be twice the amount spent in 2007. Express the dollars spent for online advertising in 2011 in terms of the dollars spent for online advertising in 2007. (Source: www.emarketer.com)
-  **25. Astronomy** The distance from Earth to the sun is approximately 390 times the distance from Earth to the moon. Express the distance from Earth to the sun in terms of the distance from Earth to the moon.
-  **26. Construction** The longest rail tunnel, from Hanshu to Hokkaido, Japan, is 23.36 mi longer than the longest road tunnel, from Goschenen to Airo, Switzerland. Express the length of the longest rail tunnel in terms of the length of the longest road tunnel.
- 27. Investments** A financial advisor has invested \$10,000 in two accounts. If one account contains x dollars, express the amount in the second account in terms of x .
- 28. Recreation** A fishing line 3 ft long is cut into two pieces, one shorter than the other. Express the length of the shorter piece in terms of the length of the longer piece.
- 29. Geometry** The measure of angle A of a triangle is twice the measure of angle B . The measure of angle C is twice the measure of angle A . Write expressions for angle A and angle C in terms of angle B .
- 30. Carpentry** A 12-foot board is cut into two pieces of different lengths. Express the length of the longer piece in terms of the length of the shorter piece.

 For Exercises 31 and 32, use the following statement: In 2010, a house sold for \$30,000 more than the same house sold for in 2005.

- 31.** If s and $s + 30,000$ represent the quantities in this statement, what is s ?
- 32.** If p and $p - 30,000$ represent the quantities in this statement, what is p ?

Applying the Concepts

For Exercises 33 to 36, write a phrase that translates into the given expression.

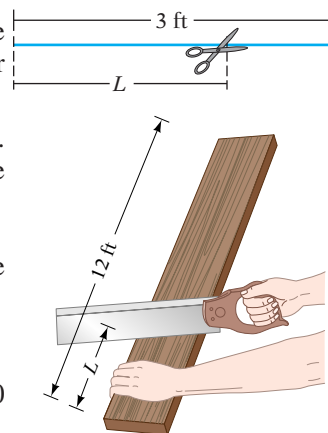
- 33.** $2x + 3$ **34.** $5y - 4$ **35.** $2(x + 3)$ **36.** $5(y - 4)$

- 37.** Translate each of the following into a variable expression. Each expression is part of a formula from the sciences, and its translation requires more than one variable.
- the product of mass (m) and acceleration (a)
 - the product of the area (A) and the square of the velocity (v)

In the News**Arctic Temperatures on the Rise**

A study conducted by scientists at the University of Washington shows that the average temperature of Arctic Ocean water is rising. Temperatures recorded for surface waters near the Bering Strait and the Chukchi Sea were 3.5 degrees higher than historical averages and 1.5 degrees higher than the historical maximum.

Source: uwnews.org



FOCUS ON PROBLEM SOLVING

Polya's Four-Step Process

Your success in mathematics and your success in the workplace are heavily dependent on your ability to solve problems. One of the foremost mathematicians to study problem solving was George Polya (1887–1985). The basic structure that Polya advocated for problem solving has four steps, as outlined below.



Point of Interest

George Polya was born in Hungary and moved to the United States in 1940. He lived in Providence, Rhode Island, where he taught at Brown University until 1942, when he moved to California. There he taught at Stanford University until his retirement. While at Stanford, he published 10 books and a number of articles for mathematics journals. Of the books Polya published, *How To Solve It* (1945) is one of his best known. In this book, Polya outlines a strategy for solving problems. This strategy, although frequently applied to mathematics, can be used to solve problems from virtually any discipline.

1. Understand the Problem

You must have a clear understanding of the problem. To help you focus on understanding the problem, here are some questions to think about.

- Can you restate the problem in your own words?
- Can you determine what is known about this type of problem?
- Is there missing information that you need in order to solve the problem?
- Is there information given that is not needed?
- What is the goal?

2. Devise a Plan

Successful problem solvers use a variety of techniques when they attempt to solve a problem. Here are some frequently used strategies.

- Make a list of the known information.
- Make a list of information that is needed to solve the problem.
- Make a table or draw a diagram.
- Work backwards.
- Try to solve a similar but simpler problem.
- Research the problem to determine whether there are known techniques for solving problems of its kind.
- Try to determine whether some pattern exists.
- Write an equation.

3. Carry Out the Plan

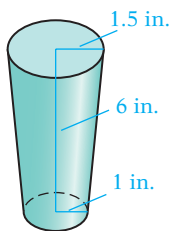
Once you have devised a plan, you must carry it out.

- Work carefully.
- Keep an accurate and neat record of all your attempts.
- Realize that some of your initial plans will not work and that you may have to return to Step 2 and devise another plan or modify your existing plan.

4. Review the Solution

Once you have found a solution, check the solution against the known facts.

- Make sure that the solution is consistent with the facts of the problem.
- Interpret the solution in the context of the problem.
- Ask yourself whether there are generalizations of the solution that could apply to other problems.
- Determine the strengths and weaknesses of your solution. For instance, is your solution only an approximation to the actual solution?
- Consider the possibility of alternative solutions.



We will use Polya's four-step process to solve the following problem.

A large soft drink costs \$1.25 at a college cafeteria. The dimensions of the cup are shown at the left. Suppose you don't put any ice in the cup. Determine the cost per ounce for the soft drink.

- 1. Understand the problem.** We must determine the cost per ounce for the soft drink. To do this, we need the dimensions of the cup (which are given), the cost of the drink (given), and a formula for the volume of the cup (unknown). Also, because the dimensions are given in inches, the volume will be in cubic inches. We need a conversion factor that will convert cubic inches to fluid ounces.
- 2. Devise a plan.** Consult a resource book that gives an equation for the volume of the figure, which is called a **frustum**. The formula for the volume is

$$V = \frac{\pi h}{3}(r^2 + rR + R^2)$$

where h is the height, r is the radius of the base, and R is the radius of the top. Also from a reference book, $1 \text{ in}^3 \approx 0.55 \text{ fl oz}$. The general plan is to calculate the volume, convert the answer to fluid ounces, and then divide the cost by the number of fluid ounces.

- 3. Carry out the plan.** Using the information from the drawing, evaluate the formula for the volume.

$$V = \frac{6\pi}{3}[1^2 + 1(1.5) + 1.5^2] = 9.5\pi \approx 29.8451 \text{ in}^3$$

$$V \approx 29.8451(0.55) \approx 16.4148 \text{ fl oz} \quad \bullet \text{ Convert to fluid ounces.}$$

$$\text{Cost per ounce} \approx \frac{1.25}{16.4148} \approx 0.07615 \quad \bullet \text{ Divide the cost by the volume.}$$

The cost of the soft drink is approximately 7.62 cents per ounce.

- 4. Review the solution.** The cost of a 12-ounce can of soda from a vending machine is generally about \$1. Therefore, the cost of canned soda is $100¢ \div 12 \approx 8.33¢$ per ounce. This is consistent with our solution. This does not mean our solution is correct, but it does indicate that it is at least reasonable. Why might soda from a cafeteria be more expensive per ounce than soda from a vending machine?

Is there an alternative way to obtain the solution? There are probably many, but one possibility is to get a measuring cup, pour the soft drink into it, and read the number of ounces. Name an advantage and a disadvantage of this method.



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Use the four-step solution process to solve Exercises 1 and 2.

- 1.** A cup dispenser next to a water cooler holds cups that have the shape of a right circular cone. The height of the cone is 4 in., and the radius of the circular top is 1.5 in. How many ounces of water can the cup hold?
- 2.** Soft drink manufacturers research the preferences of consumers with regard to the look, feel, and size of a soft drink can. Suppose a manufacturer has determined that people want to have their hands reach around approximately 75% of the can. If this preference is to be achieved, how tall should the can be if it contains 12 oz of fluid? Assume the can is a right circular cylinder.

PROJECTS AND GROUP ACTIVITIES

Water Displacement When an object is placed in water, the object displaces an amount of water that is equal to the volume of the object.

HOW TO 1

A sphere with a diameter of 4 in. is placed in a rectangular tank of water that is 6 in. long and 5 in. wide. How much does the water level rise? Round to the nearest hundredth.

$$V = \frac{4}{3}\pi r^3$$

• Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi(2^3) = \frac{32}{3}\pi$$

• $r = \frac{1}{2}d = \frac{1}{2}(4) = 2$

Let x represent the amount of the rise in water level. The volume of the sphere will equal the volume displaced by the water. As shown at the left, this volume is equal to the volume of a rectangular solid with width 5 in., length 6 in., and height x in.

$$V = LWH$$

• Use the formula for the volume of a rectangular solid.

$$\frac{32}{3}\pi = (6)(5)x$$

• Substitute $\frac{32}{3}\pi$ for V , 5 for W , and 6 for L .

$$\frac{32}{90}\pi = x$$

• The exact height that the water will fill is $\frac{32}{90}\pi$.

$$1.12 \approx x$$

• Use a calculator to find an approximation.

The water will rise approximately 1.12 in.

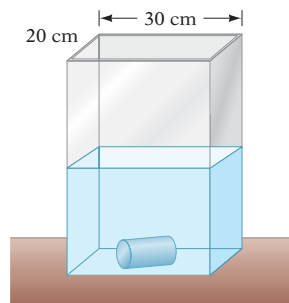
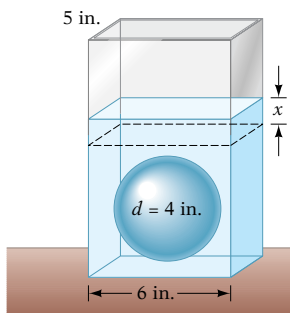


FIGURE 1

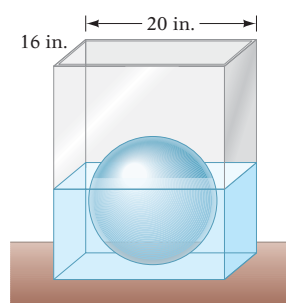


FIGURE 2

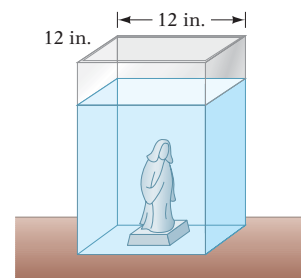


FIGURE 3

1. A cylinder with a 2-centimeter radius and a height of 10 cm is submerged in a tank of water that is 20 cm wide and 30 cm long (see Figure 1). How much does the water level rise? Round to the nearest hundredth.
2. A sphere with a radius of 6 in. is placed in a rectangular tank of water that is 16 in. wide and 20 in. long (see Figure 2). The sphere displaces water until two-thirds of the sphere is submerged. How much does the water level rise? Round to the nearest hundredth.
3. A chemist wants to know the density of a statue that weighs 15 lb. The statue is placed in a rectangular tank of water that is 12 in. long and 12 in. wide (see Figure 3). The water level rises 0.42 in. Find the density of the statue. Round to the nearest hundredth. (Hint: Density = weight $\div$ volume)

CHAPTER 1

SUMMARY

KEY WORDS

The *integers* are $\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots$. The *negative integers* are the integers $\dots, -4, -3, -2, -1$. The *positive integers*, or *natural numbers*, are the integers $1, 2, 3, 4, \dots$. The positive integers and zero are called the *whole numbers*. [1.1A, p. 2]

A *prime number* is a natural number greater than 1 that is evenly divisible only by itself and 1. A natural number that is not a prime number is a *composite number*. [1.1A, p. 2]

A *rational number* can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Every rational number can be written as either a *terminating decimal* or a *repeating decimal*. A number that cannot be written as a terminating or a repeating decimal is an *irrational number*. The rational numbers and the irrational numbers taken together are the *real numbers*. [1.1A, pp. 2–3]

The *graph of a real number* is made by placing a heavy dot directly above the number on a number line. [1.1A, p. 3]

Numbers that are the same distance from zero on the number line but are on opposite sides of zero are *additive inverses*, or *opposites*. [1.1A, p. 4]

The *absolute value* of a number is its distance from zero on the number line. [1.1A, p. 4]

A *set* is a collection of objects. The objects are called the *elements* of the set. [1.1A, p. 2]

The *roster method* of writing a set encloses a list of the elements of the set in braces. In an *infinite set*, the pattern of numbers continues without end. In a *finite set*, all the elements of the set can be listed. The set that contains no elements is the *empty set*, or *null set*, and is symbolized by $\emptyset$ or $\{ \}$. [1.1B, pp. 5–6]

Another method of representing a set is *set-builder notation*, which makes use of a variable and a certain property that only elements of that set possess. [1.1B, p. 6]

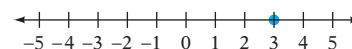
EXAMPLES

$-58, -12, 0, 7$, and 46 are integers.
 -58 and -12 are negative integers.
 7 and 46 are positive integers.

$2, 3, 5, 7, 11$, and 13 are prime numbers.
 $4, 6, 8, 9, 10$, and 12 are composite numbers.

$\frac{5}{6}, -\frac{3}{8}$, and 4 are rational numbers.
 $\frac{7}{\sqrt{2}}$ is not a rational number because $\sqrt{2}$ is not an integer. $\sqrt{2}$ is an irrational number. $-\frac{3}{8} = -0.375$, a terminating decimal. $\frac{5}{6} = 0.8\bar{3}$, a repeating decimal.

The graph of 3 is shown below.



8 and -8 are additive inverses.

The absolute value of 7 is 7 .
The absolute value of -7 is 7 .

The set of natural numbers
 $= \{1, 2, 3, 4, 5, 6, \dots\}$

$\{2, 4, 6, 8, \dots\}$ is an infinite set.
 $\{2, 4, 6, 8\}$ is a finite set.

$\{x | x < 7, x \in \text{integers}\}$ is read “the set of all x such that x is less than 7 and x is an element of the integers.”

Sets can also be expressed using *interval notation*. A parenthesis is used to indicate that a number is not included in the set. A bracket is used to indicate that a number is included in the set. An interval is said to be *closed* if it includes both endpoints. It is *open* if it does not include either endpoint. An interval is *half-open* if one endpoint is included and the other is not. To indicate an interval that extends forever in one or both directions using interval notation, use the *infinity symbol* ∞ or the *negative infinity symbol* $-\infty$. [1.1B, pp. 7–8]

The interval notation $[-4, 2)$ indicates the interval of all real numbers greater than or equal to -4 and less than 2 . The interval $[-4, 2)$ has endpoints -4 and 2 . It is an example of a half-open interval.

The interval notation $(-\infty, 5]$ indicates the interval of all real numbers less than or equal to 5 .

The *multiplicative inverse* or *reciprocal* of a nonzero real number a is $\frac{1}{a}$. [1.2A, p. 18]

The multiplicative inverse of -6 is $-\frac{1}{6}$.

The multiplicative inverse of $\frac{3}{8}$ is $\frac{8}{3}$.

The *least common multiple* (LCM) of two or more numbers is the smallest number that is a multiple of the numbers. The *greatest common factor* (GCF) of two or more numbers is the largest integer that divides evenly into all of the numbers. [1.2B, p. 20]

The LCM of 6 and 8 is 24.
The GCF of 6 and 8 is 2.

The expression a^n is in *exponential form*, where a is the base and n is the exponent. a^n is the *nth power of a* and represents the product of n factors of a . [1.2C, pp. 21–22]

In the exponential expression 5^3 , 5 is the base and 3 is the exponent.
 $5^3 = 5 \cdot 5 \cdot 5 = 125$

A *complex fraction* is a fraction in which the numerator or denominator contains one or more fractions. [1.2D, p. 23]

$\frac{\frac{3}{5} + \frac{1}{2}}{-\frac{4}{7}}$ is a complex fraction.

A *variable* is a letter of the alphabet that is used to stand for a number. [1.1A, p. 3] An expression that contains one or more variables is a *variable expression*. The *terms* of a variable expression are the addends of the expression. A *variable term* is composed of a *numerical coefficient* and a *variable part*. A *constant term* has no variable part. [1.3B, p. 31]

The variable expression $-4x^2 + 3x - 5$ has three terms: $-4x^2$, $3x$, and -5 .
 $-4x^2$ and $3x$ are variable terms.
 -5 is a constant term.
For the term $-4x^2$, the coefficient is -4 and the variable part is x^2 .

Like terms of a variable expression have the same variable part. Constant terms are also like terms. Adding the coefficients of like terms is called *combining like terms*. [1.3C, p. 32]

$-6a^3b^2$ and $4a^3b^2$ are like terms.
 $-6a^3b^2 + 4a^3b^2 = -2a^3b^2$

Replacing the variable or variables in a variable expression and then simplifying the resulting numerical expression is called *evaluating the variable expression*. [1.3B, p. 31]

Evaluate $5x^3 + |6 - 2y|$ when $x = -1$ and $y = 4$.
 $5x^3 + |6 - 2y|$
 $5(-1)^3 + |6 - 2(4)| = 5(-1)^3 + |6 - 8|$
 $= 5(-1)^3 + |-2|$
 $= 5(-1)^3 + 2$
 $= 5(-1) + 2$
 $= -5 + 2 = -3$

ESSENTIAL RULES AND PROCEDURES

EXAMPLES

Definition of Inequality Symbols [1.1A, p. 4]

If a and b are two real numbers and a is to the left of b on the number line, then a **is less than** b . This is written $a < b$.

If a and b are two real numbers and a is to the right of b on the number line, then a **is greater than** b . This is written $a > b$.

$$-19 < 36$$

$$-1 > -20$$

Absolute Value [1.1A, p. 4]

The absolute value of a positive number is the number itself.

The absolute value of a negative number is the opposite of the negative number.

The absolute value of zero is zero.

$$|18| = 18$$

$$|-18| = 18$$

$$|0| = 0$$

Union of Two Sets [1.1C, p. 10]

The **union** of two sets, written $A \cup B$, is the set of all elements that belong to either set A **or** set B . In set-builder notation, this is written

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

Given $A = \{0, 1, 2, 3, 4\}$ and

$B = \{2, 4, 6, 8\}$,

$$A \cup B = \{0, 1, 2, 3, 4, 6, 8\}.$$

Intersection of Two Sets [1.1C, p. 10]

The **intersection** of two sets, written $A \cap B$, is the set of all elements that are common to both set A **and** set B . In set-builder notation, this is written

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

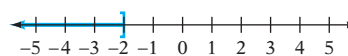
Given $A = \{0, 1, 2, 3, 4\}$ and

$B = \{2, 4, 6, 8\}$, $A \cap B = \{2, 4\}$.

Graphing Intervals on the Number Line [1.1B, pp. 6–8]

A parenthesis on a graph indicates that the number is not included in a set. A bracket indicates that the number is included in the set.

The graph of $\{x | x \leq -2\}$ is shown below.

**Rules for Addition of Real Numbers** [1.2A, p. 17]

To add numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

To add numbers with different signs, find the absolute value of each number. Subtract the lesser of the two numbers from the greater. Then attach the sign of the number with the greater absolute value.

$$-12 + (-18) = -30$$

$$12 + (-18) = -6$$

Rule for Subtraction of Real Numbers [1.2A, p. 17]

If a and b are real numbers, then $a - b = a + (-b)$.

$$6 - 9 = 6 + (-9) = -3$$

Rules for Multiplication of Real Numbers [1.2A, p. 18]

The product of two numbers with the same sign is positive.

The product of two numbers with different signs is negative.

$$-5(-9) = 45$$

$$-5(9) = -45$$

Equivalent Fractions [1.2A, p. 18]

If a and b are real numbers and $b \neq 0$, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

$$\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$$

Properties of Zero and One in Division [1.2A, pp. 18–19]

Zero divided by any number other than zero is zero.

$$0 \div 4 = 0$$

Division by zero is not defined.

$$4 \div 0 \text{ is undefined.}$$

Any number other than zero divided by itself is 1.

$$4 \div 4 = 1$$

Any number divided by 1 is the number.

$$4 \div 1 = 4$$

Division of Fractions [1.2B, p. 21]

To divide two fractions, multiply by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

$$-\frac{3}{5} \div \frac{3}{10} = -\frac{3}{5} \cdot \frac{10}{3} = -2$$

Order of Operations Agreement [1.2D, p. 22]**Step 1** Perform operations inside grouping symbols.

$$\begin{aligned} -6^2 - 3(-2 + 4) &= -6^2 - 3(2) \\ &= -36 - 3(2) \\ &= -36 - 6 \\ &= -36 + (-6) \\ &= -42 \end{aligned}$$

Step 2 Simplify exponential expressions.**Step 3** Do multiplication and division as they occur from left to right.**Step 4** Do addition and subtraction as they occur from left to right.**Properties of Real Numbers** [1.3A, pp. 29–30]**Commutative Property of Addition** $a + b = b + a$

$$3 + 8 = 8 + 3$$

Commutative Property of Multiplication $a \cdot b = b \cdot a$

$$4 \cdot 9 = 9 \cdot 4$$

Associative Property of Addition $(a + b) + c = a + (b + c)$

$$(-2 + 4) + 6 = -2 + (4 + 6)$$

Associative Property of Multiplication $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

$$(-5 \cdot 3) \cdot 6 = -5 \cdot (3 \cdot 6)$$

Addition Property of Zero $a + 0 = 0 + a = a$

$$-9 + 0 = -9$$

Multiplication Property of Zero $a \cdot 0 = 0 \cdot a = 0$

$$-6(0) = 0$$

Multiplication Property of One $a \cdot 1 = 1 \cdot a = a$

$$-12(1) = -12$$

Inverse Property of Addition $a + (-a) = (-a) + a = 0$

$$-7 + 7 = 0$$

Inverse Property of Multiplication $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1, a \neq 0$

$$8 \cdot \frac{1}{8} = 1$$

Distributive Property $a(b + c) = ab + ac$

$$-2(4x + 5) = -8x - 10$$

CHAPTER 1**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. When is the symbol $<$ used to compare two numbers?
2. How do you evaluate the absolute value of a number?
3. What is the difference between the roster method and set-builder notation?
4. How do you represent an infinite set when using the roster method?
5. When is a half-open interval used to represent a set?
6. How is the multiplicative inverse used to divide real numbers?
7. What are the steps in the Order of Operations Agreement?
8. What is the difference between the Commutative Property of Multiplication and the Associative Property of Multiplication?
9. What is the difference between union and intersection of two sets?
10. How do you simplify a variable expression?
11. What operations are represented by the phrases **a.** less than, **b.** the total of, **c.** the quotient of, and **d.** the product of?
12. Which property of multiplication is used to evaluate $7 \cdot \frac{1}{7}$?

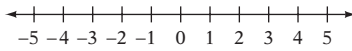
CHAPTER 1

REVIEW EXERCISES

1. Use the roster method to write the set of integers between -3 and 4 .

2. Find $A \cap B$ given $A = \{0, 1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$.

3. Graph $(-2, 4]$.



4. Identify the property that justifies the statement.
 $2(3x) = (2 \cdot 3)x$

5. Simplify: $-4.07 + 2.3 - 1.07$

6. Evaluate $(a - 2b^2) \div (ab)$ when $a = 4$ and $b = -3$.

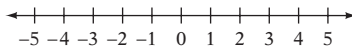
7. Simplify: $-2 \cdot (4^2) \cdot (-3)^2$

8. Simplify: $4y - 3[x - 2(3 - 2x) - 4y]$

9. Find the additive inverse of $-\frac{3}{4}$.

10. Use set-builder notation to write the set of real numbers less than -3 .

11. Graph $\{x | x < 1\}$.



12. Simplify: $-10 - (-3) - 8$

13. Simplify: $-\frac{2}{3} + \frac{3}{5} - \frac{1}{6}$

14. Use the Associative Property of Addition to complete the statement.
 $3 + (4 + y) = (3 + ?) + y$

15. Simplify: $-\frac{3}{8} \div \frac{3}{5}$

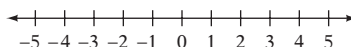
16. Let $x \in \{-4, -2, 0, 2\}$. For what values of x is $x > -1$ true?

17. Evaluate $2a^2 - \frac{3b}{a}$ when $a = -3$ and $b = 2$.

18. Simplify: $18 - |-12 + 8|$

19. Simplify: $20 \div \frac{3^2 - 2^2}{3^2 + 2^2}$

20. Graph $[-3, \infty)$.



21. Find $A \cup B$ given $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$.

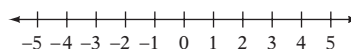
22. Simplify: $-204 \div (-17)$

23. Write $[-2, 3]$ in set-builder notation.

24. Simplify: $\frac{3}{4} \left(-\frac{8}{21} \right) \left(-\frac{7}{15} \right)$

25. Use the Distributive Property to complete the statement.
 $6x - 21y = ?(2x - 7y)$

26. Graph $\{x | x \leq -3\} \cup \{x | x > 0\}$.



27. Simplify: $-2(x - 3) + 4(2 - x)$

28. Let $p \in \{-4, 0, 7\}$. Evaluate $-|p|$ for each element of the set.

29. Identify the property that justifies the statement.
 $-4 + 4 = 0$

30. Simplify: $-3.286 \div (-1.06)$

31. Find the additive inverse of -87 .

32. Let $y \in \{-4, -1, 4\}$. For which values of y is $y > -2$ true?

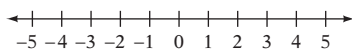
33. Use the roster method to write the set of integers between -4 and 2 .

34. Use set-builder notation to write the set of real numbers less than 7 .

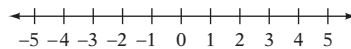
35. Given $A = \{-4, -2, 0, 2, 4\}$ and $B = \{0, 5, 10\}$, find $A \cup B$.

36. Given $A = \{-9, -6, -3\}$ and $B = \{3, 6, 9\}$, find $A \cap B$.

37. Graph $\{x | x \leq 3\} \cap \{x | x > -2\}$.



38. Graph $(-3, 4) \cup [-1, 5]$.



39. Simplify: $9 - (-3) - 7$
40. Simplify: $-\frac{2}{3} + \left(-\frac{1}{4}\right) - \left(-\frac{5}{12}\right)$
41. Simplify: $\left(\frac{2}{3}\right)^3 (-3)^4$
42. Simplify: $(-3)^3 - (2 - 6)^2 \cdot 5$
43. Evaluate $-8ac \div b^2$ when $a = -1$, $b = 2$, and $c = -3$.
44. Simplify: $-(3a + b) - 2(-4a - 5b)$
45. Translate “four times the sum of a number and four” into a variable expression. Then simplify.
46. **Travel** The total flying time for a round trip between New York and San Diego is 13 h. Because of the jet stream, the time going is not equal to the time returning. Express the flying time between New York and San Diego in terms of the flying time between San Diego and New York.
-  47. **Calories** For a 140-pound person, the number of calories burned by cross-country skiing for 1 h is 396 more than the number of calories burned by walking at 4 mph for 1 h. (*Source:* Healthstatus.com) Express the number of calories burned by cross-country skiing for 1 h in terms of the number of calories burned by walking at 4 mph for 1 h.
48. Translate “eight more than twice the difference between a number and two” into a variable expression. Then simplify.
49. A second integer is 5 more than four times the first integer. Express the second integer in terms of the first integer.
50. Translate “twelve minus the quotient of three more than a number and four” into a variable expression. Then simplify.
51. The sum of two numbers is forty. Using x to represent the smaller of the two numbers, translate “the sum of twice the smaller number and five more than the larger number” into a variable expression. Then simplify.
52. **Geometry** The length of a rectangle is 3 ft less than three times the width. Express the length of the rectangle in terms of the width.



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CHAPTER 1

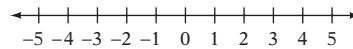
TEST

1. Simplify: $(-2)(-3)(-5)$

2. Find $A \cap B$ given $A = \{1, 3, 5, 7\}$ and $B = \{5, 7, 9, 11\}$.

3. Simplify: $(-2)^3(-3)^2$

4. Graph $(-\infty, 1]$.



5. Find $A \cap B$ given $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $B = \{-1, 0, 1\}$.

6. Evaluate $(a - b)^2 \div (2b + 1)$ when $a = 2$ and $b = -3$.

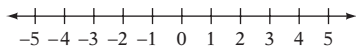
7. Simplify: $|-3 - (-5)|$

8. Simplify: $2x - 4[2 - 3(x + 4y) - 2]$

9. Find the additive inverse of -12 .

10. Simplify: $-5^2 \cdot 4$

11. Graph $\{x | x < 3\} \cap \{x | x > -2\}$.



12. Simplify: $2 - (-12) + 3 - 5$

13. Simplify: $\frac{2}{3} - \frac{5}{12} + \frac{4}{9}$

14. Use the Commutative Property of Addition to complete the statement.
 $(3 + 4) + 2 = (? + 3) + 2$

15. Simplify: $-\frac{2}{3}\left(\frac{9}{15}\right)\left(\frac{10}{27}\right)$

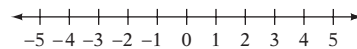
16. Let $x \in \{-5, 3, 7\}$. For what values of x is $x < -1$ true?

17. Evaluate $\frac{b^2 - c^2}{a - 2c}$ when $a = 2$, $b = 3$, and $c = -1$.

18. Simplify: $-180 \div 12$

19. Simplify: $12 - 4\left(\frac{5^2 - 1}{3}\right) \div 16$

20. Graph $(3, \infty)$.



21. Find $A \cup B$ given $A = \{1, 3, 5, 7\}$ and $B = \{2, 3, 4, 5\}$.

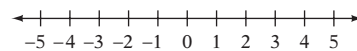
22. Simplify: $3x - 2(x - y) - 3(y - 4x)$

23. Simplify: $8 - 4(2 - 3)^2 \div 2$

24. Simplify: $\frac{3}{5}\left(-\frac{10}{21}\right)\left(-\frac{7}{15}\right)$

25. Identify the property that justifies the statement.
 $-2(x + y) = -2x - 2y$

26. Graph $\{x | x \leq 3\} \cup \{x | x < -2\}$.



27. Simplify: $4.27 - 6.98 + 1.3$

28. Find $A \cup B$ given $A = \{-2, -1, 0, 1, 2, 3\}$ and $B = \{-1, 0, 1\}$.

29. The sum of two numbers is nine. Using x to represent the larger of the two numbers, translate “the difference between one more than the larger number and twice the smaller number” into a variable expression. Then simplify.



Steven Mark Needham/Foodpix/Jupiter Images



30. **Cocoa Production** The two countries with the highest cocoa production are the Ivory Coast and Ghana. The Ivory Coast produces three times the amount of cocoa produced in Ghana. (*Source:* International Cocoa Organization) Express the amount of cocoa produced in the Ivory Coast in terms of the amount of cocoa produced in Ghana.

First-Degree Equations and Inequalities

OBJECTIVES

SECTION 2.1

- A** To solve an equation using the Addition or the Multiplication Property of Equations
- B** To solve an equation using both the Addition and the Multiplication Properties of Equations
- C** To solve an equation containing parentheses
- D** To solve a literal equation for one of the variables

SECTION 2.2

- A** To solve integer problems
- B** To solve coin and stamp problems

SECTION 2.3

- A** To solve value mixture problems
- B** To solve percent mixture problems
- C** To solve uniform motion problems

SECTION 2.4

- A** To solve an inequality in one variable
- B** To solve a compound inequality
- C** To solve application problems

SECTION 2.5

- A** To solve an absolute value equation
- B** To solve an absolute value inequality
- C** To solve application problems

ARE YOU READY?

Take the Chapter 2 Prep Test to find out if you are ready to learn to:

- Solve a first-degree equation
- Solve integer, coin and stamp, mixture, and uniform motion problems
- Solve an inequality
- Solve an absolute value equation
- Solve an absolute value inequality

PREP TEST

Do these exercises to prepare for Chapter 2.

For Exercises 1 to 5, add, subtract, multiply, or divide.

1. $8 - 12$

2. $-9 + 3$

3. $\frac{-18}{-6}$

4. $-\frac{3}{4}\left(-\frac{4}{3}\right)$

5. $-\frac{5}{8}\left(\frac{4}{5}\right)$

For Exercises 6 to 9, simplify.

6. $3x - 5 + 7x$

7. $6(x - 2) + 3$

8. $n + (n + 2) + (n + 4)$

9. $0.08x + 0.05(400 - x)$

10. A 20-ounce snack mixture contains nuts and pretzels. Let n represent the number of ounces of nuts in the mixture. Express the number of ounces of pretzels in the mixture in terms of n .

SECTION

2.1

Solving First-Degree Equations

OBJECTIVE A



Tips for Success

Before you begin a new chapter, you should take some time to review previously learned skills. One way to do this is to complete the Prep Test. See page 57. This test focuses on the particular skills that will be required for the new chapter.

To solve an equation using the Addition or the Multiplication Property of Equations



An **equation** expresses the equality of two mathematical expressions. The expressions can be either numerical or variable expressions.

$$\left. \begin{array}{l} 2 + 8 = 10 \\ x + 8 = 11 \\ x^2 + 2y = 7 \end{array} \right\} \text{Equations}$$

The equation at the right is a **conditional equation**. The equation is true if the variable is replaced by 3. The equation is false if the variable is replaced by 4. A conditional equation is true for at least one value of the variable.

$$\begin{array}{ll} x + 2 = 5 & \text{A conditional equation} \\ 3 + 2 = 5 & \text{A true equation} \\ 4 + 2 = 5 & \text{A false equation} \end{array}$$

The replacement value(s) of the variable that will make an equation true is (are) called the **root(s) of the equation** or the **solution(s) of the equation**. The solution of the equation $x + 2 = 5$ is 3 because $3 + 2 = 5$ is a true equation.

The equation at the right is an **identity**. Any replacement for x will result in a true equation.

$$x + 2 = x + 2$$

The equation at the right has no solution because there is no number that equals itself plus 1. Any replacement value for x will result in a false equation. This equation is a **contradiction**.

$$x = x + 1$$

Each of the equations at the right is a **first-degree equation in one variable**. All variables have an exponent of 1.

$$\begin{array}{l} x + 2 = 12 \\ 3y - 2 = 5y \\ 3(a + 2) = 14a \end{array}$$

Solving an equation means finding a root or solution of the equation. The simplest equation to solve is an equation of the form $\text{variable} = \text{constant}$, because the constant is the solution. If $x = 3$, then 3 is the solution of the equation, because $3 = 3$ is a true equation.

Equivalent equations are equations that have the same solution. For instance, $x + 4 = 6$ and $x = 2$ are equivalent equations because the solution of each equation is 2. In solving an equation, the goal is to produce *simpler* but equivalent equations until you reach the goal of $\text{variable} = \text{constant}$. The **Addition Property of Equations** can be used to rewrite an equation in this form.

Addition Property of Equations

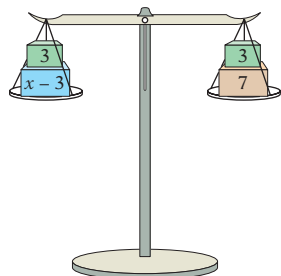
The same term can be added to each side of an equation without changing the solution of the equation. Symbolically, this is written

$$\text{If } a = b, \text{ then } a + c = b + c.$$

The Addition Property of Equations is used to remove a term from one side of the equation by adding the opposite of that term to each side of the equation.

Take Note

The model of an equation as a balance scale applies.



Adding a weight to one side of the scale (equation) requires adding the same weight to the other side of the scale (equation) so that the pans remain in balance.

Take Note

Remember to check the solution.

$$\begin{array}{r|l} x + \frac{7}{12} = \frac{1}{2} & \\ -\frac{1}{12} + \frac{7}{12} & \frac{1}{2} \\ \hline \frac{6}{12} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{array}$$

HOW TO • 1 Solve: $x - 3 = 7$

$$\begin{aligned} x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x + 0 &= 10 \\ x &= 10 \end{aligned}$$

Check: $\begin{array}{r} x - 3 = 7 \\ 10 - 3 \quad | \quad 7 \\ 7 = 7 \end{array}$

The solution is 10.

- Add 3 to each side of the equation.
- Simplify.
- The equation is in the form *variable* = *constant*.
- Check the solution. Replace x with 10.
- When simplified, the left side of the equation equals the right side. Therefore, 10 is the correct solution of the equation.

Because subtraction is defined in terms of addition, the Addition Property of Equations enables us to subtract the same number from each side of an equation.

HOW TO • 2 Solve: $x + \frac{7}{12} = \frac{1}{2}$

$$\begin{aligned} x + \frac{7}{12} &= \frac{1}{2} \\ x + \frac{7}{12} - \frac{7}{12} &= \frac{1}{2} - \frac{7}{12} \\ x + 0 &= \frac{6}{12} - \frac{7}{12} \\ x &= -\frac{1}{12} \end{aligned}$$

The solution is $-\frac{1}{12}$.

- Subtract $\frac{7}{12}$ from each side of the equation.
- Simplify.

The **Multiplication Property of Equations** is also used to produce equivalent equations.

Multiplication Property of Equations

Multiplying each side of an equation by the same nonzero number does not change the solution of the equation. Symbolically, this is written

$$\text{If } a = b \text{ and } c \neq 0, \text{ then } ac = bc.$$

Recall that the goal of solving an equation is to rewrite the equation in the form *variable* = *constant*. The Multiplication Property of Equations is used to rewrite an equation in this form by multiplying each side by the reciprocal of the coefficient.

Take Note

Remember to check the solution.

$$\begin{array}{r|l} -\frac{3}{4}x = 12 & \\ -\frac{3}{4}(-16) & 12 \\ \hline 12 = 12 \end{array}$$

HOW TO • 3 Solve: $-\frac{3}{4}x = 12$

$$\begin{aligned} -\frac{3}{4}x &= 12 \\ \left(-\frac{4}{3}\right)\left(-\frac{3}{4}\right)x &= \left(-\frac{4}{3}\right)12 \\ 1x &= -16 \\ x &= -16 \end{aligned}$$

The solution is -16 .

- Multiply each side of the equation by $-\frac{4}{3}$, the reciprocal of $-\frac{3}{4}$.
- Simplify.

Because division is defined in terms of multiplication, the Multiplication Property of Equations enables us to divide each side of an equation by the same nonzero quantity.

HOW TO • 4 Solve: $-5x = 9$

Multiplying each side of the equation by the reciprocal of -5 is equivalent to dividing each side of the equation by -5 .

$$\begin{array}{r} -5x = 9 \\ \frac{-5x}{-5} = \frac{9}{-5} \end{array}$$

• Divide each side of the equation by -5 .

$$1x = -\frac{9}{5}$$

• Simplify.

$$x = -\frac{9}{5}$$

The solution is $-\frac{9}{5}$. You should check the solution.

When using the Multiplication Property of Equations, it is usually easier to multiply each side of the equation by the reciprocal of the coefficient when the coefficient is a fraction. Divide each side of the equation by the coefficient when the coefficient is an integer or a decimal.

EXAMPLE • 1

Solve: $x - 7 = -12$

Solution

$$x - 7 = -12$$

$$x - 7 + 7 = -12 + 7$$

• Add 7 to each side.

$$x = -5$$

The solution is -5 .

YOU TRY IT • 1

Solve: $x + 4 = -3$

Your solution

EXAMPLE • 2

Solve: $-\frac{2x}{7} = \frac{4}{21}$

Solution

$$-\frac{2x}{7} = \frac{4}{21}$$

$$\left(-\frac{7}{2}\right)\left(-\frac{2}{7}x\right) = \left(-\frac{7}{2}\right)\left(\frac{4}{21}\right)$$

• Multiply each side by $-\frac{7}{2}$.

$$x = -\frac{2}{3}$$

The solution is $-\frac{2}{3}$.

YOU TRY IT • 2

Solve: $-3x = 18$

Your solution

OBJECTIVE B**To solve an equation using both the Addition and the Multiplication Properties of Equations**

In solving an equation, it is often necessary to apply both the Addition and the Multiplication Properties of Equations.

**Take Note**

You should always check your work by substituting your answer into the original equation and simplifying the resulting numerical expressions. If the left and right sides are equal, your answer is the solution of the equation.

HOW TO • 5 Solve: $-8 - 5x = 4x - 11$

$$\begin{aligned}
 -8 - 5x &= 4x - 11 \\
 -8 - 5x - 4x &= 4x - 4x - 11 && \bullet \text{ Subtract } 4x \text{ from each side of the equation.} \\
 -8 - 9x &= -11 && \bullet \text{ Simplify.} \\
 -8 + 8 - 9x &= -11 + 8 && \bullet \text{ Add } 8 \text{ to each side of the equation.} \\
 -9x &= -3 && \bullet \text{ Simplify.} \\
 \frac{-9x}{-9} &= \frac{-3}{-9} && \bullet \text{ Divide each side of the equation by } -9, \text{ the} \\
 x &= \frac{1}{3} && \text{coefficient of the variable.}
 \end{aligned}$$

Check:

$$\begin{array}{r|l}
 -8 - 5x = 4x - 11 & \\
 -8 - 5\left(\frac{1}{3}\right) & 4\left(\frac{1}{3}\right) - 11 \\
 -8 - \frac{5}{3} & \frac{4}{3} - 11 \\
 -\frac{24}{3} - \frac{5}{3} & \frac{4}{3} - \frac{33}{3} \\
 -\frac{29}{3} & = -\frac{29}{3}
 \end{array}$$

The solution is $\frac{1}{3}$.**EXAMPLE • 3**Solve: $3x - 5 = 7x - 11$ **Solution**

$$\begin{aligned}
 3x - 5 &= 7x - 11 \\
 3x - 7x - 5 &= 7x - 7x - 11 && \bullet \text{ Subtract } 7x \text{ from each side.} \\
 -4x - 5 &= -11 \\
 -4x - 5 + 5 &= -11 + 5 && \bullet \text{ Add } 5 \text{ to each side.} \\
 -4x &= -6 \\
 \frac{-4x}{-4} &= \frac{-6}{-4} && \bullet \text{ Divide each side by } -4. \\
 x &= \frac{3}{2}
 \end{aligned}$$

The solution is $\frac{3}{2}$.**YOU TRY IT • 3**Solve: $4x - 9 = 5 - 3x$ **Your solution**

OBJECTIVE C**To solve an equation containing parentheses**

When an equation contains parentheses, one of the steps required to solve the equation involves using the Distributive Property.

HOW TO 6 Solve: $6 - 4(2x + 5) = 2 - 3x$

$$\begin{aligned} 6 - 4(2x + 5) &= 2 - 3x \\ 6 - 8x - 20 &= 2 - 3x \end{aligned}$$

$$\begin{aligned} -8x - 14 &= 2 - 3x \\ -8x + 3x - 14 &= 2 - 3x + 3x \\ -5x - 14 &= 2 \\ -5x - 14 + 14 &= 2 + 14 \\ -5x &= 16 \\ \frac{-5x}{-5} &= \frac{16}{-5} \\ x &= -\frac{16}{5} \end{aligned}$$

The solution is $-\frac{16}{5}$.

- Use the **Distributive Property** to remove parentheses. Note that, by the Order of Operations Agreement, multiplication is done before subtraction.
- **Simplify.**
- **Add $3x$** to each side of the equation.
- **Simplify.**
- **Add 14** to each side of the equation.
- **Simplify.**
- **Divide each side of the equation by -5 .**

To solve an equation containing fractions, first clear the denominators by multiplying each side of the equation by the least common multiple (LCM) of the denominators.

HOW TO 7 Solve: $\frac{x}{2} - \frac{7}{9} = \frac{x}{6} + \frac{2}{3}$

Find the LCM of the denominators. The LCM of 2, 9, 6, and 3 is 18.

$$\begin{aligned} \frac{x}{2} - \frac{7}{9} &= \frac{x}{6} + \frac{2}{3} \\ 18\left(\frac{x}{2} - \frac{7}{9}\right) &= 18\left(\frac{x}{6} + \frac{2}{3}\right) \\ \frac{18x}{2} - \frac{18 \cdot 7}{9} &= \frac{18x}{6} + \frac{18 \cdot 2}{3} \\ 9x - 14 &= 3x + 12 \\ 6x - 14 &= 12 \\ 6x &= 26 \\ \frac{6x}{6} &= \frac{26}{6} \\ x &= \frac{13}{3} \end{aligned}$$

The solution is $\frac{13}{3}$.

- **Multiply each side of the equation by the LCM of the denominators.**
- **Use the Distributive Property** to remove parentheses. Then simplify.
- **Subtract $3x$** from each side of the equation. Then simplify.
- **Add 14** to each side of the equation. Then simplify.
- **Divide each side of the equation by 6 ,** the coefficient of x . Then simplify.

EXAMPLE • 4

Solve:

$$5(2x - 7) + 2 = 3(4 - x) - 12$$

Solution

$$\begin{aligned} 5(2x - 7) + 2 &= 3(4 - x) - 12 \\ 10x - 35 + 2 &= 12 - 3x - 12 \\ 10x - 33 &= -3x \\ -33 &= -13x \\ \frac{-33}{-13} &= \frac{-13x}{-13} \\ \frac{33}{13} &= x \end{aligned}$$

The solution is $\frac{33}{13}$.**YOU TRY IT • 4**

Solve:

$$6(5 - x) - 12 = 2x - 3(4 + x)$$

Your solution

Solution on p. S3

OBJECTIVE D**To solve a literal equation for one of the variables**

A **literal equation** is an equation that contains more than one variable. Some examples are shown at the right.

$$\begin{aligned} 3x - 2y &= 4 \\ v^2 &= v_0^2 + 2as \end{aligned}$$

Formulas are used to express a relationship among physical quantities. A **formula** is a literal equation that states rules about measurement. Examples are shown at the right.

$$\begin{aligned} s &= vt - 16t^2 && \text{(Physics)} \\ c^2 &= a^2 + b^2 && \text{(Geometry)} \\ I &= P(1 + r)^n && \text{(Business)} \end{aligned}$$

The Addition and Multiplication Properties of Equations can be used to solve a literal equation for one of the variables. The goal is to rewrite the equation so that the variable being solved for is alone on one side of the equation and all the other numbers and variables are on the other side.

HOW TO • 8Solve $A = P + Prt$ for t .

$$A = P + Prt$$

$$A - P = Prt$$

$$\frac{A - P}{Pr} = \frac{Prt}{Pr}$$

$$\frac{A - P}{Pr} = t$$

• Subtract P from each side of the equation.• Divide each side of the equation by Pr .**EXAMPLE • 5**Solve $C = \frac{5}{9}(F - 32)$ for F .**Solution**

$$C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F$$

- Multiply each side by $\frac{9}{5}$.
- Add 32 to each side.

YOU TRY IT • 5Solve $S = C - rC$ for r .**Your solution**

Solution on p. S3

2.1 EXERCISES**OBJECTIVE A****To solve an equation using the Addition or the Multiplication Property of Equations**

-  1. How does an equation differ from an expression?
-  2. What is the solution of an equation?
-  3. What is the Addition Property of Equations, and how is it used?
-  4. What is the Multiplication Property of Equations, and how is it used?

5. Is 1 a solution of $7 - 3m = 4$?

6. Is 5 a solution of $4y - 5 = 3y$?

7. Is -2 a solution of $6x - 1 = 7x + 1$?

8. Is 3 a solution of $x^2 = 4x - 5$?

For Exercises 9 to 39, solve and check.

9. $x - 2 = 7$

10. $x - 8 = 4$

11. $-7 = x + 8$

12. $-12 = x - 3$

13. $3x = 12$

14. $8x = 4$

15. $-3x = 2$

16. $-5a = 7$

17. $-\frac{3}{2} + x = \frac{4}{3}$

18. $\frac{2}{7} + x = \frac{17}{21}$

19. $x + \frac{2}{3} = \frac{5}{6}$

20. $\frac{5}{8} + y = \frac{1}{4}$

21. $\frac{2}{3}y = 5$

22. $\frac{3}{5}y = 12$

23. $\frac{4}{5} = -\frac{5}{8}x$

24. $\frac{7}{16} = -\frac{5}{12}x$

25. $-12 = \frac{4x}{7}$

26. $-9 = \frac{3c}{10}$

27. $-\frac{5y}{7} = \frac{10}{21}$

28. $-\frac{2d}{9} = \frac{4}{3}$

29. $-\frac{3b}{5} = -\frac{3}{5}$

30. $-\frac{7b}{12} = -\frac{7}{8}$

31. $-\frac{2}{3}x = -\frac{5}{8}$

32. $-\frac{3}{4}x = -\frac{4}{7}$

33. $1.5x = 27$

34. $-2.25y = 0.9$

35. $-0.015x = -12$

36. $-0.012x = -6$

37. $3x + 5x = 12$

38. $2x - 7x = 15$

39. $3y - 5y = 0$



40. Let r be a positive number less than 1. Is the solution of the equation $\frac{10}{9} + x = r$ positive or negative?



41. Let a be a negative number less than -5 . Is the solution of the equation $a = -5b$ less than or greater than 1?

OBJECTIVE B

To solve an equation using both the Addition and the Multiplication Properties of Equations

For Exercises 42 to 65, solve and check.

42. $5x + 9 = 6$

43. $2x - 4 = 12$

44. $2y - 9 = -9$

45. $4x - 6 = 3x$

46. $2a - 7 = 5a$

47. $7x + 12 = 9x$

48. $3x - 12 = 5x$

49. $4x + 2 = 4x$

50. $3m - 7 = 3m$

51. $2x + 2 = 3x + 5$

52. $7x - 9 = 3 - 4x$

53. $2 - 3t = 3t - 4$

54. $7 - 5t = 2t - 9$

55. $3b - 2b = 4 - 2b$

56. $3x - 5 = 6 - 9x$

57. $3x + 7 = 3 + 7x$

58. $\frac{5}{8}b - 3 = 12$

59. $\frac{1}{3} - 2b = 3$

60. $3.24a + 7.14 = 5.34a$

61. $5.3y + 0.35 = 5.02y$

62. $-1.27 - 4.6d = 7.93$

63. $\frac{2x}{3} - \frac{1}{2} = \frac{5}{6}$

64. $\frac{5x}{4} + \frac{3}{8} = -\frac{1}{4}$

65. $\frac{3}{2} - \frac{10x}{9} = -\frac{1}{6}$

66. If $3x + 1 = 2x - 3$, evaluate $5x - 8$.

67. If $2y - 6 = 3y + 2$, evaluate $7y + 1$.



68. If A is a positive number, is the solution of the equation $Ax + 8 = -3$ positive or negative?



69. If A is a negative number, is the solution of the equation $Ax - 2 = -5$ positive or negative?

OBJECTIVE C

To solve an equation containing parentheses

For Exercises 70 to 93, solve and check.

70. $2x + 2(x + 1) = 10$

71. $2x + 3(x - 5) = 15$

72. $3 - 2(y - 3) = 4y - 7$

73. $3(y - 5) - 5y = 2y + 9$

74. $2(3x - 2) - 5x = 3 - 2x$

75. $4 - 3x = 7x - 2(3 - x)$

76. $8 - 5(4 - 3x) = 2(4 - x) - 8x$

77. $-3x - 2(4 + 5x) = 14 - 3(2x - 3)$

78. $3[2 - 3(y - 2)] = 12$

79. $3y = 2[5 - 3(2 - y)]$

80. $4[3 + 5(3 - x) + 2x] = 6 - 2x$

81. $2[4 + 2(5 - x) - 2x] = 4x - 7$

82. $3[4 - 2(a - 2)] = 3(2 - 4a)$

83. $2[3 - 2(z + 4)] = 3(4 - z)$

84. $-3(x - 2) = 2[x - 4(x - 2) + x]$

85. $3[x - (2 - x) - 2x] = 3(4 - x)$

86. $\frac{5 - 2x}{5} + \frac{x - 4}{10} = \frac{3}{10}$

87. $\frac{2x - 5}{12} - \frac{3 - x}{6} = \frac{11}{12}$

88. $\frac{x - 2}{4} - \frac{x + 5}{6} = \frac{5x - 2}{9}$

89. $\frac{2x - 1}{4} + \frac{3x + 4}{8} = \frac{1 - 4x}{12}$

90. $-4.2(p + 3.4) = 11.13$

91. $-1.6(b - 2.35) = -11.28$

92. $0.08x + 0.06(200 - x) = 30$

93. $0.05(300 - x) + 0.07x = 45$

94. If $2x - 5(x + 1) = 7$, evaluate $x^2 - 1$.

95. If $3(2x + 1) = 5 - 2(x - 2)$, evaluate $2x^2 + 1$.

96. If $4 - 3(2x + 3) = 5 - 4x$, evaluate $x^2 - 2x$.

97. If $5 - 2(4x - 1) = 3x + 7$, evaluate $x^4 - x^2$.

98. How many times is the Distributive Property used to remove grouping symbols in solving the equation $-3[5 - 4(x - 2)] = 5(x - 5)$?

99. Which equation is equivalent to the equation in Exercise 98?

(i) $-15 + 12(x - 2) = 5x - 25$ (ii) $-3[x - 2] = 5(x - 5)$ (iii) $-3[5 - 4x - 8] = 5x - 25$ **OBJECTIVE D****To solve a literal equation for one of the variables**

For Exercises 100 to 115, solve the formula for the given variable.

100. $I = Prt$; r (Business)

101. $C = 2\pi r$; r (Geometry)

102. $PV = nRT$; R (Chemistry)

103. $A = \frac{1}{2}bh$; h (Geometry)

104. $V = \frac{1}{3}\pi r^2 h; h$ (Geometry)

105. $I = \frac{100M}{C}; M$ (Intelligence Quotient)

106. $P = 2L + 2W; W$ (Geometry)

107. $A = P + Prt; r$ (Business)

108. $s = V_0 t - 16t^2; V_0$ (Physics)

109. $s = \frac{1}{2}(a + b + c); c$ (Geometry)

110. $F = \frac{9}{5}C + 32; C$ (Temperature Conversion)

111. $S = 2\pi r^2 + 2\pi rh; h$ (Geometry)

112. $A = \frac{1}{2}h(b_1 + b_2); b_2$ (Geometry)

113. $P = \frac{R - C}{n}; R$ (Business)

114. $a_n = a_1 + (n - 1)d; d$ (Mathematics)

115. $A = P(1 + i); P$ (Business)



116. To solve the formula $I = Prt$ for P , do you divide each side of the equation by rt or subtract rt from each side of the equation?



117. To solve the formula $P = 2L + 2W$ for L , do you divide each side of the equation by $2W$ or subtract $2W$ from each side of the equation?

Applying the Concepts



118. The following is offered as the solution of the equation $5x + 15 = 2x + 3(2x + 5)$.

$$\begin{aligned} 5x + 15 &= 2x + 3(2x + 5) \\ 5x + 15 &= 2x + 6x + 15 && \bullet \text{ Use the Distributive Property.} \\ 5x + 15 &= 8x + 15 && \bullet \text{ Combine like terms.} \\ 5x + 15 - 15 &= 8x + 15 - 15 && \bullet \text{ Subtract 15 from each side.} \\ 5x &= 8x \\ \frac{5x}{x} &= \frac{8x}{x} && \bullet \text{ Divide each side by } x. \\ 5 &= 8 \end{aligned}$$

Because $5 = 8$ is not a true equation, the equation has no solution.

If this result is correct, so state. If not, explain why it is not correct and supply the correct answer.



119. Why, when the Multiplication Property of Equations is used, must the quantity that multiplies each side of the equation not be zero?

SECTION

2.2

Applications: Puzzle Problems

OBJECTIVE A

To solve integer problems 

Point of Interest

The Rhind papyrus, purchased by A. Henry Rhind in 1858, contains much of the historical evidence of ancient Egyptian mathematics. The papyrus was written in demotic script (a kind of written hieroglyphics). The key to deciphering this script is contained in the *Rosetta Stone*, which was discovered by an expedition of Napoleon's soldiers along the Rosetta branch of the Nile in 1799. This stone contains a passage written in hieroglyphics, demotic script, and Greek. By translating the Greek passage, archaeologists were able to determine how to translate demotic script, which they applied to translating the Rhind papyrus.

An equation states that two mathematical expressions are equal. Therefore, translating a sentence into an equation requires recognition of the words or phrases that mean “equals.” A partial list of these phrases includes “is,” “is equal to,” “amounts to,” and “represents.”

Once the sentence is translated into an equation, the equation can be solved by rewriting it in the form $\text{variable} = \text{constant}$.

Recall that an **even integer** is an integer that is divisible by 2. An **odd integer** is an integer that is not divisible by 2.

Consecutive integers are integers that follow one another in order. Examples of consecutive integers are shown at the right. (Assume that the variable n represents an integer.)

8, 9, 10
 $-3, -2, -1$
 $n, n + 1, n + 2$

Examples of **consecutive even integers** are shown at the right. (Assume that the variable n represents an even integer.)

16, 18, 20
 $-6, -4, -2$
 $n, n + 2, n + 4$

Examples of **consecutive odd integers** are shown at the right. (Assume that the variable n represents an odd integer.)

11, 13, 15
 $-23, -21, -19$
 $n, n + 2, n + 4$

HOW TO • 1

The sum of three consecutive even integers is seventy-eight. Find the integers.

Strategy for Solving an Integer Problem

- Let a variable represent one of the integers. Express each of the other integers in terms of that variable. Remember that for consecutive integer problems, consecutive integers differ by 1. Consecutive even or consecutive odd integers differ by 2.

First even integer: n

Second even integer: $n + 2$

Third even integer: $n + 4$

- Represent three consecutive even integers.

- Determine the relationship among the integers.

$$n + (n + 2) + (n + 4) = 78$$

$$3n + 6 = 78$$

$$3n = 72$$

$$n = 24$$

- The sum of the three even integers is 78.

- The first integer is 24.

$$n + 2 = 24 + 2 = 26$$

$$n + 4 = 24 + 4 = 28$$

- Find the second and third integers.

The three consecutive even integers are 24, 26, and 28.

EXAMPLE • 1

One number is four more than another number. The sum of the two numbers is sixty-six. Find the two numbers.

Strategy

- The smaller number: n
The larger number: $n + 4$
- The sum of the numbers is 66.

$$n + (n + 4) = 66$$

Solution

$$n + (n + 4) = 66$$

$$2n + 4 = 66$$

$$2n = 62$$

$$n = 31$$

$$n + 4 = 31 + 4 = 35$$

The numbers are 31 and 35.

- Combine like terms.
- Subtract 4 from each side.
- Divide each side by 2.

YOU TRY IT • 1

The sum of three numbers is eighty-one. The second number is twice the first number, and the third number is three less than four times the first number. Find the numbers.

Your strategy**Your solution****EXAMPLE • 2**

Five times the first of three consecutive even integers is five more than the product of four and the third integer. Find the integers.

Strategy

- First even integer: n
Second even integer: $n + 2$
Third even integer: $n + 4$
- Five times the first integer equals five more than the product of four and the third integer.

$$5n = 4(n + 4) + 5$$

Solution

$$5n = 4(n + 4) + 5$$

$$5n = 4n + 16 + 5$$

$$5n = 4n + 21$$

$$n = 21$$

- Distributive Property
- Combine like terms.
- Subtract $4n$ from each side.

Because 21 is not an even integer, there is no solution.

YOU TRY IT • 2

Find three consecutive odd integers such that three times the sum of the first two integers is ten more than the product of the third integer and four.

Your strategy**Your solution**

OBJECTIVE B**To solve coin and stamp problems**

In solving problems that deal with coins or stamps of different values, it is necessary to represent the value of the coins or stamps in the same unit of money. The unit of money is frequently cents. For example,

The value of five 8¢ stamps is $5 \cdot 8$, or 40 cents.

The value of four 20¢ stamps is $4 \cdot 20$, or 80 cents.

The value of n 10¢ stamps is $n \cdot 10$, or $10n$ cents.

HOW TO • 2

A collection of stamps consists of 5¢, 13¢, and 18¢ stamps.

The number of 13¢ stamps is two more than three times the number of 5¢ stamps.

The number of 18¢ stamps is five less than the number of 13¢ stamps. The total value of all the stamps is \$1.68. Find the number of 18¢ stamps.

Strategy for Solving a Stamp Problem

1. For each denomination of stamp, write a numerical or variable expression for the number of stamps, the value of the stamp, and the total value of the stamps in cents. The results can be recorded in a table.

The number of 5¢ stamps: x

The number of 13¢ stamps: $3x + 2$

The number of 18¢ stamps: $(3x + 2) - 5 = 3x - 3$

Stamp	Number of Stamps	•	Value of Stamp in Cents	=	Total Value in Cents
5¢	x	•	5	=	$5x$
13¢	$3x + 2$	•	13	=	$13(3x + 2)$
18¢	$3x - 3$	•	18	=	$18(3x - 3)$

2. Determine the relationships among the total values of the stamps. Use the fact that the sum of the total values of each denomination of stamp is equal to the total value of all the stamps.

The sum of the total values of each denomination of stamp is equal to the total value of all the stamps (168 cents).

$$5x + 13(3x + 2) + 18(3x - 3) = 168$$

$$5x + 39x + 26 + 54x - 54 = 168$$

$$98x - 28 = 168$$

$$98x = 196$$

$$x = 2$$

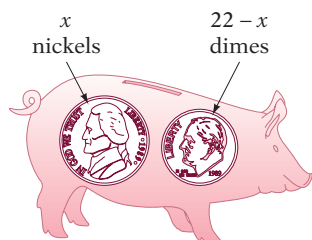
• The sum of the total values equals 168.

The number of 18¢ stamps is $3x - 3$. Replace x by 2 and evaluate.

$$3x - 3 = 3(2) - 3 = 3$$

There are three 18¢ stamps in the collection.

Some of the problems in Section 4 of the chapter “Review of Real Numbers” involved using one variable to describe two numbers whose sum was known. For example, given that the sum of two numbers is 12, we let one of the two numbers be x . Then the other number is $12 - x$. Note that the sum of these two numbers, $x + 12 - x$, equals 12.



$$x + (22 - x) = 22$$

In Example 3 below, we are told that there are only nickels and dimes in a coin bank, and that there is a total of twenty-two coins. This means that the sum of the number of nickels and the number of dimes is 22. Let the number of nickels be x . Then the number of dimes is $22 - x$. (If you let the number of dimes be x and the number of nickels be $22 - x$, the solution to the problem will be the same.)

EXAMPLE • 3

A coin bank contains \$1.80 in nickels and dimes; in all, there are twenty-two coins in the bank. Find the number of nickels and the number of dimes in the bank.

YOU TRY IT • 3

A collection of stamps contains 3¢, 10¢, and 15¢ stamps. The number of 10¢ stamps is two more than twice the number of 3¢ stamps. There are three times as many 15¢ stamps as there are 3¢ stamps. The total value of the stamps is \$1.56. Find the number of 15¢ stamps.

Strategy

- Number of nickels: x
Number of dimes: $22 - x$

Coin	Number	Value	Total Value
Nickel	x	5	$5x$
Dime	$22 - x$	10	$10(22 - x)$

- The sum of the total values of each denomination of coin equals the total value of all the coins (180 cents).

$$5x + 10(22 - x) = 180$$

Your strategy

Solution

$$\begin{aligned}
 5x + 10(22 - x) &= 180 \\
 5x + 220 - 10x &= 180 && \bullet \text{ Distributive Property} \\
 -5x + 220 &= 180 && \bullet \text{ Combine like terms.} \\
 -5x &= -40 && \bullet \text{ Subtract 220 from each side.} \\
 x &= 8 && \bullet \text{ Divide each side by } -5.
 \end{aligned}$$

$$22 - x = 22 - 8 = 14$$

The bank contains 8 nickels and 14 dimes.

Your solution

2.2 EXERCISES**OBJECTIVE A****To solve integer problems**

1. What number must be added to the numerator of $\frac{3}{10}$ to produce the fraction $\frac{4}{5}$?
2. What number must be added to the numerator of $\frac{5}{12}$ to produce the fraction $\frac{2}{3}$?
3. The sum of two integers is ten. Three times the larger integer is three less than eight times the smaller integer. Find the integers.
4. The sum of two integers is thirty. Eight times the smaller integer is six more than five times the larger integer. Find the integers.
5. One integer is eight less than another integer. The sum of the two integers is fifty. Find the integers.
6. One integer is four more than another integer. The sum of the integers is twenty-six. Find the integers.
7. The sum of three numbers is one hundred twenty-three. The second number is two more than twice the first number. The third number is five less than the product of three and the first number. Find the three numbers.
8. The sum of three numbers is forty-two. The second number is twice the first number, and the third number is three less than the second number. Find the three numbers.
9. The sum of three consecutive integers is negative fifty-seven. Find the integers.
10. The sum of three consecutive integers is one hundred twenty-nine. Find the integers.
11. Five times the smallest of three consecutive odd integers is ten more than twice the largest. Find the integers.
12. Find three consecutive even integers such that twice the sum of the first and third integers is twenty-one more than the second integer.
13. Find three consecutive odd integers such that three times the middle integer is seven more than the sum of the first and third integers.
14. Which of the following could *not* be used to represent three consecutive odd integers?

(i) $n + 1, n + 3, n + 5$	(ii) $n - 2, n, n + 2$
(iii) $n + 2, n + 4, n + 6$	(iv) $n, n + 3, n + 5$



OBJECTIVE B To solve coin and stamp problems

15. Write an expression to represent the value of n 39¢ stamps and m 41¢ stamps in
a. cents and b. dollars.
16. A collection of twenty-two coins has a value of \$4.75. The collection contains dimes and quarters. Find the number of quarters in the collection.
17. A coin bank contains twenty-two coins in nickels, dimes, and quarters. There are four times as many dimes as quarters. The value of the coins is \$2.30. How many dimes are in the bank?
18. A coin collection contains nickels, dimes, and quarters. There are twice as many dimes as quarters and seven more nickels than dimes. The total value of all the coins is \$2.00. How many quarters are in the collection?
19. A stamp collector has some 15¢ stamps and some 20¢ stamps. The number of 15¢ stamps is eight less than three times the number of 20¢ stamps. The total value is \$4. Find the number of each type of stamp in the collection.
20. An office has some 27¢ stamps and some 42¢ stamps. All together the office has 140 stamps for a total value of \$43.80. How many of each type of stamp does the office have?
21. A stamp collection consists of 3¢, 8¢, and 13¢ stamps. The number of 8¢ stamps is three less than twice the number of 3¢ stamps. The number of 13¢ stamps is twice the number of 8¢ stamps. The total value of all the stamps is \$2.53. Find the number of 3¢ stamps in the collection.
22. A stamp collector bought 330 stamps for \$79.50. The purchase included 15¢ stamps, 20¢ stamps, and 40¢ stamps. The number of 20¢ stamps is four times the number of 15¢ stamps. How many 40¢ stamps were purchased?
23. A stamp collector has 8¢, 13¢, and 18¢ stamps. The collector has twice as many 8¢ stamps as 18¢ stamps. There are three more 13¢ than 18¢ stamps. The total value of the stamps in the collection is \$3.68. Find the number of 18¢ stamps in the collection.
24. A stamp collection consists of 3¢, 12¢, and 15¢ stamps. The number of 3¢ stamps is five times the number of 12¢ stamps. The number of 15¢ stamps is four less than the number of 12¢ stamps. The total value of the stamps in the collection is \$3.18. Find the number of 15¢ stamps in the collection.

**Applying the Concepts**

25. **Integers** Find three consecutive odd integers such that the product of the second and third minus the product of the first and second is 42.
26. **Integers** The sum of the digits of a three-digit number is six. The tens digit is one less than the units digit, and the number is twelve more than one hundred times the hundreds digit. Find the number.

SECTION

2.3

Applications: Mixture and Uniform Motion Problems

OBJECTIVE A

To solve value mixture problems

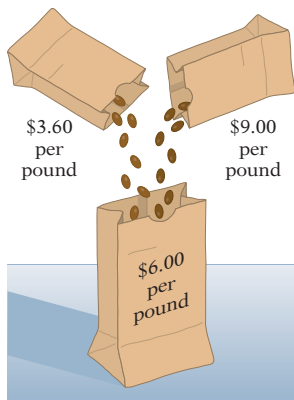
**Take Note**

The equation $AC = V$ is used to find the value of an ingredient. For example, the value of 12 lb of coffee costing \$5.25 per pound is

$$AC = V$$

$$12(\$5.25) = V$$

$$\$63 = V$$



A **value mixture problem** involves combining two ingredients that have different prices into a single blend. For example, a coffee manufacturer may blend two types of coffee into a single blend.

The solution of a value mixture problem is based on the equation $AC = V$, where A is the amount of the ingredient, C is the cost per unit of the ingredient, and V is the value of the ingredient.

HOW TO • 1

How many pounds of peanuts that cost \$3.60 per pound must be mixed with 40 lb of cashews that cost \$9.00 per pound to make a mixture that costs \$6.00 per pound?

Strategy for Solving a Value Mixture Problem

- For each ingredient in the mixture, write a numerical or variable expression for the amount of the ingredient used, the unit cost of the ingredient, and the value of the amount used. For the mixture, write a numerical or variable expression for the amount, the unit cost of the mixture, and the value of the amount. The results can be recorded in a table.

Pounds of peanuts: x
 Pounds of cashews: 40
 Pounds of mixture: $x + 40$

	Amount (A)	•	Unit Cost (C)	=	Value (V)
Peanuts	x	•	3.60	=	$3.60x$
Cashews	40	•	9.00	=	$9.00(40)$
Mixture	$x + 40$	•	6.00	=	$6.00(x + 40)$

- Determine how the values of the ingredients are related. Use the fact that the sum of the values of all the ingredients taken separately is equal to the value of the mixture.

The sum of the values of the peanuts and the cashews is equal to the value of the mixture.

$$3.60x + 9.00(40) = 6.00(x + 40)$$

$$3.60x + 360 = 6x + 240$$

$$-2.4x + 360 = 240$$

$$-2.4x = -120$$

$$x = 50$$

- Value of peanuts plus value of cashews equals value of mixture.

The mixture must contain 50 lb of peanuts.

EXAMPLE • 1

A butcher combined hamburger that costs \$3.30 per pound with hamburger that costs \$4.50 per pound. How many pounds of each were used to make a 30-pound mixture costing \$3.70 per pound?

Strategy

- Pounds of \$3.30 hamburger: x
Pounds of \$4.50 hamburger: $30 - x$
Pounds of \$3.70 mixture: 30

	<i>Amount</i>	<i>Cost</i>	<i>Value</i>
\$3.30 hamburger	x	3.30	$3.30x$
\$4.50 hamburger	$30 - x$	4.50	$4.50(30 - x)$
Mixture	30	3.70	$3.70(30)$

- The sum of the values before mixing equals the value after mixing.

$$3.30x + 4.50(30 - x) = 3.70(30)$$

Solution

$$\begin{aligned}
 3.30x + 4.50(30 - x) &= 3.70(30) \\
 3.30x + 135 - 4.50x &= 111 \\
 -1.20x + 135 &= 111 \\
 -1.20x &= -24 \\
 x &= 20
 \end{aligned}$$

The number of pounds of the \$4.50 hamburger is $30 - x$. Replace x by 20 and evaluate.

$$\begin{aligned}
 30 - x \\
 30 - 20 &= 10
 \end{aligned}$$

The butcher should use 20 pounds of the \$3.30 hamburger and 10 pounds of the \$4.50 hamburger.

YOU TRY IT • 1

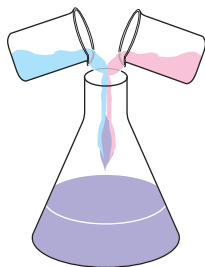
How many ounces of a gold alloy that costs \$320 per ounce must be mixed with 100 oz of an alloy that costs \$100 per ounce to make a mixture that costs \$160 per ounce?

Your strategy**Your solution**

OBJECTIVE B**To solve percent mixture problems**
Take Note

The equation $Ar = Q$ is used to find the amount of a substance in a mixture. For example, the number of grams of silver in 50 g of a 40% alloy is:

$$\begin{aligned} Ar &= Q \\ (50 \text{ g})(0.40) &= Q \\ 20 \text{ g} &= Q \end{aligned}$$



The amount of a substance in a solution or an alloy can be given as a percent of the total solution or alloy. For example, in a 10% hydrogen peroxide solution, 10% of the total solution is hydrogen peroxide. The remaining 90% is water.

The solution of a **percent mixture problem** is based on the equation $Ar = Q$, where A is the amount of solution or alloy, r is the percent of concentration, and Q is the quantity of a substance in the solution or alloy.

HOW TO • 2

A chemist mixes an 11% acid solution with a 4% acid solution. How many milliliters of each solution should the chemist use to make a 700-milliliter solution that is 6% acid?

Strategy for Solving a Percent Mixture Problem

- For each solution, use the equation $Ar = Q$. Write a numerical or variable expression for the amount of solution, the percent of concentration, and the quantity of the substance in the solution. The results can be recorded in a table.

Amount of 11% solution: x
 Amount of 4% solution: $700 - x$
 Amount of 6% mixture: 700

	Amount of Solution (A)	•	Percent of Concentration (r)	=	Quantity of Substance (Q)
11% solution	x	•	0.11	=	$0.11x$
4% solution	$700 - x$	•	0.04	=	$0.04(700 - x)$
6% solution	700	•	0.06	=	$0.06(700)$

- Determine how the quantities of the substance in each solution are related. Use the fact that the sum of the quantities of the substances being mixed is equal to the quantity of the substance after mixing.

The sum of the quantities of the substance in the 11% solution and the 4% solution is equal to the quantity of the substance in the 6% solution.

$$0.11x + 0.04(700 - x) = 0.06(700) \quad \bullet \text{ Quantity in 11\% solution plus quantity in 4\% solution equals quantity in 6\% solution.}$$

$$\begin{aligned} 0.11x + 28 - 0.04x &= 42 \\ 0.07x + 28 &= 42 \\ 0.07x &= 14 \\ x &= 200 \end{aligned}$$

The amount of 4% solution is $700 - x$. Replace x by 200 and evaluate.

$$700 - x = 700 - 200 = 500 \quad \bullet x = 200$$

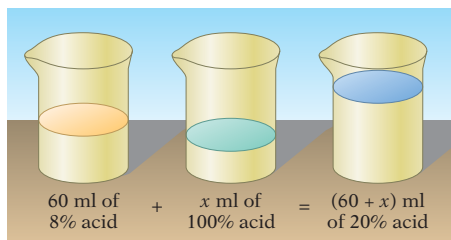
The chemist should use 200 ml of the 11% solution and 500 ml of the 4% solution.

EXAMPLE • 2

How many milliliters of pure acid must be added to 60 ml of an 8% acid solution to make a 20% acid solution?

Strategy

- Milliliters of pure acid: x



	Amount	Percent	Quantity
Pure Acid (100%)	x	1.00	x
8%	60	0.08	$0.08(60)$
20%	$60 + x$	0.20	$0.20(60 + x)$

- The sum of the quantities before mixing equals the quantity after mixing.

$$x + 0.08(60) = 0.20(60 + x)$$

Solution

$$x + 0.08(60) = 0.20(60 + x)$$

$$x + 4.8 = 12 + 0.20x$$

$$0.8x + 4.8 = 12$$

$$0.8x = 7.2$$

$$x = 9$$

- Subtract $0.20x$ from each side.
- Subtract 4.8 from each side.
- Divide each side by 0.8.

To make the 20% acid solution, 9 ml of pure acid must be used.

YOU TRY IT • 2

A butcher has some hamburger that is 22% fat and some that is 12% fat. How many pounds of each should be mixed to make 80 lb of hamburger that is 18% fat?

Your strategy**Your solution**

OBJECTIVE C**To solve uniform motion problems**

A car that travels constantly in a straight line at 55 mph is in uniform motion. **Uniform motion** means that the speed of an object does not change.

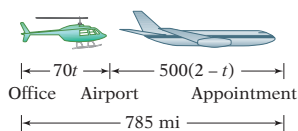
The solution of a **uniform motion problem** is based on the equation $rt = d$, where r is the rate of travel, t is the time spent traveling, and d is the distance traveled.

HOW TO 3

An executive has an appointment 785 mi from the office. The executive takes a helicopter from the office to the airport and a plane from the airport to the business appointment. The helicopter averages 70 mph and the plane averages 500 mph. The total time spent traveling is 2 h. Find the distance from the executive's office to the airport.

Strategy for Solving a Uniform Motion Problem

- For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table. It may also help to draw a diagram.



Unknown time in the helicopter: t

Time in the plane: $2 - t$

	Rate (r)	•	Time (t)	=	Distance (d)
Helicopter	70	•	t	=	$70t$
Plane	500	•	$2 - t$	=	$500(2 - t)$

- Determine how the distances traveled by each object are related. For example, the total distance traveled by both objects may be known, or it may be known that the two objects traveled the same distance.

The total distance traveled is 785 mi.

$$70t + 500(2 - t) = 785$$

• Distance by helicopter plus distance by plane equals 785.

$$70t + 1000 - 500t = 785$$

$$-430t + 1000 = 785$$

$$-430t = -215$$

$$t = 0.5$$

The time spent traveling from the office to the airport in the helicopter is 0.5 h. To find the distance between these two points, substitute the values of r and t into the equation $rt = d$.

$$rt = d$$

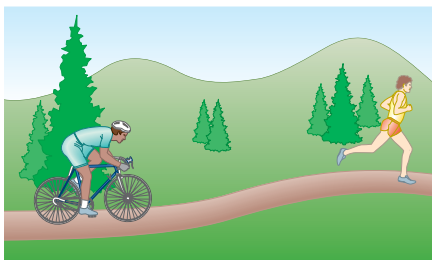
$$70 \cdot 0.5 = d \quad \bullet r = 70; t = 0.5$$

$$35 = d$$

The distance from the office to the airport is 35 mi.

EXAMPLE 3

A long-distance runner started a course running at an average speed of 6 mph. Twenty minutes later, a cyclist traveled the same course at an average speed of 10 mph. How long, in minutes, after the runner started did the cyclist overtake the runner?

Strategy

- Because the rates are given in miles per hour, let t be the time, in hours, for the cyclist.

Time for the runner: $t + \frac{1}{3}$ $\left(20 \text{ min} = \frac{1}{3} \text{ h}\right)$

	Rate	Time	Distance
Runner	6	$t + \frac{1}{3}$	$6\left(t + \frac{1}{3}\right)$
Cyclist	10	t	$10t$

- The runner and the cyclist travel the same distance.

$$6\left(t + \frac{1}{3}\right) = 10t$$

Solution

$$6\left(t + \frac{1}{3}\right) = 10t$$

$$6t + 2 = 10t \quad \bullet \text{ Distributive Property}$$

$$2 = 4t \quad \bullet \text{ Subtract } 6t \text{ from each side.}$$

$$\frac{1}{2} = t \quad \bullet \text{ Divide each side by 4.}$$

The cyclist traveled for $\frac{1}{2}$ h.

To find the time for the cyclist to overtake the runner,

evaluate $t + \frac{1}{3}$ when $t = \frac{1}{2}$.

$$t + \frac{1}{3} = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Because the problem asks for the answer in minutes,

convert $\frac{5}{6}$ h to minutes: $\frac{5}{6} \text{ h} = 50 \text{ min}$

The cyclist overtook the runner 50 min after the runner started.

YOU TRY IT 3

Two small planes start from the same point and fly in opposite directions. The first plane is flying 30 mph faster than the second plane. In 4 h the planes are 1160 mi apart. Find the rate of each plane.

Your strategy**Your solution**

2.3 EXERCISES

OBJECTIVE A

To solve value mixture problems



1. A coffee merchant mixes a dark roast coffee that costs \$10 per pound with a light roast coffee that costs \$7 per pound. Assuming the merchant wants to make a profit, which of the following are *not* possible answers for the cost per pound of the mixture? There may be more than one correct answer.

(i) \$9.40 (ii) \$7.60 (iii) \$11.00 (iv) \$6.50 (v) \$8.50



2. A snack mix is made from peanuts that cost \$3 per pound and caramel popcorn that costs \$2.20 per pound. If the mixture costs \$2.50 per pound, does the mixture contain more peanuts or more popcorn?

3. Forty pounds of cashews costing \$9.20 per pound were mixed with 100 lb of peanuts costing \$3.32 per pound. Find the cost of the resulting mixture.

4. A coffee merchant combines coffee costing \$6 per pound with coffee costing \$3.50 per pound. How many pounds of each should be used to make 25 lb of a blend costing \$5.25 per pound?

5. Adult tickets for a play cost \$10.00, and children's tickets cost \$4.00. For one performance, 460 tickets were sold. Receipts for the performance were \$3760. Find the number of adult tickets sold.

6. Tickets for a school play sold for \$7.50 for each adult and \$3.00 for each child. The total receipts for 113 tickets sold were \$663. Find the number of adult tickets sold.

7. A restaurant manager mixes 5 L of pure maple syrup that costs \$9.50 per liter with imitation maple syrup that costs \$4.00 per liter. How much imitation maple syrup is needed to make a mixture that costs \$5.00 per liter?

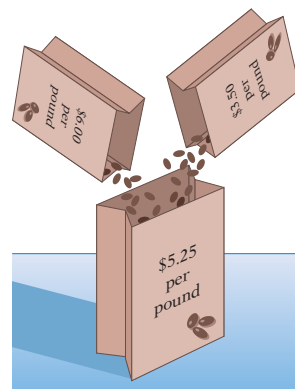
8. Succotash is made by combining corn with lima beans and costs \$1.00 per pound. If lima beans cost \$1.10 per pound and corn costs \$.60 per pound, how many pounds of each should be used to make 5 lb of succotash?

9. A goldsmith combined pure gold that cost \$890 per ounce with an alloy of gold that cost \$360 per ounce. How many ounces of each were used to make 50 oz of gold alloy costing \$519 per ounce?

10. A silversmith combined pure silver that cost \$17.80 per ounce with 50 oz of a silver alloy that cost \$8.50 per ounce. How many ounces of pure silver were used to make an alloy of silver costing \$15.30 per ounce?

11. A tea mixture was made from 40 lb of tea costing \$5.40 per pound and 60 lb of tea costing \$3.25 per pound. Find the cost of the tea mixture.

12. Find the cost per ounce of a sunscreen made from 100 oz of lotion that cost \$3.46 per ounce and 60 oz of lotion that cost \$12.50 per ounce.



13. The owner of a fruit stand combined cranberry juice that cost \$28.50 per gallon with 20 gal of apple juice that cost \$11.25 per gallon. How much cranberry juice was used to make the cranapple juice if the mixture cost \$17.00 per gallon?
14. Pecans that cost \$28.50 per kilogram were mixed with almonds that cost \$22.25 per kilogram. How many kilograms of each were used to make a 25-kilogram mixture costing \$24.25 per kilogram?

OBJECTIVE B**To solve percent mixture problems**

15. A 30% salt solution is mixed with a 50% salt solution. Which of the following are *not* possible for the percent concentration of the resulting solution? There may be more than one correct answer.
(i) 38.7% (ii) 30% (iii) 25.8% (iv) 80% (v) 50%



16. A 20% acid solution is mixed with a 60% acid solution. If the resulting solution is a 42% acid solution, which of the following statements is true?
(i) More 20% acid solution was used than 60% acid solution.
(ii) More 60% acid solution was used than 20% acid solution.
(iii) The same amount of each acid solution was used.
(iv) There is not enough information to answer the question.
17. How many pounds of a 15% aluminum alloy must be mixed with 500 lb of a 22% aluminum alloy to make a 20% aluminum alloy?
18. A hospital staff mixed a 75% disinfectant solution with a 25% disinfectant solution. How many liters of each were used to make 20 L of a 40% disinfectant solution?
19. Rubbing alcohol is typically diluted with water to 70% strength. If you need 3.5 oz of 45% rubbing alcohol, how many ounces of 70% rubbing alcohol and how much water should you combine?
20. A silversmith mixed 25 g of a 70% silver alloy with 50 g of a 15% silver alloy. What is the percent concentration of the resulting alloy?
21. How many ounces of pure water must be added to 75 oz of an 8% salt solution to make a 5% salt solution?
22. How many quarts of water must be added to 5 qt of an 80% antifreeze solution to make a 50% antifreeze solution?
23. How many milliliters of alcohol must be added to 200 ml of a 25% iodine solution to make a 10% iodine solution?
24. A butcher has some hamburger that is 21% fat and some that is 15% fat. How many pounds of each should be mixed to make 84 lb of hamburger that is 17% fat?
25. Many fruit drinks are actually only 5% real fruit juice. If you let 2 oz of water evaporate from 12 oz of a drink that is 5% fruit juice, what is the percent concentration of the remaining fruit drink?

26. How much water must be evaporated from 6 qt of a 50% antifreeze solution to produce a 75% solution?
27. A car radiator contains 12 qt of a 40% antifreeze solution. How many quarts will have to be replaced with pure antifreeze if the resulting solution is to be 60% antifreeze?

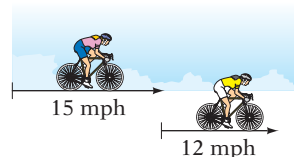
OBJECTIVE C**To solve uniform motion problems**

28. Mike and Mindy live 3 mi apart. They leave their houses at the same time and walk toward each other until they meet. Mindy walks faster than Mike.
- Is the distance Mike walks less than, equal to, or greater than the distance Mindy walks?
 - Is the time spent walking by Mike less than, equal to, or greater than the time spent walking by Mindy?
 - What is the total distance traveled by Mike and Mindy?



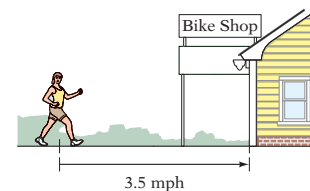
29. Eric and Ramona ride their bikes from Eric's house to school. Ramona begins biking 10 min before Eric begins. Eric bikes faster than Ramona and catches up with her just as they reach school.
- Is the distance Ramona bikes less than, equal to, or greater than the distance Eric bikes?
 - Is the time spent biking by Ramona less than, equal to, or greater than the time spent biking by Eric?

30. Angela leaves from Jocelyn's house on her bicycle traveling at 12 mph. Ten minutes later, Jocelyn leaves her house on her bicycle traveling at 15 mph to catch up with Angela. How long, in minutes, does it take Jocelyn to reach Angela?
31. A speeding car traveling at 80 mph passes a police officer. Ten seconds later, the police officer gives chase at a speed of 100 mph. How long, in minutes, does it take the police officer to catch up with the car?

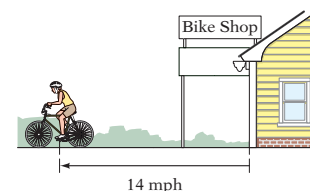


32. Two planes are 1620 mi apart and are traveling toward each other. One plane is traveling 120 mph faster than the other plane. The planes meet in 1.5 h. Find the speed of each plane.
33. Two cars are 310 mi apart and are traveling toward each other. One car travels 8 mph faster than the other car. The cars meet in 2.5 h. Find the speed of each car.

34. A ferry leaves a harbor and travels to a resort island at an average speed of 20 mph. On the return trip, the ferry travels at an average speed of 12 mph because of fog. The total time for the trip is 5 h. How far is the island from the harbor?

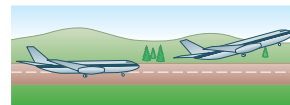


35. A commuter plane provides transportation from an international airport to the surrounding cities. One commuter plane averaged 250 mph flying to a city and 150 mph returning to the international airport. The total flying time was 4 h. Find the distance between the two airports.



36. Hana walked from her home to a bicycle repair shop at a rate of 3.5 mph and then rode her bicycle back home at a rate of 14 mph. If the total time spent traveling was 1 h, how far from Hana's home is the repair shop?

37. A passenger train leaves a depot 1.5 h after a freight train leaves the same depot. The passenger train is traveling 18 mph faster than the freight train. Find the rate of each train if the passenger train overtakes the freight train in 2.5 h.
38. A plane leaves an airport at 3 P.M. At 4 P.M. another plane leaves the same airport traveling in the same direction at a speed 150 mph faster than that of the first plane. Four hours after the first plane takes off, the second plane is 250 mi ahead of the first plane. How far does the second plane travel?
39. A jogger and a cyclist set out at 9 A.M. from the same point headed in the same direction. The average speed of the cyclist is four times the average speed of the jogger. In 2 h, the cyclist is 33 mi ahead of the jogger. How far did the cyclist ride?



Applying the Concepts

40. Uniform Motion

- If a parade 2 mi long is proceeding at 3 mph, how long will it take a runner jogging at 6 mph to travel from the front of the parade to the end of the parade?
- If a parade 2 mi long is proceeding at 3 mph, how long will it take a runner jogging at 6 mph to travel from the end of the parade to the start of the parade?



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41. **Mixtures** The concentration of gold in an alloy is measured in karats, which indicate how many parts out of 24 are pure gold. For example, 1 karat is $\frac{1}{24}$ pure gold. What amount of 12-karat gold should be mixed with 3 oz of 24-karat gold to create 14-karat gold, the most commonly used alloy?



42. **Uniform Motion** A student jogs 1 mi at a rate of 8 mph and jogs back at a rate of 6 mph. Does it seem reasonable that the average rate is 7 mph? Why or why not? Support your answer.
43. **Uniform Motion** Two cars are headed directly toward each other at rates of 40 mph and 60 mph. How many miles apart are they 2 min before impact?
44. **Mixtures**
- A radiator contains 6 qt of a 25% antifreeze solution. How much should be removed and replaced with pure antifreeze to yield a 33% solution?
 - A radiator contains 6 qt of a 25% antifreeze solution. How much should be removed and replaced with pure antifreeze to yield a 60% solution?

SECTION

2.4

First-Degree Inequalities

OBJECTIVE A

To solve an inequality in one variable



The **solution set of an inequality** is a set of numbers, each element of which, when substituted for the variable, results in a true inequality.

The inequality at the right is true if the variable is replaced

by (for instance) 3, -1.98 , or $\frac{2}{3}$.

$$\begin{aligned}x - 1 &< 4 \\3 - 1 &< 4 \\-1.98 - 1 &< 4 \\\frac{2}{3} - 1 &< 4\end{aligned}$$

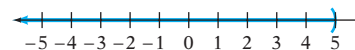


Integrating Technology

See the Keystroke Guide: Test for instructions on using a graphing calculator to graph the solution set of an inequality.

There are many values of the variable x that will make the inequality $x - 1 < 4$ true. The solution set of the inequality is any number less than 5. The solution set can be written in set-builder notation as $\{x | x < 5\}$.

The graph of the solution set of $x - 1 < 4$ is shown at the right.



When solving an inequality, we use the **Addition and Multiplication Properties of Inequalities** to rewrite the inequality in the form *variable* $<$ *constant* or in the form *variable* $>$ *constant*.

Addition Property of Inequalities

The same term can be added to each side of an inequality without changing the solution set of the inequality. Symbolically, this is written

$$\text{If } a < b, \text{ then } a + c < b + c.$$

$$\text{If } a > b, \text{ then } a + c > b + c.$$

This property is also true for an inequality that contains $\leq$ or $\geq$.

The Addition Property of Inequalities is used to remove a term from one side of an inequality by adding the additive inverse of that term to each side of the inequality. Because subtraction is defined in terms of addition, the same number can be subtracted from each side of an inequality without changing the solution set of the inequality.



Take Note

The solution set of an inequality can be written in set-builder notation or in interval notation.

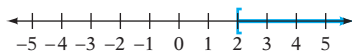
HOW TO 1

Solve and graph the solution set: $x + 2 \geq 4$

$$\begin{aligned}x + 2 &\geq 4 \\x + 2 - 2 &\geq 4 - 2 \\x &\geq 2\end{aligned}$$

- Subtract 2 from each side of the inequality.
- Simplify.

The solution set is $\{x | x \geq 2\}$ or $[2, \infty)$.



HOW TO 2 Solve: $3x - 4 < 2x - 1$

Write the solution set in set-builder notation.

$$3x - 4 < 2x - 1$$

$$3x - 4 - 2x < 2x - 1 - 2x$$

• Subtract $2x$ from each side of the inequality.

$$x - 4 < -1$$

$$x - 4 + 4 < -1 + 4$$

• Add 4 to each side of the inequality.

$$x < 3$$

The solution set is $\{x | x < 3\}$.

Care must be taken when multiplying each side of an inequality by a nonzero constant. The rule is different when multiplying each side by a *positive* number than when multiplying each side by a *negative* number.

**Take Note**

$c > 0$ means c is a positive number. Note that the inequality symbols do not change.

$c < 0$ means c is a negative number. Note that the inequality symbols are reversed.

Multiplication Property of Inequalities

Rule 1 Each side of an inequality can be multiplied by the same *positive* constant without changing the solution set of the inequality. Symbolically, this is written

$$\text{If } a < b \text{ and } c > 0, \text{ then } ac < bc.$$

$$\text{If } a > b \text{ and } c > 0, \text{ then } ac > bc.$$

Rule 2 If each side of an inequality is multiplied by the same *negative* constant and the inequality symbol is reversed, then the solution set of the inequality is not changed. Symbolically, this is written

$$\text{If } a < b \text{ and } c < 0, \text{ then } ac > bc.$$

$$\text{If } a > b \text{ and } c < 0, \text{ then } ac < bc.$$

This property is also true for an inequality that contains $\leq$ or $\geq$.

Here are examples of this property.

Rule 1: Multiply by a *positive* number.

$$2 < 5$$

$$2(4) < 5(4)$$

$$8 < 20$$

$$3 > 2$$

$$3(4) > 2(4)$$

$$12 > 8$$

Rule 2: Multiply by a *negative* number.

$$2 < 5$$

$$2(-4) > 5(-4)$$

$$-8 > -20$$

$$3 > 2$$

$$3(-4) < 2(-4)$$

$$-12 < -8$$

The Multiplication Property of Inequalities is used to remove a coefficient from one side of an inequality by multiplying each side of the inequality by the reciprocal of the coefficient.

**Take Note**

Each side of the inequality is divided by a negative number; the inequality symbol must be reversed.

HOW TO 3 Solve: $-3x > 9$

Write the solution set in interval notation.

$$-3x > 9$$

$$\frac{-3x}{-3} < \frac{9}{-3}$$

• Divide each side of the inequality by the coefficient -3 . Because -3 is a negative number, the inequality symbol must be reversed.

$$x < -3$$

The solution set is $(-\infty, -3)$.

HOW TO • 4 Solve: $3x + 2 < -4$

Write the solution set in set-builder notation.

$$3x + 2 < -4$$

$$3x < -6$$

$$\frac{3x}{3} < \frac{-6}{3}$$

$$x < -2$$

- Subtract 2 from each side of the inequality.
- Divide each side of the inequality by the coefficient 3.
- Because 3 is a positive number, the inequality symbol remains the same.

The solution set is $\{x | x < -2\}$.**HOW TO • 5** Solve: $2x - 9 > 4x + 5$

Write the solution set in set-builder notation.

$$2x - 9 > 4x + 5$$

$$-2x - 9 > 5$$

$$-2x > 14$$

$$\frac{-2x}{-2} < \frac{14}{-2}$$

$$x < -7$$

- Subtract 4x from each side of the inequality.
- Add 9 to each side of the inequality.
- Divide each side of the inequality by the coefficient -2. Because -2 is a negative number, reverse the inequality symbol.

The solution set is $\{x | x < -7\}$.**HOW TO • 6** Solve: $5(x - 2) \geq 9x - 3(2x - 4)$

Write the solution set in interval notation.

$$5(x - 2) \geq 9x - 3(2x - 4)$$

$$5x - 10 \geq 9x - 6x + 12$$

$$5x - 10 \geq 3x + 12$$

$$2x - 10 \geq 12$$

$$2x \geq 22$$

$$\frac{2x}{2} \geq \frac{22}{2}$$

$$x \geq 11$$

- Use the Distributive Property to remove parentheses.
- Subtract 3x from each side of the inequality.
- Add 10 to each side of the inequality.
- Divide each side of the inequality by the coefficient 2.

The solution set is $[11, \infty)$.**EXAMPLE • 1**Solve and graph the solution set: $\frac{1}{6} - \frac{3}{4}x > \frac{11}{12}$

Write the solution set in set-builder notation.

Solution

$$\frac{1}{6} - \frac{3}{4}x > \frac{11}{12}$$

$$12\left(\frac{1}{6}\right) - 12\left(\frac{3}{4}x\right) > 12\left(\frac{11}{12}\right)$$

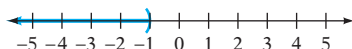
• Clear fractions by multiplying each side of the inequality by 12.

$$2 - 9x > 11$$

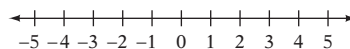
$$-9x > 9$$

$$\frac{-9x}{-9} < \frac{9}{-9}$$

$$x < -1$$

The solution set is $\{x | x < -1\}$.**YOU TRY IT • 1**Solve and graph the solution set: $2x - 1 < 6x + 7$

Write the solution set in set-builder notation.

Your solution

EXAMPLE • 2

Solve: $3x - 5 \leq 3 - 2(3x + 1)$
Write the solution set in interval notation.

Solution

$$\begin{aligned} 3x - 5 &\leq 3 - 2(3x + 1) \\ 3x - 5 &\leq 3 - 6x - 2 \\ 3x - 5 &\leq 1 - 6x \\ 9x - 5 &\leq 1 \\ 9x &\leq 6 \\ \frac{9x}{9} &\leq \frac{6}{9} \\ x &\leq \frac{2}{3} \end{aligned}$$

$$\left(-\infty, \frac{2}{3}\right]$$

YOU TRY IT • 2

Solve: $5x - 2 \leq 4 - 3(x - 2)$
Write the solution set in interval notation.

Your solution

Solution on p. S5

OBJECTIVE B**To solve a compound inequality**

A **compound inequality** is formed by joining two inequalities with a connective word such as *and* or *or*. The inequalities at the right are compound inequalities.

$$2x < 4 \text{ and } 3x - 2 > -8$$

$$2x + 3 > 5 \text{ or } x + 2 < 5$$

The solution set of a compound inequality with the connective word *and* is the set of all elements that are common to the solution sets of both inequalities. Therefore, it is the intersection of the solution sets of the two inequalities.

HOW TO • 7Solve: $2x < 6$ and $3x + 2 > -4$

$$\begin{array}{ll} 2x < 6 & \text{and} \quad 3x + 2 > -4 \\ x < 3 & 3x > -6 \\ \{x | x < 3\} & x > -2 \\ & \{x | x > -2\} \end{array}$$

• Solve each inequality.

The solution set of a compound inequality with *and* is the intersection of the solution sets of the two inequalities.

$$\{x | x < 3\} \cap \{x | x > -2\} = \{x | -2 < x < 3\} \text{ or, in interval notation, } (-2, 3).$$

HOW TO • 8Solve: $-3 < 2x + 1 < 5$

This inequality is equivalent to the compound inequality $-3 < 2x + 1$ and $2x + 1 < 5$.

$$\begin{array}{ll} -3 < 2x + 1 & \text{and} \quad 2x + 1 < 5 \\ -4 < 2x & 2x < 4 \\ -2 < x & x < 2 \\ \{x | x > -2\} & \{x | x < 2\} \end{array}$$

• Solve each inequality.

The solution set of a compound inequality with *and* is the intersection of the solution sets of the two inequalities.

$$\{x | x > -2\} \cap \{x | x < 2\} = \{x | -2 < x < 2\} \text{ or, in interval notation, } (-2, 2).$$

There is an alternative method for solving the inequality in the last example.

HOW TO • 9 Solve: $-3 < 2x + 1 < 5$

$$-3 < 2x + 1 < 5$$

$$-3 - 1 < 2x + 1 - 1 < 5 - 1$$

$$-4 < 2x < 4$$

$$\frac{-4}{2} < \frac{2x}{2} < \frac{4}{2}$$

$$-2 < x < 2$$

• **Subtract 1** from each of the three parts of the inequality.

• **Divide** each of the three parts of the inequality by the coefficient **2**.

The solution set is $\{x \mid -2 < x < 2\}$ or, in interval notation, $(-2, 2)$.

The solution set of a compound inequality with the connective word *or* is the union of the solution sets of the two inequalities.

HOW TO • 10 Solve: $2x + 3 > 7$ or $4x - 1 < 3$

$$2x + 3 > 7$$

or

$$4x - 1 < 3$$

$$2x > 4$$

$$4x < 4$$

$$x > 2$$

$$x < 1$$

$$\{x \mid x > 2\}$$

$$\{x \mid x < 1\}$$

• **Solve each inequality.**

Find the union of the solution sets.

$\{x \mid x > 2\} \cup \{x \mid x < 1\} = \{x \mid x > 2 \text{ or } x < 1\}$ or, in interval notation, $(-\infty, 1) \cup (2, \infty)$.

EXAMPLE • 3

Solve: $1 < 3x - 5 < 4$

Write the solution set in interval notation.

Solution

$$1 < 3x - 5 < 4$$

$$1 + 5 < 3x - 5 + 5 < 4 + 5$$

$$6 < 3x < 9$$

$$\frac{6}{3} < \frac{3x}{3} < \frac{9}{3}$$

$$2 < x < 3$$

$(2, 3)$

• **Add 5** to each of the three parts.

• **Divide** each of the three parts by **3**.

YOU TRY IT • 3

Solve: $-2 \leq 5x + 3 \leq 13$

Write the solution set in interval notation.

Your solution

EXAMPLE • 4

Solve: $11 - 2x > -3$ and $7 - 3x < 4$

Write the solution set in set-builder notation.

Solution

$$11 - 2x > -3 \quad \text{and} \quad 7 - 3x < 4$$

$$-2x > -14$$

$$-3x < -3$$

$$x < 7$$

$$x > 1$$

$$\{x \mid x < 7\}$$

$$\{x \mid x > 1\}$$

$$\{x \mid x < 7\} \cap \{x \mid x > 1\} = \{x \mid 1 < x < 7\}$$

YOU TRY IT • 4

Solve: $2 - 3x > 11$ or $5 + 2x > 7$

Write the solution set in set-builder notation.

Your solution

OBJECTIVE C**To solve application problems****EXAMPLE • 5**

A cellular phone company advertises two pricing plans. The first costs \$19.95 per month with 20 free minutes and \$.39 per minute thereafter. The second costs \$23.95 per month with 20 free minutes and \$.30 per minute thereafter. How many minutes can you talk per month for the first plan to cost less than the second?

Strategy

To find the number of minutes, write and solve an inequality using N to represent the number of minutes. Then $N - 20$ is the number of minutes for which you are charged after the first free 20 min.

Solution

$$\begin{aligned}
 &\text{Cost of first plan} < \text{cost of second plan} \\
 19.95 + 0.39(N - 20) &< 23.95 + 0.30(N - 20) \\
 19.95 + 0.39N - 7.8 &< 23.95 + 0.30N - 6 \\
 12.15 + 0.39N &< 17.95 + 0.30N \\
 12.15 + 0.09N &< 17.95 \\
 0.09N &< 5.8 \\
 N &< 64.\overline{4}
 \end{aligned}$$

The first plan costs less if you talk less than 65 min.

EXAMPLE • 6

Find three consecutive positive odd integers whose sum is between 27 and 51.

Strategy

To find the three integers, write and solve a compound inequality using n to represent the first odd integer.

Solution

$$\begin{aligned}
 &\text{Lower limit of the sum} < \text{sum} < \text{upper limit of the sum} \\
 27 &< n + (n + 2) + (n + 4) < 51 \\
 27 &< 3n + 6 < 51 \\
 27 - 6 &< 3n + 6 - 6 < 51 - 6 \\
 21 &< 3n < 45 \\
 \frac{21}{3} &< \frac{3n}{3} < \frac{45}{3} \\
 7 &< n < 15
 \end{aligned}$$

The three odd integers are 9, 11, and 13; or 11, 13, and 15; or 13, 15, and 17.

YOU TRY IT • 5

The base of a triangle is 12 in., and the height is $(x + 2)$ in. Express as an integer the maximum height of the triangle when the area is less than 50 in².

Your strategy**Your solution****YOU TRY IT • 6**

An average score of 80 to 89 in a history course receives a B. Luisa Montez has grades of 72, 94, 83, and 70 on four exams. Find the range of scores on the fifth exam that will give Luisa a B for the course.

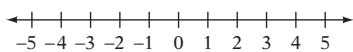
Your strategy**Your solution**

2.4 EXERCISES**OBJECTIVE A****To solve an inequality in one variable**

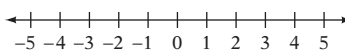
-  1. State the Addition Property of Inequalities, and give numerical examples of its use.
-  2. State the Multiplication Property of Inequalities, and give numerical examples of its use.
3. Which numbers are solutions of the inequality $x + 7 \leq -3$?
(i) -17 (ii) 8 (iii) -10 (iv) 0
4. Which numbers are solutions of the inequality $2x - 1 > 5$?
(i) 6 (ii) -4 (iii) 3 (iv) 5

For Exercises 5 to 31, solve. Write the solution in set-builder notation. For Exercises 5 to 10, graph the solution set.

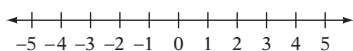
5. $x - 3 < 2$



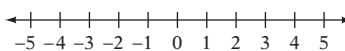
6. $x + 4 \geq 2$



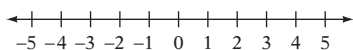
7. $4x \leq 8$



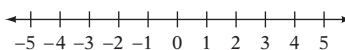
8. $6x > 12$



9. $-2x > 8$



10. $-3x \leq -9$



11. $3x - 1 > 2x + 2$

12. $5x + 2 \geq 4x - 1$

13. $2x - 1 > 7$

14. $3x + 2 < 8$

15. $5x - 2 \leq 8$

16. $4x + 3 \leq -1$

17. $6x + 3 > 4x - 1$

18. $7x + 4 < 2x - 6$

19. $8x + 1 \geq 2x + 13$

20. $5x - 4 < 2x + 5$

21. $4 - 3x < 10$

22. $2 - 5x > 7$

23. $7 - 2x \geq 1$

24. $3 - 5x \leq 18$

25. $-3 - 4x > -11$

26. $-2 - x < 7$

27. $4x - 2 < x - 11$

28. $6x + 5 \leq x - 10$

29. $x + 7 \geq 4x - 8$

30. $3x + 1 \leq 7x - 15$

31. $3x + 2 \leq 7x + 4$



For Exercises 32 to 35, state whether the solution set of an inequality of the given form contains only negative numbers, only positive numbers, or both positive and negative numbers.

32. $x + n > a$, where both n and a are positive, and $n < a$
33. $nx > a$, where both n and a are negative
34. $nx > a$, where n is negative and a is positive
35. $x - n > -a$, where both n and a are positive, and $n < a$

For Exercises 36 to 51, solve. Write the solution in interval notation.

36. $3x - 5 \geq -2x + 5$
37. $\frac{3}{5}x - 2 < \frac{3}{10} - x$
38. $\frac{5}{6}x - \frac{1}{6} < x - 4$
39. $\frac{2}{3}x - \frac{3}{2} < \frac{7}{6} - \frac{1}{3}x$
40. $\frac{7}{12}x - \frac{3}{2} < \frac{2}{3}x + \frac{5}{6}$
41. $\frac{1}{2}x - \frac{3}{4} < \frac{7}{4}x - 2$
42. $6 - 2(x - 4) \leq 2x + 10$
43. $4(2x - 1) > 3x - 2(3x - 5)$
44. $2(1 - 3x) - 4 > 10 + 3(1 - x)$
45. $2 - 5(x + 1) \geq 3(x - 1) - 8$
46. $2 - 2(7 - 2x) < 3(3 - x)$
47. $3 + 2(x + 5) \geq x + 5(x + 1) + 1$
48. $10 - 13(2 - x) < 5(3x - 2)$
49. $3 - 4(x + 2) \leq 6 + 4(2x + 1)$
50. $3x - 2(3x - 5) \leq 2 - 5(x - 4)$
51. $12 - 2(3x - 2) \geq 5x - 2(5 - x)$

OBJECTIVE B**To solve a compound inequality**

52. a. Which set operation is used when a compound inequality is combined with *or*?
b. Which set operation is used when a compound inequality is combined with *and*?



53. Explain why the inequality $-3 > x > 4$ does not make sense.

For Exercises 54 to 67, solve. Write the solution set in interval notation.

54. $3x < 6$ and $x + 2 > 1$
55. $x - 3 \leq 1$ and $2x \geq -4$
56. $x + 2 \geq 5$ or $3x \leq 3$
57. $2x < 6$ or $x - 4 > 1$

58. $-2x > -8$ and $-3x < 6$

59. $\frac{1}{2}x > -2$ and $5x < 10$

60. $\frac{1}{3}x < -1$ or $2x > 0$

61. $\frac{2}{3}x > 4$ or $2x < -8$

62. $x + 4 \geq 5$ and $2x \geq 6$

63. $3x < -9$ and $x - 2 < 2$

64. $-5x > 10$ and $x + 1 > 6$

65. $2x - 3 > 1$ and $3x - 1 < 2$

66. $7x < 14$ and $1 - x < 4$

67. $4x + 1 < 5$ and $4x + 7 > -1$



For Exercises 68 to 71, state whether the inequality describes the empty set, all real numbers, two intervals of real numbers, or one interval of real numbers.

68. $x > -3$ and $x > 2$

69. $x > -3$ or $x < 2$

70. $x < -3$ and $x > 2$

71. $x < -3$ or $x > 2$

For Exercises 72 to 91, solve. Write the solution set in set-builder notation.

72. $3x + 7 < 10$ or $2x - 1 > 5$

73. $6x - 2 < -14$ or $5x + 1 > 11$

74. $-5 < 3x + 4 < 16$

75. $5 < 4x - 3 < 21$

76. $0 < 2x - 6 < 4$

77. $-2 < 3x + 7 < 1$

78. $4x - 1 > 11$ or $4x - 1 \leq -11$

79. $3x - 5 > 10$ or $3x - 5 < -10$

80. $9x - 2 < 7$ and $3x - 5 > 10$

81. $8x + 2 \leq -14$ and $4x - 2 > 10$

82. $3x - 11 < 4$ or $4x + 9 \geq 1$

83. $5x + 12 \geq 2$ or $7x - 1 \leq 13$

84. $-6 \leq 5x + 14 \leq 24$

85. $3 \leq 7x - 14 \leq 31$

86. $3 - 2x > 7$ and $5x + 2 > -18$

87. $1 - 3x < 16$ and $1 - 3x > -16$

88. $5 - 4x > 21$ or $7x - 2 > 19$

89. $6x + 5 < -1$ or $1 - 2x < 7$

90. $3 - 7x \leq 31$ and $5 - 4x > 1$

91. $9 - x \geq 7$ and $9 - 2x < 3$

OBJECTIVE C**To solve application problems**

Exercises 92 to 95 make statements about temperatures t on a particular day. Match each statement with one of the following inequalities. Some inequalities may be used more than once.

$t > 21$

$t < 21$

$t \geq 21$

$t \leq 21$

$21 \leq t \leq 42$

$t < 42$

$t > 42$

$t \leq 42$

$t \geq 42$

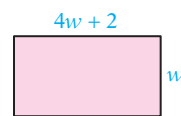
$21 < t < 42$

92. The low temperature was 21°F .93. The temperature did not go above 42°F .94. The temperature ranged from 21°F to 42°F .95. The high temperature was 42°F .

96. **Integers** Five times the difference between a number and two is greater than the quotient of two times the number and three. Find the smallest integer that will satisfy the inequality.

97. **Integers** Two times the difference between a number and eight is less than or equal to five times the sum of the number and four. Find the smallest number that will satisfy the inequality.

98. **Geometry** The length of a rectangle is 2 ft more than four times the width. Express as an integer the maximum width of the rectangle when the perimeter is less than 34 ft.



99. **Geometry** The length of a rectangle is 5 cm less than twice the width. Express as an integer the maximum width of the rectangle when the perimeter is less than 60 cm.



- 100. Aquariums** The following is a rule-of-thumb for making sure fish kept in an aquarium are not too crowded: The surface area of the water should be at least 12 times the total length of fish kept in the aquarium. (*Source:* www.tacomapet.com) Your 10-gallon aquarium has a water surface area of 288 in² and houses the following fish: one 2-inch odessa barb, three 1-inch gold tetra, three 1.75-inch cobra guppies, and five 1-inch neon tetra.

- Find the total length of all the fish in your aquarium.
- Write and solve an inequality to find the greatest number n of 2-inch black hatchetfish that you can safely add to your aquarium without overcrowding the fish.

- 101. Advertising** To run an advertisement on a certain website, the website owner charges a setup fee of \$250 and \$12 per day to display the advertisement. If a marketing group has a budget of \$1500 for an advertisement, what is the maximum number of days the advertisement can run on the site?

- 102. Consumerism** The entry fee to a state fair is \$25 and includes five tickets for carnival rides at the fair. Additional tickets for carnival rides cost \$1.50 each. If Alisha wants to spend a maximum of \$45 for the entry fee and rides, how many additional carnival ride tickets can she purchase?



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- 103. Consumerism** A homeowner has a budget of \$100 to paint a room that has 320 ft² of wall space. Drop cloths, masking tape, and paint brushes cost \$24. If 1 gal of paint will cover 100 ft² of wall space, what is the maximum cost per gallon of paint that the homeowner can pay?

- 104. Temperature** The temperature range for a week was between 14°F and 77°F. Find the temperature range in degrees Celsius. $F = \frac{9}{5}C + 32$

- 105. Temperature** The temperature range for a week in a mountain town was between 0°C and 30°C. Find the temperature range in degrees Fahrenheit.

$$C = \frac{5(F - 32)}{9}$$

- 106. Compensation** You are a sales account executive earning \$1200 per month plus 6% commission on the amount of sales. Your goal is to earn a minimum of \$6000 per month. What amount of sales will enable you to earn \$6000 or more per month?

- 107. Compensation** George Stoia earns \$1000 per month plus 5% commission on the amount of sales. George's goal is to earn a minimum of \$3200 per month. What amount of sales will enable George to earn \$3200 or more per month?

- 108. Banking** Heritage National Bank offers two different checking accounts. The first charges \$3 per month and \$.50 per check after the first 10 checks. The second account charges \$8 per month with unlimited check writing. How many checks can be written per month if the first account is to be less expensive than the second account?

- 109. Banking** Glendale Federal Bank offers a checking account to small businesses. The charge is \$8 per month plus \$.12 per check after the first 100 checks. A competitor is offering an account for \$5 per month plus \$.15 per check after the first 100 checks. If a business chooses the first account, how many checks does the business write monthly if it is assumed that the first account will cost less than the competitor's account?
- 110. Education** An average score of 90 or above in a history class receives an A grade. You have scores of 95, 89, and 81 on three exams. Find the range of scores on the fourth exam that will give you an A grade for the course.
- 111. Education** An average of 70 to 79 in a mathematics class receives a C grade. A student has scores of 56, 91, 83, and 62 on four tests. Find the range of scores on the fifth test that will give the student a C for the course.
- 112. Integers** Find four consecutive integers whose sum is between 62 and 78.
- 113. Integers** Find three consecutive even integers whose sum is between 30 and 51.

Applying the Concepts

- 114.** Let $-2 \leq x \leq 3$ and $a \leq 2x + 1 \leq b$.
- Find the largest possible value of a .
 - Find the smallest possible value of b .
- 115.** Determine whether the following statements are always true, sometimes true, or never true.
- If $a > b$, then $-a < -b$.
 - If $a < b$ and $a \neq 0, b \neq 0$, then $\frac{1}{a} < \frac{1}{b}$.
 - When dividing both sides of an inequality by an integer, we must reverse the inequality symbol.
 - If $a < 1$, then $a^2 < a$.
 - If $a < b < 0$ and $c < d < 0$, then $ac > bd$.
- 116.** The following is offered as the solution of $2 + 3(2x - 4) < 6x + 5$.
- $$\begin{aligned} 2 + 3(2x - 4) &< 6x + 5 \\ 2 + 6x - 12 &< 6x + 5 \\ 6x - 10 &< 6x + 5 \\ 6x - 6x - 10 &< 6x - 6x + 5 \\ -10 &< 5 \end{aligned}$$
- Use the Distributive Property.
 - Simplify.
 - Subtract $6x$ from each side.

Because $-10 < 5$ is a true inequality, the solution set is all real numbers.

If this result is correct, so state. If it is not correct, explain the incorrect step and supply the correct answer.

SECTION

2.5

Absolute Value Equations and Inequalities

OBJECTIVE A

To solve an absolute value equation



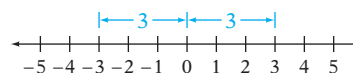
Tips for Success

Before the class meeting in which your professor begins a new section, you should read each objective statement for that section. Next, browse through the objective material. The purpose of browsing through the material is to set the stage for your brain to accept and organize new information when it is presented to you. See *AIM for Success* in the Preface.

The *absolute value* of a number is its distance from zero on the number line. Distance is always a positive number or zero. Therefore, the absolute value of a number is always a positive number or zero.

The distance from 0 to 3
or from 0 to -3 is 3 units.

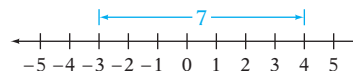
$$|3| = 3 \qquad |-3| = 3$$



Absolute value can be used to represent the distance between any two points on the number line. The **distance between two points** on the number line is the absolute value of the difference between the coordinates of the two points.

The distance between point a and point b is given by $|b - a|$.

The distance between 4 and -3 on the number line is 7 units. Note that the order in which the coordinates are subtracted does not affect the distance.



$$\begin{aligned} \text{Distance} &= |-3 - 4| & \text{Distance} &= |4 - (-3)| \\ &= |-7| & &= |7| \\ &= 7 & &= 7 \end{aligned}$$

For any two numbers a and b , $|b - a| = |a - b|$.

An equation containing an absolute value symbol is called an **absolute value equation**. Here are three examples.

$$|x| = 3 \qquad |x + 2| = 8 \qquad |3x - 4| = 5x - 9$$

Solutions of an Absolute Value Equation

If $a \geq 0$ and $|x| = a$, then $x = a$ or $x = -a$.

For instance, given $|x| = 3$, then $x = 3$ or $x = -3$ because $|3| = 3$ and $|-3| = 3$. We can solve this equation as follows:

$$\begin{aligned} |x| &= 3 \\ x &= 3 \qquad x = -3 \end{aligned}$$

- Remove the absolute value sign from $|x|$ and let x equal 3 and the opposite of 3.

$$\begin{array}{rcl} \text{Check:} & |x| = 3 & |x| = 3 \\ & \underline{|3| = 3} & \underline{|-3| = 3} \\ & 3 = 3 & 3 = 3 \end{array}$$

The solutions are 3 and -3 .

HOW TO • 1 Solve: $|x + 2| = 8$

$$\begin{array}{rcl}
 |x + 2| & = & 8 \\
 x + 2 = 8 & & x + 2 = -8 \\
 x = 6 & & x = -10
 \end{array}$$

$$\begin{array}{rcl}
 \text{Check: } |x + 2| & = & 8 \\
 |6 + 2| & = & 8 \\
 |8| & = & 8 \\
 8 & = & 8
 \end{array}
 \qquad
 \begin{array}{rcl}
 |x + 2| & = & 8 \\
 |-10 + 2| & = & 8 \\
 |-8| & = & 8 \\
 8 & = & 8
 \end{array}$$

The solutions are 6 and -10 .

- Remove the absolute value sign and rewrite as two equations.
- Solve each equation.

HOW TO • 2 Solve: $|5 - 3x| - 8 = -4$

$$\begin{array}{rcl}
 |5 - 3x| - 8 & = & -4 \\
 |5 - 3x| & = & 4 \\
 5 - 3x = 4 & & 5 - 3x = -4 \\
 -3x = -1 & & -3x = -9 \\
 x = \frac{1}{3} & & x = 3
 \end{array}$$

$$\begin{array}{rcl}
 \text{Check: } |5 - 3x| - 8 & = & -4 \\
 \left| 5 - 3\left(\frac{1}{3}\right) \right| - 8 & = & -4 \\
 |5 - 1| - 8 & = & -4 \\
 4 - 8 & = & -4 \\
 -4 & = & -4
 \end{array}
 \qquad
 \begin{array}{rcl}
 |5 - 3x| - 8 & = & -4 \\
 |5 - 3(3)| - 8 & = & -4 \\
 |5 - 9| - 8 & = & -4 \\
 4 - 8 & = & -4 \\
 -4 & = & -4
 \end{array}$$

The solutions are $\frac{1}{3}$ and 3.

- Solve for the absolute value.
- Remove the absolute value sign and rewrite as two equations.
- Solve each equation.

EXAMPLE • 1

Solve: $|2 - x| = 12$

Solution

$$\begin{array}{rcl}
 |2 - x| & = & 12 \\
 2 - x = 12 & & 2 - x = -12 \\
 -x = 10 & & -x = -14 \\
 x = -10 & & x = 14
 \end{array}$$

The solutions are -10 and 14 .

- Subtract 2.
- Multiply by -1 .

YOU TRY IT • 1

Solve: $|2x - 3| = 5$

Your solution**EXAMPLE • 2**

Solve: $|2x| = -4$

Solution

$|2x| = -4$

There is no solution to this equation because the absolute value of a number must be nonnegative.

YOU TRY IT • 2

Solve: $|x - 3| = -2$

Your solution

EXAMPLE 3

Solve: $3 - |2x - 4| = -5$

Solution

$$\begin{array}{rcl}
 3 - |2x - 4| & = & -5 \\
 -|2x - 4| & = & -8 \quad \bullet \text{ Subtract 3.} \\
 |2x - 4| & = & 8 \quad \bullet \text{ Multiply by } -1. \\
 2x - 4 = 8 & & 2x - 4 = -8 \\
 2x = 12 & & 2x = -4 \\
 x = 6 & & x = -2
 \end{array}$$

The solutions are 6 and -2 .**YOU TRY IT 3**

Solve: $5 - |3x + 5| = 3$

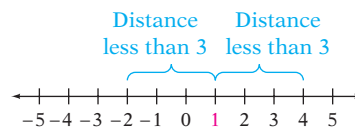
Your solution

Solution on p. S6

OBJECTIVE B**To solve an absolute value inequality**

Recall that absolute value represents the distance between two points. For example, the solutions of the absolute value equation $|x - 1| = 3$ are the numbers whose distance from 1 is 3. Therefore, the solutions are -2 and 4 . An **absolute value inequality** is an inequality that contains an absolute value symbol.

The solutions of the absolute value inequality $|x - 1| < 3$ are the numbers whose distance from 1 is less than 3. Therefore, the solutions are the numbers greater than -2 and less than 4 . The solution set is $\{x | -2 < x < 4\}$.



To solve an absolute value inequality of the form $|ax + b| < c$, solve the equivalent compound inequality $-c < ax + b < c$.

HOW TO 3

Solve: $|3x - 1| < 5$

$$\begin{array}{l}
 |3x - 1| < 5 \\
 -5 < 3x - 1 < 5 \\
 -5 + 1 < 3x - 1 + 1 < 5 + 1 \\
 -4 < 3x < 6 \\
 \frac{-4}{3} < \frac{3x}{3} < \frac{6}{3} \\
 -\frac{4}{3} < x < 2
 \end{array}$$

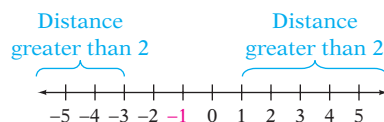
• Solve the equivalent compound inequality.

Take Note

In this objective, we will write all solution sets in set-builder notation.

The solution set is $\left\{x \mid -\frac{4}{3} < x < 2\right\}$.

The solutions of the absolute value inequality $|x + 1| > 2$ are the numbers whose distance from -1 is greater than 2. Therefore, the solutions are the numbers that are less than -3 or greater than 1 . The solution set of $|x + 1| > 2$ is $\{x | x < -3 \text{ or } x > 1\}$.



**Take Note**

Carefully observe the difference between the solution method of $|ax + b| > c$ shown here and that of $|ax + b| < c$ shown on the preceding page.

To solve an absolute value inequality of the form $|ax + b| > c$, solve the equivalent compound inequality $ax + b < -c$ or $ax + b > c$.

HOW TO • 4 Solve: $|3 - 2x| > 1$

$$\begin{array}{rcl} 3 - 2x < -1 & \text{or} & 3 - 2x > 1 \\ -2x < -4 & & -2x > -2 \\ x > 2 & & x < 1 \\ \{x | x > 2\} & & \{x | x < 1\} \end{array}$$

• Solve each inequality.

The solution set of a compound inequality with *or* is the union of the solution sets of the two inequalities.

$$\{x | x > 2\} \cup \{x | x < 1\} = \{x | x > 2 \text{ or } x < 1\}$$

The rules for solving absolute value inequalities are summarized below.

Solutions of Absolute Value Inequalities

To solve an absolute value inequality of the form $|ax + b| < c$, $c > 0$, solve the equivalent compound inequality $-c < ax + b < c$.

To solve an absolute value inequality of the form $|ax + b| > c$, solve the equivalent compound inequality $ax + b < -c$ or $ax + b > c$.

EXAMPLE • 4

Solve: $|4x - 3| < 5$

Solution

Solve the equivalent compound inequality.

$$\begin{array}{l} -5 < 4x - 3 < 5 \\ -5 + 3 < 4x - 3 + 3 < 5 + 3 \\ -2 < 4x < 8 \\ \frac{-2}{4} < \frac{4x}{4} < \frac{8}{4} \\ -\frac{1}{2} < x < 2 \\ \left\{x \mid -\frac{1}{2} < x < 2\right\} \end{array}$$

YOU TRY IT • 4

Solve: $|3x + 2| < 8$

Your solution

EXAMPLE • 5

Solve: $|x - 3| < 0$

Solution

The absolute value of a number is greater than or equal to zero, since it measures the number's distance from zero on the number line. Therefore, the solution set of $|x - 3| < 0$ is the empty set.

YOU TRY IT • 5

Solve: $|3x - 7| < 0$

Your solution

EXAMPLE • 6Solve: $|x + 4| > -2$ **Solution**

The absolute value of a number is greater than or equal to zero. Therefore, the solution set of $|x + 4| > -2$ is the set of real numbers.

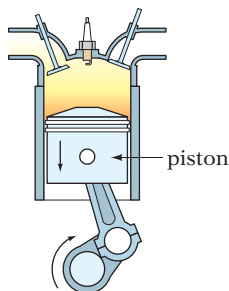
YOU TRY IT • 6Solve: $|2x + 7| \geq -1$ **Your solution****EXAMPLE • 7**Solve: $|2x - 1| > 7$ **Solution**

Solve the equivalent compound inequality.

$$\begin{aligned}
 2x - 1 &< -7 & \text{or} & & 2x - 1 &> 7 \\
 2x &< -6 & & & 2x &> 8 \\
 x &< -3 & & & x &> 4 \\
 \{x|x < -3\} & & & & \{x|x > 4\} \\
 \{x|x < -3\} \cup \{x|x > 4\} \\
 &= \{x|x < -3 \text{ or } x > 4\}
 \end{aligned}$$

YOU TRY IT • 7Solve: $|5x + 3| > 8$ **Your solution**

Solutions on pp. S6–S7

OBJECTIVE C**To solve application problems**

The **tolerance** of a component, or part, is the amount by which it is acceptable for the component to vary from a given measurement. For example, the diameter of a piston may vary from the given measurement of 9 cm by 0.001 cm. This is written $9 \text{ cm} \pm 0.001 \text{ cm}$ and is read “9 centimeters plus or minus 0.001 centimeter.” The maximum diameter, or **upper limit**, of the piston is $9 \text{ cm} + 0.001 \text{ cm} = 9.001 \text{ cm}$. The minimum diameter, or **lower limit**, is $9 \text{ cm} - 0.001 \text{ cm} = 8.999 \text{ cm}$.

The lower and upper limits of the diameter of the piston could also be found by solving the absolute value inequality $|d - 9| \leq 0.001$, where d is the diameter of the piston.

$$\begin{aligned}
 |d - 9| &\leq 0.001 \\
 -0.001 &\leq d - 9 \leq 0.001 \\
 -0.001 + 9 &\leq d - 9 + 9 \leq 0.001 + 9 \\
 8.999 &\leq d \leq 9.001
 \end{aligned}$$

The lower and upper limits of the diameter of the piston are 8.999 cm and 9.001 cm.

EXAMPLE • 8

The diameter of a piston for an automobile is $3\frac{5}{16}$ in., with a tolerance of $\frac{1}{64}$ in. Find the lower and upper limits of the diameter of the piston.

**Strategy**

To find the lower and upper limits of the diameter of the piston, let d represent the diameter of the piston, T the tolerance, and L the lower and upper limits of the diameter. Solve the absolute value inequality $|L - d| \leq T$ for L .

YOU TRY IT • 8

A machinist must make a bushing that has a tolerance of 0.003 in. The diameter of the bushing is 2.55 in. Find the lower and upper limits of the diameter of the bushing.

Your strategy**Solution**

$$\begin{aligned}
 |L - d| &\leq T \\
 \left| L - 3\frac{5}{16} \right| &\leq \frac{1}{64} \\
 -\frac{1}{64} &\leq L - 3\frac{5}{16} \leq \frac{1}{64} \\
 -\frac{1}{64} + 3\frac{5}{16} &\leq L - 3\frac{5}{16} + 3\frac{5}{16} \leq \frac{1}{64} + 3\frac{5}{16} \\
 3\frac{19}{64} &\leq L \leq 3\frac{21}{64}
 \end{aligned}$$

The lower and upper limits of the diameter of the piston are $3\frac{19}{64}$ in. and $3\frac{21}{64}$ in.

Your solution

2.5 EXERCISES**OBJECTIVE A****To solve an absolute value equation**

1. Is 2 a solution of $|x - 8| = 6$? 2. Is -2 a solution of $|2x - 5| = 9$? 3. Is -1 a solution of $|3x - 4| = 7$? 4. Is 1 a solution of $|6x - 1| = -5$?

For Exercises 5 to 64, solve.

5. $|x| = 7$ 6. $|a| = 2$ 7. $|b| = 4$ 8. $|c| = 12$
9. $|-y| = 6$ 10. $|-t| = 3$ 11. $|-a| = 7$ 12. $|-x| = 3$
13. $|x| = -4$ 14. $|y| = -3$ 15. $|-t| = -3$ 16. $|-y| = -2$
17. $|x + 2| = 3$ 18. $|x + 5| = 2$ 19. $|y - 5| = 3$ 20. $|y - 8| = 4$
21. $|a - 2| = 0$ 22. $|a + 7| = 0$ 23. $|x - 2| = -4$ 24. $|x + 8| = -2$
25. $|3 - 4x| = 9$ 26. $|2 - 5x| = 3$ 27. $|2x - 3| = 0$ 28. $|5x + 5| = 0$
29. $|3x - 2| = -4$ 30. $|2x + 5| = -2$ 31. $|x - 2| - 2 = 3$
32. $|x - 9| - 3 = 2$ 33. $|3a + 2| - 4 = 4$ 34. $|2a + 9| + 4 = 5$
35. $|2 - y| + 3 = 4$ 36. $|8 - y| - 3 = 1$ 37. $|2x - 3| + 3 = 3$
38. $|4x - 7| - 5 = -5$ 39. $|2x - 3| + 4 = -4$ 40. $|3x - 2| + 1 = -1$

41. $|6x - 5| - 2 = 4$

42. $|4b + 3| - 2 = 7$

43. $|3t + 2| + 3 = 4$

44. $|5x - 2| + 5 = 7$

45. $3 - |x - 4| = 5$

46. $2 - |x - 5| = 4$

47. $8 - |2x - 3| = 5$

48. $8 - |3x + 2| = 3$

49. $|2 - 3x| + 7 = 2$

50. $|1 - 5a| + 2 = 3$

51. $|8 - 3x| - 3 = 2$

52. $|6 - 5b| - 4 = 3$

53. $|2x - 8| + 12 = 2$

54. $|3x - 4| + 8 = 3$

55. $2 + |3x - 4| = 5$

56. $5 + |2x + 1| = 8$

57. $5 - |2x + 1| = 5$

58. $3 - |5x + 3| = 3$

59. $6 - |2x + 4| = 3$

60. $8 - |3x - 2| = 5$

61. $8 - |1 - 3x| = -1$

62. $3 - |3 - 5x| = -2$

63. $5 + |2 - x| = 3$

64. $6 + |3 - 2x| = 2$



For Exercises 65 to 68, assume that a and b are positive numbers such that $a < b$. State whether the given equation has no solution, two negative solutions, two positive solutions, or one positive and one negative solution.

65. $|x - b| = a$

66. $|x - b| = -a$

67. $|x + b| = a$

68. $|x + a| = b$

OBJECTIVE B**To solve an absolute value inequality**

For Exercises 69 to 98, solve.

69. $|x| > 3$

70. $|x| < 5$

71. $|x + 1| > 2$

72. $|x - 2| > 1$

73. $|x - 5| \leq 1$

74. $|x - 4| \leq 3$

75. $|2 - x| \geq 3$

76. $|3 - x| \geq 2$

77. $|2x + 1| < 5$

78. $|3x - 2| < 4$

79. $|5x + 2| > 12$

80. $|7x - 1| > 13$

81. $|4x - 3| \leq -2$

82. $|5x + 1| \leq -4$

83. $|2x + 7| > -5$

84. $|3x - 1| > -4$

85. $|4 - 3x| \geq 5$

86. $|7 - 2x| > 9$

87. $|5 - 4x| \leq 13$

88. $|3 - 7x| < 17$

89. $|6 - 3x| \leq 0$

90. $|10 - 5x| \geq 0$

91. $|2 - 9x| > 20$

92. $|5x - 1| < 16$

93. $|2x - 3| + 2 < 8$

94. $|3x - 5| + 1 < 7$

95. $|2 - 5x| - 4 > -2$

96. $|4 - 2x| - 9 > -3$

97. $8 - |2x - 5| < 3$

98. $12 - |3x - 4| > 7$

 For Exercises 99 and 100, assume that a and b are positive numbers such that $a < b$. State whether the given inequality has no solution, all negative solutions, all positive solutions, or both positive and negative solutions.

99. $|x + b| < a$

100. $|x + a| < b$

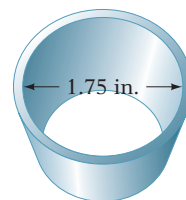
OBJECTIVE C**To solve application problems**

 101. A dosage of medicine may safely range from 2.8 ml to 3.2 ml. What is the desired dosage of the medicine? What is the tolerance?

 102. The tolerance, in inches, for the diameter of a piston is described by the absolute value inequality $|d - 5| \leq 0.01$. What is the desired diameter of the piston? By how much can the actual diameter of the piston vary from the desired diameter?

103. **Mechanics** The diameter of a bushing is 1.75 in. The bushing has a tolerance of 0.008 in. Find the lower and upper limits of the diameter of the bushing.

104. **Mechanics** A machinist must make a bushing that has a tolerance of 0.004 in. The diameter of the bushing is 3.48 in. Find the lower and upper limits of the diameter of the bushing.





- 105. Political Polling** Read the article at the right. For the poll described, the pollsters are 95% sure that the percent of American voters who felt the economy was the most important election issue lies between what lower and upper limits?



- 106. Aquatic Environments** Different species of fish have different requirements for the temperature and pH of the water in which they live. The gold swordtail requires a temperature of 73°F plus or minus 9°F and a pH level of 7.65 plus or minus 0.65. Find the upper and lower limits of **a.** the temperature and **b.** the pH level for the water in which a gold swordtail lives. (Source: www.tacomapet.com)

- 107. Automobiles** A piston rod for an automobile is $9\frac{5}{8}$ in. long, with a tolerance of $\frac{1}{32}$ in. Find the lower and upper limits of the length of the piston rod.



- 108. Football Manufacturing** An NCAA football must conform to the measurements shown in the diagram, with tolerances of $\frac{1}{4}$ in. for the girth, $\frac{3}{8}$ in. for the circumference, and $\frac{5}{32}$ in. for the length. Find the upper and lower limits for **a.** the girth, **b.** the circumference, and **c.** the length of an NCAA football. (Source: www.ncaa.org)

Electronics The tolerance of the resistors used in electronics is given as a percent. Use your calculator for Exercises 109 and 110.



- 109.** Find the lower and upper limits of a 29,000-ohm resistor with a 2% tolerance.



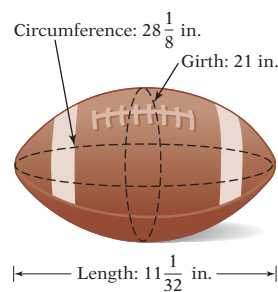
- 110.** Find the lower and upper limits of a 15,000-ohm resistor with a 10% tolerance.

In the News

Economy Is Number-One Issue

A *Washington Post*/ABC News poll conducted between April 10 and April 13, 2008, showed that 41% of American voters felt the economy was the most important election issue. The results of the poll had a margin of error of plus or minus 3 percentage points. Note: *Margin of error* is a measure of the pollsters' confidence in their results. If the pollsters conduct this poll many times, they expect that 95% of the time they will get results that fall within the margin of error of the reported results.

Source: www.washingtonpost.com



Applying the Concepts

- 111.** For what values of the variable is the equation true? Write the solution set in set-builder notation.

a. $|x + 3| = x + 3$

b. $|a - 4| = 4 - a$

- 112.** Write an absolute value inequality to represent all real numbers within 5 units of 2.

- 113.** Replace the question mark with $\leq$, $\geq$, or $=$.

a. $|x + y| ? |x| + |y|$

b. $|x - y| ? |x| - |y|$

c. $||x| - |y|| ? |x| - |y|$

d. $|xy| ? |x||y|$

- 114.** Let $|x| \leq 2$ and $|3x - 2| \leq a$. Find the smallest possible value of a .

FOCUS ON PROBLEM SOLVING

Understand the Problem

The first of the four steps that Polya advocated to solve problems is to *understand the problem*. This aspect of problem solving is frequently not given enough attention. There are various exercises that you can try to help you achieve a good understanding of a problem. Some of these are stated in the Focus on Problem Solving in the chapter entitled “Review of Real Numbers” and are reviewed here.

- Try to restate the problem in your own words.
- Determine what is known about this type of problem.
- Determine what information is given.
- Determine what information is unknown.
- Determine whether any of the information given is unnecessary.
- Determine the goal.

To illustrate this aspect of problem solving, consider the following famous ancient limerick.

As I was going to St. Ives,
I met a man with seven wives;
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits:
Kits, cats, sacks, and wives,
How many were going to St. Ives?

To answer the question in the limerick, we will ask and answer some of the questions listed above.

1. What is the goal?

The goal is to determine how many were going to St. Ives. (We know this from reading the last line of the limerick.)

2. What information is necessary and what information is unnecessary?

The first line indicates that the poet was going to St. Ives. The next five lines describe a man the poet *met* on the way. This information is irrelevant.

The answer to the question, then, is 1. Only the poet was going to St. Ives.

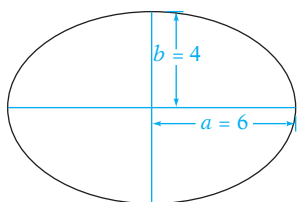
There are many other examples of the importance, in problem solving, of recognizing *irrelevant information*. One that frequently makes the college circuit is posed in the form of a test. The first line of a 100-question test states, “Read the entire test before you begin.” The last line of the test reads, “Choose any *one* question to answer.” Many people ignore the information given in the first line and just begin the test, only to find out much later that they did a lot more work than was necessary.

To illustrate another aspect of Polya’s first step in the problem-solving process, consider the problem of finding the area of the oval-shaped region (called an *ellipse*) shown in the diagram at the left. This problem can be solved by doing some research to determine *what information is known about this type of problem*. Mathematicians have found a formula for the area of an ellipse. That formula is $A = \pi ab$, where a and b are as shown in the diagram. Therefore, $A = \pi(6)(4) = 24\pi \approx 75.40$ square units. Without the formula, this problem is difficult to solve. With the formula, it is fairly easy.



Point of Interest

Try this brain teaser: You have two U.S. coins that add up to \$.55. One is not a nickel. What are the two coins?



For Exercises 1 to 5, examine the problem in terms of the first step in Polya's problem-solving method. *Do not solve the problem.*



Stephen Chernin/Getty Images

1. Johanna spent one-third of her allowance on a book. She then spent \$5 for a sandwich and iced tea. The cost of the iced tea was one-fifth the cost of the sandwich. Find the cost of the iced tea.
2. A flight from Los Angeles to Boston took 6 h. What was the average speed of the plane?
3. A major league baseball is approximately 5 in. in diameter and is covered with cowhide. Approximately how much cowhide is used to cover 10 baseballs?
4. How many donuts are in seven baker's dozen?
5. The smallest prime number is 2. Twice the difference between the eighth and the seventh prime numbers is two more than the smallest prime number. How large is the smallest prime number?

PROJECTS AND GROUP ACTIVITIES

Electricity

Since the Industrial Revolution at the turn of the century, technology has been a far-reaching and ever-changing phenomenon. Most of the technological advances that have been made, however, could not have been accomplished without electricity, and mathematics plays an integral part in the science of electricity.

Central to the study of electricity is **Ohm's law**, one part of which states that $V = IR$, where V is voltage, I is current, and R is resistance.

The word *electricity* comes from the same root word as the word *electron*. Electrons are tiny particles in atoms. Each electron has an electric charge, and this is the fundamental cause of electricity. In order to move, electrons need a source of energy—for example, light, heat, pressure, or a chemical reaction. This is where voltage comes in. Basically, **voltage** is a measure of the amount of energy in a flow of electricity. Voltage is measured in units called **volts**.

Current is a measure of how many electrons pass a given point in a fixed amount of time in a flow of electricity. This means that current is a measure of the strength of the flow of electrons through a wire—that is, their speed. Picture a faucet: The more you turn the handle, the more water you get. Similarly, the more current in a wire, the stronger the flow of electricity. Current is measured in **amperes**, often simply called **amps**. A current of 1 ampere would be sufficient to light the bulb in a flashlight.

Resistance is a measure of the amount of resistance, or opposition, to the flow of electricity. You might think of resistance as friction. The more friction, the slower the speed. Resistance is measured in **ohms**.

Watts measure the **power** in an electrical system, measuring both the strength and the speed of the flow of electrons. The power in an electrical system is equal to the voltage times the current, written $P = VI$, where P is the power measured in watts, V is voltage measured in volts, and I is current measured in amperes.



Point of Interest

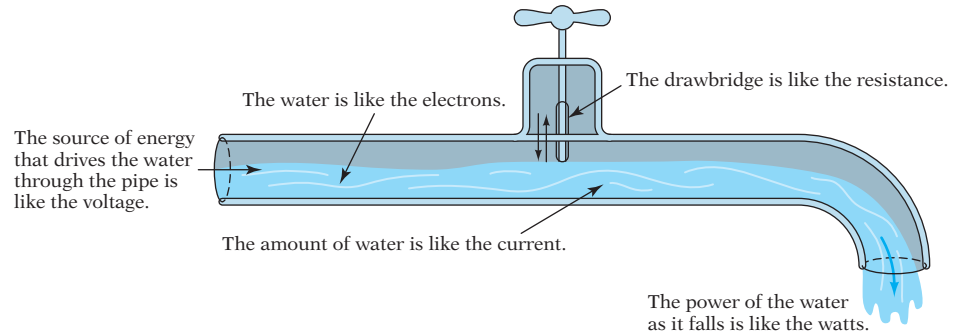
Ohm's law is named after Georg Simon Ohm (1789–1854), a German physicist whose work contributed to mathematics, acoustics, and the measurement of electrical resistance.



Point of Interest

The ampere is named after André Marie Ampère (1775–1836), a French physicist and mathematician who formulated Ampère's law, a mathematical description of the magnetic field produced by a current-carrying conductor.

In an electrical circuit, a source of electricity, such as a battery or generator, drives electrons through a wire to the part of the machine that produces the output. Because we cannot see electrons, it may help to think of an electrical system as similar to water flowing through a pipe.



Here are the formulas introduced above, along with the meaning of each variable and the unit in which it is measured:

Formulas for Voltage and Power

$$V = IR$$

Voltage = Current $\times$ Resistance
(volts) (amperes) (ohms)

$$P = VI$$

Power = Voltage $\times$ Current
(watts) (volts) (amperes)

HOW TO • 1 How many amperes of current pass through a wire in which the voltage is 90 volts and the resistance is 5 ohms?

$$V = IR$$

$$90 = I \cdot 5$$

$$\frac{90}{5} = \frac{I \cdot 5}{5}$$

$$18 = I$$

- V is the voltage, measured in volts.
- R is the resistance, measured in ohms.
- Solve for I , the current, measured in amperes.

18 amperes of current pass through the wire.

HOW TO • 2 How much voltage does a 120-watt light bulb with a current of 20 amperes require?

$$P = VI$$

$$120 = V \cdot 20$$

$$\frac{120}{20} = \frac{V \cdot 20}{20}$$

$$6 = V$$

- P is the power, measured in watts.
- I is the current, measured in amperes.
- Solve for V , the voltage, measured in volts.

The light bulb requires 6 volts.

HOW TO 3 Find the power in an electrical system that has a voltage of 120 volts and a resistance of 150 ohms.

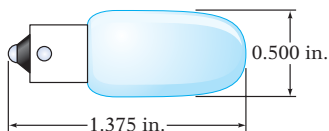
First solve the formula $V = IR$ for I . This will give us the amperes.
Then solve the formula $P = VI$ for P . This will give us the watts.

$$\begin{array}{rcl} V & = & IR \\ 120 & = & I \cdot 150 \\ \frac{120}{150} & = & \frac{I \cdot 150}{150} \\ 0.8 & = & I \end{array} \qquad \begin{array}{rcl} P & = & VI \\ P & = & 120(0.8) \\ P & = & 96 \end{array}$$

The power in the electrical system is 96 watts.

For Exercises 1 to 14, solve.

- How many volts pass through a wire in which the current is 20 amperes and the resistance is 100 ohms?
- Find the voltage in a system in which the current is 4.5 amperes and the resistance is 150 ohms.
- How many amperes of current pass through a wire in which the voltage is 100 volts and the resistance is 10 ohms?
- The lamp pictured at the left has a resistance of 70 ohms. How much current flows through the lamp when it is connected to a 115-volt circuit? Round to the nearest hundredth.
- What is the resistance of a semiconductor that passes 0.12 ampere of current when 0.48 volt is applied to it?
- Find the resistance when the current is 120 amperes and the voltage is 1.5 volts.
- Determine the power in a light bulb when the current is 10 amperes and the voltage is 12 volts.
- Find the power in a hand-held dryer that operates from a voltage of 115 volts and draws 2.175 amperes of current.
- How much current flows through a 110-volt, 850-watt lamp? Round to the nearest hundredth.
- A heating element in a clothes dryer is rated at 4500 watts. How much current is used by the dryer when the voltage is 240 volts?
- A miniature lamp pulls 0.08 ampere of current while lighting a 0.5-watt bulb. What voltage battery is needed for the lamp?
- The power of a car sound system that pulls 15 amperes is 180 watts. Find the voltage of the system.
- Find the power of a lamp that has a voltage of 160 volts and a resistance of 80 ohms.
- Find the power in an electrical system that has a voltage of 105 volts and a resistance of 70 ohms.



Tips for Success

Five important features of this text that can be used to prepare for a test are the following:

- Section Exercises
- Chapter Summary
- Concept Review
- Chapter Review Exercises
- Chapter Test

See *AIM for Success* in the Preface.

CHAPTER 2

SUMMARY

KEY WORDS

EXAMPLES

An *equation* expresses the equality of two mathematical expressions. [2.1A, p. 58]

$$3 + 2 = 5$$

$$2x - 5 = 4$$

A *conditional equation* is one that is true for at least one value of the variable but not for all values of the variable. An *identity* is an equation that is true for all values of the variable. A *contradiction* is an equation for which no value of the variable produces a true equation. [2.1A, p. 58]

$x + 3 = 7$ is a conditional equation.
 $x + 4 = x + 4$ is an identity.
 $x = x + 2$ is a contradiction.

An equation of the form $ax + b = c$, $a \neq 0$, is called a *first-degree equation* because all variables have an exponent of 1. [2.1A, p. 58]

$6x + 5 = -7$ is a first-degree equation.
 $a = 6$, $b = 5$, and $c = -7$.

The *solution*, or *root*, of an *equation* is a replacement value for the variable that will make the equation true. [2.1A, p. 58]

The solution, or root, of the equation $x + 3 = 7$ is 4 because $4 + 3 = 7$.

To *solve an equation* means to find its solutions. The goal is to rewrite the equation in the form *variable = constant* because the constant is the solution. [2.1A, p. 58]

The equation $x = 12$ is in the form *variable = constant*. The constant 12 is the solution of the equation.

Equivalent equations are equations that have the same solution. [2.1A, p. 58]

$x + 3 = 7$ and $x = 4$ are equivalent equations because the solution of each equation is 4.

A *literal equation* is an equation that contains more than one variable. A *formula* is a literal equation that states a rule about measurement. [2.1D, p. 63]

$4x - 5y = 20$ is a literal equation.
 $A = \pi r^2$ is the formula for the area of a circle. It is also a literal equation.

The *solution set of an inequality* is a set of numbers, each element of which, when substituted in the inequality, results in a true inequality. [2.4A, p. 84]

Any number greater than 4 is a solution of the inequality $x > 4$.

A *compound inequality* is formed by joining two inequalities with a connective word such as *and* or *or*. [2.4B, p. 87]

$$3x > 6 \quad \text{and} \quad 2x + 5 < 7$$

$$2x + 1 < 3 \quad \text{or} \quad x + 2 > 4$$

An *absolute value equation* is an equation that contains an absolute value symbol. [2.5A, p. 96]

$$|x - 2| = 3$$

An *absolute value inequality* is an inequality that contains an absolute value symbol. [2.5B, p. 98]

$$|x - 4| < 5$$

$$|2x - 3| > 6$$

The *tolerance* of a component or part is the amount by which it is acceptable for the component to vary from a given measurement. The maximum measurement is the *upper limit*. The minimum measurement is the *lower limit*. [2.5C, p. 100]

The diameter of a bushing is 1.5 in., with a tolerance of 0.005 in. The lower and upper limits of the diameter of the bushing are 1.5 in. $\pm$ 0.005 in.

ESSENTIAL RULES AND PROCEDURES

Addition Property of Equations [2.1A, p. 58]

If $a = b$, then $a + c = b + c$.

$$\begin{aligned}x + 5 &= -3 \\x + 5 - 5 &= -3 - 5 \\x &= -8\end{aligned}$$

Multiplication Property of Equations [2.1A, p. 59]

If $a = b$ and $c \neq 0$, then $ac = bc$.

$$\begin{aligned}\frac{2}{3}x &= 4 \\ \left(\frac{3}{2}\right)\left(\frac{2}{3}x\right) &= \left(\frac{3}{2}\right)4 \\ x &= 6\end{aligned}$$

Consecutive Integers [2.2A, p. 68]

$n, n + 1, n + 2, \dots$

The sum of three consecutive integers is 57.

$$n + (n + 1) + (n + 2) = 57$$

Consecutive Even or Consecutive Odd Integers [2.2A, p. 68]

$n, n + 2, n + 4, \dots$

The sum of three consecutive even integers is 132.

$$n + (n + 2) + (n + 4) = 132$$

Coin and Stamp Equation [2.2B, p. 70]

Number of items	·	Value of each item	=	Total Value of the items

A collection of stamps consists of 17¢ and 27¢ stamps. In all there are 15 stamps, with a value of \$3.55. How many 17¢ stamps are in the collection?

$$17n + 27(15 - n) = 355$$

Value Mixture Equation [2.3A, p. 74]

Amount · Unit Cost = Value
 $AC = V$

A merchant combines coffee that costs \$6 per pound with coffee that costs \$3.20 per pound. How many pounds of each should be used to make 60 lb of a blend that costs \$4.50 per pound?

$$6x + 3.20(60 - x) = 4.50(60)$$

Percent Mixture Problems [2.3B, p. 76]

Amount of solution $\cdot$ percent of concentration = quantity of substance

$$Ar = Q$$

A silversmith mixed 120 oz of an 80% silver alloy with 240 oz of a 30% silver alloy. Find the percent concentration of the resulting silver alloy.

$$0.80(120) + 0.30(240) = x(360)$$

Uniform Motion Equation [2.3C, p. 78]

Rate $\cdot$ Time = Distance

$$rt = d$$

Two planes are 1640 mi apart and are traveling toward each other. One plane is traveling 60 mph faster than the other plane. The planes pass each other in 2 h. Find the speed of each plane.

$$2r + 2(r + 60) = 1640$$

Addition Property of Inequalities [2.4A, p. 84]

If $a > b$, then $a + c > b + c$.

If $a < b$, then $a + c < b + c$.

$$\begin{aligned} x + 3 &> -2 \\ x + 3 - 3 &> -2 - 3 \\ x &> -5 \end{aligned}$$

Multiplication Property of Inequalities [2.4A, p. 85]

Rule 1 If $a > b$ and $c > 0$, then $ac > bc$.

If $a < b$ and $c > 0$, then $ac < bc$.

$$\begin{aligned} 3x &> 12 \\ \left(\frac{1}{3}\right)(3x) &> \left(\frac{1}{3}\right)12 \\ x &> 4 \end{aligned}$$

Rule 2 If $a > b$ and $c < 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

$$\begin{aligned} -2x &< 8 \\ \frac{-2x}{-2} &> \frac{8}{-2} \\ x &> -4 \end{aligned}$$

Solutions of an Absolute Value Equation [2.5A, p. 96]

If $a \geq 0$ and $|x| = a$, then $x = a$ or $x = -a$.

$$\begin{aligned} |x - 3| &= 7 \\ x - 3 &= 7 & x - 3 &= -7 \\ x &= 10 & x &= -4 \end{aligned}$$

Solutions of Absolute Value Inequalities [2.5B, p. 99]

To solve an absolute value inequality of the form $|ax + b| < c$, $c > 0$, solve the equivalent compound inequality $-c < ax + b < c$.

$$\begin{aligned} |x - 5| &< 9 \\ -9 &< x - 5 < 9 \\ -9 + 5 &< x - 5 + 5 < 9 + 5 \\ -4 &< x < 14 \end{aligned}$$

To solve an absolute value inequality of the form $|ax + b| > c$, solve the equivalent compound inequality $ax + b < -c$ or $ax + b > c$.

$$\begin{aligned} |x - 5| &> 9 \\ x - 5 &< -9 & \text{or} & x - 5 > 9 \\ x &< -4 & \text{or} & x > 14 \end{aligned}$$

CHAPTER 2**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How is the Addition Property of Equations used to solve an equation?
2. What is the difference between the root of an equation and the solution of an equation?
3. How do you check the solution of an equation?
4. How do you solve an equation containing parentheses?
5. What is the difference between a consecutive integer and a consecutive even integer?
6. How is the value of a stamp used in writing an expression for the total value of a number of stamps?
7. What formula is used in solving a percent mixture problem?
8. What formula is used to solve a uniform motion problem?
9. How is the Multiplication Property of Inequalities different from the Multiplication Property of Equations?
10. What compound inequality is equivalent to an absolute value inequality of the form $|ax + b| < c, c > 0$?
11. What compound inequality is equivalent to an absolute value inequality of the form $|ax + b| > c$?
12. How do you check the solution to an absolute value equation?

CHAPTER 2

REVIEW EXERCISES

1. Solve: $3t - 3 + 2t = 7t - 15$
2. Solve: $3x - 7 > -2$
Write the solution set in interval notation.
3. Solve $P = 2L + 2W$ for L .
4. Solve: $x + 4 = -5$
5. Solve: $3x < 4$ and $x + 2 > -1$
Write the solution set in set-builder notation.
6. Solve: $\frac{3}{5}x - 3 = 2x + 5$
7. Solve: $-\frac{2}{3}x = \frac{4}{9}$
8. Solve: $|x - 4| - 8 = -3$
9. Solve: $|2x - 5| < 3$
10. Solve: $\frac{2x - 3}{3} + 2 = \frac{2 - 3x}{5}$
11. Solve: $2(a - 3) = 5(4 - 3a)$
12. Solve: $5x - 2 > 8$ or $3x + 2 < -4$
Write the solution set in set-builder notation.
13. Solve: $|4x - 5| \geq 3$
14. Solve $P = \frac{R - C}{n}$ for C .
15. Solve: $\frac{1}{2}x - \frac{5}{8} = \frac{3}{4}x + \frac{3}{2}$
16. Solve: $6 + |3x - 3| = 2$
17. Solve: $3x - 2 > x - 4$ or $7x - 5 < 3x + 3$
Write the solution set in interval notation.
18. Solve: $2x - (3 - 2x) = 4 - 3(4 - 2x)$

19. Solve: $x + 9 = -6$

20. Solve: $\frac{2}{3} = x + \frac{3}{4}$

21. Solve: $-3x = -21$

22. Solve: $\frac{2}{3}a = \frac{4}{9}$

23. Solve: $3y - 5 = 3 - 2y$

24. Solve: $4x - 5 + x = 6x - 8$

25. Solve: $3(x - 4) = -5(6 - x)$

26. Solve: $\frac{3x - 2}{4} + 1 = \frac{2x - 3}{2}$

27. Solve: $5x - 8 < -3$
Write the solution set in interval notation.

28. Solve: $2x - 9 \leq 8x + 15$
Write the solution set in interval notation.

29. Solve: $\frac{2}{3}x - \frac{5}{8} \geq \frac{3}{4}x + 1$
Write the solution set in set-builder notation.

30. Solve: $2 - 3(2x - 4) \leq 4x - 2(1 - 3x)$
Write the solution set in set-builder notation.

31. Solve: $-5 < 4x - 1 < 7$
Write the solution set in interval notation.

32. Solve: $|2x - 3| = 8$

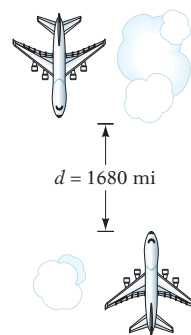
33. Solve: $|5x + 8| = 0$

34. Solve: $|5x - 4| < -2$

35. **Uniform Motion** A ferry leaves a dock and travels to an island at an average speed of 16 mph. On the return trip, the ferry travels at an average speed of 12 mph. The total time for the trip is $2\frac{1}{3}$ h. How far is the island from the dock?



36. **Mixtures** A grocer mixed apple juice that costs \$12.50 per gallon with 25 gal of cranberry juice that costs \$31.50 per gallon. How much apple juice was used to make cranapple juice costing \$25.00 per gallon?
37. **Compensation** A sales executive earns \$1200 per month plus 8% commission on the amount of sales. The executive's goal is to earn \$5000 per month. What amount of sales will enable the executive to earn \$5000 or more per month?
38. **Coins** A coin collection contains thirty coins in nickels, dimes, and quarters. There are three more dimes than nickels. The value of the coins is \$3.55. Find the number of quarters in the collection.
39. **Mechanics** The diameter of a bushing is 2.75 in. The bushing has a tolerance of 0.003 in. Find the lower and upper limits of the diameter of the bushing.
40. **Integers** The sum of two integers is twenty. Five times the smaller integer is two more than twice the larger integer. Find the two integers.
41. **Education** An average score of 80 to 90 in a psychology class receives a B grade. A student has scores of 92, 66, 72, and 88 on four tests. Find the range of scores on the fifth test that will give the student a B for the course.
42. **Uniform Motion** Two planes are 1680 mi apart and are traveling toward each other. One plane is traveling 80 mph faster than the other plane. The planes pass each other in 1.75 h. Find the speed of each plane.
43. **Mixtures** An alloy containing 30% tin is mixed with an alloy containing 70% tin. How many pounds of each were used to make 500 lb of an alloy containing 40% tin?
44. **Automobiles** A piston rod for an automobile is $10\frac{3}{8}$ in. long, with a tolerance of $\frac{1}{32}$ in. Find the lower and upper limits of the length of the piston rod.

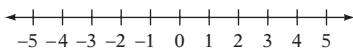


CHAPTER 2**TEST**

1. Solve: $x - 2 = -4$
2. Solve: $b + \frac{3}{4} = \frac{5}{8}$
3. Solve: $-\frac{3}{4}y = -\frac{5}{8}$
4. Solve: $3x - 5 = 7$
5. Solve: $\frac{3}{4}y - 2 = 6$
6. Solve: $2x - 3 - 5x = 8 + 2x - 10$
7. Solve: $2[a - (2 - 3a) - 4] = a - 5$
8. Solve $E = IR + Ir$ for R .
9. Solve: $\frac{2x + 1}{3} - \frac{3x + 4}{6} = \frac{5x - 9}{9}$
10. Solve: $3x - 2 \geq 6x + 7$
Write the solution set in set-builder notation.
11. Solve: $4 - 3(x + 2) < 2(2x + 3) - 1$
Write the solution set in interval notation.
12. Solve: $4x - 1 > 5$ or $2 - 3x < 8$
Write the solution set in set-builder notation.
13. Solve: $4 - 3x \geq 7$ and $2x + 3 \geq 7$
Write the solution set in set-builder notation.
14. Solve: $|3 - 5x| = 12$
15. Solve: $2 - |2x - 5| = -7$
16. Solve: $|3x - 5| \leq 4$
17. Solve: $|4x - 3| > 5$

18. **Consumerism** Gambelli Agency rents cars for \$40 per day plus 25¢ for every mile driven. McDougal Rental rents cars for \$58 per day with unlimited mileage. How many miles a day can you drive a Gambelli Agency car if it is to cost you less than a McDougal Rental car?
19. **Mechanics** A machinist must make a bushing that has a tolerance of 0.002 in. The diameter of the bushing is 2.65 in. Find the lower and upper limits of the diameter of the bushing.
20. **Integers** The sum of two integers is fifteen. Eight times the smaller integer is one less than three times the larger integer. Find the integers.
21. **Stamps** A stamp collection contains 11¢, 15¢, and 24¢ stamps. There are twice as many 11¢ stamps as 15¢ stamps. There are thirty stamps in all, with a value of \$4.40. How many 24¢ stamps are in the collection?
22. **Mixtures** A butcher combines 100 lb of hamburger that costs \$3.10 per pound with 60 lb of hamburger that costs \$4.38 per pound. Find the cost of the hamburger mixture.
23. **Uniform Motion** A jogger runs a distance at a speed of 8 mph and returns the same distance running at a speed of 6 mph. Find the total distance that the jogger ran if the total time running was 1 h 45 min.
24. **Uniform Motion** Two trains are 250 mi apart and are traveling toward each other. One train is traveling 5 mph faster than the other train. The trains pass each other in 2 h. Find the speed of each train.
25. **Mixtures** How many ounces of pure water must be added to 60 oz of an 8% salt solution to make a 3% salt solution?

CUMULATIVE REVIEW EXERCISES

1. Simplify: $-4 - (-3) - 8 + (-2)$
2. Simplify: $-2^2 \cdot 3^3$
3. Simplify: $4 - (2 - 5)^2 \div 3 + 2$
4. Simplify: $4 \div \frac{\frac{3}{8} - 1}{5} \cdot 2$
5. Evaluate $2a^2 - (b - c)^2$ when $a = 2$, $b = 3$, and $c = -1$.
6. Evaluate $\frac{a - b^2}{b - c}$ when $a = 2$, $b = -3$, and $c = 4$.
7. Identify the property that justifies the statement.
 $(2x + 3y) + 2 = (3y + 2x) + 2$
8. Translate and simplify “the sum of three times a number and six added to the product of three and the number.”
9. Solve $F = \frac{evB}{c}$ for B .
10. Simplify: $5[y - 2(3 - 2y) + 6]$
11. Find $A \cap B$, given $A = \{-4, -2, 0, 2\}$ and $B = \{-4, 0, 4, 8\}$.
12. Graph the solution set of $\{x | x \leq 3\} \cap \{x | x > -1\}$.

13. Solve $Ax + By + C = 0$ for y .
14. Solve: $-\frac{5}{6}b = -\frac{5}{12}$
15. Solve: $2x + 5 = 5x + 2$
16. Solve: $\frac{5}{12}x - 3 = 7$
17. Solve: $2[3 - 2(3 - 2x)] = 2(3 + x)$
18. Solve: $3[2x - 3(4 - x)] = 2(1 - 2x)$
19. Solve: $\frac{1}{2}y - \frac{2}{3}y + \frac{5}{12} = \frac{3}{4}y - \frac{1}{2}$
20. Solve: $\frac{3x - 1}{4} - \frac{4x - 1}{12} = \frac{3 + 5x}{8}$

21. Solve: $3 - 2(2x - 1) \geq 3(2x - 2) + 1$
Write the solution set in interval notation.
22. Solve: $3x + 2 \leq 5$ and $x + 5 > 1$
Write the solution set in set-builder notation.
23. Solve: $|3 - 2x| = 5$
24. Solve: $3 - |2x - 3| = -8$
25. Solve: $|3x - 1| > 5$
26. Solve: $|2x - 4| < 8$
27. **Banking** A bank offers two types of checking accounts. One account has a charge of \$5 per month plus 4¢ for each check. The second account has a charge of \$2 per month plus 10¢ for each check. How many checks can a customer who has the second type of account write if it is to cost the customer less than the first type of account?
28. **Integers** Four times the sum of the first and third of three consecutive odd integers is one less than seven times the middle integer. Find the first integer.
29. **Coins** A coin purse contains dimes and quarters. The number of dimes is five less than twice the number of quarters. The total value of the coins is \$4.00. Find the number of dimes in the coin purse.
30. **Mixtures** A silversmith combined pure silver that costs \$15.78 per ounce with 100 oz of a silver alloy that costs \$8.26 per ounce. How many ounces of pure silver were used to make an alloy of silver costing \$11.78 per ounce?
31. **Uniform Motion** Two planes are 1400 mi apart and are traveling toward each other. One plane is traveling 120 mph faster than the other plane. The planes pass each other in 2.5 h. Find the speed of the slower plane.
32. **Mechanics** The diameter of a bushing is 2.45 in. The bushing has a tolerance of 0.001 in. Find the lower and upper limits of the diameter of the bushing.
33. **Mixtures** How many liters of a 12% acid solution must be mixed with 4 L of a 5% acid solution to make an 8% acid solution?

Linear Functions and Inequalities in Two Variables

OBJECTIVES

SECTION 3.1

- A** To graph points in a rectangular coordinate system
- B** To find the length and midpoint of a line segment
- C** To graph a scatter diagram

SECTION 3.2

- A** To evaluate a function

SECTION 3.3

- A** To graph a linear function
- B** To graph an equation of the form $Ax + By = C$
- C** To find the x - and the y -intercepts of a straight line
- D** To solve application problems

SECTION 3.4

- A** To find the slope of a line given two points
- B** To graph a line given a point and the slope

SECTION 3.5

- A** To find the equation of a line given a point and the slope
- B** To find the equation of a line given two points
- C** To solve application problems

SECTION 3.6

- A** To find parallel and perpendicular lines

SECTION 3.7

- A** To graph the solution set of an inequality in two variables

ARE YOU READY?

Take the Chapter 3 Prep Test to find out if you are ready to learn to:

- Evaluate a function
- Graph equations of the form $y = mx + b$ and of the form $Ax + By = C$
- Find the slope of a line
- Find the equation of a line given a point and the slope or given two points
- Find parallel and perpendicular lines
- Graph the solution set of an inequality in two variables

PREP TEST

Do these exercises to prepare for Chapter 3.

For Exercises 1 to 3, simplify.

1. $-4(x - 3)$
2. $\sqrt{(-6)^2 + (-8)^2}$
3. $\frac{3 - (-5)}{2 - 6}$
4. Evaluate $-2x + 5$ for $x = -3$.
5. Evaluate $\frac{2r}{r - 1}$ for $r = 5$.
6. Evaluate $2p^3 - 3p + 4$ for $p = -1$.
7. Evaluate $\frac{x_1 + x_2}{2}$ for $x_1 = 7$ and $x_2 = -5$.
8. Given $3x - 4y = 12$, find the value of x when $y = 0$.
9. Solve $2x - y = 7$ for y .

SECTION

3.1

The Rectangular Coordinate System

OBJECTIVE A

To graph points in a rectangular coordinate system

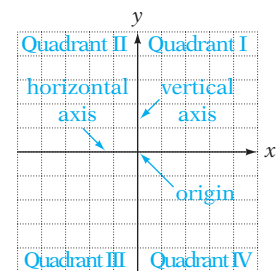


Point of Interest

A rectangular coordinate system is also called a **Cartesian coordinate system**, in honor of Descartes.

Before the 15th century, geometry and algebra were considered separate branches of mathematics. That all changed when René Descartes, a French mathematician who lived from 1596 to 1650, founded analytic geometry. In this geometry, a *coordinate system* is used to study relationships between variables.

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the **origin**. The two lines are called **coordinate axes**, or simply **axes**.



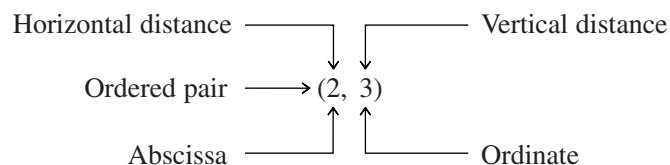
The axes determine a **plane**, which can be thought of as a large, flat sheet of paper. The two axes divide the plane into four regions called **quadrants**. The quadrants are numbered counterclockwise from I to IV.



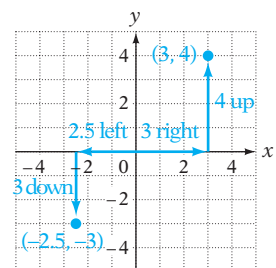
Point of Interest

Gottfried Leibnitz introduced the words *abscissa* and *ordinate*. *Abscissa* is from Latin, meaning “to cut off.” Originally, Leibnitz used the phrase *abscissa linea*, “cut off a line” (axis). The root of *ordinate* is also a Latin word used to suggest a sense of order.

Each point in the plane can be identified by a pair of numbers called an **ordered pair**. The first number of the pair measures a horizontal distance and is called the **abscissa**. The second number of the pair measures a vertical distance and is called the **ordinate**. The **coordinates of a point** are the numbers in the ordered pair associated with the point. The abscissa is also called the **first coordinate** of the ordered pair, and the ordinate is also called the **second coordinate** of the ordered pair.



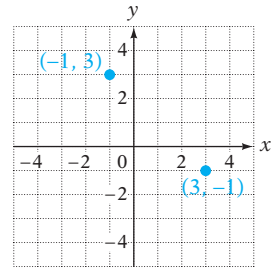
Graphing, or plotting, an ordered pair in the plane means placing a dot at the location given by the ordered pair. The **graph of an ordered pair** is the dot drawn at the coordinates of the point in the plane. The points whose coordinates are $(3, 4)$ and $(-2.5, -3)$ are graphed in the figure at the right.



**Take Note**

The concept of *ordered pair* is an important concept. Remember: There are two numbers (a *pair*), and the *order* in which they are given is important.

The points whose coordinates are $(3, -1)$ and $(-1, 3)$ are graphed at the right. Note that the graphs are in different locations. The *order* of the coordinates of an ordered pair is important.



When drawing a rectangular coordinate system, we often label the horizontal axis x and the vertical axis y . In this case, the coordinate system is called an **xy -coordinate system**. The coordinates of the points are given by ordered pairs (x, y) , where the abscissa is called the **x -coordinate** and the ordinate is called the **y -coordinate**.

The xy -coordinate system is used to graph **equations in two variables**. Examples of equations in two variables are shown at the right. A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

$$\begin{aligned} y &= 3x + 7 \\ y &= x^2 - 4x + 3 \\ x^2 + y^2 &= 25 \\ x &= \frac{y}{y^2 + 4} \end{aligned}$$

HOW TO • 1

Is the ordered pair $(-3, 7)$ a solution of the equation $y = -2x + 1$?

$$\begin{array}{rcl} y & = & -2x + 1 \\ 7 & | & -2(-3) + 1 \\ & & 6 + 1 \\ 7 & = & 7 \end{array}$$

Yes, the ordered pair $(-3, 7)$ is a solution of the equation.

- Replace x by -3 and y by 7 .
- Simplify.
- Compare the results. If the resulting equation is a true statement, the ordered pair is a solution of the equation. If it is not a true statement, the ordered pair is not a solution of the equation.

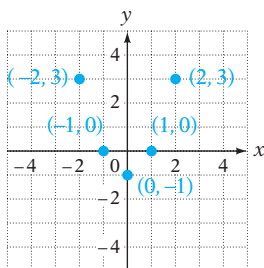
Besides the ordered pair $(-3, 7)$, there are many other ordered-pair solutions of the equation $y = -2x + 1$. For example, $(-5, 11)$, $(0, 1)$, $(-\frac{3}{2}, 4)$, and $(4, -7)$ are also solutions of the equation.

In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value for y . The resulting ordered-pair solution (x, y) of the equation can be graphed in a rectangular coordinate system.

HOW TO • 2

Graph the solutions (x, y) of $y = x^2 - 1$ when x equals $-2, -1, 0, 1,$ and 2 .

Substitute each value of x into the equation and solve for y . It is convenient to record the ordered-pair solutions in a table similar to the one shown below. Then graph the ordered pairs, as shown at the left.



x	$y = x^2 - 1$	y	(x, y)
-2	$y = (-2)^2 - 1$	3	$(-2, 3)$
-1	$y = (-1)^2 - 1$	0	$(-1, 0)$
0	$y = 0^2 - 1$	-1	$(0, -1)$
1	$y = 1^2 - 1$	0	$(1, 0)$
2	$y = 2^2 - 1$	3	$(2, 3)$

EXAMPLE • 1

Determine the ordered-pair solution of $y = |x - 3|$ corresponding to $x = -1$.

Solution

$y = |x - 3| = |-1 - 3| = |-4| = 4$ • Replace x by -1 and solve for y .
The ordered-pair solution is $(-1, 4)$.

YOU TRY IT • 1

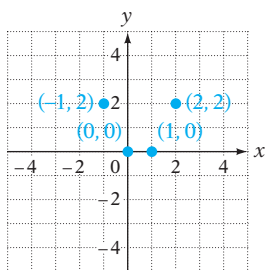
Determine the ordered-pair solution of $y = -x^2 + 5$ corresponding to $x = -3$.

Your solution**EXAMPLE • 2**

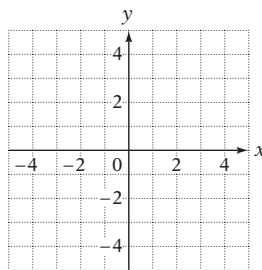
Graph the ordered-pair solutions of $y = x^2 - x$ when $x = -1, 0, 1$, and 2 .

Solution

x	y
-1	2
0	0
1	0
2	2

**YOU TRY IT • 2**

Graph the ordered-pair solutions of $y = |x + 1|$ when $x = -3, -2, -1, 0$, and 1 .

Your solution

Solutions on p. S7

OBJECTIVE B**To find the length and midpoint of a line segment**

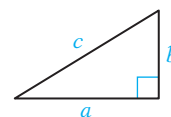
The distance between two points in an xy -coordinate system can be calculated by using the Pythagorean Theorem.

Take Note

A **right triangle** contains one 90° angle. The side opposite the 90° angle is the **hypotenuse**. The other two sides are called **legs**.

Pythagorean Theorem

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

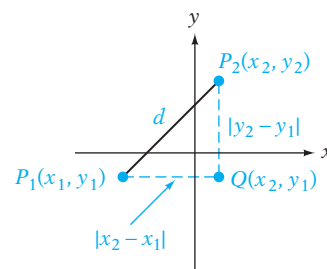


Consider the points P_1 and P_2 and the right triangle shown at the right. The vertical distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $|y_2 - y_1|$.

The horizontal distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $|x_2 - x_1|$.

The quantity d^2 is calculated by applying the Pythagorean Theorem to the right triangle.

The distance d is the square root of d^2 .



$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, the distance formula is usually written in the following form.

The Distance Formula

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points in the plane, then the distance d between the two points is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



Take Note

When asked to find the distance between two points, it does not matter which point is selected as P_1 and which is selected as P_2 . We could have labeled the points $P_1(-2, 4)$ and $P_2(-6, 1)$, where we have reversed the naming of P_1 and P_2 . Then

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{[-2 - (-6)]^2 + (4 - 1)^2} \\ &= \sqrt{4^2 + 3^2} = \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

The distance is the same.

HOW TO 3

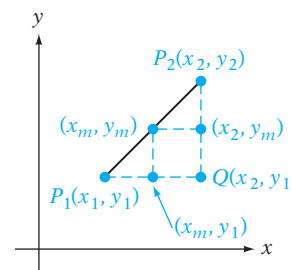
Find the distance between the points whose coordinates are $(-6, 1)$ and $(-2, 4)$.

Choose P_1 (point 1) and P_2 (point 2). We will choose $P_1(-6, 1)$ and $P_2(-2, 4)$. From $P_1(-6, 1)$, we have $x_1 = -6$, $y_1 = 1$. From $P_2(-2, 4)$, we have $x_2 = -2$, $y_2 = 4$. Now use the distance formula.

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{[-6 - (-2)]^2 + (1 - 4)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \end{aligned}$$

The distance between the two points is 5 units. We could also say “The length of the line segment between the two points is 5 units.”

The **midpoint of a line segment** is equidistant from its endpoints. The coordinates of the midpoint of the line segment P_1P_2 are (x_m, y_m) . The intersection of the horizontal line segment through P_1 and the vertical line segment through P_2 is Q , with coordinates (x_2, y_1) .



The x -coordinate x_m of the midpoint of the line segment P_1P_2 is the same as the x -coordinate of the midpoint of the line segment P_1Q . It is the average of the x -coordinates of the points P_1 and P_2 .

$$x_m = \frac{x_1 + x_2}{2}$$

Similarly, the y -coordinate y_m of the midpoint of the line segment P_1P_2 is the same as the y -coordinate of the midpoint of the line segment P_2Q . It is the average of the y -coordinates of the points P_1 and P_2 .

$$y_m = \frac{y_1 + y_2}{2}$$

The Midpoint Formula

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are the endpoints of a line segment, then the coordinates of the midpoint (x_m, y_m) of the line segment are given by

$$x_m = \frac{x_1 + x_2}{2} \quad \text{and} \quad y_m = \frac{y_1 + y_2}{2}$$

HOW TO 4 Find the coordinates of the midpoint of the line segment with endpoints whose coordinates are $(-3, 5)$ and $(1, -7)$.

Choose $P_1(-3, 5)$ and $P_2(1, -7)$.

$$\begin{aligned}x_m &= \frac{x_1 + x_2}{2} & y_m &= \frac{y_1 + y_2}{2} \\&= \frac{-3 + 1}{2} & &= \frac{5 + (-7)}{2} \\&= -1 & &= -1\end{aligned}$$

• Use the midpoint formula.

• Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (1, -7)$.

The coordinates of the midpoint are $(-1, -1)$.

EXAMPLE 3

Find the distance, to the nearest hundredth, between the points whose coordinates are $(-3, 2)$ and $(4, -1)$.

Solution

Choose $P_1(-3, 2)$ and $P_2(4, -1)$.

$$\begin{aligned}d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(-3 - 4)^2 + [2 - (-1)]^2} \\&= \sqrt{(-7)^2 + 3^2} = \sqrt{49 + 9} \\&= \sqrt{58} \approx 7.62\end{aligned}$$

EXAMPLE 4

Find the coordinates of the midpoint of the line segment with endpoints whose coordinates are $(-5, 4)$ and $(-3, 7)$.

Solution

Choose $P_1(-5, 4)$ and $P_2(-3, 7)$.

$$\begin{aligned}x_m &= \frac{x_1 + x_2}{2} & y_m &= \frac{y_1 + y_2}{2} \\&= \frac{-5 + (-3)}{2} & &= \frac{4 + 7}{2} \\&= -4 & &= \frac{11}{2}\end{aligned}$$

The midpoint is $(-4, \frac{11}{2})$.

YOU TRY IT 3

Find the distance, to the nearest hundredth, between the points whose coordinates are $(5, -2)$ and $(-4, 3)$.

Your solution

YOU TRY IT 4

Find the coordinates of the midpoint of the line segment with endpoints whose coordinates are $(-3, -5)$ and $(-2, 3)$.

Your solution

Solutions on p. S7

OBJECTIVE C

To graph a scatter diagram



© Craig Tuttle/Corbis

Finding a relationship between two variables is an important task in the study of mathematics. These relationships occur in many forms and in a wide variety of applications. Here are some examples.

- A botanist wants to know the relationship between the number of bushels of wheat yielded per acre and the amount of watering per acre.
- An environmental scientist wants to know the relationship between the incidence of skin cancer and the amount of ozone in the atmosphere.
- A business analyst wants to know the relationship between the price of a product and the number of products that are sold at that price.



Integrating Technology

See the Keystroke Guide: *Scatter Diagrams* for instructions on using a graphing calculator to create a scatter diagram.

A researcher may investigate the relationship between two variables by means of *regression analysis*, which is a branch of statistics. The study of the relationship between the two variables may begin with a **scatter diagram**, which is a graph of the ordered pairs of the known data.

The following table shows randomly selected data for a recent Boston Marathon. It shows times (in minutes) for various participants 40 years old and older.

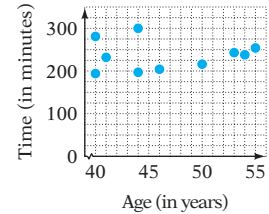
Age (x)	55	46	53	40	40	44	54	44	41	50
Time (y)	254	204	243	194	281	197	238	300	232	216



Take Note

The jagged portion of the horizontal axis in the figure at the right indicates that the numbers between 0 and 40 are missing.

The scatter diagram for these data is shown at the right. Each ordered pair represents the age and time for a participant. For instance, the ordered pair (53, 243) indicates that a 53-year-old participant ran the marathon in 243 min.



EXAMPLE 5



Life expectancy depends on a number of factors, including age. The table below, based on data from the Census Bureau, shows the life expectancy (in years) for people up to age 70 in the United States. Draw a scatter diagram for these data.

Age	0	10	20	30	40	50	60	70
Life Expectancy	77.8	78.6	78.8	79.3	79.9	80.9	82.5	85.2

YOU TRY IT 5

A cup of tea is placed in a microwave oven and its temperature (in $^{\circ}\text{F}$) recorded at 10-second intervals. The results are shown in the table below. Draw a scatter diagram for these data.

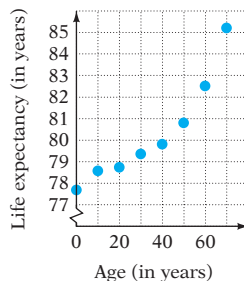
Time (in seconds)	0	10	20	30	40	50	60
Temperature (in $^{\circ}\text{F}$)	72	76	88	106	126	140	160

Strategy

To draw a scatter diagram:

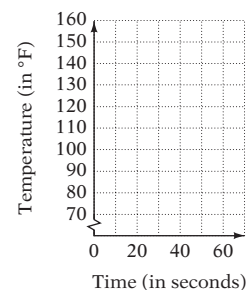
- Draw a coordinate grid with the horizontal axis representing the age of the person and the vertical axis representing life expectancy. Because the life expectancies start at 77.8, it is more convenient to start labeling the vertical axis at 77 (any number less than 77.8 could be used). A jagged line on the vertical axis indicates that the graph does not start at zero.
- Graph the ordered pairs (0, 77.8), (10, 78.6), (20, 78.8), (30, 79.3), (40, 79.9), (50, 80.9), (60, 82.5), and (70, 85.2).

Solution



Your strategy

Your solution



Solution on p. S7

3.1 EXERCISES

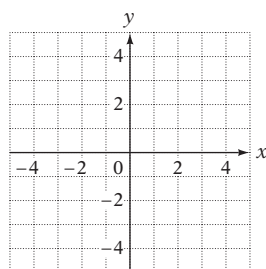
OBJECTIVE A

To graph points in a rectangular coordinate system

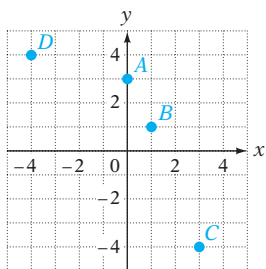


1. a. If the x -coordinate of an ordered pair is positive, in which quadrants could the graph of the ordered pair lie?
- b. If the ordinate of an ordered pair is negative, in which quadrants could the graph of the ordered pair lie?

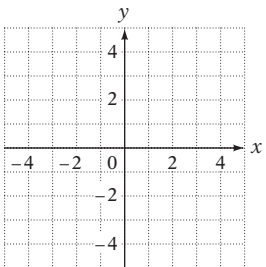
3. Graph the ordered pairs $(1, -1)$, $(2, 0)$, $(3, 2)$, and $(-1, 4)$.



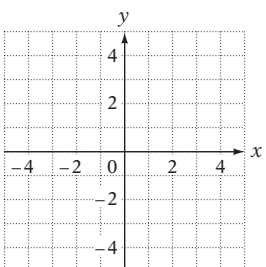
5. Find the coordinates of each of the points.



7. Draw a line through all points with an abscissa of 2.

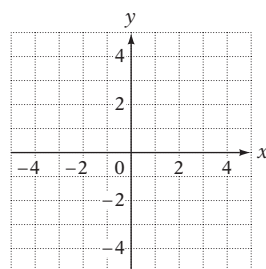


9. Draw a line through all points with an ordinate of -3 .

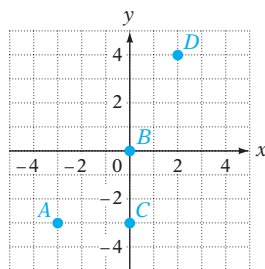


2. a. What is the x -coordinate of any point on the y -axis?
- b. What is the y -coordinate of any point on the x -axis?

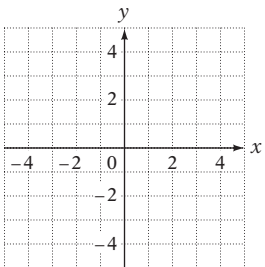
4. Graph the ordered pairs $(-1, -3)$, $(0, -4)$, $(0, 4)$, and $(3, -2)$.



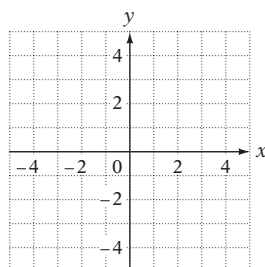
6. Find the coordinates of each of the points.



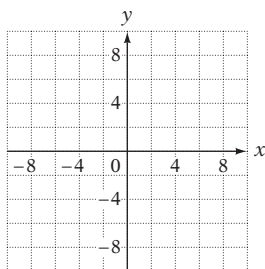
8. Draw a line through all points with an abscissa of -3 .



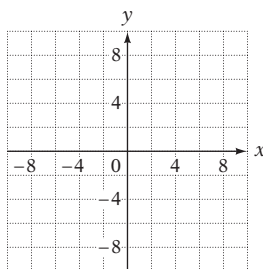
10. Draw a line through all points with an ordinate of 4.



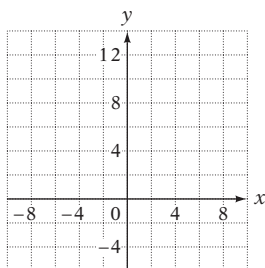
11. Graph the ordered-pair solutions of $y = x^2$ when $x = -2, -1, 0, 1,$ and 2 .



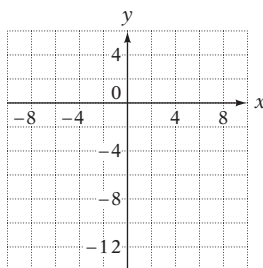
12. Graph the ordered-pair solutions of $y = -x^2 + 1$ when $x = -2, -1, 0, 1,$ and 2 .



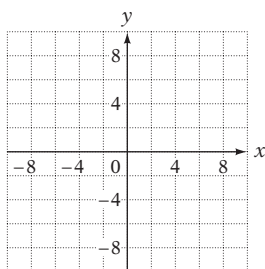
13. Graph the ordered-pair solutions of $y = |x + 1|$ when $x = -5, -3, 0, 3,$ and 5 .



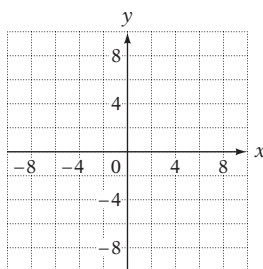
14. Graph the ordered-pair solutions of $y = -2|x|$ when $x = -3, -1, 0, 1,$ and 3 .



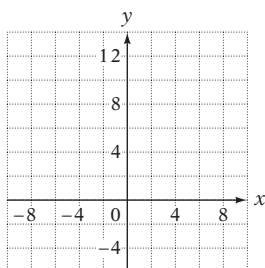
15. Graph the ordered-pair solutions of $y = -x^2 + 2$ when $x = -2, -1, 0, 1,$ and 2 .



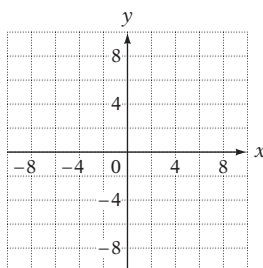
16. Graph the ordered-pair solutions of $y = -x^2 + 4$ when $x = -3, -1, 0, 1,$ and 3 .



17. Graph the ordered-pair solutions of $y = x^3 - 2$ when $x = -1, 0, 1,$ and 2 .



18. Graph the ordered-pair solutions of $y = -x^3 + 1$ when $x = -1, 0, 1,$ and $\frac{3}{2}$.



OBJECTIVE B To find the length and midpoint of a line segment

For Exercises 19 to 30, find the distance, to the nearest hundredth, between the given points. Then find the coordinates of the midpoint of the line segment connecting the points.

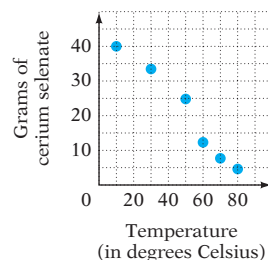
19. $P_1(3, 5)$ and $P_2(5, 1)$
20. $P_1(-2, 3)$ and $P_2(4, -1)$
21. $P_1(0, 3)$ and $P_2(-2, 4)$
22. $P_1(6, -1)$ and $P_2(-3, -2)$
23. $P_1(-3, -5)$ and $P_2(2, -4)$
24. $P_1(-7, -5)$ and $P_2(-2, -1)$
25. $P_1(5, -2)$ and $P_2(-2, 5)$
26. $P_1(3, -6)$ and $P_2(6, 0)$
27. $P_1(5, -5)$ and $P_2(2, -5)$
28. $P_1(-2, -3)$ and $P_2(-2, 5)$
29. $P_1\left(\frac{3}{2}, -\frac{4}{3}\right)$ and $P_2\left(-\frac{1}{2}, \frac{7}{3}\right)$
30. $P_1(4.5, -6.3)$ and $P_2(-1.7, -4.5)$

 31. If the distance between two points on a line equals the difference in the y -coordinates of the points, what can be said about the x -coordinates of the two points?

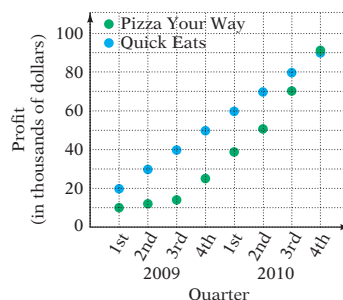
 32. If the midpoint of a line segment is on the x -axis, what can be said about the y -coordinates of the endpoints of the line segment?

OBJECTIVE C To graph a scatter diagram

33. **Chemistry** The amount of a substance that can be dissolved in a fixed amount of water usually increases as the temperature of the water increases. Cerium selenate, however, does not behave in this manner. The graph at the right shows the number of grams of cerium selenate that will dissolve in 100 mg of water for various temperatures, in degrees Celsius.
- a. Determine the temperature at which 25 g of cerium selenate will dissolve.
 - b. Determine the number of grams of cerium selenate that will dissolve when the temperature is 80°C .



34. The scatter diagram at the right shows the quarterly profits for two fast food restaurants, Quick Eats and Pizza Your Way.
- a. Which company had the greater profit in the first quarter of 2010?
 - b. If the trend shown in the scatter diagram were to continue, which restaurant would have the larger profit in 2011?





35. **Meteorology** Draw a scatter diagram for the data in the article.

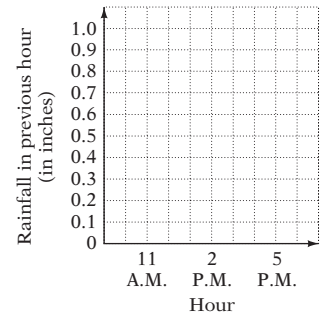
In the News

Tropical Storm Fay Lashes Coast

Tropical storm Fay hit the Florida coast today, with heavy rain and high winds. Here's a look at the amount of rainfall over the course of the afternoon.

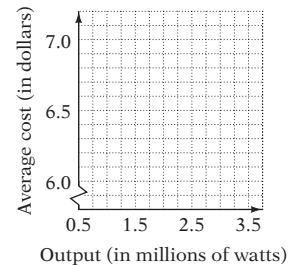
Hour	11 A.M.	12 P.M.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.
Inches of rain in preceding hour	0.25	0.69	0.85	1.05	0.70	0	0.08

Source: www.weather.gov



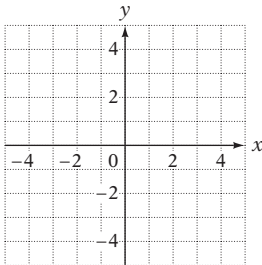
36. **Utilities** A power company suggests that a larger power plant can produce energy more efficiently and therefore at lower cost to consumers. The table below shows the output and average cost for power plants of various sizes. Draw a scatter diagram for these data.

Output (in millions of watts)	0.7	2.2	2.6	3.2	2.8	3.5
Average Cost (in dollars)	6.9	6.5	6.3	6.4	6.5	6.1

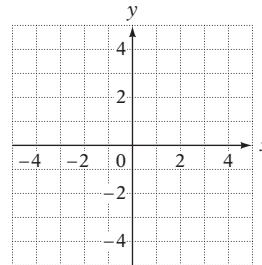


Applying the Concepts

37. Graph the ordered pairs (x, x^2) , where $x \in \{-2, -1, 0, 1, 2\}$.



38. Graph the ordered pairs $\left(x, \frac{1}{x}\right)$, where $x \in \left\{-2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 1, 2\right\}$.

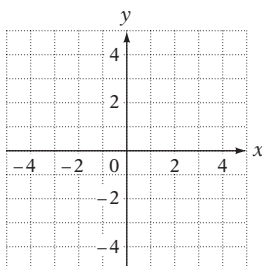


39. Describe the graph of all the ordered pairs (x, y) that are 5 units from the origin.

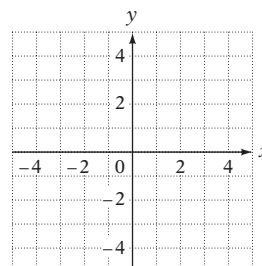


40. Consider two distinct fixed points in the plane. Describe the graph of all the points (x, y) that are equidistant from these fixed points.

41. Draw a line passing through every point whose abscissa equals its ordinate.



42. Draw a line passing through every point whose ordinate is the additive inverse of its abscissa.



SECTION

3.2

Introduction to Functions

OBJECTIVE A

To evaluate a function



In mathematics and its applications, there are many times when it is necessary to investigate a relationship between two quantities. Here is a financial application: Consider a person who is planning to finance the purchase of a car. If the current interest rate for a 5-year loan is 5%, the equation that describes the relationship between the amount that is borrowed B and the monthly payment P is $P = 0.018871B$.

For each amount the purchaser may borrow (B), there is a certain monthly payment (P). The relationship between the amount borrowed and the payment can be recorded as ordered pairs, where the first coordinate is the amount borrowed and the second coordinate is the monthly payment. Some of these ordered pairs are shown at the right.

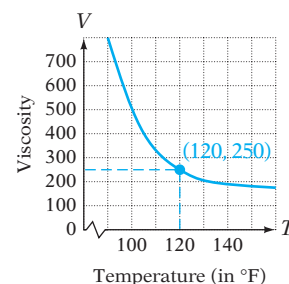
$$0.018871B = P$$

$\swarrow \quad \searrow$
 (6000, 113.23)
 (7000, 132.10)
 (8000, 150.97)
 (9000, 169.84)

A relationship between two quantities is not always given by an equation. The table at the right describes a grading scale that defines a relationship between a score on a test and a letter grade. For each score, the table assigns only one letter grade. The ordered pair (84, B) indicates that a score of 84 receives a letter grade of B.

Score	Grade
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

The graph at the right also shows a relationship between two quantities. It is a graph of the viscosity V of SAE 40 motor oil at various temperatures T . Ordered pairs can be approximated from the graph. The ordered pair (120, 250) indicates that the viscosity of the oil at 120°F is 250 units.



In each of these examples, there is a rule (an equation, a table, or a graph) that determines a certain set of ordered pairs.

Definition of Relation

A **relation** is a set of ordered pairs.

Here are some of the ordered pairs for the relations given above.

Relation	Some of the Ordered Pairs of the Relation
Car Payment	(7500, 141.53), (8750, 165.12), (9390, 177.20)
Grading Scale	(78, C), (98, A), (70, C), (81, B), (94, A)
Oil Viscosity	(100, 500), (120, 250), (130, 200), (150, 180)



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Tips for Success

Have you considered joining a study group? Getting together regularly with other students in the class to go over material and quiz each other can be very beneficial. See *AIM for Success* in the Preface.

Each of these three relations is actually a special type of relation called a *function*. Functions play an important role in mathematics and its applications.

Definition of Function

A **function** is a relation in which no two ordered pairs have the same first coordinate and different second coordinates.

The **domain** of a function is the set of the first coordinates of all the ordered pairs of the function. The **range** is the set of the second coordinates of all the ordered pairs of the function.

For the function defined by the ordered pairs

$$\{(2, 3), (4, 5), (6, 7), (8, 9)\}$$

the domain is $\{2, 4, 6, 8\}$ and the range is $\{3, 5, 7, 9\}$.

HOW TO • 1

Find the domain and range of the function

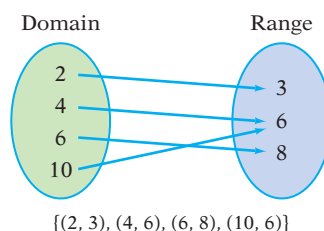
$$\{(2, 3), (4, 6), (6, 8), (10, 6)\}.$$

The domain is $\{2, 4, 6, 10\}$.

The range is $\{3, 6, 8\}$.

- The domain of the function is the set of the first coordinates of the ordered pairs.
- The range of the function is the set of the second coordinates of the ordered pairs.

For each element of the domain of a function there is a corresponding element in the range of the function. A possible diagram for the function above is



Functions defined by tables or graphs, such as those described at the beginning of this section, have important applications. However, a major focus of this text is functions defined by equations in two variables.

The **square function**, which pairs each real number with its square, can be defined by the equation

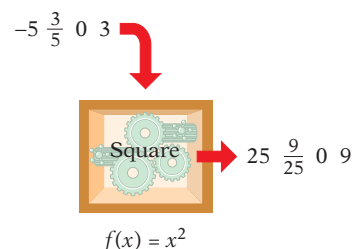
$$y = x^2$$

This equation states that for a given value of x in the domain, the corresponding value of y in the range is the square of x . For instance, if $x = 6$, then $y = 36$ and if $x = -7$, then $y = 49$. Because the value of y *depends* on the value of x , y is called the **dependent variable** and x is called the **independent variable**.

Take Note

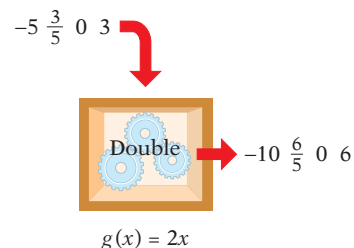
A pictorial representation of the square function is shown at the right. The function acts as a machine that changes a number from the domain into the square of the number.

A function can be thought of as a rule that pairs one number with another number. For instance, the *square function* pairs a number with its square. The ordered pairs for the values shown at the right are $(-5, 25)$, $(\frac{3}{5}, \frac{9}{25})$, $(0, 0)$, and $(3, 9)$. For this function, the second coordinate is the square of the first coordinate. If we let x represent the first coordinate, then the second coordinate is x^2 and we have the ordered pair (x, x^2) .



A function cannot have two ordered pairs with *different* second coordinates and the same first coordinate. However, a function may contain ordered pairs with the *same* second coordinate. For instance, the square function has the ordered pairs $(-3, 9)$ and $(3, 9)$; the second coordinates are the same but the first coordinates are different.

The **double function** pairs a number with twice that number. The ordered pairs for the values shown at the right are $(-5, -10)$, $(\frac{3}{5}, \frac{6}{5})$, $(0, 0)$, and $(3, 6)$. For this function, the second coordinate is twice the first coordinate. If we let x represent the first coordinate, then the second coordinate is $2x$ and we have the ordered pair $(x, 2x)$.



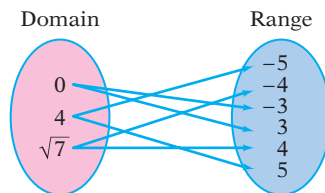
Not every equation in two variables defines a function. For instance, consider the equation

$$y^2 = x^2 + 9$$

Because

$$5^2 = 4^2 + 9 \quad \text{and} \quad (-5)^2 = 4^2 + 9$$

the ordered pairs $(4, 5)$ and $(4, -5)$ are both solutions of the equation. Consequently, there are two ordered pairs that have the same first coordinate (4) but *different* second coordinates (5 and -5). Therefore, the equation does not define a function. Other ordered pairs for this equation are $(0, -3)$, $(0, 3)$, $(\sqrt{7}, -4)$, and $(\sqrt{7}, 4)$. A graphical representation of these ordered pairs is shown below.



Note from this graphical representation that each element from the domain has two arrows pointing to two different elements in the range. Any time this occurs, the situation does not represent a function. However, this diagram *does* represent a relation. The relation for the values shown is $\{(0, -3), (0, 3), (4, -5), (4, 5), (\sqrt{7}, -4), (\sqrt{7}, 4)\}$.

The phrase “ y is a function of x ,” or the same phrase with different variables, is used to describe an equation in two variables that defines a function. To emphasize that the equation represents a function, **function notation** is used.

**Take Note**

The dependent variable y and the notation $f(x)$ can be used interchangeably.

This is the value of the function.
It is the number that is paired with x .

$$f(x) = x^2$$

The name of the function is f .

This is an algebraic expression that defines the relationship between the dependent and independent variables.

The symbol $f(x)$ is read “the value of f at x ” or “ f of x .”

It is important to note that $f(x)$ does *not* mean f times x . The symbol $f(x)$ is the **value of the function** and represents the value of the dependent variable for a given value of the independent variable. We often write $y = f(x)$ to emphasize the relationship between the independent variable x and the dependent variable y . Remember that y and $f(x)$ are different symbols for the same number.

The letters used to represent a function are somewhat arbitrary. All of the following equations represent the same function.

$$\left. \begin{array}{l} f(x) = x^2 \\ s(t) = t^2 \\ P(v) = v^2 \end{array} \right\} \text{Each equation represents the square function.}$$

The process of determining $f(x)$ for a given value of x is called **evaluating a function**. For instance, to evaluate $f(x) = x^2$ when $x = 4$, replace x by 4 and simplify.

$$\begin{aligned} f(x) &= x^2 \\ f(4) &= 4^2 = 16 \end{aligned}$$

The *value* of the function is 16 when $x = 4$. An ordered pair of the function is $(4, 16)$.

**Integrating Technology**

See the Projects and Group Activities at the end of this chapter for instructions on using a graphing calculator to evaluate a function. Instructions are also provided in the Keystroke Guide: *Evaluating Functions*.

HOW TO • 2 Evaluate $g(t) = 3t^2 - 5t + 1$ when $t = -2$.

$$\begin{aligned} g(t) &= 3t^2 - 5t + 1 \\ g(-2) &= 3(-2)^2 - 5(-2) + 1 && \bullet \text{ Replace } t \text{ by } -2 \text{ and then simplify.} \\ &= 3(4) - 5(-2) + 1 \\ &= 12 + 10 + 1 = 23 \end{aligned}$$

When t is -2 , the value of the function is 23.

Therefore, an ordered pair of the function is $(-2, 23)$.

It is possible to evaluate a function for a variable expression.

HOW TO • 3 Evaluate $P(z) = 3z - 7$ when $z = 3 + h$.

$$\begin{aligned} P(z) &= 3z - 7 \\ P(3 + h) &= 3(3 + h) - 7 && \bullet \text{ Replace } z \text{ by } 3 + h \text{ and then simplify.} \\ &= 9 + 3h - 7 \\ &= 3h + 2 \end{aligned}$$

When z is $3 + h$, the value of the function is $3h + 2$.

Therefore, an ordered pair of the function is $(3 + h, 3h + 2)$.

Recall that the range of a function is found by applying the function to each element of the domain. If the domain contains an infinite number of elements, it may be difficult to find the range. However, if the domain has a finite number of elements, then the range can be found by evaluating the function for each element in the domain.

HOW TO • 4 Find the range of $f(x) = x^3 + x$ if the domain is $\{-2, -1, 0, 1, 2\}$.

$$\begin{aligned}f(x) &= x^3 + x \\f(-2) &= (-2)^3 + (-2) = -10 \\f(-1) &= (-1)^3 + (-1) = -2 \\f(0) &= 0^3 + 0 = 0 \\f(1) &= 1^3 + 1 = 2 \\f(2) &= 2^3 + 2 = 10\end{aligned}$$

• Replace x by each member of the domain. The range includes the values of $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(2)$.

The range is $\{-10, -2, 0, 2, 10\}$.

When a function is represented by an equation, the domain of the function is all real numbers for which the value of the function is a real number. For instance:

- The domain of $f(x) = x^2$ is all real numbers, because the square of every real number is a real number.
- The domain of $g(x) = \frac{1}{x-2}$ is all real numbers except 2, because when $x = 2$,
 $g(2) = \frac{1}{2-2} = \frac{1}{0}$, which is not a real number.

The domain of the grading-scale function is the set of whole numbers from 0 to 100. In set-builder notation, this is written $\{x \mid 0 \leq x \leq 100, x \in \text{whole numbers}\}$. The range is $\{A, B, C, D, F\}$.

Score	Grade
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

HOW TO • 5 What values, if any, are excluded from the domain of

$$f(x) = 2x^2 - 7x + 1?$$

Because the value of $2x^2 - 7x + 1$ is a real number for any value of x , the domain of the function is all real numbers. No values are excluded from the domain of $f(x) = 2x^2 - 7x + 1$.

EXAMPLE • 1

Find the domain and range of the function $\{(5, 3), (9, 7), (13, 7), (17, 3)\}$.

Solution

Domain: $\{5, 9, 13, 17\}$;
Range: $\{3, 7\}$

- The domain is the set of first coordinates.

YOU TRY IT • 1

Find the domain and range of the function $\{(-1, 5), (3, 5), (4, 5), (6, 5)\}$.

Your solution

EXAMPLE • 2

Given $p(r) = 5r^3 - 6r - 2$, find $p(-3)$.

Solution

$$\begin{aligned} p(r) &= 5r^3 - 6r - 2 \\ p(-3) &= 5(-3)^3 - 6(-3) - 2 \\ &= 5(-27) + 18 - 2 \\ &= -135 + 18 - 2 = -119 \end{aligned}$$

YOU TRY IT • 2

Evaluate $G(x) = \frac{3x}{x+2}$ when $x = -4$.

Your solution**EXAMPLE • 3**

Evaluate $Q(r) = 2r + 5$ when $r = h + 3$.

Solution

$$\begin{aligned} Q(r) &= 2r + 5 \\ Q(h + 3) &= 2(h + 3) + 5 \\ &= 2h + 6 + 5 \\ &= 2h + 11 \end{aligned}$$

YOU TRY IT • 3

Evaluate $f(x) = x^2 - 11$ when $x = 3h$.

Your solution**EXAMPLE • 4**

Find the range of $f(x) = x^2 - 1$ if the domain is $\{-2, -1, 0, 1, 2\}$.

Solution

To find the range, evaluate the function at each element of the domain.

$$\begin{aligned} f(x) &= x^2 - 1 \\ f(-2) &= (-2)^2 - 1 = 4 - 1 = 3 \\ f(-1) &= (-1)^2 - 1 = 1 - 1 = 0 \\ f(0) &= 0^2 - 1 = 0 - 1 = -1 \\ f(1) &= 1^2 - 1 = 1 - 1 = 0 \\ f(2) &= 2^2 - 1 = 4 - 1 = 3 \end{aligned}$$

The range is $\{-1, 0, 3\}$. Note that 0 and 3 are listed only once.

YOU TRY IT • 4

Find the range of $h(z) = 3z + 1$ if the domain is $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$.

Your solution**EXAMPLE • 5**

What is the domain of $f(x) = 3x^2 - 5x + 2$?

Solution

Because $3x^2 - 5x + 2$ evaluates to a real number for any value of x , the domain of the function is all real numbers.

YOU TRY IT • 5

What value is excluded from the domain of $f(x) = \frac{2}{x-5}$?

Your solution

3.2 EXERCISES

OBJECTIVE A

To evaluate a function



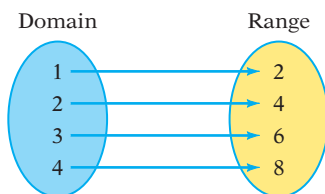
1. In your own words, explain what a function is.



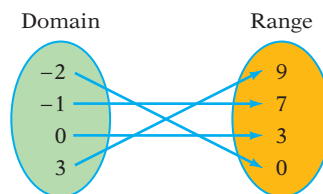
2. What is the domain of a function? What is the range of a function?



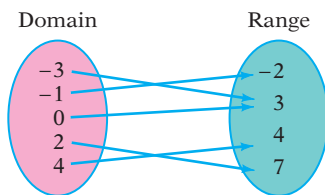
3. Does the diagram below represent a function? Explain your answer.



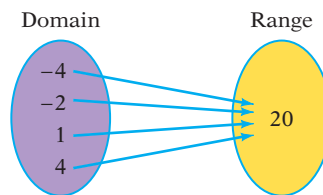
4. Does the diagram below represent a function? Explain your answer.



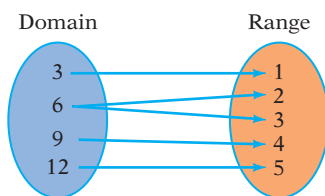
5. Does the diagram below represent a function? Explain your answer.



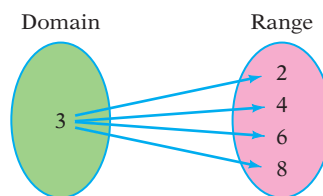
6. Does the diagram below represent a function? Explain your answer.



7. Does the diagram below represent a function? Explain your answer.



8. Does the diagram below represent a function? Explain your answer.



For Exercises 9 to 16, state whether the relation is a function.

9. $\{(0, 0), (2, 4), (3, 6), (4, 8), (5, 10)\}$

10. $\{(1, 3), (3, 5), (5, 7), (7, 9)\}$

11. $\{(-2, -1), (-4, -5), (0, -1), (3, 5)\}$

12. $\{(-3, -1), (-1, -1), (0, 1), (2, 6)\}$

13. $\{(-2, 3), (-1, 3), (0, -3), (1, 3), (2, 3)\}$

14. $\{(0, 0), (1, 0), (2, 0), (3, 0), (4, 0)\}$

15. $\{(1, 1), (4, 2), (9, 3), (1, -1), (4, -2)\}$

16. $\{(3, 1), (3, 2), (3, 3), (3, 4)\}$



17. **Traffic Safety** The table in the article at the right shows the speeding fines that went into effect in the state of Nebraska in 2008.
- Does this table define a function?
 - Given $x = 18$ mph, find y .



18. **Shipping** The table at the right shows the cost to send a Priority Mail package using the U.S. Postal Service.
- Does this table define a function?
 - Given $x = 2$ lb, find y .

Weight in pounds (x)	Cost (y)
$0 < x \leq 1$	\$4.95
$1 < x \leq 2$	\$5.20
$2 < x \leq 3$	\$6.25
$3 < x \leq 4$	\$7.10
$4 < x \leq 5$	\$8.15



19. True or false? If f is a function, then it is possible that $f(0) = -2$ and $f(3) = -2$.



20. True or false? If f is a function, then it is possible that $f(4) = 3$ and $f(4) = 2$.

For Exercises 21 to 24, given $f(x) = 5x - 4$, evaluate.

21. $f(3)$ 22. $f(-2)$ 23. $f(0)$ 24. $f(-1)$

For Exercises 25 to 28, given $G(t) = 4 - 3t$, evaluate.

25. $G(0)$ 26. $G(-3)$ 27. $G(-2)$ 28. $G(4)$

For Exercises 29 to 32, given $q(r) = r^2 - 4$, evaluate.

29. $q(3)$ 30. $q(4)$ 31. $q(-2)$ 32. $q(-5)$

For Exercises 33 to 36, given $F(x) = x^2 + 3x - 4$, evaluate.

33. $F(4)$ 34. $F(-4)$ 35. $F(-3)$ 36. $F(-6)$

For Exercises 37 to 40, given $H(p) = \frac{3p}{p+2}$, evaluate.

37. $H(1)$ 38. $H(-3)$ 39. $H(t)$ 40. $H(v)$

For Exercises 41 to 44, given $s(t) = t^3 - 3t + 4$, evaluate.

41. $s(-1)$ 42. $s(2)$ 43. $s(a)$ 44. $s(w)$

45. Given $P(x) = 4x + 7$, write $P(-2 + h) - P(-2)$ in simplest form.

46. Given $G(t) = 9 - 2t$, write $G(-3 + h) - G(-3)$ in simplest form.

In the News

State Raises Speeding Fines

Beginning next week, you will pay more to speed in Nebraska.

Miles per hour over limit, x	Fine (in dollars), y
$1 \leq x \leq 5$	10
$5 < x \leq 10$	25
$10 < x \leq 15$	75
$15 < x \leq 20$	125
$20 < x \leq 35$	200
$x > 35$	300

Source: *The Grand Island Independent*



- 47. Automotive Technology** Read the article below. If you change the tires on your car to low-rolling-resistance tires that increase your car's fuel efficiency by 5%, what is your annual cost savings? Round to the nearest cent.

In the News

Lower Rolling Resistance Means Lower Cost

Carmakers, seeking ways to meet consumer demands for better gas mileage, are using low-rolling-resistance tires that increase fuel efficiency by as much as 10%. If you drive 12,000 miles a year and the price of gas is \$4.00 per gallon, increasing your fuel efficiency by p

percent can give you an annual cost savings of $S = \frac{2400p}{1+p}$ dollars.

Source: *The Detroit News*

- 48. Airports** Airport administrators have a tendency to price airport parking at a rate that discourages people from using the parking lot for long periods of time. The rate structure for an airport is given in the table at the right.
- Evaluate this function when $t = 2.5$ h.
 - Evaluate this function when $t = 7$ h.

Hours Parked	Cost
$0 < t \leq 1$	\$1.00
$1 < t \leq 2$	\$3.00
$2 < t \leq 4$	\$6.50
$4 < t \leq 7$	\$10.00
$7 < t \leq 12$	\$14.00

- 49. Business** Game Engineering has just completed the programming and testing for a new computer game. The cost to manufacture and package the game depends on the number of units Game Engineering plans to sell. The table at the right shows the cost per game for packaging various quantities.
- Evaluate this function when $x = 7000$.
 - Evaluate this function when $x = 20,000$.

Number of Games Manufactured	Cost to Manufacture One Game
$0 < x \leq 2500$	\$6.00
$2500 < x \leq 5000$	\$5.50
$5000 < x \leq 10,000$	\$4.75
$10,000 < x \leq 20,000$	\$4.00
$20,000 < x \leq 40,000$	\$3.00

- 50. Real Estate** A real estate appraiser charges a fee that depends on the estimated value V of the property. A table giving the fees charged for various estimated values of real estate appears at the right.
- Evaluate this function when $V = \$5,000,000$.
 - Evaluate this function when $V = \$767,000$.

Value of Property	Appraisal Fee
$V < 100,000$	\$350
$100,000 \leq V < 500,000$	\$525
$500,000 \leq V < 1,000,000$	\$950
$1,000,000 \leq V < 5,000,000$	\$2500
$5,000,000 \leq V < 10,000,000$	\$3000

For Exercises 51 to 60, find the domain and range of the function.

- $\{(1, 1), (2, 4), (3, 7), (4, 10), (5, 13)\}$
- $\{(2, 6), (4, 18), (6, 38), (8, 66), (10, 102)\}$
- $\{(0, 1), (2, 2), (4, 3), (6, 4)\}$
- $\{(0, 1), (1, 2), (4, 3), (9, 4)\}$
- $\{(1, 0), (3, 0), (5, 0), (7, 0), (9, 0)\}$
- $\{(-2, -4), (2, 4), (-1, 1), (1, 1), (-3, 9), (3, 9)\}$

57. $\{(0, 0), (1, 1), (-1, 1), (2, 2), (-2, 2)\}$

58. $\{(0, -5), (5, 0), (10, 5), (15, 10)\}$

59. $\{(-2, -3), (-1, 6), (0, 7), (2, 3), (1, 9)\}$

60. $\{(-8, 0), (-4, 2), (-2, 4), (0, -4), (4, 4)\}$

For Exercises 61 to 78, what values are excluded from the domain of the function?

61. $f(x) = \frac{1}{x-1}$

62. $g(x) = \frac{1}{x+4}$

63. $h(x) = \frac{x+3}{x+8}$

64. $F(x) = \frac{2x-5}{x-4}$

65. $f(x) = 3x + 2$

66. $g(x) = 4 - 2x$

67. $G(x) = x^2 + 1$

68. $H(x) = \frac{1}{2}x^2$

69. $f(x) = \frac{x-1}{x}$

70. $g(x) = \frac{2x+5}{7}$

71. $H(x) = x^2 - x + 1$

72. $f(x) = 3x^2 + x + 4$

73. $f(x) = \frac{2x-5}{3}$

74. $g(x) = \frac{3-5x}{5}$

75. $H(x) = \frac{x-2}{x+2}$

76. $h(x) = \frac{3-x}{6-x}$

77. $f(x) = \frac{x-2}{2}$

78. $G(x) = \frac{2}{x-2}$

For Exercises 79 to 92, find the range of the function defined by the equation and the given domain.

79. $f(x) = 4x - 3$; domain = $\{0, 1, 2, 3\}$

80. $G(x) = 3 - 5x$; domain = $\{-2, -1, 0, 1, 2\}$

81. $g(x) = 5x - 8$; domain = $\{-3, -1, 0, 1, 3\}$

82. $h(x) = 3x - 7$; domain = $\{-4, -2, 0, 2, 4\}$

83. $h(x) = x^2$; domain = $\{-2, -1, 0, 1, 2\}$

84. $H(x) = 1 - x^2$; domain = $\{-2, -1, 0, 1, 2\}$

85. $f(x) = 2x^2 - 2x + 2$;
domain = $\{-4, -2, 0, 4\}$

86. $G(x) = -2x^2 + 5x - 2$;
domain = $\{-3, -1, 0, 1, 3\}$

87. $H(x) = \frac{5}{1-x}$; domain = $\{-2, 0, 2\}$

88. $g(x) = \frac{4}{4-x}$; domain = $\{-5, 0, 3\}$

89. $f(x) = \frac{2}{x-4}$; domain = $\{-2, 0, 2, 6\}$

90. $g(x) = \frac{x}{3-x}$; domain = $\{-2, -1, 0, 1, 2\}$

91. $H(x) = 2 - 3x - x^2$; domain = $\{-5, 0, 5\}$

92. $G(x) = 4 - 3x - x^3$; domain = $\{-3, 0, 3\}$

Applying the Concepts

 93. Explain the meanings of the words *relation* and *function*. Include in your explanation how the meanings of the two words differ.

 94. Give a real-world example of a relation that is not a function. Is it possible to give an example of a function that is not a relation? If so, give one. If not, explain why it is not possible.

95. a. Find the set of ordered pairs (x, y) determined by the equation $y = x^3$, where $x \in \{-2, -1, 0, 1, 2\}$.

 b. Does the set of ordered pairs define a function? Why or why not?

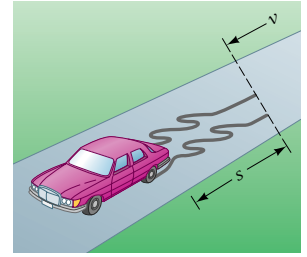
96. a. Find the set of ordered pairs (x, y) determined by the equation $|y| = x$, where $x \in \{0, 1, 2, 3\}$.

 b. Does the set of ordered pairs define a function? Why or why not?

97. **Energy** The power a windmill can generate is a function of the velocity of the wind. The function can be approximated by $P = f(v) = 0.015v^3$, where P is the power in watts and v is the velocity of the wind in meters per second. How much power will be produced by a windmill when the velocity of the wind is 15 m/s?

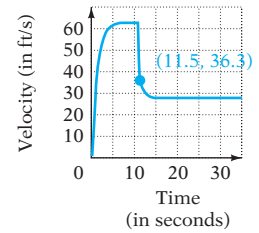


98. **Automotive Technology** The distance s (in feet) a car will skid on a certain road surface after the brakes are applied is a function of the car's velocity v (in miles per hour). The function can be approximated by $s = f(v) = 0.017v^2$. How far will a car skid after its brakes are applied if it is traveling 60 mph?

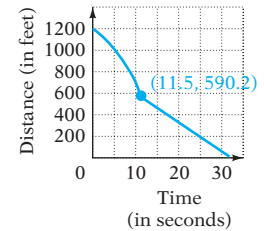


For Exercises 99 to 102, each graph defines a function.

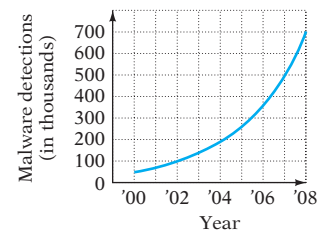
99. **Parachuting** The graph at the right shows the descent speed of a paratrooper after making a low-level training jump.
- The ordered pair $(11.5, 36.3)$ gives the coordinates of a point on the graph. Write a sentence that explains the meaning of this ordered pair.
 - Estimate the speed at which the paratrooper is falling 1 s after jumping from the plane.



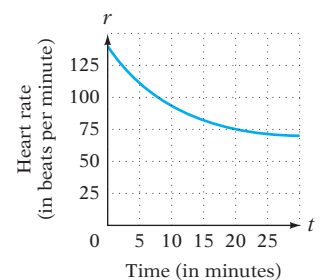
100. **Parachuting** The graph at the right shows the distance above ground of a paratrooper after making a low-level training jump.
- The ordered pair $(11.5, 590.2)$ gives the coordinates of a point on the graph. Write a sentence that explains the meaning of this ordered pair.
 - Estimate the time from the beginning of the jump to the end of the jump.



101. **Computer Science** The graph at the right is based on data from *PC Magazine* and shows the trend in the number of malware (malicious software) attacks, in thousands, for some recent years. (Source: *PC Magazine*, June 2008)
- Estimate the number of malware attacks in 2005.
 - Estimate the number of malware attacks in 2008.



102. **Athletics** The graph at the right shows the decrease in the heart rate r of a runner (in beats per minute) t minutes after the completion of a race.
- Estimate the heart rate of a runner when $t = 5$ min.
 - Estimate the heart rate of a runner when $t = 20$ min.



SECTION

3.3 Linear Functions

OBJECTIVE A

To graph a linear function

Recall that the ordered pairs of a function can be written as $(x, f(x))$ or (x, y) . The **graph of a function** is a graph of the ordered pairs (x, y) that belong to the function. Certain functions have characteristic graphs. A function that can be written in the form $f(x) = mx + b$ (or $y = mx + b$) is called a **linear function** because its graph is a straight line.

Examples of linear functions are shown at the right. Note that the exponent on each variable is 1.

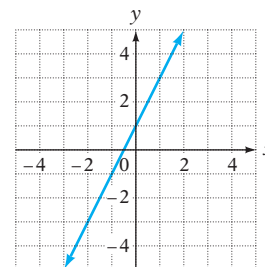
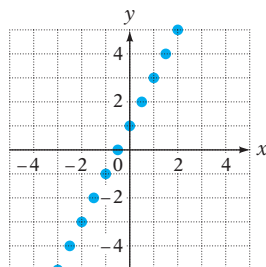
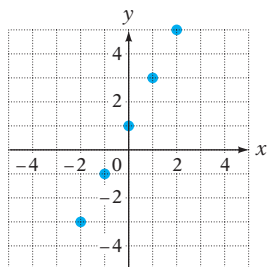
$$\begin{array}{ll} f(x) = 2x + 5 & (m = 2, b = 5) \\ P(t) = 3t - 2 & (m = 3, b = -2) \\ y = -2x & (m = -2, b = 0) \\ y = -\frac{2}{3}x + 1 & \left(m = -\frac{2}{3}, b = 1\right) \\ g(z) = z - 2 & (m = 1, b = -2) \end{array}$$

The equation $y = x^2 + 4x + 3$ is not a linear function because it includes a term with a variable squared. The equation $f(x) = \frac{3}{x-2}$ is not a linear function because a variable occurs in the denominator. Another example of an equation that is not a linear function is $y = \sqrt{x} + 4$; this equation contains a variable within a radical and so is not a linear function.

Consider $f(x) = 2x + 1$. Evaluating the linear function when $x = -2, -1, 0, 1$, and 2 produces some of the ordered pairs of the function. It is convenient to record the results in a table similar to the one at the right. The graph of the ordered pairs is shown in the leftmost figure below.

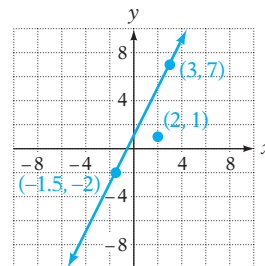
x	$f(x) = 2x + 1$	y	(x, y)
-2	$2(-2) + 1$	-3	$(-2, -3)$
-1	$2(-1) + 1$	-1	$(-1, -1)$
0	$2(0) + 1$	1	$(0, 1)$
1	$2(1) + 1$	3	$(1, 3)$
2	$2(2) + 1$	5	$(2, 5)$

Evaluating the function when x is not an integer produces more ordered pairs to graph, such as $\left(-\frac{5}{2}, -4\right)$ and $\left(\frac{3}{2}, 4\right)$, as shown in the middle figure below. Evaluating the function for still other values of x would result in more and more ordered pairs being graphed. The result would be so many dots that the graph would look like the straight line shown in the rightmost figure, which is the graph of $f(x) = 2x + 1$.



No matter what value of x is chosen, $2x + 1$ is a real number. This means the domain of $f(x) = 2x + 1$ is all real numbers. Therefore, we can use any real number when evaluating the function. Normally, however, values such as π or $\sqrt{5}$ are not used because it is difficult to graph the resulting ordered pairs.

Note from the graph of $f(x) = 2x + 1$ shown at the right that $(-1.5, -2)$ and $(3, 7)$ are the coordinates of points on the graph and that $f(-1.5) = -2$ and $f(3) = 7$. Note also that the point whose coordinates are $(2, 1)$ is not a point on the graph and that $f(2) \neq 1$. Every point on the graph is an ordered pair that belongs to the function, and every ordered pair that belongs to the function corresponds to a point on the graph.



Integrating Technology

See the Projects and Group Activities at the end of this chapter for instructions on using a graphing calculator to graph a linear function. Instructions are also provided in the Keystroke Guide: *Graph*.

Whether an equation is written as $f(x) = mx + b$ or as $y = mx + b$, the equation represents a linear function, and the graph of the equation is a straight line.

Because the graph of a linear function is a straight line, and a straight line is determined by two points, the graph of a linear function can be drawn by finding only two of the ordered pairs of the function. However, it is recommended that you find at least *three* ordered pairs to ensure accuracy.



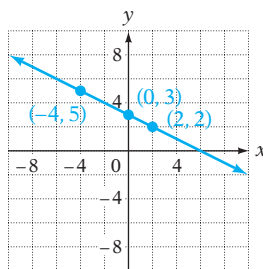
Take Note

When the coefficient of x is a fraction, choose values of x that are multiples of the denominator of the fraction. This will result in coordinates that are integers.

HOW TO • 1

Graph: $f(x) = -\frac{1}{2}x + 3$

x	$y = f(x)$
-4	5
0	3
2	2



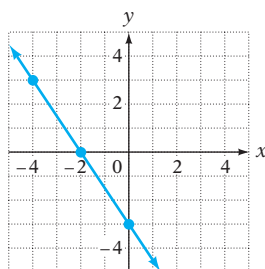
- Find at least three ordered pairs. When the coefficient of x is a fraction, choose values of x that will simplify the calculations. The ordered pairs can be displayed in a table.
- Graph the ordered pairs and draw a line through the points.

EXAMPLE • 1

Graph: $f(x) = -\frac{3}{2}x - 3$

Solution

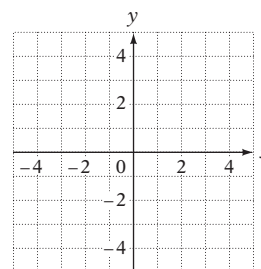
x	$y = f(x)$
0	-3
-2	0
-4	3



YOU TRY IT • 1

Graph: $f(x) = \frac{3}{5}x - 4$

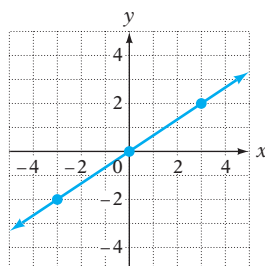
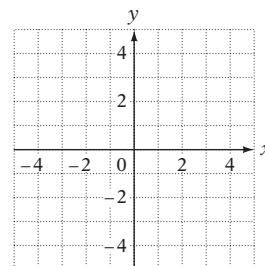
Your solution



Solution on p. S8

EXAMPLE • 2Graph: $y = \frac{2}{3}x$ **Solution**

x	y
0	0
3	2
-3	-2

**YOU TRY IT • 2**Graph: $y = -\frac{3}{4}x$ **Your solution**

Solution on p. S8

OBJECTIVE B**To graph an equation of the form $Ax + By = C$** 

An equation of the form $Ax + By = C$, where A and B are coefficients and C is a constant, is also a *linear equation in two variables*. This equation can be written in the form $y = mx + b$.

HOW TO • 2Write $4x - 3y = 6$ in the form $y = mx + b$.

$$\begin{aligned} 4x - 3y &= 6 \\ -3y &= -4x + 6 \\ y &= \frac{4}{3}x - 2 \end{aligned}$$

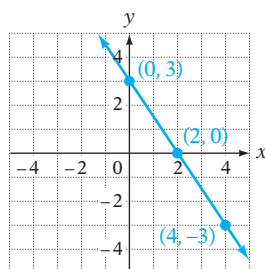
- Subtract $4x$ from each side of the equation.
- Divide each side of the equation by -3 . This is the form $y = mx + b$. $m = \frac{4}{3}$, $b = -2$

To graph an equation of the form $Ax + By = C$, first solve the equation for y . Then follow the same procedure used for graphing an equation of the form $y = mx + b$.

HOW TO • 3Graph: $3x + 2y = 6$

$$\begin{aligned} 3x + 2y &= 6 \\ 2y &= -3x + 6 \\ y &= -\frac{3}{2}x + 3 \end{aligned}$$

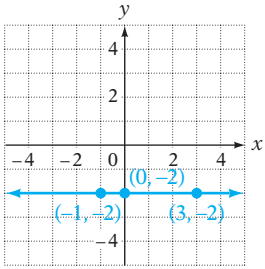
x	y
0	3
2	0
4	-3



- Solve the equation for y .
- Find at least three solutions.
- Graph the ordered pairs in a rectangular coordinate system. Draw a straight line through the points.

If one of the coefficients A or B is zero, the graph of $Ax + By = C$ is a horizontal or vertical line. Consider the equation $y = -2$, where A , the coefficient of x , is 0. We can write this equation in *two* variables as $0 \cdot x + y = -2$. No matter what value of x is selected, $0 \cdot x = 0$. Therefore, y equals -2 for all values of x . The table below shows some of the ordered-pair solutions of $y = -2$. The graph is shown to the right of the table.

x	$0 \cdot x + y = -2$	y	(x, y)
-1	$0 \cdot (-1) + y = -2$	-2	$(-1, -2)$
0	$0 \cdot 0 + y = -2$	-2	$(0, -2)$
3	$0 \cdot 3 + y = -2$	-2	$(3, -2)$



Graph of $y = b$

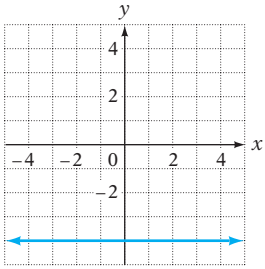
The **graph of $y = b$** is a horizontal line passing through the point $(0, b)$.

HOW TO 4 Graph: $y + 4 = 0$

Solve for y .

$$\begin{aligned} y + 4 &= 0 \\ y &= -4 \end{aligned}$$

The graph of $y = -4$ is a horizontal line passing through $(0, -4)$.



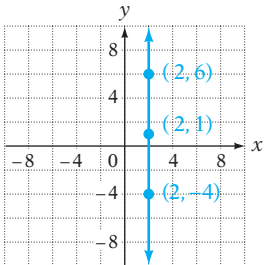
The equation $y = -2$ represents a function. Some of the ordered pairs of this function are $(-1, -2)$, $(0, -2)$, and $(3, -2)$. In function notation, we write $f(x) = -2$. This function is an example of a *constant function*. For every value of x , the value of the function is the constant -2 . For instance, we have $f(17) = -2$, $f(\sqrt{2}) = -2$, and $f(\pi) = -2$.

Constant Function

A function given by $f(x) = b$, where b is a constant, is a **constant function**. The graph of a constant function is a horizontal line passing through $(0, b)$.

Now consider the equation $x = 2$, where B , the coefficient of y , is zero. We write this equation in two variables by writing $x + 0 \cdot y = 2$, where the coefficient of y is 0. No matter what value of y is selected, $0 \cdot y = 0$. Therefore, x equals 2 for all values of y . The following table shows some of the ordered-pair solutions of $x = 2$. The graph is shown to the right of the table.

y	$x + 0 \cdot y = 2$	x	(x, y)
6	$x + 0 \cdot 6 = 2$	2	$(2, 6)$
1	$x + 0 \cdot 1 = 2$	2	$(2, 1)$
-4	$x + 0 \cdot (-4) = 2$	2	$(2, -4)$



Take Note

The equation $y = b$ represents a function. The equation $x = a$ does *not* represent a function. Remember, not all equations represent functions.

Graph of $x = a$

The **graph of $x = a$** is a vertical line passing through the point $(a, 0)$.

Recall that a function is a set of ordered pairs in which no two ordered pairs have the same first coordinate and different second coordinates. Because $(2, 6)$, $(2, 1)$, and $(2, -4)$ are ordered pairs belonging to the equation $x = 2$, this equation does not represent a function, and the graph is not the graph of a function.

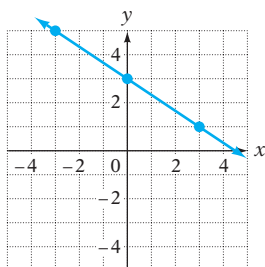
EXAMPLE 3

Graph: $2x + 3y = 9$

Solution Solve the equation for y .

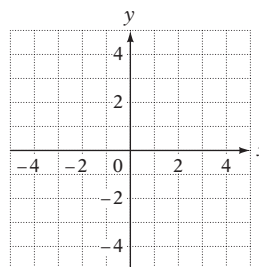
$$\begin{aligned} 2x + 3y &= 9 \\ 3y &= -2x + 9 \\ y &= -\frac{2}{3}x + 3 \end{aligned}$$

x	y
-3	5
0	3
3	1

**YOU TRY IT 3**

Graph: $-3x + 2y = 4$

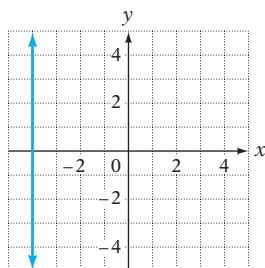
Your solution

**EXAMPLE 4**

Graph: $x = -4$

Solution

The graph of an equation of the form $x = a$ is a vertical line passing through the point whose coordinates are $(a, 0)$.

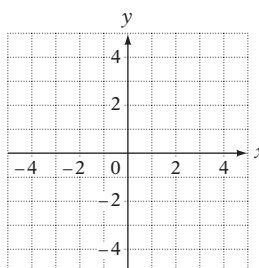


- The graph of $x = -4$ goes through the point $(-4, 0)$.

YOU TRY IT 4

Graph: $y - 3 = 0$

Your solution



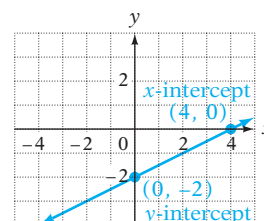
Solutions on p. S8

OBJECTIVE C

To find the x - and the y -intercepts of a straight line



The graph of the equation $x - 2y = 4$ is shown at the right. The graph crosses the x -axis at the point $(4, 0)$. This point is called the **x -intercept**. The graph also crosses the y -axis at the point $(0, -2)$. This point is called the **y -intercept**.



**Take Note**

The x -intercept occurs when $y = 0$.

The y -intercept occurs when $x = 0$.

HOW TO 5 Find the x - and y -intercepts of the graph of the equation

$$3x + 4y = -12.$$

To find the x -intercept, let $y = 0$.
(Any point on the x -axis has y -coordinate 0.)

$$\begin{aligned} 3x + 4y &= -12 \\ 3x + 4(0) &= -12 \\ 3x &= -12 \\ x &= -4 \end{aligned}$$

The x -intercept is $(-4, 0)$.

To find the y -intercept, let $x = 0$. (Any point on the y -axis has x -coordinate 0.)

$$\begin{aligned} 3x + 4y &= -12 \\ 3(0) + 4y &= -12 \\ 4y &= -12 \\ y &= -3 \end{aligned}$$

The y -intercept is $(0, -3)$.

A linear equation can be graphed by finding the x - and y -intercepts and then drawing a line through the two points.

HOW TO 6 Graph $3x - 2y = 6$ by using the x - and y -intercepts.

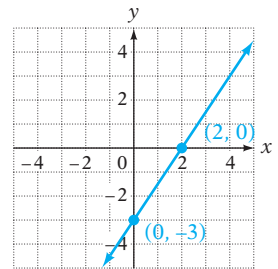
$$\begin{aligned} 3x - 2y &= 6 \\ 3x - 2(0) &= 6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

• To find the x -intercept, let $y = 0$.
Then solve for x .

$$\begin{aligned} 3x - 2y &= 6 \\ 3(0) - 2y &= 6 \\ -2y &= 6 \\ y &= -3 \end{aligned}$$

• To find the y -intercept, let $x = 0$.
Then solve for y .

The x -intercept is $(2, 0)$. The y -intercept is $(0, -3)$.

**EXAMPLE 5**

Graph $4x - y = 4$ by using the x - and y -intercepts.

Solution

x -intercept:

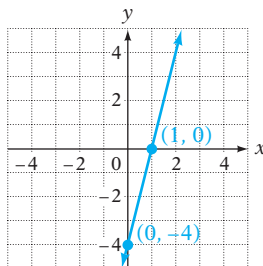
$$\begin{aligned} 4x - y &= 4 \\ 4x - 0 &= 4 && \bullet \text{ Let } y = 0. \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

$(1, 0)$

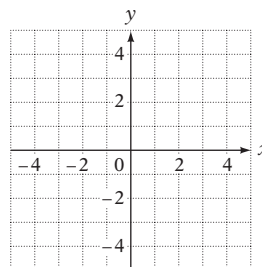
y -intercept:

$$\begin{aligned} 4x - y &= 4 \\ 4(0) - y &= 4 && \bullet \text{ Let } x = 0. \\ -y &= 4 \\ y &= -4 \end{aligned}$$

$(0, -4)$

**YOU TRY IT 5**

Graph $3x - y = 2$ by using the x - and y -intercepts.

Your solution

OBJECTIVE D**To solve application problems**

There are a variety of applications of linear functions.

**Take Note**

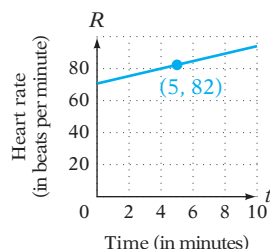
In many applications, the domain of the variable is given such that the equation makes sense. For this application, it would not be sensible to have values of t that are less than 0. This would indicate negative time! The number 10 is somewhat arbitrary, but after 10 min most people's heart rates would level off, and a linear function would no longer apply.

HOW TO 7

The heart rate R after t minutes for a person taking a brisk walk can be approximated by the equation $R = 2t + 72$.

- Graph this equation for $0 \leq t \leq 10$.
- The point whose coordinates are $(5, 82)$ is on the graph. Write a sentence that describes the meaning of this ordered pair.

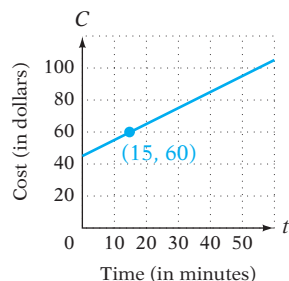
a.



- The ordered pair $(5, 82)$ means that after 5 min the person's heart rate is 82 beats per minute.

EXAMPLE 6

An electronics technician charges \$45 plus \$1 per minute to repair defective wiring in a home or apartment. The equation that describes the total cost C to have defective wiring repaired is given by $C = t + 45$, where t is the number of minutes the technician works. Graph this equation for $0 \leq t \leq 60$. The point whose coordinates are $(15, 60)$ is on this graph. Write a sentence that describes the meaning of this ordered pair.

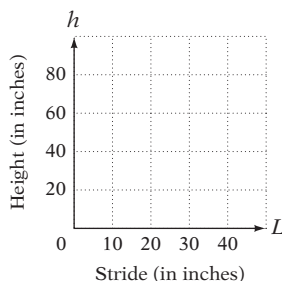
Solution

- Graph $C = t + 45$.
When $t = 0$, $C = 45$.
When $t = 50$, $C = 95$.

The ordered pair $(15, 60)$ means that it costs \$60 for the technician to work 15 min.

YOU TRY IT 6

The height h (in inches) of a person and the length L (in inches) of that person's stride while walking are related. The equation $h = \frac{3}{4}L + 50$ approximates this relationship. Graph this equation for $15 \leq L \leq 40$. The point whose coordinates are $(32, 74)$ is on this graph. Write a sentence that describes the meaning of this ordered pair.

Your solution

3.3 EXERCISES

OBJECTIVE A

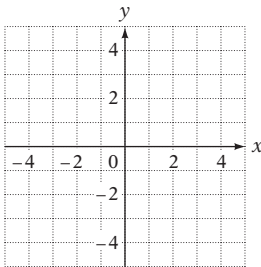
To graph a linear function



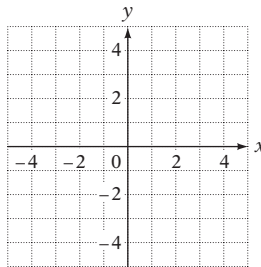
- When finding ordered pairs to graph a line, why do we recommend that you find three ordered pairs?
- To graph points on the graph of $y = \frac{5}{3}x + 4$, it is helpful to choose values of x that are divisible by what number?

For Exercises 3 to 14, graph.

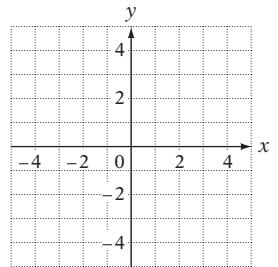
3. $y = x - 3$



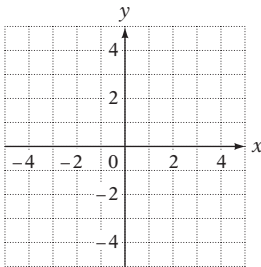
4. $y = -x + 2$



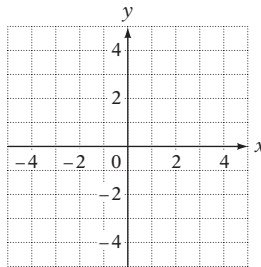
5. $y = -3x + 2$



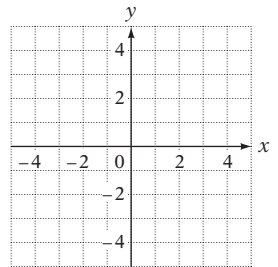
6. $y = -2x + 3$



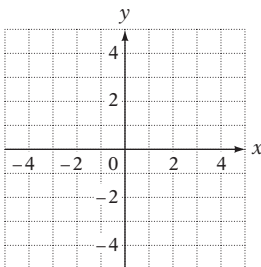
7. $f(x) = 3x - 4$



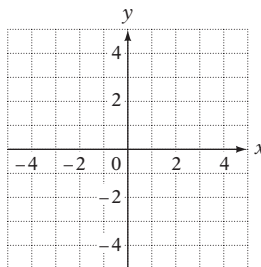
8. $f(x) = \frac{3}{2}x$



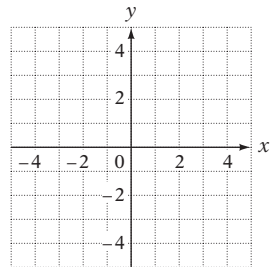
9. $f(x) = -\frac{2}{3}x$



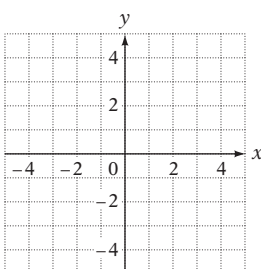
10. $f(x) = \frac{3}{4}x + 2$



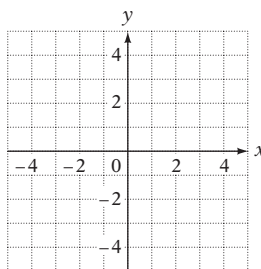
11. $y = \frac{2}{3}x - 4$



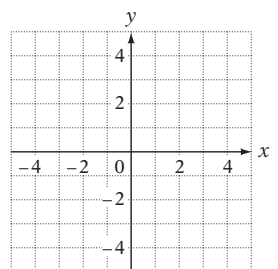
12. $y = -\frac{3}{2}x - 3$



13. $f(x) = -\frac{1}{3}x + 2$



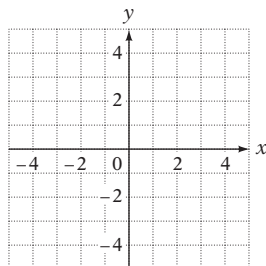
14. $f(x) = \frac{3}{5}x - 1$



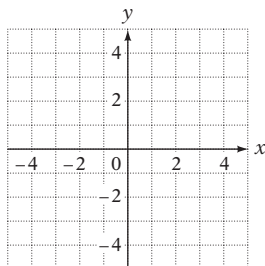
OBJECTIVE B**To graph an equation of the form $Ax + By = C$**

For Exercises 15 to 29, graph.

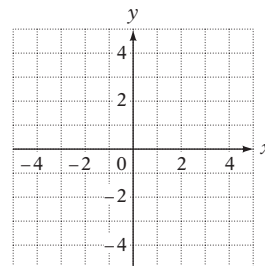
15. $2x + y = -3$



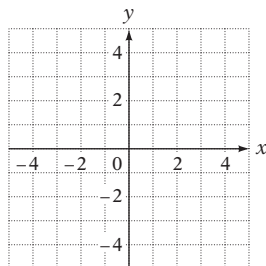
16. $2x - y = 3$



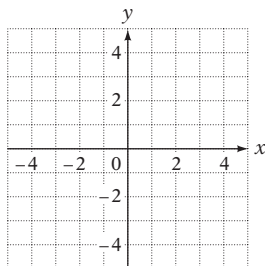
17. $x - 4y = 8$



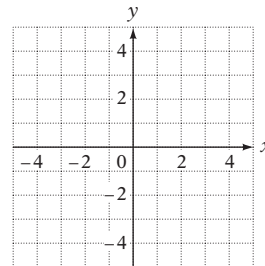
18. $2x + 5y = 10$



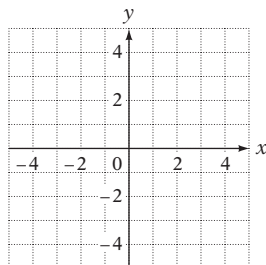
19. $4x + 3y = 12$



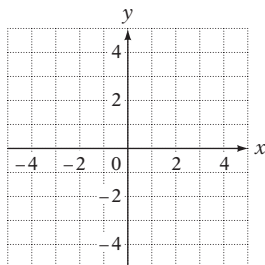
20. $2x - 5y = 10$



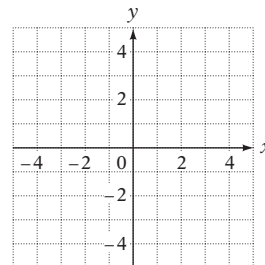
21. $x - 3y = 0$



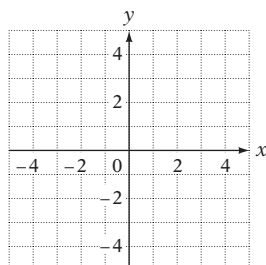
22. $x + 3y = 9$



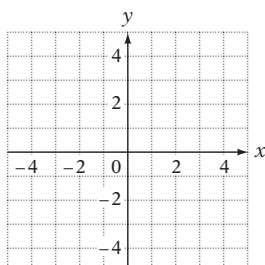
23. $y = -2$



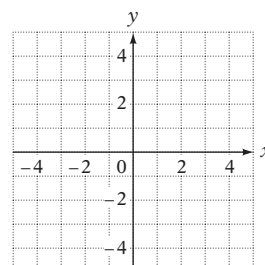
24. $f(x) = 3$



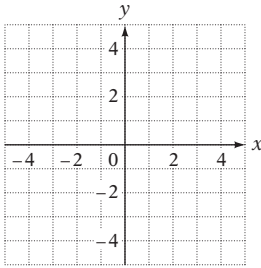
25. $x = -3$



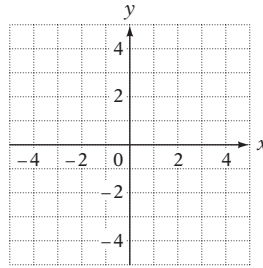
26. $x = 1$



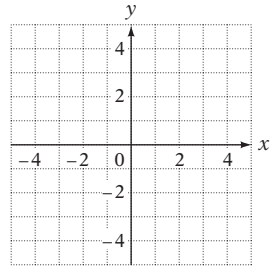
27. $3x - y = -2$



28. $2x - 3y = 12$



29. $3x - 2y = 8$



30. Suppose the graph of
- $Ax + By = C$
- is a horizontal line. Which of the numbers
- A
- ,
- B
- , or
- C
- must be zero?

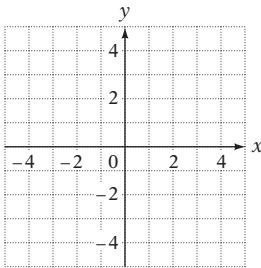


31. Is it always possible to solve
- $Ax + By = C$
- for
- y
- and write the equation in the form
- $y = mx + b$
- ? If not, explain.

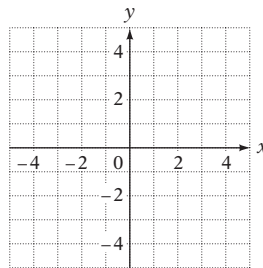
OBJECTIVE C**To find the x - and y -intercepts of a straight line**

For Exercises 32 to 40, find the x - and y -intercepts and graph.

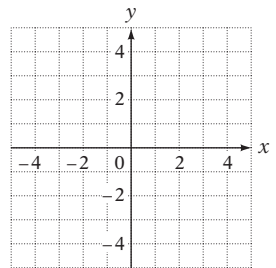
32. $3x + y = 3$



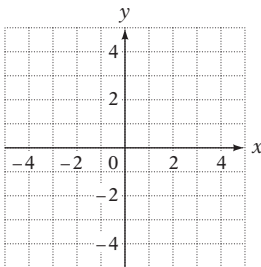
33. $x - 2y = -4$



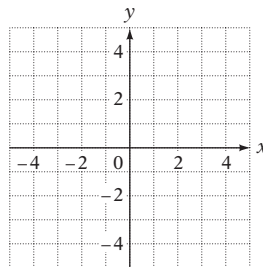
34. $2x - y = 4$



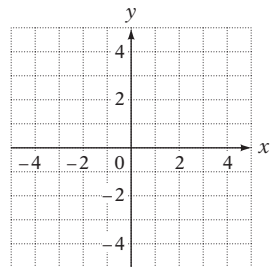
35. $2x - 3y = 9$



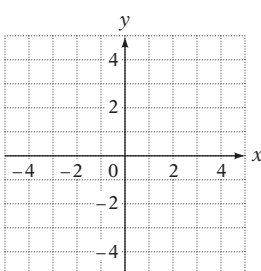
36. $4x - 3y = 8$



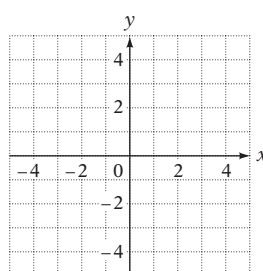
37. $2x + y = 3$



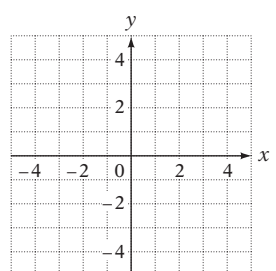
38. $2x - 3y = 4$



39. $3x + 2y = 4$



40. $4x - 3y = 6$



41. Does the graph of every straight line have a
- y
- intercept? Explain.

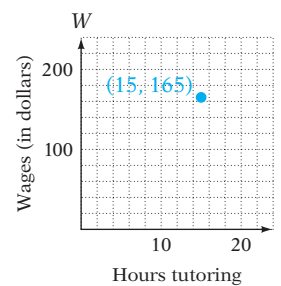


42. Why is it not possible to graph an equation of the form
- $Ax + By = 0$
- by using only the
- x
- and
- y
- intercepts?

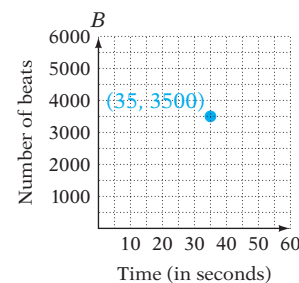
OBJECTIVE D To solve application problems

- 43. Biology** The heart of a ruby-throated hummingbird beats about 1200 times per minute while in flight. The equation $B = 1200t$ gives the total number of heartbeats B of the hummingbird in t minutes of flight. How many times will this hummingbird's heart beat in 7 min of flight?
- 44. Telecommunications** The monthly cost of a wireless telephone plan is \$39.99 for up to 450 min of calling time plus \$.45 per minute for each minute over 450 min. The equation that describes the cost of this plan is $C = 0.45x + 39.99$, where x is the number of minutes over 450. What is the cost of this plan if a person uses **a.** 398 min and **b.** 475 min of calling time?

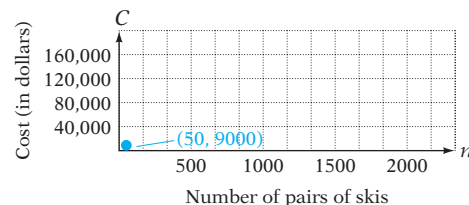
-  **45. Compensation** Marlys receives \$11 per hour as a mathematics department tutor. The equation that describes Marlys's wages is $W = 11t$, where t is the number of hours she spends tutoring. Graph this equation for $0 \leq t \leq 20$. The ordered pair (15, 165) is on the graph. Write a sentence that describes the meaning of this ordered pair.



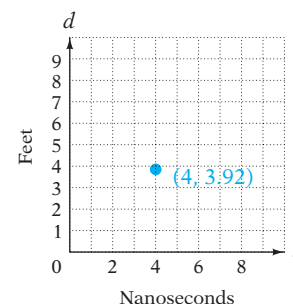
-  **46. Animal Science** A bee beats its wings approximately 100 times per second. The equation that describes the total number of times a bee beats its wings is given by $B = 100t$. Graph this equation for $0 \leq t \leq 60$. The point (35, 3500) is on this graph. Write a sentence that describes the meaning of this ordered pair.



-  **47. Manufacturing** The cost of manufacturing skis is \$5000 for startup costs and \$80 per pair of skis manufactured. The equation that describes the cost of manufacturing n pairs of skis is $C = 80n + 5000$. Graph this equation for $0 \leq n \leq 2000$. The point (50, 9000) is on the graph. Write a sentence that describes the meaning of this ordered pair.



-  **48. Atomic Physics** The Large Hadron Collider, or LHC, is a machine that is capable of accelerating a proton to a velocity that is 99.99% the speed of light. At this speed, a proton will travel approximately 0.98 ft in a billionth of a second (one nanosecond). (Source: news.yahoo.com) The equation $d = 0.98t$ gives the distance d , in feet, traveled by a proton in t nanoseconds. Graph this equation for $0 \leq t \leq 10$. The ordered pair (4, 3.92) is on the graph. Write a sentence that explains the meaning of this ordered pair.



Applying the Concepts



Oceanography For Exercises 49–51, read the article below about the small submarine *Alvin*, used by scientists to explore the ocean for more than 40 years.

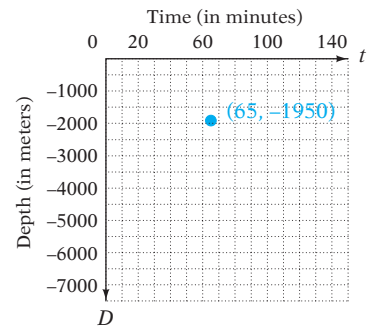
In the News

Alvin, First Viewer of the Titanic, to Retire

Woods Hole Oceanographic Institute announced plans for a replacement for *Alvin*, the original Human Occupied Vehicle (HOV), built in 1964 for deep-sea exploration. *Alvin* can descend at a rate of 30 m/min to a maximum depth of 4500 m. The replacement HOV will be able to descend at a rate of 48 m/min to a maximum depth of 6500 m.

Source: Woods Hole Oceanographic Institute

49. a. The equation that describes *Alvin*'s depth D , in meters, is $D = -30t$, where t is the number of minutes *Alvin* has been descending. Graph this equation for $0 \leq t \leq 150$. The ordered pair $(65, -1950)$ is on the graph. Write a sentence that describes the meaning of this ordered pair.
- b. The equation that describes the replacement HOV's depth D , in meters, is $D = -48t$, where t is the number of minutes the HOV has been descending. On the same set of axes you used in part (a), graph this equation for $0 \leq t \leq 150$. Is the ordered pair with x -coordinate 65 above or below the ordered pair $(65, -1950)$ from part (a)?



50. Does your graph from Exercise 49(a) include any points that do not represent depths that *Alvin* can descend to? If so, describe them.



51. Does your graph from Exercise 49(b) include any points that do not represent depths that the replacement HOV can descend to? If so, describe them.



52. Explain what the graph of an equation represents.



53. Explain how to graph the equation of a straight line by plotting points.



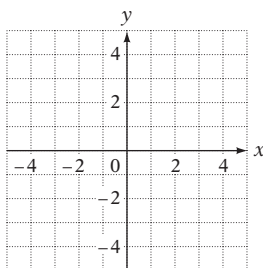
54. Explain how to graph the equation of a straight line by using its x - and y -intercepts.



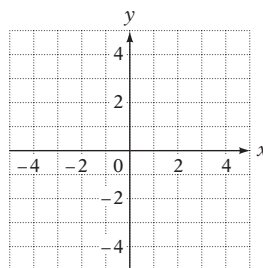
55. Explain why you cannot graph the equation $4x + 3y = 0$ by using just its intercepts.

An equation of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a \neq 0$ and $b \neq 0$, is called the *intercept form of a straight line* because $(a, 0)$ and $(0, b)$ are the x - and y -intercepts of the graph of the equation. Graph the equations in Exercises 56 to 58.

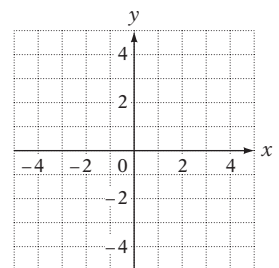
56. $\frac{x}{3} + \frac{y}{5} = 1$



57. $\frac{x}{2} + \frac{y}{3} = 1$



58. $\frac{x}{3} - \frac{y}{2} = 1$

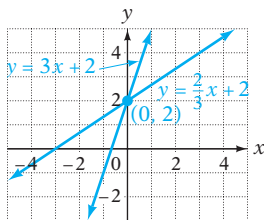


SECTION

3.4 Slope of a Straight Line

OBJECTIVE A

To find the slope of a line given two points



The graphs of $y = 3x + 2$ and $y = \frac{2}{3}x + 2$ are shown at the left. Each graph crosses the y -axis at the point $P(0, 2)$, but the graphs have different slants. The **slope** of a line is a measure of the slant of the line. The symbol for slope is m .

The slope of a line containing two points is the ratio of the change in the y values of the two points to the change in the x values. The line containing the points whose coordinates are $(-1, -3)$ and $(5, 2)$ is shown below.

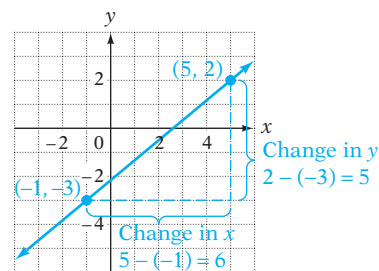
The change in the y values is the difference between the y -coordinates of the two points.

$$\text{Change in } y = 2 - (-3) = 5$$

The change in the x values is the difference between the x -coordinates of the two points.

$$\text{Change in } x = 5 - (-1) = 6$$

The slope of the line between the two points is the ratio of the change in y to the change in x .



$$\text{Slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{5}{6} \qquad m = \frac{2 - (-3)}{5 - (-1)} = \frac{5}{6}$$

In general, if $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points on a line, then

$$\text{Change in } y = y_2 - y_1 \qquad \text{Change in } x = x_2 - x_1$$

Using these ideas, we can state a formula for slope.

Slope Formula

The slope of the line containing the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Frequently, the Greek letter Δ is used to designate the change in a variable. Using this notation, we can write equations for the change in y and the change in x as follows:

$$\text{Change in } y = \Delta y = y_2 - y_1 \qquad \text{Change in } x = \Delta x = x_2 - x_1$$

Using this notation, the slope formula is written $m = \frac{\Delta y}{\Delta x}$.

**Take Note**

When asked to find the slope of a line between two points on a graph, it does not matter which is selected as P_1 and which is selected as P_2 . For the graph at the far right, we could have labeled the points $P_1(4, 5)$ and $P_2(-2, 0)$, where we have reversed the naming of P_1 and P_2 . Then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 5}{-2 - 4} = \frac{-5}{-6} = \frac{5}{6}$$

The value of the slope is the same.

HOW TO • 1

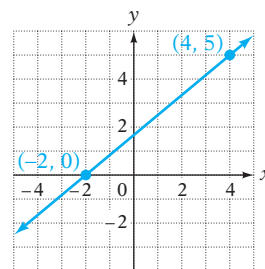
Find the slope of the line passing through the points whose coordinates are $(-2, 0)$ and $(4, 5)$.

Choose P_1 (point 1) and P_2 (point 2). We will choose $P_1(-2, 0)$ and $P_2(4, 5)$. From $P_1(-2, 0)$, we have $x_1 = -2$, $y_1 = 0$. From $P_2(4, 5)$, we have $x_2 = 4$, $y_2 = 5$. Now use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{4 - (-2)} = \frac{5}{6}$$

The slope of the line is $\frac{5}{6}$.

A line that slants upward to the right has a **positive slope**.



Positive slope

HOW TO • 2

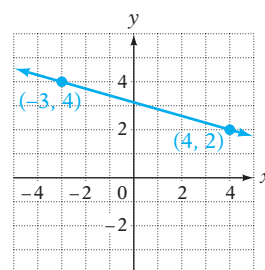
Find the slope of the line passing through the points whose coordinates are $(-3, 4)$ and $(4, 2)$.

We will choose $P_1(-3, 4)$ and $P_2(4, 2)$. From $P_1(-3, 4)$, we have $x_1 = -3$, $y_1 = 4$. From $P_2(4, 2)$, we have $x_2 = 4$, $y_2 = 2$. Now use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{4 - (-3)} = \frac{-2}{7} = -\frac{2}{7}$$

The slope of the line is $-\frac{2}{7}$.

A line that slants downward to the right has a **negative slope**.



Negative slope

HOW TO • 3

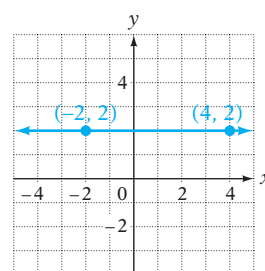
Find the slope of the line passing through the points whose coordinates are $(-2, 2)$ and $(4, 2)$.

We will choose $P_1(-2, 2)$ and $P_2(4, 2)$. From $P_1(-2, 2)$, we have $x_1 = -2$, $y_1 = 2$. From $P_2(4, 2)$, we have $x_2 = 4$, $y_2 = 2$. Now use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{4 - (-2)} = \frac{0}{6} = 0$$

The slope of the line is 0.

A horizontal line has **zero slope**.



Zero slope

**Tips for Success**

To learn mathematics, you must be an active participant. Listening to and watching your professor do mathematics is not enough. Take notes in class, mentally think through every question your instructor asks, and try to answer it even if you are not called on to answer it verbally. Ask questions when you have them. See *AIM for Success* in the Preface for other ways to be an active learner.

HOW TO • 4

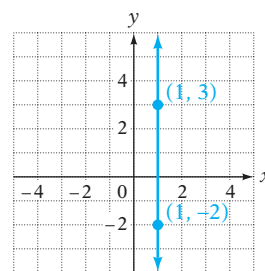
Find the slope of the line passing through the points whose coordinates are $(1, -2)$ and $(1, 3)$.

We will choose $P_1(1, -2)$ and $P_2(1, 3)$. From $P_1(1, -2)$, we have $x_1 = 1$, $y_1 = -2$. From $P_2(1, 3)$, we have $x_2 = 1$, $y_2 = 3$. Now use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{1 - 1} = \frac{5}{0} \quad \text{Not a real number}$$

The slope of the line is **undefined**.

A vertical line has **undefined slope**. Sometimes a vertical line is said to have **no slope**.

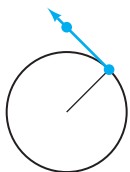


Undefined

Point of Interest

One of the motivations for the discovery of calculus was the desire to solve a more complicated version of the distance-rate problem at the right.

You may be familiar with twirling a ball on the end of a string. If you release the string, the ball flies off in a path as shown below.



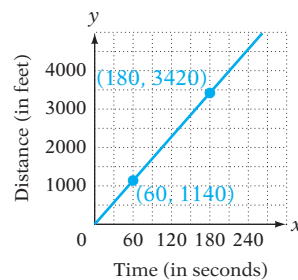
The question that mathematicians tried to answer was essentially, “What is the slope of the line represented by the arrow?”

Answering questions similar to this led to the development of one aspect of calculus.

There are many applications of slope. Here are two examples.

 The first record for the 1-mile run was recorded in 1865 in England. Richard Webster ran the mile in 4 min 36.5 s. His average speed was approximately 19 ft/s.

The graph at the right shows the distance Webster ran during that run. From the graph, note that after 60 s (1 min) he had traveled 1140 ft and that after 180 s (3 min) he had traveled 3420 ft.



We will choose $P_1(60, 1140)$ and $P_2(180, 3420)$. The slope of the line between these two points is

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3420 - 1140}{180 - 60} = \frac{2280}{120} = 19 \end{aligned}$$

Note that the slope of the line is the same as Webster's average speed, 19 ft/s.

Average speed is related to slope.

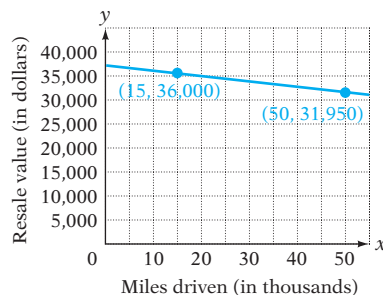
The following example is related to the automotive industry.

 The resale value of a 2006 Chevrolet Corvette declines as the number of miles the car is driven increases. (Source: Edmunds.com, July 2008)

We will choose $P_1(15, 36,000)$ and $P_2(50, 31,950)$. The slope of the line between the two points is

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{31,950 - 36,000}{50 - 15} = \frac{-4050}{35} \approx -115.71 \end{aligned}$$

If we interpret negative slope as decreasing value, then the slope of the line represents the dollar decline of the value of the car for each 1000 mi driven. Thus the value of the car declines approximately \$115.71 for each 1000 mi driven.



In general, any quantity that is expressed by using the word *per* is represented mathematically as slope. In the first example, the slope represented the average speed, 19 ft/s. In the second example, the slope represented the rate at which the value of the car was decreasing, \$115.71 for each 1000 mi driven.

EXAMPLE • 1

Find the slope of the line containing the points whose coordinates are $(2, -5)$ and $(-4, 2)$.

Solution

Choose $P_1 = (2, -5)$ and $P_2 = (-4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{-4 - 2} = \frac{7}{-6}$$

The slope is $-\frac{7}{6}$.

EXAMPLE • 2

Find the slope of the line containing the points whose coordinates are $(-3, 4)$ and $(5, 4)$.

Solution

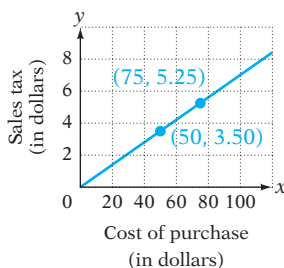
Choose $P_1 = (-3, 4)$ and $P_2 = (5, 4)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 4}{5 - (-3)} \\ &= \frac{0}{8} = 0 \end{aligned}$$

The slope of the line is zero.

EXAMPLE • 3

The graph below shows the relationship between the cost of an item and the sales tax. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

**Solution**

$$\begin{aligned} m &= \frac{5.25 - 3.50}{75 - 50} \\ &= \frac{1.75}{25} \\ &= 0.07 \end{aligned}$$

• Choose $P_1(50, 3.50)$ and $P_2(75, 5.25)$.

A slope of 0.07 means that the sales tax is \$.07 per dollar.

YOU TRY IT • 1

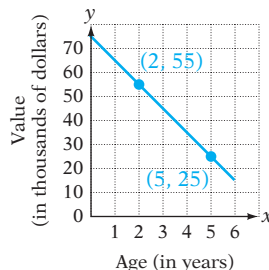
Find the slope of the line containing the points whose coordinates are $(4, -3)$ and $(2, 7)$.

Your solution**YOU TRY IT • 2**

Find the slope of the line containing the points whose coordinates are $(6, -1)$ and $(6, 7)$.

Your solution**YOU TRY IT • 3**

The graph below shows the decrease in the value of a recycling truck for a period of 6 years. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

**Your solution**

OBJECTIVE B**To graph a line given a point and the slope**

The graph of the equation $y = -\frac{3}{4}x + 4$ is shown at the right. The points with coordinates $(-4, 7)$ and $(4, 1)$ are on the graph. The slope of the line is

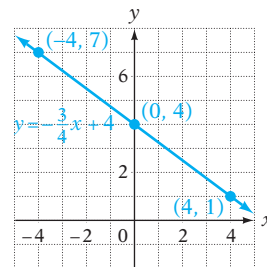
$$m = \frac{7 - 1}{-4 - 4} = \frac{6}{-8} = -\frac{3}{4}$$

The slope of the line has the same value as the coefficient of x .

Recall that the y -intercept is found by replacing x by zero and solving for y .

$$y = -\frac{3}{4}x + 4 \qquad y = -\frac{3}{4}(0) + 4 = 4$$

The y -intercept is $(0, 4)$. The y -coordinate of the y -intercept is the constant term of the equation.

**Slope-Intercept Form of a Straight Line**

The equation $y = mx + b$ is called the **slope-intercept form of a straight line**. The slope of the line is m , the coefficient of x . The y -intercept is $(0, b)$.

When the equation of a straight line is in the form $y = mx + b$, its graph can be drawn by using the slope and y -intercept. First locate the y -intercept. Use the slope to find a second point on the line. Then draw a line through the two points.

 Take Note

When graphing a line by using its slope and y -intercept, *always* start at the y -intercept.

HOW TO 5

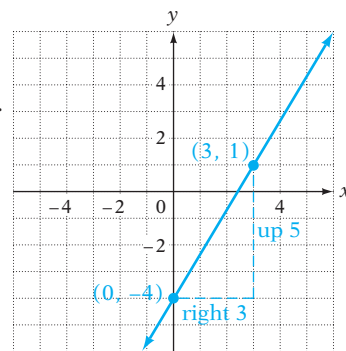
Graph $y = \frac{5}{3}x - 4$ by using the slope and y -intercept.

The slope is the coefficient of x :

$$m = \frac{5}{3} = \frac{\text{change in } y}{\text{change in } x} \quad \text{The } y\text{-intercept is } (0, -4).$$

Beginning at the y -intercept $(0, -4)$, move **right 3** units (change in x) and then **up 5** units (change in y).

The point whose coordinates are $(3, 1)$ is a second point on the graph. Draw a line through the points whose coordinates are $(0, -4)$ and $(3, 1)$.

**HOW TO 6**

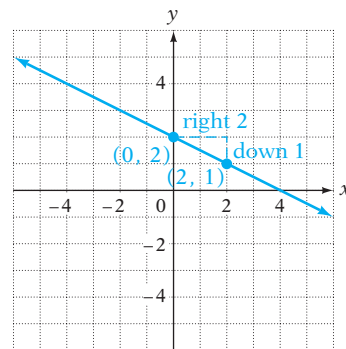
Graph $x + 2y = 4$ by using the slope and y -intercept.

Solve the equation for y .

$$\begin{aligned} x + 2y &= 4 \\ 2y &= -x + 4 \\ y &= -\frac{1}{2}x + 2 \end{aligned} \quad \bullet \quad m = -\frac{1}{2} = \frac{-1}{2}, \quad y\text{-intercept} = (0, 2)$$

Beginning at the y -intercept $(0, 2)$, move **right 2** units (change in x) and then **down 1** unit (change in y).

The point whose coordinates are $(2, 1)$ is a second point on the graph. Draw a line through the points whose coordinates are $(0, 2)$ and $(2, 1)$.



**Take Note**

This example differs from the preceding two in that a point other than the y -intercept is used. In this case, start at the given point.

HOW TO • 7

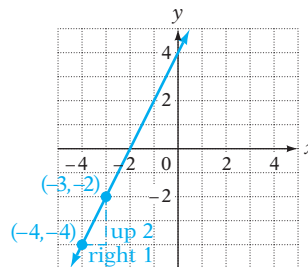
Graph the line that passes through the point whose coordinates are $(-4, -4)$ and has slope 2.

When the slope is an integer, write it as a fraction with denominator 1.

$$m = 2 = \frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$$

Locate $(-4, -4)$ in the coordinate plane. Beginning at that point, move **right 1** unit (change in x) and then **up 2** units (change in y).

The point whose coordinates are $(-3, -2)$ is a second point on the graph. Draw a line through the points $(-4, -4)$ and $(-3, -2)$.

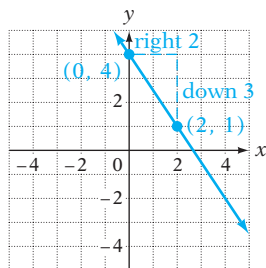
**EXAMPLE • 4**

Graph $y = -\frac{3}{2}x + 4$ by using the slope and y -intercept.

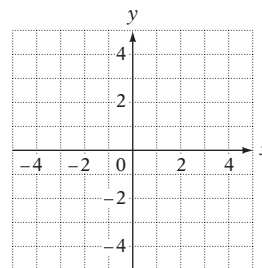
Solution

$$m = -\frac{3}{2} = \frac{-3}{2}$$

y -intercept = $(0, 4)$

**YOU TRY IT • 4**

Graph $2x + 3y = 6$ by using the slope and y -intercept.

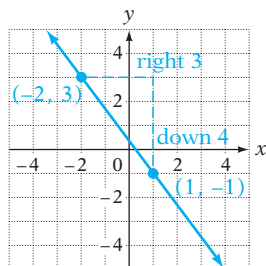
Your solution**EXAMPLE • 5**

Graph the line that passes through the point whose coordinates are $(-2, 3)$ and has slope $-\frac{4}{3}$.

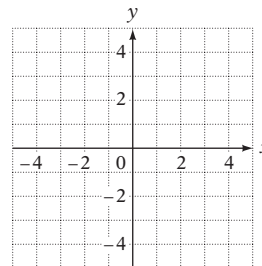
Solution

Locate $(-2, 3)$.

$$m = -\frac{4}{3} = \frac{-4}{3}$$

**YOU TRY IT • 5**

Graph the line that passes through the point whose coordinates are $(-3, -2)$ and has slope 3.

Your solution

Solutions on p. S9

3.4 EXERCISES

OBJECTIVE A

To find the slope of a line given two points

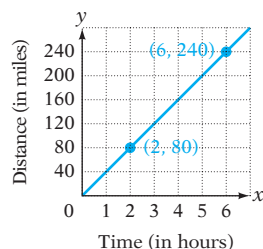
For Exercises 1 to 18, find the slope of the line containing the points with the given coordinates.

1. $(1, 3), (3, 1)$
2. $(2, 3), (5, 1)$
3. $(-1, 4), (2, 5)$
4. $(3, -2), (1, 4)$
5. $(-1, 3), (-4, 5)$
6. $(-1, -2), (-3, 2)$
7. $(0, 3), (4, 0)$
8. $(-2, 0), (0, 3)$
9. $(2, 4), (2, -2)$
10. $(4, 1), (4, -3)$
11. $(2, 5), (-3, -2)$
12. $(4, 1), (-1, -2)$
13. $(2, 3), (-1, 3)$
14. $(3, 4), (0, 4)$
15. $(0, 4), (-2, 5)$
16. $(-2, 3), (-2, 5)$
17. $(-3, -1), (-3, 4)$
18. $(-2, -5), (-4, -1)$

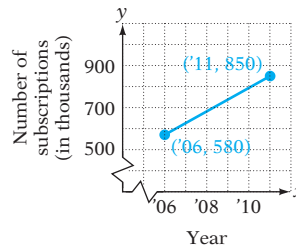
-  19. Let l be a line passing through the points (a, b) and (c, d) . Which two of a, b, c , and d are equal if the slope of l is undefined?

-  20. Let l be a line passing through the points (a, b) and (c, d) . Which two of a, b, c , and d are equal if the slope of l is zero?

-  21. **Travel** The graph below shows the relationship between the distance traveled by a motorist and the time of travel. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

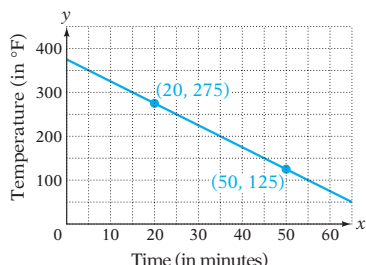


-  22. **Media** The graph below shows the number of people subscribing to a sports magazine of increasing popularity. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

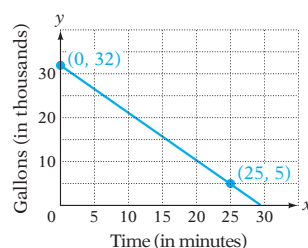




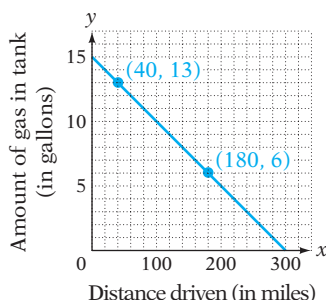
- 23. Temperature** The graph below shows the relationship between the temperature inside an oven and the time since the oven was turned off. Find the slope of the line. Write a sentence that states the meaning of the slope.



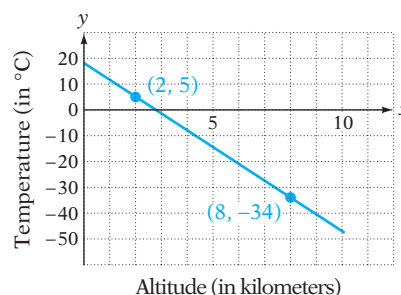
- 24. Home Maintenance** The graph below shows the number of gallons of water remaining in a pool x minutes after a valve is opened to drain the pool. Find the slope of the line. Write a sentence that states the meaning of the slope.



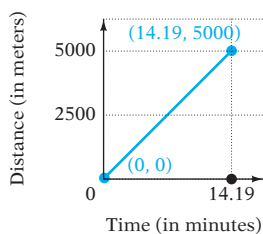
- 25. Fuel Consumption** The graph below shows how the amount of gas in the tank of a car decreases as the car is driven. Find the slope of the line. Write a sentence that states the meaning of the slope.



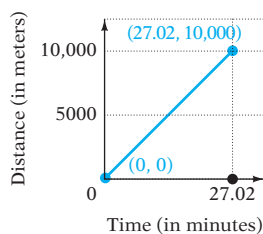
- 26. Meteorology** The troposphere extends from Earth's surface to an elevation of about 11 km. The graph below shows the decrease in the temperature of the troposphere as altitude increases. Find the slope of the line. Write a sentence that states the meaning of the slope.



- 27. Sports** The graph below shows the relationship between distance and time for the 5000-meter run for the world record by Tirunesh Dibaba in 2008. Find the slope of the line between the two points shown on the graph. Round to the nearest tenth. Write a sentence that states the meaning of the slope.



- 28. Sports** The graph below shows the relationship between distance and time for the 10,000-meter run for the Olympic record by Kenenisa Bekele in 2008. Find the slope of the line between the two points shown on the graph. Round to the nearest tenth. Write a sentence that states the meaning of the slope.



- 29. Construction** The American National Standards Institute (ANSI) states that the slope for a wheelchair ramp must not exceed $\frac{1}{12}$.

- Does a ramp that is 6 in. high and 5 ft long meet the requirements of ANSI?
- Does a ramp that is 12 in. high and 170 in. long meet the requirements of ANSI?

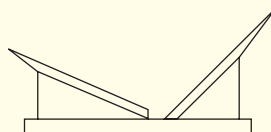


- 30. Solar Roof** Look at the butterfly roof design shown in the article below. Which side of the roof, the left or the right, has a slope of approximately 1? Is the slope of the other side of the roof greater than 1 or less than 1? (*Note:* Consider both slopes to be positive.)

In the News**Go Green with Power Pod's Butterfly Roof**

PowerHouse Enterprises has designed a modular home with a butterfly roof. The roof design combines two sections that slant toward each other at different angles, and is ideal for the use of solar panels.

Source: www.powerhouse-enterprises.com

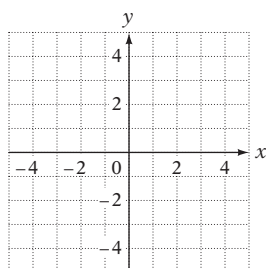
**OBJECTIVE B** To graph a line given a point and the slope

For Exercises 31 to 34, complete the table.

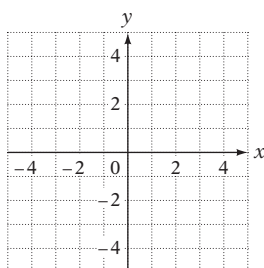
	Equation	Value of m	Value of b	Slope	y -intercept
31.	$y = -3x + 5$				
32.	$y = \frac{2}{5}x - 8$				
33.	$y = 4x$				
34.	$y = -7$				

For Exercises 35 to 46, graph by using the slope and the y -intercept.

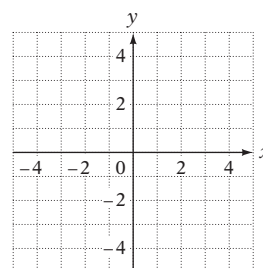
35. $y = \frac{1}{2}x + 2$



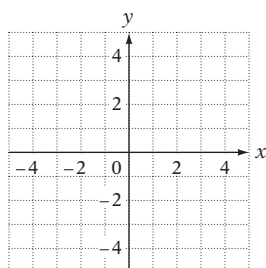
36. $y = \frac{2}{3}x - 3$



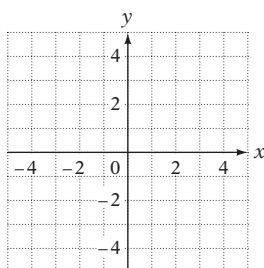
37. $y = -\frac{3}{2}x$



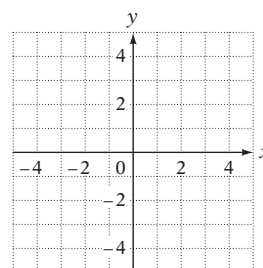
38. $y = \frac{3}{4}x$



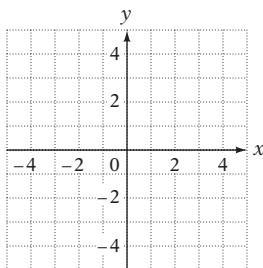
39. $y = -\frac{1}{2}x + 2$



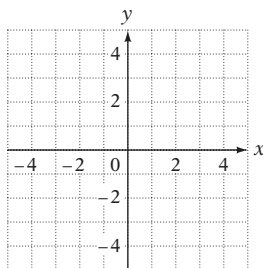
40. $y = \frac{2}{3}x - 1$



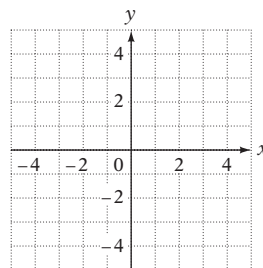
41. $y = 2x - 4$



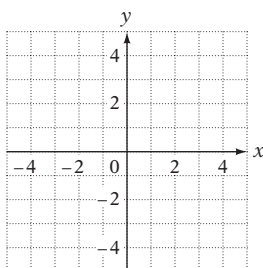
42. $y = -3x + 1$



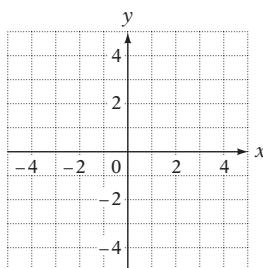
43. $4x - y = 1$



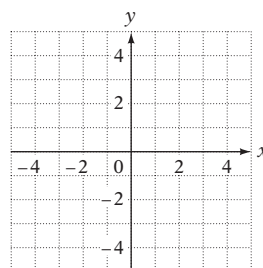
44. $4x + y = 2$



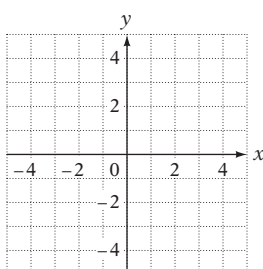
45. $x - 3y = 3$



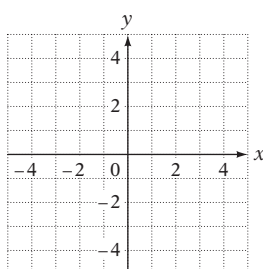
46. $3x + 2y = 8$



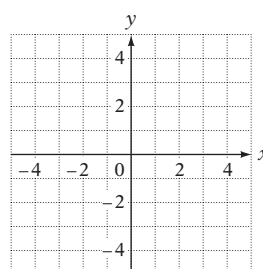
47. Graph the line that passes through the point $(-1, -3)$ and has slope $\frac{4}{3}$.



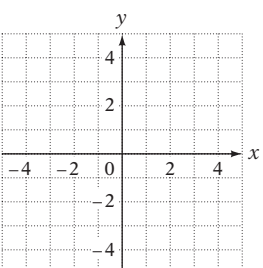
48. Graph the line that passes through the point $(-2, -3)$ and has slope $\frac{5}{4}$.



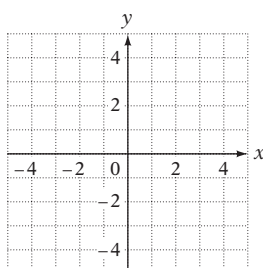
49. Graph the line that passes through the point $(-3, 0)$ and has slope -3 .



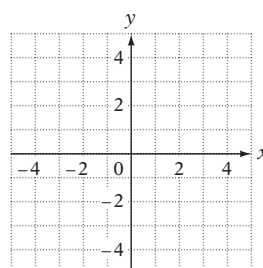
50. Graph the line that passes through the point $(2, -4)$ and has slope $-\frac{1}{2}$.



51. Graph the line that passes through the point $(-4, 1)$ and has slope $\frac{2}{3}$.



52. Graph the line that passes through the point $(1, 5)$ and has slope -4 .



For Exercises 53 and 54, for the given conditions, state **a.** whether the y -intercept of the graph of $Ax + By = C$ lies above or below the x -axis, and **b.** whether the graph of $Ax + By = C$ has positive or negative slope.

53. A and B are positive numbers, and C is a negative number.

54. A and C are positive numbers, and B is a negative number.

Applying the Concepts

Complete the following sentences.

55. If a line has a slope of 2, then the value of y increases/decreases by _____ as the value of x increases by 1.
56. If a line has a slope of -3 , then the value of y increases/decreases by _____ as the value of x increases by 1.
57. If a line has a slope of -2 , then the value of y increases/decreases by _____ as the value of x decreases by 1.
58. If a line has a slope of 3, then the value of y increases/decreases by _____ as the value of x decreases by 1.
59. If a line has a slope of $-\frac{2}{3}$, then the value of y increases/decreases by _____ as the value of x increases by 1.
60. If a line has a slope of $\frac{1}{2}$, then the value of y increases/decreases by _____ as the value of x increases by 1.

61. Match each equation with its graph.

i. $y = -2x + 4$

ii. $y = 2x - 4$

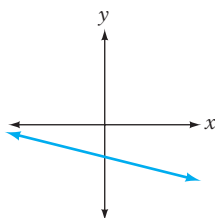
iii. $y = 2$

iv. $2x + 4y = 0$

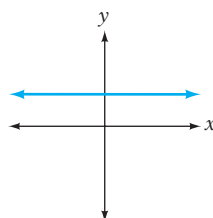
v. $y = \frac{1}{2}x + 4$

vi. $y = -\frac{1}{4}x - 2$

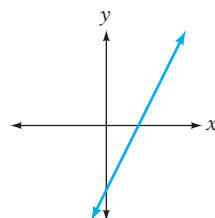
A.



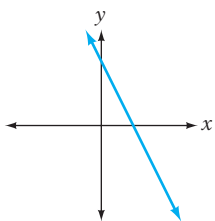
B.



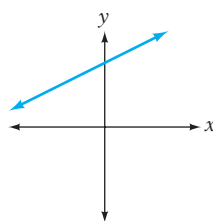
C.



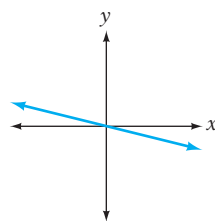
D.



E.



F.



62. Explain how you can use the slope of a line to determine whether three given points lie on the same line. Then use your procedure to determine whether each of the following sets of points lie on the same line.

a. $(2, 5), (-1, -1), (3, 7)$

b. $(-1, 5), (0, 3), (-3, 4)$

For Exercises 63 to 66, determine the value of k such that the points whose coordinates are given lie on the same line.

63. $(3, 2), (4, 6), (5, k)$

64. $(-2, 3), (1, 0), (k, 2)$

65. $(k, 1), (0, -1), (2, -2)$

66. $(4, -1), (3, -4), (k, k)$

SECTION

3.5

Finding Equations of Lines

OBJECTIVE A

To find the equation of a line given a point and the slope



When the slope of a line and a point on the line are known, the equation of the line can be determined. If the particular point is the y -intercept, use the slope-intercept form, $y = mx + b$, to find the equation.

HOW TO • 1

Find the equation of the line that contains the point $P(0, 3)$ and has slope $\frac{1}{2}$.

The known point is the y -intercept, $P(0, 3)$.

$$y = mx + b$$

- Use the slope-intercept form.

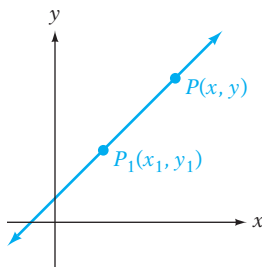
$$y = \frac{1}{2}x + 3$$

- Replace m with $\frac{1}{2}$, the given slope. Replace b with 3, the y -coordinate of the y -intercept.

The equation of the line is $y = \frac{1}{2}x + 3$.

One method of finding the equation of a line when the slope and *any* point on the line are known involves using the *point-slope formula*. This formula is derived from the formula for the slope of a line as follows.

Let $P_1(x_1, y_1)$ be the given point on the line, and let $P(x, y)$ be any other point on the line. See the graph at the left.



$$\frac{y - y_1}{x - x_1} = m$$

- Use the formula for the slope of a line.

$$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1)$$

- Multiply each side by $(x - x_1)$.

$$y - y_1 = m(x - x_1)$$

- Simplify.

Point-Slope Formula

Let m be the slope of a line, and let $P_1(x_1, y_1)$ be the coordinates of a point on the line. The equation of the line can be found from the **point-slope formula**:

$$y - y_1 = m(x - x_1)$$

HOW TO • 2

Find the equation of the line that contains the point whose coordinates are $P(4, -1)$ and has slope $-\frac{3}{4}$.

$$y - y_1 = m(x - x_1)$$

- Use the point-slope formula.

$$y - (-1) = \left(-\frac{3}{4}\right)(x - 4)$$

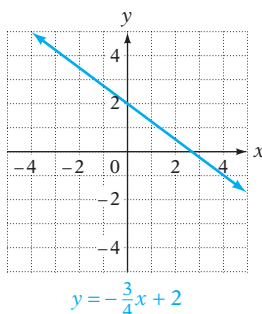
- $m = -\frac{3}{4}$, $(x_1, y_1) = (4, -1)$

$$y + 1 = -\frac{3}{4}x + 3$$

- Simplify.

$$y = -\frac{3}{4}x + 2$$

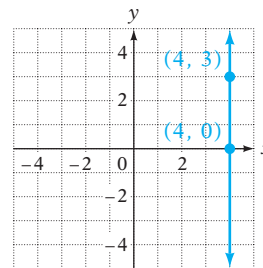
- Write the equation in the form $y = mx + b$.



HOW TO • 3 Find the equation of the line that passes through the point whose coordinates are $(4, 3)$ and whose slope is undefined.

Because the slope is undefined, the point-slope formula cannot be used to find the equation. Instead, recall that when the slope of a line is undefined, the line is vertical. The equation of a vertical line is $x = a$, where a is the x -coordinate of the x -intercept. Because the line is vertical and passes through $(4, 3)$, the x -intercept is $(4, 0)$.

The equation of the line is $x = 4$.



EXAMPLE • 1

Find the equation of the line that contains the point $P(3, 0)$ and has slope -4 .

Solution

$$m = -4 \quad (x_1, y_1) = (3, 0)$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -4(x - 3) \\ y &= -4x + 12 \end{aligned}$$

The equation of the line is $y = -4x + 12$.

YOU TRY IT • 1

Find the equation of the line that contains the point $P(-3, -2)$ and has slope $-\frac{1}{3}$.

Your solution

EXAMPLE • 2

Find the equation of the line that contains the point $P(-2, 4)$ and has slope 2 .

Solution

$$m = 2 \quad (x_1, y_1) = (-2, 4)$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 2[x - (-2)] \\ y - 4 &= 2(x + 2) \\ y - 4 &= 2x + 4 \\ y &= 2x + 8 \end{aligned}$$

The equation of the line is $y = 2x + 8$.

YOU TRY IT • 2

Find the equation of the line that contains the point $P(4, -3)$ and has slope -3 .

Your solution

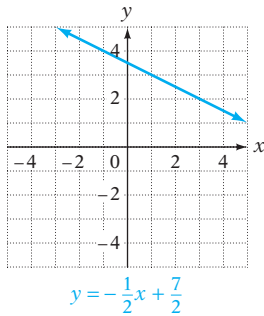
Solutions on p. S9

OBJECTIVE B

To find the equation of a line given two points



The point-slope formula and the formula for slope are used to find the equation of a line when two points are known.



HOW TO • 4 Find the equation of the line containing the points whose coordinates are (3, 2) and (-5, 6).

To use the point-slope formula, we must know the slope. Use the formula for slope to determine the slope of the line between the two given points. We will choose $P_1(3, 2)$ and $P_2(-5, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{-5 - 3} = \frac{4}{-8} = -\frac{1}{2}$$

Now use the point-slope formula with $m = -\frac{1}{2}$ and $(x_1, y_1) = (3, 2)$.

$$y - y_1 = m(x - x_1)$$

• Use the point-slope formula.

$$y - 2 = \left(-\frac{1}{2}\right)(x - 3)$$

• $m = -\frac{1}{2}, (x_1, y_1) = (3, 2)$

$$y - 2 = -\frac{1}{2}x + \frac{3}{2}$$

• Simplify.

$$y = -\frac{1}{2}x + \frac{7}{2}$$

The equation of the line is $y = -\frac{1}{2}x + \frac{7}{2}$.

EXAMPLE • 3

Find the equation of the line passing through the points whose coordinates are (2, 3) and (4, 1).

Solution

Choose $P_1(2, 3)$ and $P_2(4, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 2} = \frac{-2}{2} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1(x - 2)$$

$$y - 3 = -x + 2$$

$$y = -x + 5$$

The equation of the line is $y = -x + 5$.

YOU TRY IT • 3

Find the equation of the line passing through the points whose coordinates are (2, 0) and (5, 3).

Your solution

EXAMPLE • 4

Find the equation of the line containing the points whose coordinates are (2, -3) and (2, 5).

Solution

Choose $P_1(2, -3)$ and $P_2(2, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{2 - 2} = \frac{8}{0}$$

The slope is undefined, so the graph of the line is vertical.

The equation of the line is $x = 2$.

YOU TRY IT • 4

Find the equation of the line containing the points whose coordinates are (2, 3) and (-5, 3).

Your solution

OBJECTIVE C**To solve application problems**

Linear functions can be used to model a variety of applications in science and business. For each application, data are collected and the independent and dependent variables are selected. Then a linear function is determined that models the data.

EXAMPLE 5

Suppose a manufacturer has determined that at a price of \$150, consumers will purchase 1 million 8 GB digital media players and that at a price of \$125, consumers will purchase 1.25 million 8 GB digital media players. Describe this situation with a linear function. Use this function to predict how many 8 GB digital media players consumers will purchase if the price is \$80.

Strategy

- Select the independent and dependent variables. Because you are trying to determine the number of 8 GB digital media players, that quantity is the *dependent* variable, y . The price of the 8 GB digital media players is the *independent* variable, x .
- From the given data, two ordered pairs are (150, 1) and (125, 1.25). (The ordinates are in millions of units.) Use these ordered pairs to determine the linear function.
- Evaluate this function when $x = 80$ to predict how many 8 GB digital media players consumers will purchase if the price is \$80.

Solution

Choose $P_1(150, 1)$ and $P_2(125, 1.25)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.25 - 1}{125 - 150} = \frac{0.25}{-25} = -0.01$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -0.01(x - 150)$$

$$y = -0.01x + 2.50$$

The linear function is $f(x) = -0.01x + 2.50$.

$$f(80) = -0.01(80) + 2.50 = 1.7$$

Consumers will purchase 1.7 million 8 GB digital media players at a price of \$80.

YOU TRY IT 5

Gabriel Daniel Fahrenheit invented the mercury thermometer in 1717. In terms of readings on this thermometer, water freezes at 32°F and boils at 212°F. In 1742, Anders Celsius invented the Celsius temperature scale. On this scale, water freezes at 0°C and boils at 100°C. Determine a linear function that can be used to predict the Celsius temperature when the Fahrenheit temperature is known.

Your strategy**Your solution**

3.5 EXERCISES

OBJECTIVE A

To find the equation of a line given a point and the slope



1. Explain how to find the equation of a line given its slope and its y-intercept.



2. What is the point-slope formula and how is it used?



3. After you find an equation of a line given the coordinates of a point on the line and its slope, how can you determine whether you have the correct equation?

4. What point must the graph of the equation $y = mx$ pass through?

For Exercises 5 to 40, find the equation of the line that contains the given point and has the given slope.

5. $P(0, 5), m = 2$

6. $P(0, 3), m = 1$

7. $P(2, 3), m = \frac{1}{2}$

8. $P(5, 1), m = \frac{2}{3}$

9. $P(-1, 4), m = \frac{5}{4}$

10. $P(-2, 1), m = \frac{3}{2}$

11. $P(3, 0), m = -\frac{5}{3}$

12. $P(-2, 0), m = \frac{3}{2}$

13. $P(2, 3), m = -3$

14. $P(1, 5), m = -\frac{4}{5}$

15. $P(-1, 7), m = -3$

16. $P(-2, 4), m = -4$

17. $P(-1, -3), m = \frac{2}{3}$

18. $P(-2, -4), m = \frac{1}{4}$

19. $P(0, 0), m = \frac{1}{2}$

20. $P(0, 0), m = \frac{3}{4}$

21. $P(2, -3), m = 3$

22. $P(4, -5), m = 2$

23. $P(3, 5), m = -\frac{2}{3}$

24. $P(5, 1), m = -\frac{4}{5}$

25. $P(0, -3), m = -1$

26. $P(2, 0), m = \frac{5}{6}$

27. $P(1, -4), m = \frac{7}{5}$

28. $P(3, 5), m = -\frac{3}{7}$

29. $P(4, -1), m = -\frac{2}{5}$

30. $P(-3, 5), m = -\frac{1}{4}$

31. $P(3, -4)$, slope is undefined

32. $P(-2, 5)$, slope is undefined

33. $P(-2, -5), m = -\frac{5}{4}$

34. $P(-3, -2), m = -\frac{2}{3}$

35. $P(-2, -3), m = 0$

36. $P(-3, -2), m = 0$

37. $P(4, -5), m = -2$

38. $P(-3, 5), m = 3$

39. $P(-5, -1)$, slope is undefined

40. $P(0, 4)$, slope is undefined

OBJECTIVE B**To find the equation of a line given two points**


41. After you find an equation of a line given the coordinates of two points on the line, how can you determine whether you have the correct equation?



42. If you are asked to find the equation of a line through two given points, does it matter which point is selected as (x_1, y_1) and which point is selected as (x_2, y_2) ?

For Exercises 43 to 78, find the equation of the line that contains the points with the given coordinates.

43. $(0, 2), (3, 5)$

44. $(0, 4), (1, 5)$

45. $(0, -3), (-4, 5)$

46. $(0, -2), (-3, 4)$

47. $(2, 3), (5, 5)$

48. $(4, 1), (6, 3)$

49. $(-1, 3), (2, 4)$

50. $(-1, 1), (4, 4)$

51. $(-1, -2), (3, 4)$

52. $(-3, -1), (2, 4)$

53. $(0, 3), (2, 0)$

54. $(0, 4), (2, 0)$

55. $(-3, -1), (2, -1)$

56. $(-3, -5), (4, -5)$

57. $(-2, -3), (-1, -2)$

58. $(4, 1), (3, -2)$

59. $(-2, 3), (2, -1)$

60. $(3, 1), (-3, -2)$

61. $(2, 3), (5, -5)$

62. $(7, 2), (4, 4)$

63. $(2, 0), (0, -1)$

64. $(0, 4), (-2, 0)$

65. $(3, -4), (-2, -4)$

66. $(-3, 3), (-2, 3)$

67. $(0, 0), (4, 3)$

68. $(2, -5), (0, 0)$

69. $(2, -1), (-1, 3)$

70. $(3, -5), (-2, 1)$

71. $(-2, 5), (-2, -5)$

72. $(3, 2), (3, -4)$

73. $(2, 1), (-2, -3)$

74. $(-3, -2), (1, -4)$

75. $(-4, -3), (2, 5)$

76. $(4, 5), (-4, 3)$

77. $(0, 3), (3, 0)$

78. $(1, -3), (-2, 4)$

OBJECTIVE C**To solve application problems**

79. **Aviation** The pilot of a Boeing 747 jet takes off from Boston's Logan Airport, which is at sea level, and climbs to a cruising altitude of 32,000 ft at a constant rate of 1200 ft/min.

- Write a linear function for the height of the plane in terms of the time after takeoff.
- Use your function to find the height of the plane 11 min after takeoff.



© David Frazier/Corbis

80. **Calories** A jogger running at 9 mph burns approximately 14 Calories per minute.

- Write a linear function for the number of Calories burned by the jogger in terms of the number of minutes run.
- Use your function to find the number of Calories that the jogger has burned after jogging for 32 min.



- 81. Ecology** Use the information in the article at the right.
- Determine a linear function for the percent of trees at 2600 ft that are hardwoods in terms of the year.
 - Use your function from part (a) to predict the percent of trees at 2600 ft that will be hardwoods in 2012.
- 82. Telecommunications** A cellular phone company offers several different service options. One option, for people who plan on using the phone only in emergencies, costs the user \$4.95 per month plus \$.59 per minute for each minute the phone is used.
- Write a linear function for the monthly cost of the phone in terms of the number of minutes the phone is used.
 - Use your function to find the cost of using the cellular phone for 13 min in 1 month.
- 83. Fuel Consumption** The gas tank of a certain car contains 16 gal when the driver of the car begins a trip. Each mile driven by the driver decreases the amount of gas in the tank by 0.032 gal.
- Write a linear function for the number of gallons of gas in the tank in terms of the number of miles driven.
 - Use your function to find the number of gallons in the tank after 150 mi are driven.



- 84. Boiling Points** At sea level, the boiling point of water is 100°C . At an altitude of 2 km, the boiling point of water is 93°C .
- Write a linear function for the boiling point of water in terms of the altitude above sea level.
 - Use your function to predict the boiling point of water on top of Mount Everest, which is approximately 8.85 km above sea level. Round to the nearest degree.
- 85. Business** A manufacturer of motorcycles has determined that 50,000 motorcycles per month can be sold at a price of \$9000. At a price of \$8750, the number of motorcycles sold per month would increase to 55,000.
- Determine a linear function that will predict the number of motorcycles that would be sold each month at a given price.
 - Use this model to predict the number of motorcycles that would be sold at a price of \$8500.
- 86. Business** A manufacturer of graphing calculators has determined that 10,000 calculators per week will be sold at a price of \$95. At a price of \$90, it is estimated that 12,000 calculators would be sold.
- Determine a linear function that will predict the number of calculators that would be sold each week at a given price.
 - Use this model to predict the number of calculators that would be sold each week at a price of \$75.
- 87. Calories** There are approximately 126 Calories in a 2-ounce serving of lean hamburger and approximately 189 Calories in a 3-ounce serving.
- Determine a linear function for the number of Calories in lean hamburger in terms of the size of the serving.
 - Use your function to estimate the number of Calories in a 5-ounce serving of lean hamburger.
- 88. Compensation** An account executive receives a base salary plus a commission. On \$20,000 in monthly sales, the account executive receives \$1800. On \$50,000 in monthly sales, the account executive receives \$3000.
- Determine a linear function that will yield the compensation of the sales executive for a given amount of monthly sales.
 - Use this model to determine the account executive's compensation for \$85,000 in monthly sales.

In the News

Is Global Warming Moving Mountains?

In the mountains of Vermont, maples, beeches, and other hardwood trees that thrive in warm climates are gradually taking over areas that once supported more cold-loving trees, such as balsam and fir. Ecologists report that in 2004, 82% of the trees at an elevation of 2600 ft were hardwoods, as compared to only 57% in 1964.

Source: *The Boston Globe*



Mt. Everest

© David Keaton/Corbis



89. Refer to Exercise 86. Describe how you could use the linear function found in part (a) to find the price at which the manufacturer should sell the calculators in order to sell 15,000 calculators a week.



90. Refer to Exercise 88. Describe how you could use the linear function found in part (a) to find the monthly sales the executive would need to make in order to earn a commission of \$6000 a month.

91. Let f be a linear function. If $f(2) = 5$ and $f(0) = 3$, find $f(x)$.
92. Let f be a linear function. If $f(-3) = 4$ and $f(1) = -8$, find $f(x)$.
93. Given that f is a linear function for which $f(1) = 3$ and $f(-1) = 5$, determine $f(4)$.
94. Given that f is a linear function for which $f(-3) = 2$ and $f(2) = 7$, determine $f(0)$.
95. A line with slope $\frac{4}{3}$ passes through the point $(3, 2)$.
- What is y when $x = -6$?
 - What is x when $y = 6$?
96. A line with slope $-\frac{3}{4}$ passes through the point $(8, -2)$.
- What is y when $x = -4$?
 - What is x when $y = 1$?

Applying the Concepts



97. Explain why the point-slope formula cannot be used to find the equation of a line that is parallel to the y -axis.



98. Refer to Example 5 in this section for each of the following.
- Explain the meaning of the slope of the graph of the linear function given in the example.
 - Explain the meaning of the y -intercept.
 - Explain the meaning of the x -intercept.
99. A line contains the points $(-3, 6)$ and $(6, 0)$. Find the coordinates of three other points that are on this line.
100. A line contains the points $(4, -1)$ and $(2, 1)$. Find the coordinates of three other points that are on this line.
101. Find the equation of the line that passes through the midpoint of the line segment between $P_1(2, 5)$ and $P_2(-4, 1)$ and has slope -2 .

SECTION

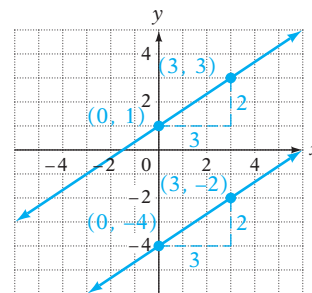
3.6

Parallel and Perpendicular Lines

OBJECTIVE A

To find parallel and perpendicular lines Two lines that have the same slope do not intersect and are called **parallel lines**.The slope of each of the lines at the right is $\frac{2}{3}$.

The lines are parallel.

**Slopes of Parallel Lines**Two nonvertical lines with slopes of m_1 and m_2 are parallel if and only if $m_1 = m_2$. Any two vertical lines are parallel.**HOW TO • 1**Is the line containing the points whose coordinates are $(-2, 1)$ and $(-5, -1)$ parallel to the line that contains the points whose coordinates are $(1, 0)$ and $(4, 2)$?

$$m_1 = \frac{-1 - 1}{-5 - (-2)} = \frac{-2}{-3} = \frac{2}{3}$$

- Find the slope of the line through $(-2, 1)$ and $(-5, -1)$.

$$m_2 = \frac{2 - 0}{4 - 1} = \frac{2}{3}$$

- Find the slope of the line through $(1, 0)$ and $(4, 2)$.

Because $m_1 = m_2$, the lines are parallel.**HOW TO • 2**Find the equation of the line that contains the point whose coordinates are $(2, 3)$ and is parallel to the graph of $y = \frac{1}{2}x - 4$.The slope of the given line is $\frac{1}{2}$. Because parallel lines have the same slope, the slope of the unknown line is also $\frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2}(x - 2)$$

$$y - 3 = \frac{1}{2}x - 1$$

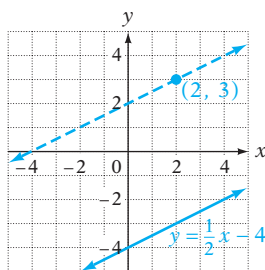
$$y = \frac{1}{2}x + 2$$

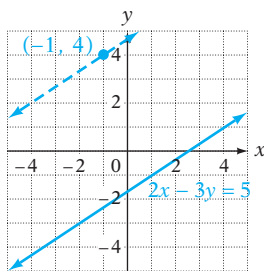
- Use the point-slope formula.

- $m = \frac{1}{2}, (x_1, y_1) = (2, 3)$

- Simplify.

- Write the equation in the form $y = mx + b$.

The equation of the line is $y = \frac{1}{2}x + 2$.

**HOW TO 3**

Find the equation of the line that contains the point whose coordinates are $(-1, 4)$ and is parallel to the graph of $2x - 3y = 5$.

Because the lines are parallel, the slope of the unknown line is the same as the slope of the given line. Solve $2x - 3y = 5$ for y and determine its slope.

$$\begin{aligned} 2x - 3y &= 5 \\ -3y &= -2x + 5 \\ y &= \frac{2}{3}x - \frac{5}{3} \end{aligned}$$

The slope of the given line is $\frac{2}{3}$. Because the lines are parallel, this is the slope of the unknown line. Use the point-slope formula to determine the equation.

$$y - y_1 = m(x - x_1)$$

• Use the point-slope formula.

$$y - 4 = \frac{2}{3}[x - (-1)]$$

• $m = \frac{2}{3}, (x_1, y_1) = (-1, 4)$

$$y - 4 = \frac{2}{3}x + \frac{2}{3}$$

• Simplify.

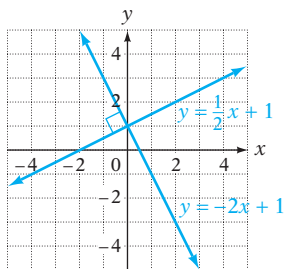
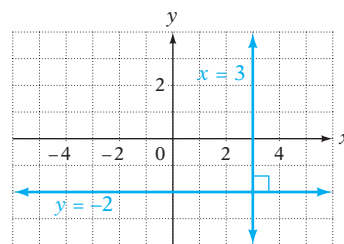
$$y = \frac{2}{3}x + \frac{14}{3}$$

• Write the equation in the form $y = mx + b$.

The equation of the line is $y = \frac{2}{3}x + \frac{14}{3}$.

Two lines that intersect at right angles are **perpendicular lines**.

Any horizontal line is perpendicular to any vertical line. For example, $x = 3$ is perpendicular to $y = -2$.

**Slopes of Perpendicular Lines**

If m_1 and m_2 are the slopes of two lines, neither of which is vertical, then the lines are perpendicular if and only if $m_1 \cdot m_2 = -1$.

A vertical line is perpendicular to a horizontal line.

Solving $m_1 \cdot m_2 = -1$ for m_1 gives $m_1 = -\frac{1}{m_2}$. This last equation states that the slopes of perpendicular lines are **negative reciprocals** of each other.

HOW TO 4

Is the line that contains the points whose coordinates are $(4, 2)$ and $(-2, 5)$ perpendicular to the line that contains the points whose coordinates are $(-4, 3)$ and $(-3, 5)$?

$$m_1 = \frac{5 - 2}{-2 - 4} = \frac{3}{-6} = -\frac{1}{2}$$

• Find the slope of the line through $(4, 2)$ and $(-2, 5)$.

$$m_2 = \frac{5 - 3}{-3 - (-4)} = \frac{2}{1} = 2$$

• Find the slope of the line through $(-4, 3)$ and $(-3, 5)$.

$$m_1 \cdot m_2 = -\frac{1}{2}(2) = -1$$

• Find the product of the two slopes.

Because $m_1 \cdot m_2 = -1$, the lines are perpendicular.

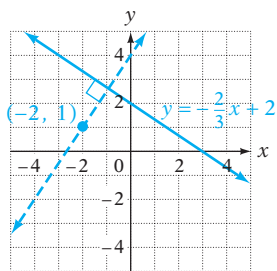
HOW TO • 5 Are the graphs of the lines whose equations are $3x + 4y = 8$ and $8x + 6y = 5$ perpendicular?

To determine whether the lines are perpendicular, solve each equation for y and find the slope of each line. Then use the equation $m_1 \cdot m_2 = -1$.

$$\begin{array}{ll} 3x + 4y = 8 & 8x + 6y = 5 \\ 4y = -3x + 8 & 6y = -8x + 5 \\ y = -\frac{3}{4}x + 2 & y = -\frac{4}{3}x + \frac{5}{6} \\ m_1 = -\frac{3}{4} & m_2 = -\frac{4}{3} \end{array}$$

$$m_1 \cdot m_2 = \left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 1$$

Because $m_1 \cdot m_2 = 1 \neq -1$, the lines are not perpendicular.



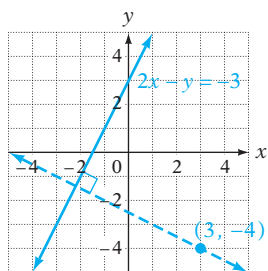
HOW TO • 6 Find the equation of the line that contains the point whose coordinates are $(-2, 1)$ and is perpendicular to the graph of $y = -\frac{2}{3}x + 2$.

The slope of the given line is $-\frac{2}{3}$. The slope of the line perpendicular to the given line is the negative reciprocal of $-\frac{2}{3}$, which is $\frac{3}{2}$. Substitute this slope and the coordinates of the given point, $(-2, 1)$, into the point-slope formula.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{3}{2}[x - (-2)] \\ y - 1 &= \frac{3}{2}x + 3 \\ y &= \frac{3}{2}x + 4 \end{aligned}$$

- The point-slope formula
- $m = \frac{3}{2}, (x_1, y_1) = (-2, 1)$
- Simplify.
- Write the equation in the form $y = mx + b$.

The equation of the perpendicular line is $y = \frac{3}{2}x + 4$.



HOW TO • 7 Find the equation of the line that contains the point whose coordinates are $(3, -4)$ and is perpendicular to the graph of $2x - y = -3$.

$$\begin{array}{ll} 2x - y = -3 & \bullet \text{ Determine the slope of the given line by} \\ -y = -2x - 3 & \text{solving the equation for } y. \\ y = 2x + 3 & \bullet \text{ The slope is 2.} \end{array}$$

The slope of the line perpendicular to the given line is $-\frac{1}{2}$, the negative reciprocal of 2. Now use the point-slope formula to find the equation of the line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= -\frac{1}{2}(x - 3) \\ y + 4 &= -\frac{1}{2}x + \frac{3}{2} \\ y &= -\frac{1}{2}x - \frac{5}{2} \end{aligned}$$

- The point-slope formula
- $m = -\frac{1}{2}, (x_1, y_1) = (3, -4)$
- Simplify.
- Write the equation in the form $y = mx + b$.

The equation of the perpendicular line is $y = -\frac{1}{2}x - \frac{5}{2}$.

EXAMPLE • 1

Is the line that contains the points whose coordinates are $(-4, 2)$ and $(1, 6)$ parallel to the line that contains the points whose coordinates are $(2, -4)$ and $(7, 0)$?

Solution

$$m_1 = \frac{6 - 2}{1 - (-4)} = \frac{4}{5}$$

$$m_2 = \frac{0 - (-4)}{7 - 2} = \frac{4}{5}$$

$$m_1 = m_2 = \frac{4}{5}$$

- $(x_1, y_1) = (-4, 2),$
 $(x_2, y_2) = (1, 6)$
- $(x_1, y_1) = (2, -4),$
 $(x_2, y_2) = (7, 0)$

The lines are parallel.

YOU TRY IT • 1

Is the line that contains the points whose coordinates are $(-2, -3)$ and $(7, 1)$ perpendicular to the line that contains the points whose coordinates are $(4, 1)$ and $(6, -5)$?

Your solution**EXAMPLE • 2**

Are the lines $4x - y = -2$ and $x + 4y = -12$ perpendicular?

Solution

$$4x - y = -2$$

$$-y = -4x - 2$$

$$y = 4x + 2$$

$$m_1 = 4$$

$$x + 4y = -12$$

$$4y = -x - 12$$

$$y = -\frac{1}{4}x - 3$$

$$m_2 = -\frac{1}{4}$$

$$m_1 \cdot m_2 = 4 \left(-\frac{1}{4} \right) = -1$$

The lines are perpendicular.

YOU TRY IT • 2

Are the lines $5x + 2y = 2$ and $5x + 2y = -6$ parallel?

Your solution**EXAMPLE • 3**

Find the equation of the line that contains the point whose coordinates are $(3, -1)$ and is parallel to the graph of $3x - 2y = 4$.

Solution

$$3x - 2y = 4$$

$$-2y = -3x + 4$$

$$y = \frac{3}{2}x - 2$$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{3}{2}(x - 3)$$

$$y + 1 = \frac{3}{2}x - \frac{9}{2}$$

$$y = \frac{3}{2}x - \frac{11}{2}$$

$$m = \frac{3}{2}$$

$$(x_1, y_1) = (3, -1)$$

The equation of the line is $y = \frac{3}{2}x - \frac{11}{2}$.

YOU TRY IT • 3

Find the equation of the line that contains the point whose coordinates are $(-2, 2)$ and is perpendicular to the graph of $x - 4y = 3$.

Your solution

3.6 EXERCISES

OBJECTIVE A

To find parallel and perpendicular lines



1. Explain how to determine whether the graphs of two lines are parallel.



2. Explain how to determine whether the graphs of two lines are perpendicular.



3. Is it possible for two lines to be perpendicular and for the slope of each to be a positive number? Explain.



4. Is it possible for two lines to be parallel and for the slope of one line to be positive and the slope of the other line to be negative? Explain.

5. The slope of a line is -5 . What is the slope of any line parallel to this line?

6. The slope of a line is $\frac{3}{2}$. What is the slope of any line parallel to this line?

7. The slope of a line is 4 . What is the slope of any line perpendicular to this line?

8. The slope of a line is $-\frac{4}{5}$. What is the slope of any line perpendicular to this line?

9. Is the graph of $x = -2$ perpendicular to the graph of $y = 3$?

10. Is the graph of $y = \frac{1}{2}$ perpendicular to the graph of $y = -2$?

11. Is the graph of $x = -3$ parallel to the graph of $y = \frac{1}{3}$?

12. Is the graph of $x = 4$ parallel to the graph of $x = -4$?

13. Is the graph of $y = \frac{2}{3}x - 4$ parallel to the graph of $y = -\frac{3}{2}x - 4$?

14. Is the graph of $y = -2x + \frac{2}{3}$ parallel to the graph of $y = -2x + 3$?

15. Is the graph of $y = \frac{4}{3}x - 2$ perpendicular to the graph of $y = -\frac{3}{4}x + 2$?

16. Is the graph of $y = \frac{1}{2}x + \frac{3}{2}$ perpendicular to the graph of $y = -\frac{1}{2}x + \frac{3}{2}$?

17. Are the graphs of $2x + 3y = 2$ and $2x + 3y = -4$ parallel?

18. Are the graphs of $2x - 4y = 3$ and $2x + 4y = -3$ parallel?

19. Are the graphs of $x - 4y = 2$ and $4x + y = 8$ perpendicular?

20. Are the graphs of $4x - 3y = 2$ and $4x + 3y = -7$ perpendicular?

21. Is the line that contains the points whose coordinates are $(3, 2)$ and $(1, 6)$ parallel to the line that contains the points whose coordinates are $(-1, 3)$ and $(-1, -1)$?

22. Is the line that contains the points whose coordinates are $(4, -3)$ and $(2, 5)$ parallel to the line that contains the points whose coordinates are $(-2, -3)$ and $(-4, 1)$?

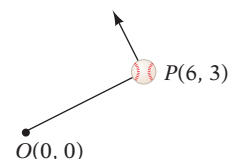
23. Is the line that contains the points whose coordinates are $(-3, 2)$ and $(4, -1)$ perpendicular to the line that contains the points whose coordinates are $(1, 3)$ and $(-2, -4)$?

24. Is the line that contains the points whose coordinates are $(-1, 2)$ and $(3, 4)$ perpendicular to the line that contains the points whose coordinates are $(-1, 3)$ and $(-4, 1)$?

25. Find the equation of the line that contains the point with coordinates $(3, -2)$ and is parallel to the graph of $y = 2x + 1$.
26. Find the equation of the line that contains the point with coordinates $(-1, 3)$ and is parallel to the graph of $y = -x + 3$.
27. Find the equation of the line that contains the point with coordinates $(-2, -1)$ and is perpendicular to the graph of $y = -\frac{2}{3}x - 2$.
28. Find the equation of the line that contains the point with coordinates $(-4, 1)$ and is perpendicular to the graph of $y = 2x - 5$.
29. Find the equation of the line containing the point whose coordinates are $(-2, -4)$ and parallel to the graph of $2x - 3y = 2$.
30. Find the equation of the line containing the point whose coordinates are $(3, 2)$ and parallel to the graph of $3x + y = -3$.
31. Find the equation of the line containing the point whose coordinates are $(4, 1)$ and perpendicular to the graph of $y = -3x + 4$.
32. Find the equation of the line containing the point whose coordinates are $(2, -5)$ and perpendicular to the graph of $y = \frac{5}{2}x - 4$.
33. Find the equation of the line containing the point whose coordinates are $(-1, -3)$ and perpendicular to the graph of $3x - 5y = 2$.
34. Find the equation of the line containing the point whose coordinates are $(-1, 3)$ and perpendicular to the graph of $2x + 4y = -1$.

Applying the Concepts

Physics For Exercises 35 and 36, suppose a ball is being twirled at the end of a string and the center of rotation is the origin of a coordinate system. If the string breaks, the initial path of the ball is on a line that is perpendicular to the radius of the circle.



35. Suppose the string breaks when the ball is at the point $P(6, 3)$. Find the equation of the line on which the initial path lies.
36. Suppose the string breaks when the ball is at the point $P(2, 8)$. Find the equation of the line on which the initial path lies.
37. If the graphs of $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ are parallel, express $\frac{A_1}{B_1}$ in terms of A_2 and B_2 .
38. If the graphs of $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ are perpendicular, express $\frac{A_1}{B_1}$ in terms of A_2 and B_2 .

SECTION

3.7

Inequalities in Two Variables

OBJECTIVE A

To graph the solution set of an inequality in two variables

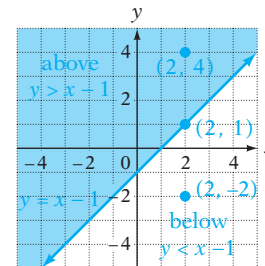


The graph of the linear equation $y = x - 1$ separates the plane into three sets: the set of points on the line, the set of points above the line, and the set of points below the line.

The point whose coordinates are $(2, 1)$ is a solution of $y = x - 1$ and is a point on the line.

The point whose coordinates are $(2, 4)$ is a solution of $y > x - 1$ and is a point above the line.

The point whose coordinates are $(2, -2)$ is a solution of $y < x - 1$ and is a point below the line.



The set of points **on** the line is the solution of the equation $y = x - 1$. The set of points **above** the line is the solution of the inequality $y > x - 1$. These points form a **half-plane**. The set of points **below** the line is the solution of the inequality $y < x - 1$. These points also form a half-plane.

An inequality of the form $y > mx + b$ or $Ax + By > C$ is a **linear inequality in two variables**. (The inequality symbol $>$ could be replaced by $\geq$, $<$, or $\leq$.) The solution set of a linear inequality in two variables is a half-plane.

The following illustrates the procedure for graphing the solution set of a linear inequality in two variables.

Take Note

When solving the inequality at the right for y , both sides of the inequality are divided by -4 , so the inequality symbol must be reversed.

HOW TO • 1

Graph the solution set of $3x - 4y < 12$.

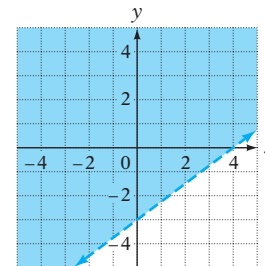
$$\begin{aligned} 3x - 4y &< 12 \\ -4y &< -3x + 12 \\ y &> \frac{3}{4}x - 3 \end{aligned}$$

• **Solve the inequality for y .**

Change the inequality $y > \frac{3}{4}x - 3$ to the equality $y = \frac{3}{4}x - 3$, and graph the line.

If the inequality contains $\leq$ or $\geq$, the line belongs to the solution set and is shown by a **solid line**. If the inequality contains $<$ or $>$, the line is not part of the solution set and is shown by a **dashed line**.

If the inequality contains $>$ or $\geq$, shade the upper half-plane. If the inequality contains $<$ or $\leq$, shade the lower half-plane.


Take Note

As shown below, $(0, 0)$ is a solution of the inequality in the example at the right.

$$\begin{aligned} y &> \frac{3}{4}x - 3 \\ 0 &> \frac{3}{4}(0) - 3 \\ 0 &> 0 - 3 \\ 0 &> -3 \end{aligned}$$

Because $(0, 0)$ is a solution of the inequality, $(0, 0)$ should be in the shaded region. The solution set as graphed is correct.

As a check, use the ordered pair $(0, 0)$ to determine whether the correct region of the plane has been shaded. If $(0, 0)$ is a solution of the inequality, then $(0, 0)$ should be in the shaded region. If $(0, 0)$ is not a solution of the inequality, then $(0, 0)$ should not be in the shaded region.



Integrating Technology

See the Keystroke Guide: *Graphing Inequalities* for instructions on using a graphing calculator to graph the solution set of an inequality in two variables.

If the line passes through the point $(0, 0)$, then another point, such as $(0, 1)$, must be used as a check.

From the graph of $y > \frac{3}{4}x - 3$, note that for a given value of x , more than one value of y can be paired with that value of x . For instance, $(4, 1)$, $(4, 3)$, $(5, 1)$, and $(5, \frac{9}{4})$ are all ordered pairs that belong to the graph. Because there are ordered pairs with the same first coordinate and different second coordinates, the inequality does not represent a function. The inequality is a relation but not a function.

EXAMPLE • 1

Graph the solution set of $x + 2y \leq 4$.

Solution

$$\begin{aligned} x + 2y &\leq 4 \\ 2y &\leq -x + 4 \\ y &\leq -\frac{1}{2}x + 2 \end{aligned}$$

Graph $y = -\frac{1}{2}x + 2$ as a solid line.

Shade the lower half-plane.

Check:

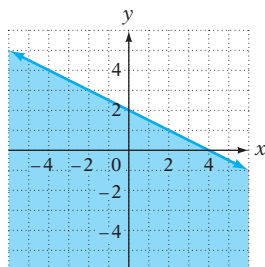
$$y \leq -\frac{1}{2}x + 2$$

$$0 \leq -\frac{1}{2}(0) + 2$$

$$0 \leq 0 + 2$$

$$0 \leq 2$$

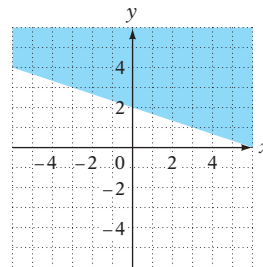
The point $(0, 0)$ should be in the shaded region.



YOU TRY IT • 1

Graph the solution set of $x + 3y > 6$.

Your solution

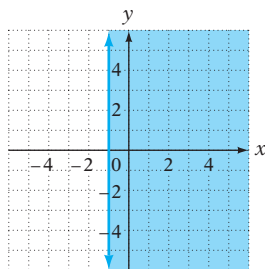


EXAMPLE • 2

Graph the solution set of $x \geq -1$.

Solution Graph $x = -1$ as a solid line.

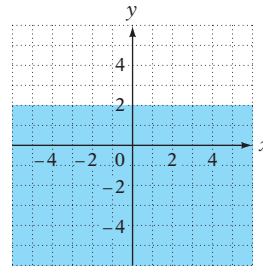
Shade the half-plane to the right of the line.



YOU TRY IT • 2

Graph the solution set of $y < 2$.

Your solution



3.7 EXERCISES

OBJECTIVE A

To graph the solution set of an inequality in two variables



1. What is a half-plane?

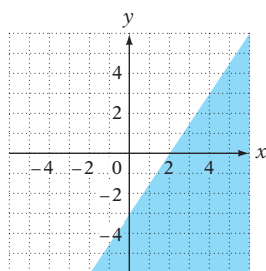


2. Explain a method you can use to check that the graph of a linear inequality in two variables has been shaded correctly.

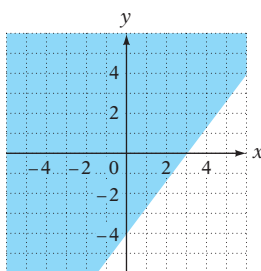
3. Is $(0, 0)$ a solution of $y > 2x - 7$?4. Is $(0, 0)$ a solution of $y < 5x + 3$?5. Is $(0, 0)$ a solution of $y \leq -\frac{2}{3}x - 8$?6. Is $(0, 0)$ a solution of $y \geq -\frac{3}{4}x + 9$?

For Exercises 7 to 24, graph the solution set.

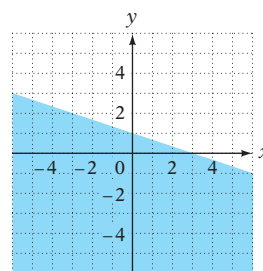
7. $y \leq \frac{3}{2}x - 3$



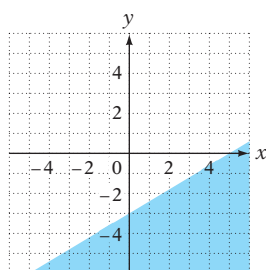
8. $y \geq \frac{4}{3}x - 4$



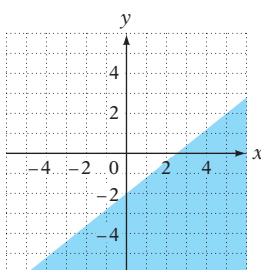
9. $y < -\frac{1}{3}x + 1$



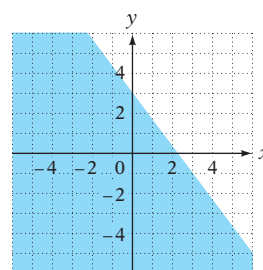
10. $y < \frac{3}{5}x - 3$



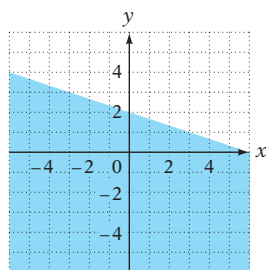
11. $4x - 5y > 10$



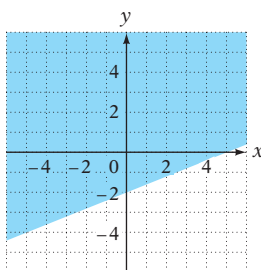
12. $4x + 3y < 9$



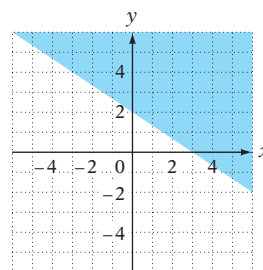
13. $x + 3y < 6$



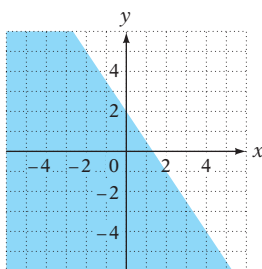
14. $2x - 5y \leq 10$



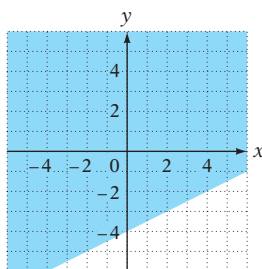
15. $2x + 3y \geq 6$



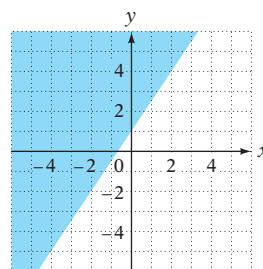
16. $3x + 2y < 4$



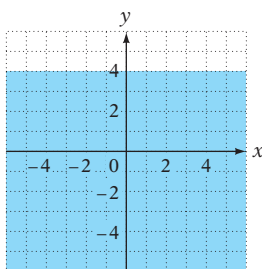
17. $-x + 2y > -8$



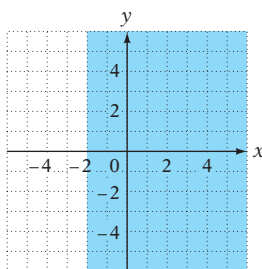
18. $-3x + 2y > 2$



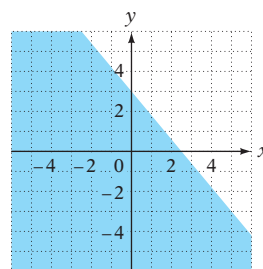
19. $y - 4 < 0$



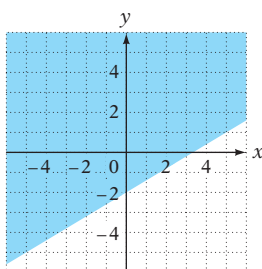
20. $x + 2 \geq 0$



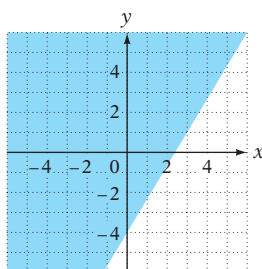
21. $6x + 5y < 15$



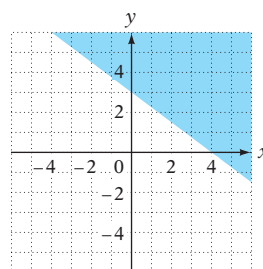
22. $3x - 5y < 10$



23. $-5x + 3y \geq -12$



24. $3x + 4y \geq 12$

25. What quadrant is represented by the two linear inequalities $x > 0$ and $y > 0$?26. What quadrant is represented by the two linear inequalities $x < 0$ and $y < 0$?

Applying the Concepts

27. Does the inequality $y < 3x - 1$ represent a function? Explain your answer.28. Are there any points whose coordinates satisfy both $y \leq x + 3$ and $y \geq -\frac{1}{2}x + 1$? If so, give the coordinates of three such points. If not, explain why not.29. Are there any points whose coordinates satisfy both $y \leq x - 1$ and $y \geq x + 2$? If so, give the coordinates of three such points. If not, explain why not.

FOCUS ON PROBLEM SOLVING

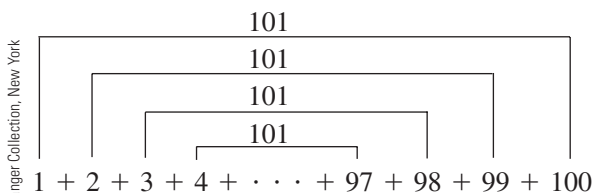
Find a Pattern Polya's four recommended problem-solving steps are stated below.

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review the solution.

One of the several ways of devising a plan is first to try to find a pattern. Karl Friedrich Gauss supposedly used this method to solve a problem that was given to his math class when he was in elementary school. As the story goes, his teacher wanted to grade some papers while the class worked on a math problem. The problem given to the class was to find the sum

$$1 + 2 + 3 + 4 + \cdots + 100$$

Gauss quickly solved the problem by seeing a pattern. Here is what he saw.



Note that

$$\begin{aligned} 1 + 100 &= 101 \\ 2 + 99 &= 101 \\ 3 + 98 &= 101 \\ 4 + 97 &= 101 \end{aligned}$$

Gauss noted that there were 50 sums of 101. Therefore, the sum of the first 100 natural numbers is

$$1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100 = 50(101) = 5050$$

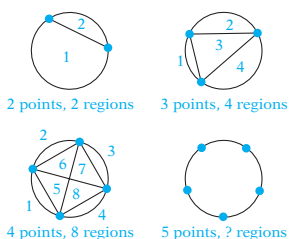
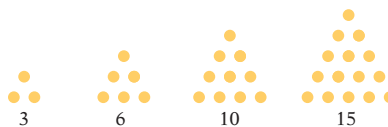
Try to solve Exercises 1 to 6 by finding a pattern.

1. Find the sum $2 + 4 + 6 + \cdots + 96 + 98 + 100$.
2. Find the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$.
3. Find another method of finding the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$ given in the preceding exercise.

4. Find the sum $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{49 \cdot 50}$.

Hint: $\frac{1}{1 \cdot 2} = \frac{1}{2}, \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} = \frac{2}{3}, \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{3}{4}$

5. A *polynomial number* is a number that can be represented by arranging that number of dots in rows to form a geometric figure such as a triangle, square, pentagon, or hexagon. For instance, the first four *triangular numbers*, 3, 6, 10, and 15, are shown below. What are the next two triangular numbers?



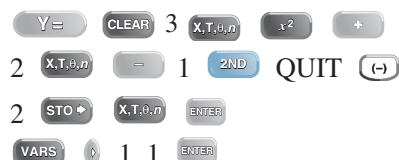
6. The following problem shows that checking a few cases does not always result in a conjecture that is true for *all* cases. Select any two points on a circle (see the drawing in the left margin) and draw a *chord*, a line connecting the points. The chord divides the circle into two regions. Now select three different points and draw chords connecting each of the three points with every other point. The chords divide the circle into four regions. Now select four points and connect each of the points with every other point. Make a conjecture as to the relationship between the number of regions and the number of points on the circle. Does your conjecture work for five points? Six points?

PROJECTS AND GROUP ACTIVITIES

Evaluating a Function with a Graphing Calculator



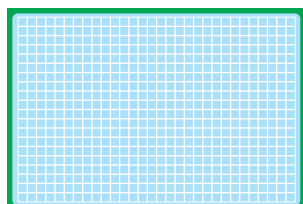
You can use a graphing calculator to evaluate some functions. Shown below are the keystrokes needed to evaluate $f(x) = 3x^2 + 2x - 1$ for $x = -2$ on a TI-84. (There are other methods of evaluating functions. This is just one of them.) Try these keystrokes. The calculator should display 7 as the value of the function.



Introduction to Graphing Calculators



There are a variety of computer programs and calculators that can graph an equation. A computer or graphing calculator screen is divided into pixels. Depending on the computer or calculator, there are approximately 6000 to 790,000 pixels available on the screen. The greater the number of pixels, the smoother the graph will appear. A portion of a screen is shown at the left. Each little rectangle represents one pixel.



Calculator Screen

A graphing calculator draws a graph in a manner similar to the method we have used in this chapter. Values of x are chosen and ordered pairs calculated. Then a graph is drawn through those points by illuminating pixels (an abbreviation for “picture element”) on the screen.

Graphing utilities can display only a portion of the xy -plane, called a window. The window $[Xmin, Xmax]$ by $[Ymin, Ymax]$ consists of those points (x, y) that satisfy both of the following inequalities:

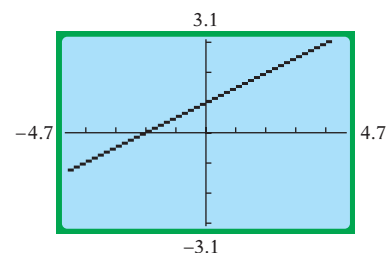
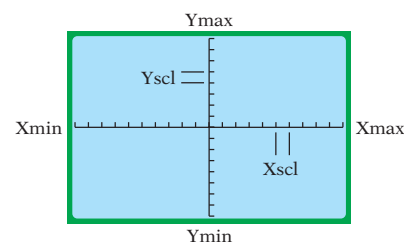
$$Xmin \leq x \leq Xmax \quad \text{and} \quad Ymin \leq y \leq Ymax$$

The user sets these values before a graph is drawn.

The numbers $Xscl$ and $Yscl$ are the distances between the tick marks that are drawn on the x - and y -axes. If you do not want tick marks on the axes, set $Xscl = 0$ and $Yscl = 0$.

The graph at the right is a portion of the graph of $y = \frac{1}{2}x + 1$ as it was drawn with a graphing calculator. The window is

$$\begin{aligned} Xmin &= -4.7, Xmax = 4.7 \\ Ymin &= -3.1, Ymax = 3.1 \\ Xscl &= 1 \text{ and } Yscl = 1 \end{aligned}$$



Using interval notation, this is written $[-4.7, 4.7]$ by $[-3.1, 3.1]$. The window $[-4.7, 4.7]$ by $[-3.1, 3.1]$ gives “nice” coordinates in the sense that each time the $\odot$ or the $\ominus$ is pressed, the change in x is 0.1. The reason for this is that the horizontal distance from the middle of the first pixel to the middle of the last pixel is 94 units. By using $X_{\min} = -4.7$ and $X_{\max} = 4.7$, we have¹

$$\text{Change in } x = \frac{X_{\max} - X_{\min}}{94} = \frac{4.7 - (-4.7)}{94} = \frac{9.4}{94} = 0.1$$

Similarly, the vertical distance from the middle of the first pixel to the middle of the last pixel is 62 units. Therefore, using $Y_{\min} = -3.1$ and $Y_{\max} = 3.1$ will give nice coordinates in the vertical direction.

Graph the equations in Exercises 1 to 6 by using a graphing calculator.



Integrating Technology

See the Keystroke Guide: *Windows* for instructions on changing the viewing window.

1. $y = 2x + 1$ For $2x$, you may enter $2 \times x$ or just $2x$. Use of the times sign $\times$ is not necessary on many graphing calculators.
2. $y = -x + 2$ Many calculators use the $(\leftarrow)$ key to enter a negative sign.
3. $3x + 2y = 6$ Solve for y . Then enter the equation.
4. $y = 50x$ You must adjust the viewing window. Try the window $[-4.7, 4.7]$ by $[-250, 250]$, with $Y_{\text{scl}} = 50$.
5. $y = \frac{2}{3}x - 3$ You may enter $\frac{2}{3}x$ as $2x/3$ or $(2/3)x$. Although entering $2/3x$ works on some calculators, it is not recommended.
6. $4x + 3y = 75$ You must adjust the viewing window.

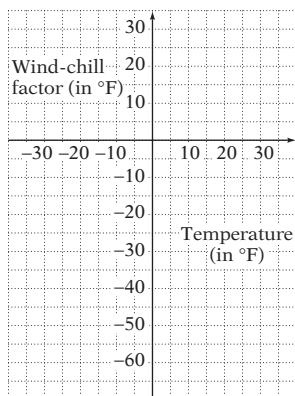
Wind-Chill Index



The wind-chill index is the temperature of still air that would have the same effect on exposed human skin as a given combination of wind speed and air temperature. For example, given a wind speed of 10 mph and a temperature reading of 20°F , the wind-chill index is 9°F .

In the fall of 2001, the U.S. National Weather Service began using a new wind-chill formula. The following ordered pairs are derived from the new formula. In each ordered pair, the abscissa is the air temperature in degrees Fahrenheit, and the ordinate is the wind-chill index when the wind speed is 10 mph.

$\{(35, 27), (30, 21), (25, 15), (20, 9), (15, 3), (10, -4), (5, -10), (0, -16), (-5, -22), (-10, -28), (-15, -35), (-20, -41), (-25, -47), (-30, -53), (-35, -59)\}$



1. Use the coordinate axes at the left to graph the ordered pairs.
2. List the domain of the relation. List the range of the relation.
3. Is the set of ordered pairs a function?
4. The equation of the line that approximately models the graph of the ordered pairs above is $y = 1.2357x - 16$, where x is the air temperature in degrees Fahrenheit and y is the wind-chill index. Evaluate the function $f(x) = 1.2357x - 16$ to determine whether any of the ordered pairs listed above do not satisfy this function. Round to the nearest integer.
5. What does the model predict for the wind-chill index when the air temperature is -40°F and the wind speed is 10 mph?

¹Some calculators have screen widths of 126 pixels. For those calculators, use $X_{\min} = -6.3$ and $X_{\max} = 6.3$ to obtain nice coordinates.

6. a. What is the x -intercept of the graph of $f(x) = 1.2357x - 16$? Round to the nearest integer. What does the x -intercept represent?
- b. What is the y -intercept of the graph of $f(x) = 1.2357x - 16$? What does the y -intercept represent?

In the set of ordered pairs that follows, the abscissa is the air temperature in degrees Fahrenheit, and the ordinate is the wind-chill index when the wind speed is 20 mph. Note that the abscissa in each case is the same as in the ordered pairs given on the previous page. However, the wind speed has increased to 20 mph.

$$\{(35, 24), (30, 17), (25, 11), (20, 4), (15, -2), (10, -9), (5, -15), (0, -22), (-5, -29), (-10, -35), (-15, -42), (-20, -48), (-25, -55), (-30, -61), (-35, -68)\}$$



Stan Honda/AFP/Getty Images

7. Use the same coordinate axes to graph these ordered pairs.
8. Where do the points representing this relation lie in relation to the points drawn for Exercise 1? What does this mean with respect to the temperature remaining constant and the wind speed increasing?
9. The equation of the line that approximately models the graph of the ordered pairs in this second relation is $y = 1.3114x - 22$, where x is the air temperature in degrees Fahrenheit and y is the wind-chill index. Compare this with the model for the first relation. Are the lines parallel? Explain why that might be the case.
10. Using the old wind-chill formula, a wind speed of 10 mph resulted in a wind-chill index of -10°F when the air temperature was 20°F and a wind-chill index of -67°F when the air temperature was -20°F .
 - a. Find the linear equation that models these data. [Hint: Use the ordered pairs $(20, -10)$ and $(-20, -67)$.]
 - b. Compare the y -intercept of your equation to the y -intercept of the equation $y = 1.2357x - 16$, which models the new wind-chill index for a wind speed of 10 mph. Use the difference to determine whether the new formula yields a wind-chill index that is warmer or colder, given the same air temperature and wind speed.

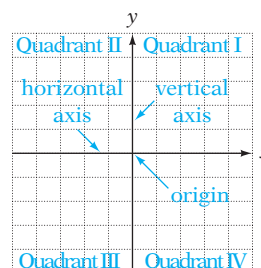
CHAPTER 3

SUMMARY

KEY WORDS

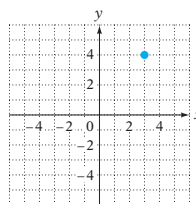
A *rectangular coordinate system* is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the *origin*. The number lines that make up a rectangular coordinate system are called *coordinate axes*. A rectangular coordinate system divides the plane into four regions called *quadrants*. [3.1A, p. 122]

EXAMPLES



An *ordered pair* (x, y) is used to locate a point in a rectangular coordinate system. The first number of the pair measures a horizontal distance and is called the *abscissa* or *x-coordinate*. The second number of the pair measures a vertical distance and is called the *ordinate* or *y-coordinate*. The *coordinates* of the point are the numbers in the ordered pair associated with the point. To *graph*, or *plot*, a point in the plane, place a dot at the location given by the ordered pair. The *graph of an ordered pair* is the dot drawn at the coordinates of the point in the plane. [3.1A, pp. 122–123]

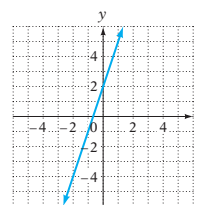
$(3, 4)$ is an ordered pair.
3 is the abscissa.
4 is the ordinate.
The graph of $(3, 4)$ is shown below.



An equation of the form $y = mx + b$, where m and b are constants, is a *linear equation in two variables*. A solution of a linear equation in two variables is an ordered pair (x, y) whose coordinates make the equation a true statement. The graph of a linear equation in two variables is a straight line. [3.1A, p. 123]

$y = 3x + 2$ is a linear equation in two variables; $m = 3$ and $b = 2$. Ordered-pair solutions of $y = 3x + 2$ are shown below, along with the graph of the equation.

x	y
1	5
0	2
-1	-1



A *scatter diagram* is a graph of ordered-pair data. [3.1C, p. 127]

A *relation* is a set of ordered pairs. [3.2A, p. 132]

$\{(2, 3), (2, 4), (3, 4), (5, 7)\}$

A *function* is a relation in which no two ordered pairs have the same first coordinate and different second coordinates. The *domain* of a function is the set of the first coordinates of all the ordered pairs of the function. The *range* is the set of the second coordinates of all the ordered pairs of the function. [3.2A, p. 133]

$\{(2, 3), (3, 5), (5, 7), (6, 9)\}$
The domain is $\{2, 3, 5, 6\}$.
The range is $\{3, 5, 7, 9\}$.

Function notation is used for those equations that represent functions. For the equation at the right, x is the *independent variable* and y is the *dependent variable*. The symbol $f(x)$ is the *value of the function* and represents the value of the dependent variable for a given value of the independent variable. [3.2A, pp. 133–135]

In function notation, $y = 3x + 7$ is written as $f(x) = 3x + 7$.

The process of determining $f(x)$ for a given value of x is called *evaluating the function*. [3.2A, p. 135]

Evaluate $f(x) = 2x - 3$ when $x = 4$.

$$\begin{aligned} f(x) &= 2x - 3 \\ f(4) &= 2(4) - 3 \\ f(4) &= 5 \end{aligned}$$

The *graph of a function* is a graph of the ordered pairs (x, y) that belong to the function. A function that can be written in the form $f(x) = mx + b$ (or $y = mx + b$) is a *linear function* because its graph is a straight line. [3.3A, p. 144]

$f(x) = -\frac{2}{3}x + 3$ is an example of a linear function. In this case, $m = -\frac{2}{3}$ and $b = 3$.

The point at which a graph crosses the x -axis is called the *x-intercept*, and the point at which a graph crosses the y -axis is called the *y-intercept*. [3.3C, p. 148]

The x -intercept of $x + y = 4$ is $(4, 0)$.
The y -intercept of $x + y = 4$ is $(0, 4)$.

The *slope* of a line is a measure of the slant, or tilt, of the line. The symbol for slope is m . A line that slants upward to the right has a *positive slope*, and a line that slants downward to the right has a *negative slope*. A horizontal line has *zero slope*. The slope of a vertical line is *undefined*. [3.4A, pp. 156–157]

The line $y = 2x - 3$ has a slope of 2 and slants upward to the right.
The line $y = -5x + 2$ has a slope of -5 and slants downward to the right.
The line $y = 4$ has a slope of 0.

An inequality of the form $y > mx + b$ or of the form $Ax + By > C$ is a *linear inequality in two variables*. (The symbol $>$ can be replaced by $\geq$, $<$, or $\leq$.) The solution set of an inequality in two variables is a *half-plane*. [3.7A, p. 182]

$4x - 3y < 12$ and $y \geq 2x + 6$ are linear inequalities in two variables.

ESSENTIAL RULES AND PROCEDURES

Pythagorean Theorem [3.1B, p. 124]

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

A triangle with legs that measure 3 in. and 4 in. and a hypotenuse that measures 5 in. is a right triangle because 3, 4, and 5 satisfy the Pythagorean Theorem.

$$\begin{aligned}a^2 + b^2 &= c^2 \\3^2 + 4^2 &= 5^2 \\9 + 16 &= 25 \\25 &= 25\end{aligned}$$

Distance Formula [3.1B, p. 125]

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points in the plane, then the distance between the two points is given by $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$$\begin{aligned}(x_1, y_1) &= (-3, 5), (x_2, y_2) = (2, 4) \\d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(-3 - 2)^2 + (5 - 4)^2} \\&= \sqrt{25 + 1} \\&= \sqrt{26}\end{aligned}$$

Midpoint Formula [3.1B, p. 125]

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are the endpoints of a line segment, then the coordinates of the midpoint of the line segment $P_m(x_m, y_m)$ are given by $x_m = \frac{x_1 + x_2}{2}$ and $y_m = \frac{y_1 + y_2}{2}$.

$$\begin{aligned}(x_1, y_1) &= (2, 3), (x_2, y_2) = (-6, -1) \\x_m &= \frac{x_1 + x_2}{2} = \frac{2 + (-6)}{2} = -2 \\y_m &= \frac{y_1 + y_2}{2} = \frac{3 + (-1)}{2} = 1 \\(x_m, y_m) &= (-2, 1)\end{aligned}$$

Graph of $y = b$ [3.3B, p. 147]

The graph of $y = b$ is a horizontal line passing through the point $(0, b)$.

The graph of $y = -5$ is a horizontal line passing through the point $(0, -5)$.

EXAMPLES

Graph of a Constant Function [3.3B, p. 147]

A function given by $f(x) = b$, where b is a constant, is a *constant function*. The graph of the constant function is a horizontal line passing through $(0, b)$.

The graph of $f(x) = -5$ is a horizontal line passing through the point $(0, -5)$. Note that this is the same as the graph of $y = -5$.

Graph of $x = a$ [3.3B, p. 148]

The graph of $x = a$ is a vertical line passing through the point $(a, 0)$.

The graph of $x = 4$ is a vertical line passing through the point $(4, 0)$.

Finding Intercepts of Graphs of Linear Equations

[3.3C, p. 149]

To find the x -intercept, let $y = 0$.

To find the y -intercept, let $x = 0$.

For any equation of the form $y = mx + b$, the y -intercept is $(0, b)$.

$$3x + 4y = 12$$

Let $y = 0$:

$$3x + 4(0) = 12$$

$$3x = 12$$

$$x = 4$$

The x -intercept is $(4, 0)$.

Let $x = 0$:

$$3(0) + 4y = 12$$

$$4y = 12$$

$$y = 3$$

The y -intercept is $(0, 3)$.

Slope Formula [3.4A, p. 156]

The slope of the line containing the two points $P_1(x_1, y_1)$ and

$P_2(x_2, y_2)$ is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.

$$(x_1, y_1) = (-3, 2), (x_2, y_2) = (1, 4)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{1 - (-3)} = \frac{2}{4} = \frac{1}{2}$$

The slope of the line through the points $(-3, 2)$ and $(1, 4)$ is $\frac{1}{2}$.

Slope-Intercept Form of a Straight Line [3.4B, p. 160]

The equation $y = mx + b$ is called the *slope-intercept form* of a straight line. The slope of the line is m , the coefficient of x . The y -intercept is $(0, b)$.

For the equation $y = -3x + 2$, the slope is -3 and the y -intercept is $(0, 2)$.

Point-Slope Formula [3.5A, p. 167]

Let m be the slope of a line, and let (x_1, y_1) be the coordinates of a point on the line. The equation of the line can be found from the point-slope formula: $y - y_1 = m(x - x_1)$.

The equation of the line that passes through the point $(4, 2)$ and has slope -3 is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -3(x - 4)$$

$$y - 2 = -3x + 12$$

$$y = -3x + 14$$

Slopes of Parallel Lines [3.6A, p. 176]

Two nonvertical lines with slopes of m_1 and m_2 are parallel if and only if $m_1 = m_2$. Any two vertical lines are parallel.

$$y = 3x - 4, \quad m_1 = 3$$

$$y = 3x + 2, \quad m_2 = 3$$

Because $m_1 = m_2$, the lines are parallel.

Slopes of Perpendicular Lines [3.6A, p. 177]

If m_1 and m_2 are the slopes of two lines, neither of which is vertical, then the lines are perpendicular if and only if $m_1 \cdot m_2 = -1$. This states that the slopes of perpendicular lines are *negative reciprocals* of each other. A vertical line is perpendicular to a horizontal line.

$$y = \frac{1}{2}x - 1, \quad m_1 = \frac{1}{2}$$

$$y = -2x + 2, \quad m_2 = -2$$

Because $m_1 \cdot m_2 = -1$, the lines are perpendicular.

CHAPTER 3**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How do you find the midpoint of a line segment?
2. What is the difference between the dependent variable and the independent variable?
3. How do you find any values excluded from the domain of a function?
4. How do you find the y -intercept of a constant function?
5. How do you graph the equation of a line using the x - and y -intercepts?
6. In finding the slope of a line, why is the answer one number?
7. What is the slope of a vertical line?
8. Where do you start when graphing a line using the slope and y -intercept?
9. What is the equation of a line when the slope is undefined?
10. What is the slope of a line that is perpendicular to a horizontal line?
11. After graphing the line of a linear inequality, how do you determine which half-plane to shade?
12. Given two points, what is the first step in finding the equation of the line?

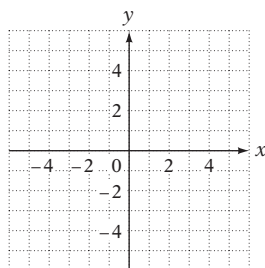
CHAPTER 3

REVIEW EXERCISES

1. Determine the ordered-pair solution of

$$y = \frac{x}{x-2} \text{ that corresponds to } x = 4.$$

3. Graph the ordered-pair solutions of
- $y = 2x^2 - 5$
- when
- $x = -2, -1, 0, 1,$
- and
- 2
- .

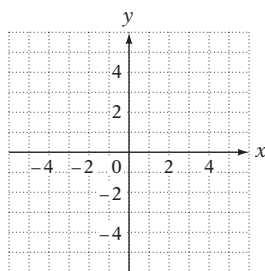


5. Find the range of
- $f(x) = x^2 + x - 1$
- if the domain is
- $\{-2, -1, 0, 1, 2\}$
- .

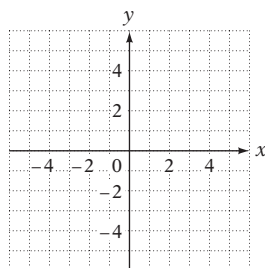
7. Find the midpoint and the length (to the nearest hundredth) of the line segment with endpoints
- $(-2, 4)$
- and
- $(3, 5)$
- .

9. Find the
- x
- and
- y
- intercepts and graph

$$y = -\frac{2}{3}x - 2.$$



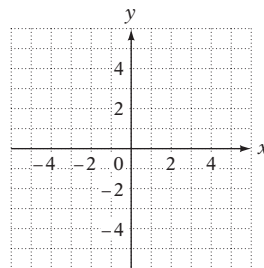
11. Graph:
- $y = -2x + 2$



13. Find the slope of the line that contains the points whose coordinates are
- $(3, -2)$
- and
- $(-1, 2)$
- .

2. Given
- $P(x) = 3x + 4$
- , evaluate
- $P(-2)$
- and
- $P(a)$
- .

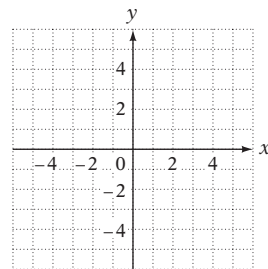
4. Draw a line through all the points with an abscissa of
- -3
- .



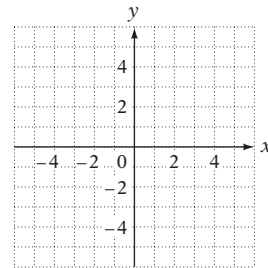
6. Find the domain and range of the function
- $\{(-1, 0), (0, 2), (1, 2), (5, 4)\}$
- .

8. What value of
- x
- is excluded from the domain of
- $f(x) = \frac{x}{x+4}$
- ?

10. Graph
- $3x + 2y = -6$
- by using the
- x
- and
- y
- intercepts.

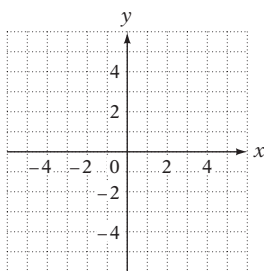


12. Graph:
- $4x - 3y = 12$



14. Find the equation of the line that contains the point with coordinates
- $(-3, 4)$
- and has slope
- $\frac{5}{2}$
- .

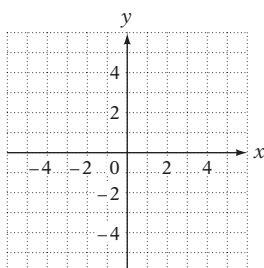
15. Draw a line through all points with an ordinate of -2 .



17. Find the range of $f(x) = x^2 - 2$ if the domain is $\{-2, -1, 0, 1, 2\}$.

19. Find the equation of the line that contains the point whose coordinates are $(-2, 3)$ and is parallel to the graph of $y = -4x + 3$.

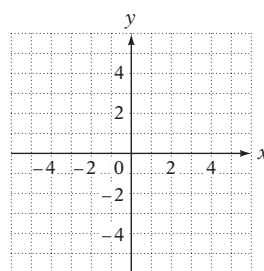
21. Graph $y = 1$.



23. Find the equation of the line that contains the point whose coordinates are $(-3, 3)$ and has slope $-\frac{2}{3}$.

25. Find the distance between the points whose coordinates are $(4, -5)$ and $(-2, 3)$.

16. Graph the line that passes through the point with coordinates $(-2, 3)$ and has slope $-\frac{1}{4}$.

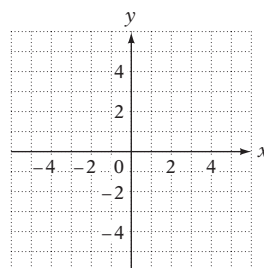


18. **The Hospitality Industry** The manager of a hotel determines that 200 rooms will be occupied if the rate is \$95 per night. For each \$10 increase in the rate, 10 fewer rooms will be occupied.

- Determine a linear function that predicts the number of rooms that will be occupied at a given rate.
- Use the model to predict occupancy when the rate is \$120.

20. Find the equation of the line that contains the point whose coordinates are $(-2, 3)$ and is perpendicular to the graph of $y = -\frac{2}{5}x - 3$.

22. Graph $x = -1$.



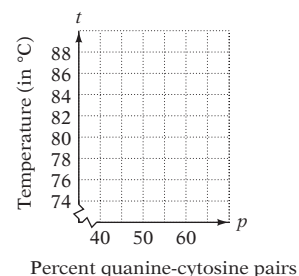
24. Find the equation of the line that contains the points whose coordinates are $(-8, 2)$ and $(4, 5)$.

26. Find the coordinates of the midpoint of the line segment with endpoints $(-3, 8)$ and $(5, -2)$.

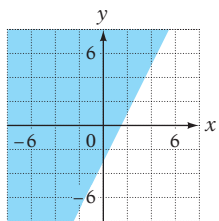


27. **Biology** The melting point t , in degrees Celsius, of a DNA molecule is the temperature at which the bonds holding the double helix together are broken. The table below shows the temperature at which some DNA molecules melt and the percent p of guanine-cytosine pairs in the DNA molecule. (Source: www.biology.arizona.edu) Graph a scatter diagram for these data.

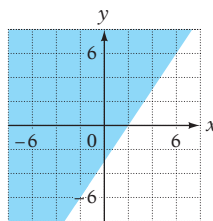
Percent guanine-cytosine pairs	40	45	50	55	60	65
Temperature, $^{\circ}\text{C}$	78	80	81	83	85	87



28. Graph the solution set of
- $y \geq 2x - 3$
- .



29. Graph the solution set of
- $3x - 2y < 6$
- .



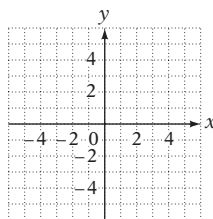
30. Find the equation of the line that contains the points whose coordinates are
- $(-2, 4)$
- and
- $(4, -3)$
- .

31. Find the equation of the line that contains the point with coordinates
- $(-2, -4)$
- and is parallel to the graph of
- $4x - 2y = 7$
- .

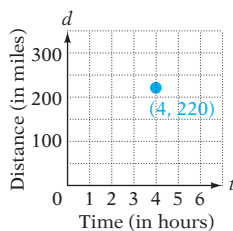
32. Find the equation of the line that contains the point with coordinates
- $(3, -2)$
- and is parallel to the graph of
- $y = -3x + 4$
- .

33. Find the equation of the line that contains the point with coordinates
- $(2, 5)$
- and is perpendicular to the graph of
- $y = -\frac{2}{3}x + 6$
- .

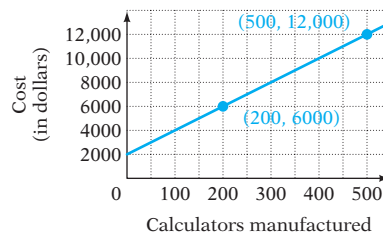
34. Graph the line that passes through the point whose coordinates are
- $(-1, 4)$
- and has slope
- $-\frac{1}{3}$
- .



- 35.
- Travel**
- A car is traveling at 55 mph. The equation that describes the distance traveled is
- $d = 55t$
- . Graph this equation for
- $0 \leq t \leq 6$
- . The point whose coordinates are
- $(4, 220)$
- is on the graph. Write a sentence that explains the meaning of this ordered pair.



- 36.
- Manufacturing**
- The graph at the right shows the relationship between the cost of manufacturing calculators and the number of calculators manufactured. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.

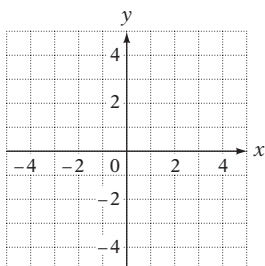


37. **Construction** A building contractor estimates that the cost to build a new home is \$25,000 plus \$80 for each square foot of floor space.
- Determine a linear function that will give the cost to build a house that contains a given number of square feet of floor space.
 - Use the model to determine the cost to build a house that contains 2000 ft² of floor space.

CHAPTER 3

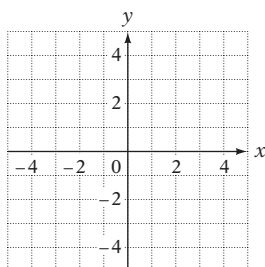
TEST

1. Graph the ordered-pair solutions of $P(x) = 2 - x^2$ when $x = -2, -1, 0, 1,$ and 2 .

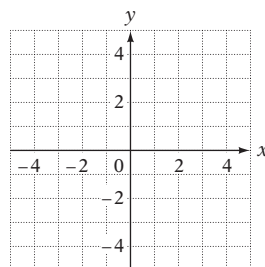


2. Find the ordered-pair solution of $y = 2x + 6$ that corresponds to $x = -3$.

3. Graph: $y = \frac{2}{3}x - 4$



4. Graph: $2x + 3y = -3$



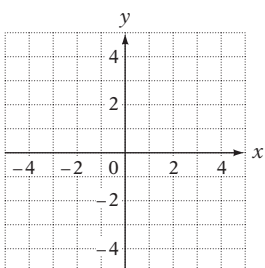
5. Find the equation of the vertical line that contains the point $(-2, 3)$.

6. Find the length, to the nearest hundredth, and the midpoint of the line segment with endpoints $(4, 2)$ and $(-5, 8)$.

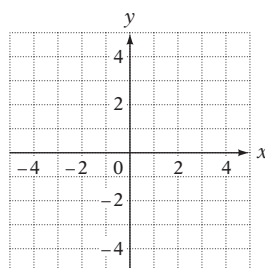
7. Find the slope of the line that contains the points whose coordinates are $(-2, 3)$ and $(4, 2)$.

8. Given $P(x) = 3x^2 - 2x + 1$, evaluate $P(2)$.

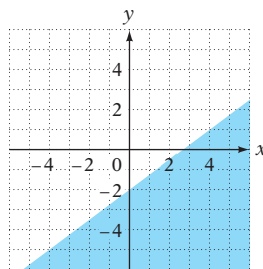
9. Graph $2x - 3y = 6$ by using the x - and the y -intercepts.



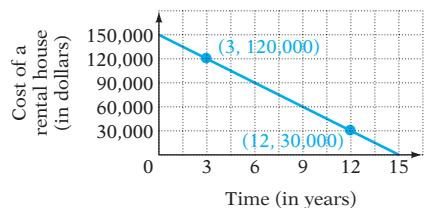
10. Graph the line that passes through the point with coordinates $(-2, 3)$ and has slope $-\frac{3}{2}$.



11. Find the equation of the line that contains the point with coordinates $(-5, 2)$ and has slope $\frac{2}{5}$.
12. What value of x is excluded from the domain of $f(x) = \frac{2x+1}{x}$?
13. Find the equation of the line that contains the points whose coordinates are $(3, -4)$ and $(-2, 3)$.
14. Find the equation of the horizontal line that contains the point with coordinates $(4, -3)$.
15. Find the domain and range of the function $\{(-4, 2), (-2, 2), (0, 0), (3, 5)\}$.
16. Find the equation of the line that contains the point with coordinates $(1, 2)$ and is parallel to the graph of $y = -\frac{3}{2}x - 6$.
17. Find the equation of the line that contains the point with coordinates $(-2, -3)$ and is perpendicular to the graph of $y = -\frac{1}{2}x - 3$.
18. Graph the solution set of $3x - 4y > 8$.

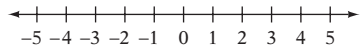


19. **Depreciation** The graph below shows the relationship between the cost of a rental house and the depreciation allowed for income tax purposes. Find the slope between the two points shown on the graph. Write a sentence that states the meaning of the slope.



20. **Summer Camp** The director of a baseball camp estimates that 100 students will enroll if the tuition is \$250. For each \$20 increase in tuition, six fewer students will enroll.
- Determine a linear function that will predict the number of students who will enroll at a given tuition.
 - Use this model to predict enrollment when the tuition is \$300.

CUMULATIVE REVIEW EXERCISES

1. Identify the property that justifies the statement $(x + y) \cdot 2 = 2 \cdot (x + y)$.
2. Solve: $3 - \frac{x}{2} = \frac{3}{4}$
3. Solve:
 $2[y - 2(3 - y) + 4] = 4 - 3y$
4. Solve:
 $\frac{1 - 3x}{2} + \frac{7x - 2}{6} = \frac{4x + 2}{9}$
5. Solve:
 $x - 3 < -4$ or $2x + 2 > 3$
Write the solution set in set-builder notation.
6. Solve:
 $8 - |2x - 1| = 4$
7. Solve: $|3x - 5| < 5$
8. Simplify: $4 - 2(4 - 5)^3 + 2$
9. Evaluate $(a - b)^2 \div (ab)$ when $a = 4$ and $b = -2$.
10. Graph: $\{x | x < -2\} \cup \{x | x > 0\}$

11. Solve $P = \frac{R - C}{n}$ for C .
12. Solve $2x + 3y = 6$ for x .
13. Solve: $3x - 1 < 4$ and $x - 2 > 2$
14. Given $P(x) = x^2 + 5$, evaluate $P(-3)$.
15. Find the ordered-pair solution of
 $y = -\frac{5}{4}x + 3$ that corresponds to $x = -8$.
16. Find the slope of the line that contains the points $(-1, 3)$ and $(3, -4)$.
17. Find the equation of the line that contains the point with coordinates $(-1, 5)$ and has slope $\frac{3}{2}$.
18. Find the equation of the line that contains the points whose coordinates are $(4, -2)$ and $(0, 3)$.

19. Find the equation of the line that contains the point with coordinates (2, 4) and is parallel to the graph of $y = -\frac{3}{2}x + 2$.

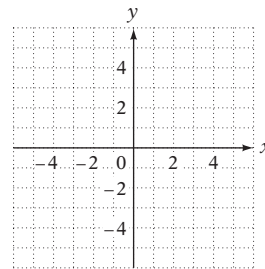
21. **Coins** A coin purse contains 17 coins with a total value of \$1.60. The purse contains nickels, dimes, and quarters. There are four times as many nickels as quarters. Find the number of dimes in the purse.

23. **Mixtures** A grocer combines coffee costing \$9 per pound with coffee costing \$6 per pound. How many pounds of each should be used to make 60 lb of a blend costing \$8 per pound?

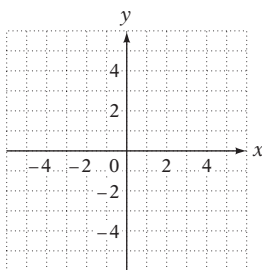
20. Find the equation of the line that contains the point with coordinates (4, 0) and is perpendicular to the graph of $3x - 2y = 5$.

22. **Uniform Motion** Two planes are 1800 mi apart and are traveling toward each other. One plane is traveling twice as fast as the other plane. The planes pass each other in 3 h. Find the speed of each plane.

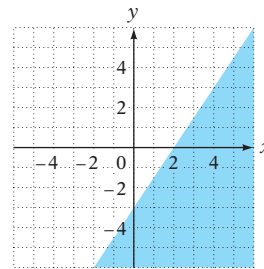
24. Graph $3x - 5y = 15$ by using the x - and y -intercepts.



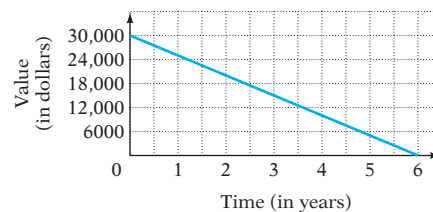
25. Graph the line that passes through the point with coordinates $(-3, 1)$ and has slope $-\frac{3}{2}$.



26. Graph the solution set of $3x - 2y \geq 6$.



27. **Depreciation** The relationship between the value of a truck and the depreciation allowed for income tax purposes is shown in the graph at the right.
- Write the equation for the line that represents the depreciated value of the truck.
 - Write a sentence that states the meaning of the slope.



Systems of Linear Equations and Inequalities

OBJECTIVES

SECTION 4.1

- A** To solve a system of linear equations by graphing
- B** To solve a system of linear equations by the substitution method
- C** To solve investment problems

SECTION 4.2

- A** To solve a system of two linear equations in two variables by the addition method
- B** To solve a system of three linear equations in three variables by the addition method

SECTION 4.3

- A** To evaluate a determinant
- B** To solve a system of equations by using Cramer's Rule

SECTION 4.4

- A** To solve rate-of-wind or rate-of-current problems
- B** To solve application problems

SECTION 4.5

- A** To graph the solution set of a system of linear inequalities

ARE YOU READY?

Take the Chapter 4 Prep Test to find out if you are ready to learn to:

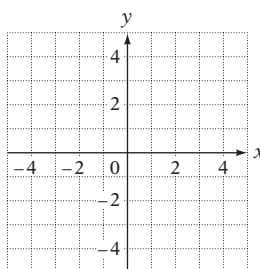
- Solve a system of two linear equations in two variables by graphing, by the substitution method, and by the addition method
- Solve a system of three linear equations in three variables by the addition method
- Solve a system of equations by using Cramer's Rule
- Solve investment problems and rate-of-wind or rate-of-current problems
- Graph the solution set of a system of linear inequalities

PREP TEST

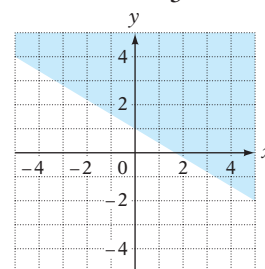
Do these exercises to prepare for Chapter 4.

1. Simplify: $10\left(\frac{3}{5}x + \frac{1}{2}y\right)$
2. Evaluate $3x + 2y - z$ for $x = -1$, $y = 4$, and $z = -2$.
3. Given $3x - 2z = 4$, find the value of x when $z = -2$.
4. Solve: $3x + 4(-2x - 5) = -5$
5. Solve: $0.45x + 0.06(-x + 4000) = 630$

6. Graph: $3x - 2y = 6$



7. Graph: $y > -\frac{3}{5}x + 1$



SECTION

4.1

Solving Systems of Linear Equations by Graphing and by the Substitution Method

OBJECTIVE A

To solve a system of linear equations by graphing



A **system of equations** is two or more equations considered together. The system at the right is a system of two linear equations in two variables. The graphs of the equations are straight lines.

$$\begin{aligned} 3x + 4y &= 7 \\ 2x - 3y &= 6 \end{aligned}$$

A **solution of a system of equations in two variables** is an ordered pair that is a solution of each equation of the system.

HOW TO 1

Is $(3, -2)$ a solution of the system

$$2x - 3y = 12$$

$$5x + 2y = 11$$

$$\begin{array}{r|l} 2x - 3y = 12 & \\ 2(3) - 3(-2) & 12 \\ 6 - (-6) & 12 \\ 12 & 12 \end{array}$$

$$\begin{array}{r|l} 5x + 2y = 11 & \\ 5(3) + 2(-2) & 11 \\ 15 + (-4) & 11 \\ 11 & 11 \end{array}$$

• Replace x by 3 and y by -2 .

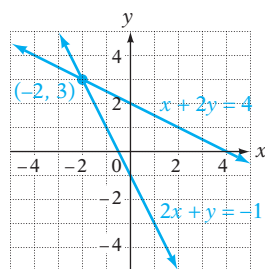
Yes, because $(3, -2)$ is a solution of each equation, it is a solution of the system of equations.

A solution of a system of linear equations can be found by graphing the lines of the system on the same set of coordinate axes. Three examples of linear equations in two variables are shown below, along with the graphs of the equations of the systems.

System I

$$x + 2y = 4$$

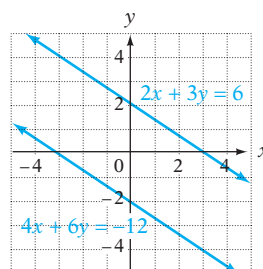
$$2x + y = -1$$



System II

$$2x + 3y = 6$$

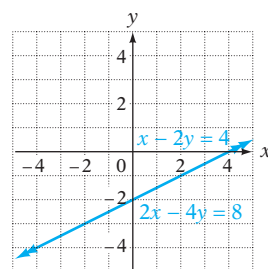
$$4x + 6y = -12$$



System III

$$x - 2y = 4$$

$$2x - 4y = 8$$



Check:

$$\begin{array}{r|l} x + 2y = 4 & \\ -2 + 2(3) & 4 \\ -2 + 6 & 4 \\ 4 & 4 \end{array}$$

$$\begin{array}{r|l} 2x + y = -1 & \\ 2(-2) + 3 & -1 \\ -4 + 3 & -1 \\ -1 & -1 \end{array}$$

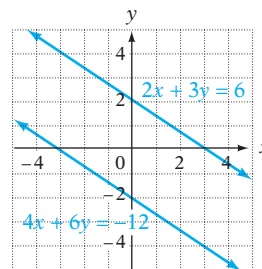
In System I, the two lines intersect at a single point whose coordinates are $(-2, 3)$. Because this point lies on both lines, it is a solution of each equation of the system of equations. We can check this by replacing x by -2 and y by 3 . The check is shown at the left. The ordered pair $(-2, 3)$ is a solution of System I.

When the graphs of a system of equations intersect at only one point, the system is called an **independent system of equations**. System I is an independent system of equations.

System II from the preceding page and the graph of the equations of that system are shown again at the right. Note in this case that the graphs of the lines are parallel and do not intersect. Since the graphs do not intersect, there is no point that is on both lines. Therefore, the system of equations has no solution.

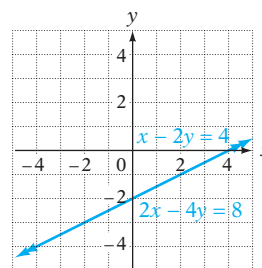
When a system of equations has no solution, it is called an **inconsistent system of equations**. System II is an inconsistent system of equations.

$$\begin{aligned} 2x + 3y &= 6 \\ 4x + 6y &= -12 \end{aligned}$$



System III from the preceding page and the graph of the equations of that system are shown again at the right. Note that the graph of $x - 2y = 4$ lies directly on top of the graph of $2x - 4y = 8$. Thus **the two lines intersect at an infinite number of points**. Because the graphs intersect at an infinite number of points, **there is an infinite number of solutions** of this system of equations. Since **each equation represents the same set of points**, the solutions of the system of equations can be stated by using the ordered pairs of either one of the equations. Therefore, we can say, “The solutions are the ordered pairs that satisfy $x - 2y = 4$,” or we can solve the equation for y and say, “The solutions are the ordered pairs that satisfy $y = \frac{1}{2}x - 2$.” We normally state this solution using ordered pairs—“The solutions are the ordered pairs $(x, \frac{1}{2}x - 2)$.”

$$\begin{aligned} x - 2y &= 4 \\ 2x - 4y &= 8 \end{aligned}$$



When the two equations in a system of equations represent the same line, the system is called a **dependent system of equations**. System III is a dependent system of equations.



Take Note

Keep in mind the differences among independent, dependent, and inconsistent systems of equations. You should be able to express your understanding of these terms by using graphs.

Summary of the Three Possibilities for a System of Linear Equations in Two Variables

1. The graphs intersect at one point.

The solution of the system of equations is the ordered pair (x, y) whose coordinates name the point of intersection.

The system of equations is independent.

2. The lines are parallel and never intersect.

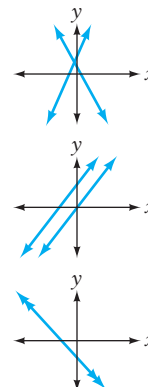
There is no solution of the system of equations.

The system of equations is inconsistent.

3. The graphs are the same line, and they intersect at infinitely many points.

There are infinitely many solutions of the system of equations.

The system of equations is dependent.





Integrating Technology

See the Projects and Group Activities at the end of this chapter for instructions on using a graphing calculator to solve a system of equations. Instructions are also provided in the Keystroke Guide: *Intersect*.

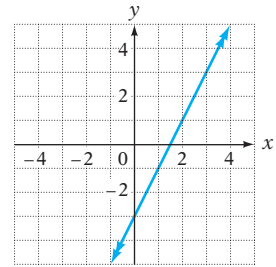
HOW TO • 2

Solve by graphing: $2x - y = 3$
 $4x - 2y = 6$

Graph each line.
 The system of equations is dependent.
 Solve one of the equations for y .

$$\begin{aligned} 2x - y &= 3 \\ -y &= -2x + 3 \\ y &= 2x - 3 \end{aligned}$$

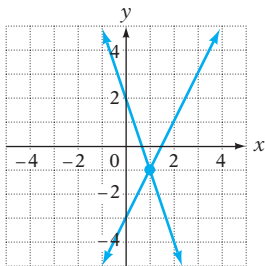
The solutions are the ordered pairs $(x, 2x - 3)$.



EXAMPLE • 1

Solve by graphing:
 $2x - y = 3$
 $3x + y = 2$

Solution



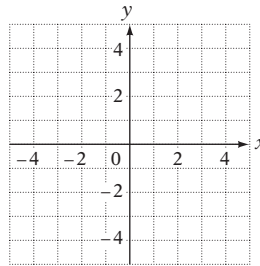
The solution is $(1, -1)$.

- Find the point of intersection of the graphs of the equations.

YOU TRY IT • 1

Solve by graphing:
 $x + y = 1$
 $2x + y = 0$

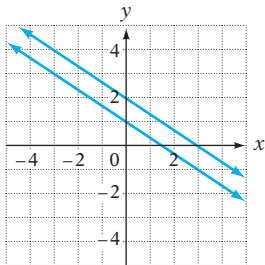
Your solution



EXAMPLE • 2

Solve by graphing:
 $2x + 3y = 6$
 $y = -\frac{2}{3}x + 1$

Solution



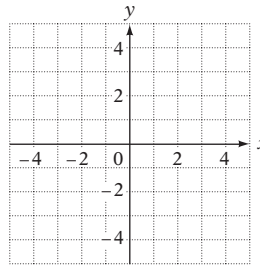
The lines are parallel and therefore do not intersect.
 The system of equations has no solution. The system of equations is inconsistent.

- Graph the two equations.

YOU TRY IT • 2

Solve by graphing:
 $2x + 5y = 10$
 $y = -\frac{2}{5}x - 2$

Your solution

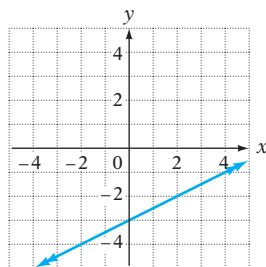


EXAMPLE 3

Solve by graphing:

$x - 2y = 6$

$y = \frac{1}{2}x - 3$

Solution

The system of equations is dependent. The solutions are the ordered pairs $(x, \frac{1}{2}x - 3)$.

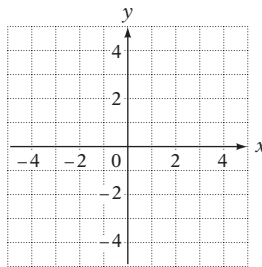
- Graph the two equations.

YOU TRY IT 3

Solve by graphing:

$3x - 4y = 12$

$y = \frac{3}{4}x - 3$

Your solution

Solution on p. S10

OBJECTIVE B**To solve a system of linear equations by the substitution method**

The graphical solution of a system of equations is based on approximating the coordinates of a point of intersection. An algebraic method called the **substitution method** can be used to find an exact solution of a system of equations. To use the substitution method, we must write one of the equations of the system in terms of x or in terms of y .

HOW TO 3

Solve by the substitution method: (1) $3x + y = 5$
(2) $4x + 5y = 3$

$$(3) \quad \begin{aligned} 3x + y &= 5 \\ y &= -3x + 5 \end{aligned}$$

- Solve Equation (1) for y . This is Equation (3).

$$(2) \quad \begin{aligned} 4x + 5y &= 3 \\ 4x + 5(-3x + 5) &= 3 \end{aligned}$$

- This is Equation (2).
- Equation (3) states that $y = -3x + 5$. Substitute $-3x + 5$ for y in Equation (2).
- Solve for x .

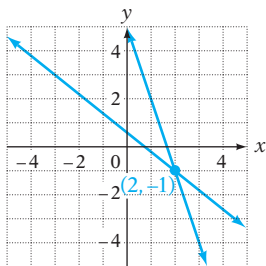
$$\begin{aligned} 4x - 15x + 25 &= 3 \\ -11x + 25 &= 3 \\ -11x &= -22 \\ x &= 2 \end{aligned}$$

$$(3) \quad \begin{aligned} y &= -3x + 5 \\ y &= -3(2) + 5 \\ y &= -6 + 5 \\ y &= -1 \end{aligned}$$

- Substitute the value of x into Equation (3) and solve for y .

The solution is the ordered pair $(2, -1)$.

The graph of the system of equations is shown at the left. Note that the graphs intersect at the point whose coordinates are $(2, -1)$, the solution of the system of equations.



HOW TO • 4

Solve by the substitution method:

$$(1) 6x + 2y = 8$$

$$(2) 3x + y = 2$$

$$(3) \quad \begin{aligned} 3x + y &= 2 \\ y &= -3x + 2 \end{aligned}$$

- We will solve Equation (2) for y .
- This is Equation (3).

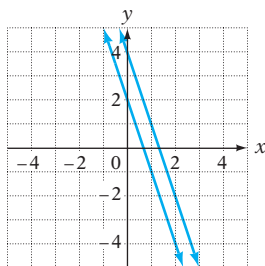
$$(1) \quad \begin{aligned} 6x + 2y &= 8 \\ 6x + 2(-3x + 2) &= 8 \end{aligned}$$

- This is Equation (1).
- Equation (3) states that $y = -3x + 2$. Substitute $-3x + 2$ for y in Equation (1).
- Solve for x .

$$\begin{aligned} 6x - 6x + 4 &= 8 \\ 0x + 4 &= 8 \\ 4 &= 8 \end{aligned}$$

This is not a true equation.
The system of equations has no solution.
The system of equations is inconsistent.

The graph of the system of equations is shown at the left. Note that the lines are parallel.

**EXAMPLE • 4**

Solve by substitution:

$$(1) 3x - 2y = 4$$

$$(2) -x + 4y = -3$$

Solution

Solve Equation (2) for x .

$$\begin{aligned} -x + 4y &= -3 \\ -x &= -4y - 3 \\ x &= 4y + 3 \quad \bullet \text{ Equation (3)} \end{aligned}$$

Substitute $4y + 3$ for x in Equation (1).

$$\begin{aligned} 3x - 2y &= 4 \quad \bullet \text{ Equation (1)} \\ 3(4y + 3) - 2y &= 4 \quad \bullet x = 4y + 3 \\ 12y + 9 - 2y &= 4 \\ 10y + 9 &= 4 \\ 10y &= -5 \\ y &= -\frac{5}{10} = -\frac{1}{2} \end{aligned}$$

Substitute the value of y into Equation (3).

$$\begin{aligned} x &= 4y + 3 \quad \bullet \text{ Equation (3)} \\ &= 4\left(-\frac{1}{2}\right) + 3 \quad \bullet y = -\frac{1}{2} \\ &= -2 + 3 = 1 \end{aligned}$$

The solution is $\left(1, -\frac{1}{2}\right)$.

YOU TRY IT • 4

Solve by substitution:

$$3x - y = 3$$

$$6x + 3y = -4$$

Your solution

EXAMPLE • 5

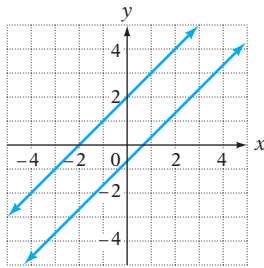
Solve by substitution and graph:

$$\begin{aligned} 3x - 3y &= 2 \\ y &= x + 2 \end{aligned}$$

Solution

$$\begin{aligned} 3x - 3y &= 2 \\ 3x - 3(x + 2) &= 2 & \bullet y = x + 2 \\ 3x - 3x - 6 &= 2 \\ -6 &= 2 \end{aligned}$$

This is not a true equation. The lines are parallel, so the system is inconsistent. The system does not have a solution.

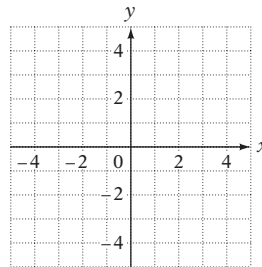


• Graph the two equations.

YOU TRY IT • 5

Solve by substitution and graph:

$$\begin{aligned} y &= 2x - 3 \\ 3x - 2y &= 6 \end{aligned}$$

Your solution**EXAMPLE • 6**

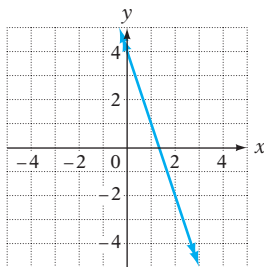
Solve by substitution and graph:

$$\begin{aligned} 9x + 3y &= 12 \\ y &= -3x + 4 \end{aligned}$$

Solution

$$\begin{aligned} 9x + 3y &= 12 \\ 9x + 3(-3x + 4) &= 12 & \bullet y = -3x + 4 \\ 9x - 9x + 12 &= 12 \\ 12 &= 12 \end{aligned}$$

This is a true equation. The system is dependent. The solutions are the ordered pairs $(x, -3x + 4)$.

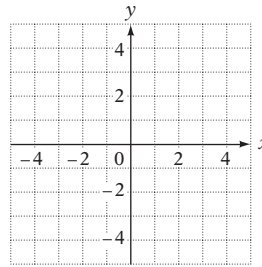


• Graph the two equations.

YOU TRY IT • 6

Solve by substitution and graph:

$$\begin{aligned} 6x - 3y &= 6 \\ 2x - y &= 2 \end{aligned}$$

Your solution

OBJECTIVE C**To solve investment problems**

The annual simple interest that an investment earns is given by the equation $Pr = I$, where P is the principal, or the amount invested, r is the simple interest rate, and I is the simple interest.

For instance, if you invest \$500 at a simple interest rate of 5%, then the interest earned after one year is calculated as follows:

$$\begin{aligned} Pr &= I \\ 500(0.05) &= I && \bullet \text{ Replace } P \text{ by } 500 \text{ and } r \text{ by } 0.05 \text{ (5\%).} \\ 25 &= I && \bullet \text{ Simplify.} \end{aligned}$$

The amount of interest earned is \$25.

**Tips for Success**

Note that solving a word problem includes stating a strategy and using the strategy to find a solution. If you have difficulty with a word problem, write down the known information. Be very specific. Write out a phrase or sentence that states what you are trying to find. See *AIM for Success* in the Preface.

HOW TO • 5

You have a total of \$5000 to invest in two simple interest accounts. On one account, the money market fund, the annual simple interest rate is 3.5%. On the second account, the bond fund, the annual simple interest rate is 7.5%. If you earn \$245 per year from these two investments, how much do you have invested in each account?

Strategy for Solving Simple-Interest Investment Problems

1. For each amount invested, use the equation $Pr = I$. Write a numerical or variable expression for the principal, the interest rate, and the interest earned.

Amount invested at 3.5%: x

Amount invested at 7.5%: y

	Principal, P	$\cdot$	Interest rate, r	$=$	Interest earned, I
Amount at 3.5%	x	$\cdot$	0.035	$=$	$0.035x$
Amount at 7.5%	y	$\cdot$	0.075	$=$	$0.075y$

2. Write a system of equations. One equation will express the relationship between the amounts invested. The second equation will express the relationship between the amounts of interest earned by the investments.

The total amount invested is \$5000: $x + y = 5000$

The total annual interest earned is \$245: $0.035x + 0.075y = 245$

$$\begin{aligned} \text{Solve the system of equations.} \quad (1) \quad & x + y = 5000 \\ (2) \quad & 0.035x + 0.075y = 245 \end{aligned}$$

$$\begin{aligned} \text{Solve Equation (1) for } y: \quad (3) \quad & y = -x + 5000 \\ \text{Substitute into Equation (2):} \quad (2) \quad & 0.035x + 0.075(-x + 5000) = 245 \\ & 0.035x - 0.075x + 375 = 245 \\ & -0.04x = -130 \\ & x = 3250 \end{aligned}$$

Substitute the value of x into Equation (3) and solve for y .

$$\begin{aligned} y &= -x + 5000 \\ y &= -3250 + 5000 = 1750 \end{aligned}$$

The amount invested at 3.5% is \$3250.

The amount invested at 7.5% is \$1750.

EXAMPLE • 7

An investment of \$4000 is made at an annual simple interest rate of 4.9%. How much additional money must be invested at an annual simple interest rate of 7.4% so that the total interest earned is 6.4% of the total investment?

Strategy

- Amount invested at 4.9%: 4000
Amount invested at 7.4%: x
Amount invested at 6.4%: y

	<i>Principal</i>	<i>Rate</i>	<i>Interest</i>
Amount at 4.9%	4000	0.049	$0.049(4000)$
Amount at 7.4%	x	0.074	$0.074x$
Amount at 6.4%	y	0.064	$0.064y$

- The amount invested at 6.4% (y) is \$4000 more than the amount invested at 7.4% (x):
 $y = x + 4000$
- The sum of the interest earned at 4.9% and the interest earned at 7.4% equals the interest earned at 6.4%:
 $0.049(4000) + 0.074x = 0.064y$

Solution

$$y = x + 4000 \quad (1)$$

$$0.049(4000) + 0.074x = 0.064y \quad (2)$$

Replace y in Equation (2) by $x + 4000$ from Equation (1). Then solve for x .

$$\begin{aligned} 0.049(4000) + 0.074x &= 0.064(x + 4000) \\ 196 + 0.074x &= 0.064x + 256 \\ 0.01x &= 60 \\ x &= 6000 \end{aligned}$$

\$6000 must be invested at an annual simple interest rate of 7.4%.

YOU TRY IT • 7

An investment club invested \$13,600 in two simple interest accounts. On one account, the annual simple interest rate is 4.2%. On the other, the annual simple interest rate is 6%. How much should be invested in each account so that both accounts earn the same annual interest?

Your strategy**Your solution**

4.1 EXERCISES

OBJECTIVE A

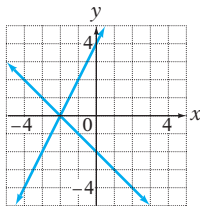
To solve a system of linear equations by graphing

For Exercises 1 to 4, determine whether the ordered pair is a solution of the system of equations.

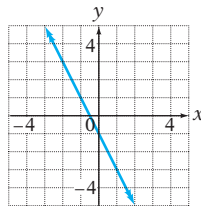
1. $(0, -1)$	2. $(2, 1)$	3. $(-3, -5)$	4. $(1, -1)$
$3x - 2y = 2$	$x + y = 3$	$x + y = -8$	$3x - y = 4$
$x + 2y = 6$	$2x - 3y = 1$	$2x + 5y = -31$	$7x + 2y = -5$

For Exercises 5 to 8, state whether the system of equations is independent, inconsistent, or dependent.

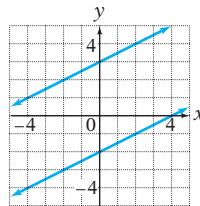
5.



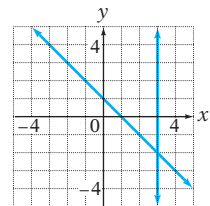
6.



7.



8.



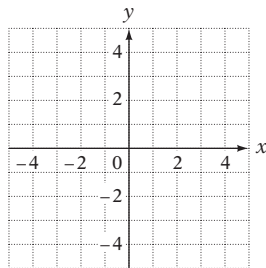
9. In a system of linear equations in two variables, the slope of one line is twice the slope of the other line. Is the system of equations independent, dependent, or inconsistent? If there is not enough information to answer the question, so state.



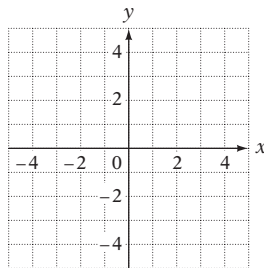
10. In an inconsistent system of linear equations in two variables, what is the relationship between the slopes of the lines? If the graphs of the lines are not vertical, what is the relationship between the y-intercepts of the two lines?

For Exercises 11 to 28, solve by graphing.

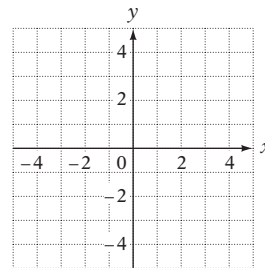
11. $x + y = 2$
 $x - y = 4$



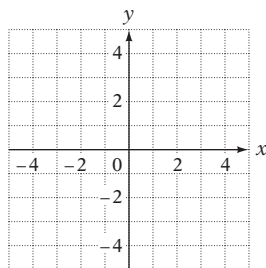
12. $x + y = 1$
 $3x - y = -5$



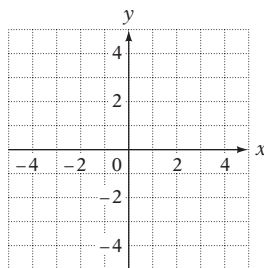
13. $x - y = -2$
 $x + 2y = 10$



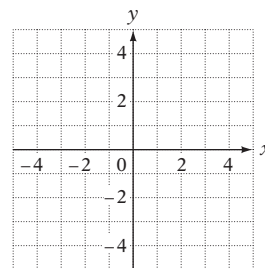
14. $2x - y = 5$
 $3x + y = 5$



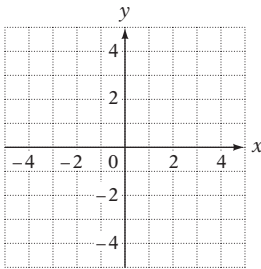
15. $3x - 2y = 6$
 $y = 3$



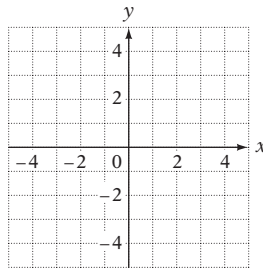
16. $x = 4$
 $3x - 2y = 4$



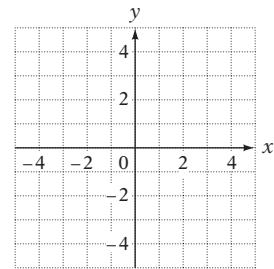
17. $x = 4$
 $y = -1$



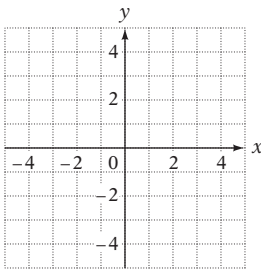
18. $x + 2 = 0$
 $y - 1 = 0$



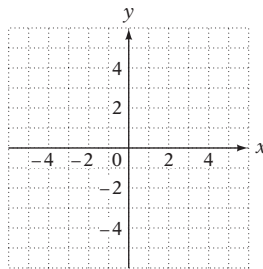
19. $2x + y = 3$
 $x - 2 = 0$



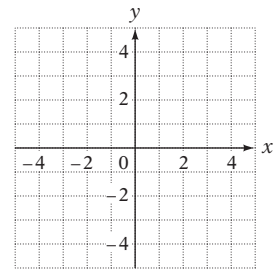
20. $x - 3y = 6$
 $y + 3 = 0$



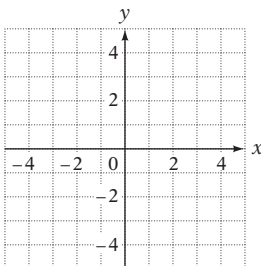
21. $x - y = 6$
 $x + y = 2$



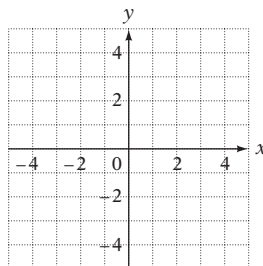
22. $2x + y = 2$
 $-x + y = 5$



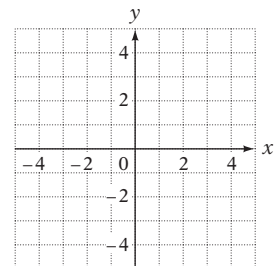
23. $y = x - 5$
 $2x + y = 4$



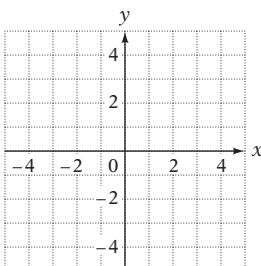
24. $2x - 5y = 4$
 $y = x + 1$



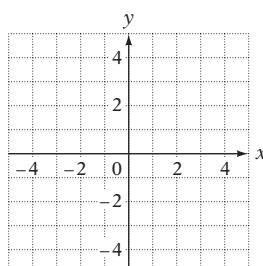
25. $y = \frac{1}{2}x - 2$
 $x - 2y = 8$



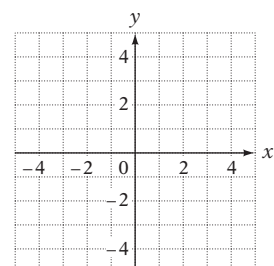
26. $2x + 3y = 6$
 $y = -\frac{2}{3}x + 1$



27. $2x - 5y = 10$
 $y = \frac{2}{5}x - 2$



28. $3x - 2y = 6$
 $y = \frac{3}{2}x - 3$



OBJECTIVE B**To solve a system of linear equations by the substitution method**

For Exercises 29 to 58, solve by the substitution method.

29.
$$\begin{aligned} y &= -x + 1 \\ 2x - y &= 5 \end{aligned}$$

32.
$$\begin{aligned} 4x - 3y &= 5 \\ y &= 2x - 3 \end{aligned}$$

35.
$$\begin{aligned} 4x - 3y &= 2 \\ y &= 2x + 1 \end{aligned}$$

38.
$$\begin{aligned} 5x + 4y &= -1 \\ y &= 2 - 2x \end{aligned}$$

41.
$$\begin{aligned} 5x + 2y &= 15 \\ x &= 6 - y \end{aligned}$$

44.
$$\begin{aligned} 2x + 2y &= 7 \\ y &= 4x + 1 \end{aligned}$$

47.
$$\begin{aligned} 3x - y &= 10 \\ 6x - 2y &= 5 \end{aligned}$$

50.
$$\begin{aligned} 5x + 3y &= 8 \\ 3x + y &= 8 \end{aligned}$$

53.
$$\begin{aligned} 2x - 4y &= 16 \\ -x + 2y &= -8 \end{aligned}$$

56.
$$\begin{aligned} y &= 3x - 7 \\ y &= 2x - 5 \end{aligned}$$

30.
$$\begin{aligned} x &= 3y + 1 \\ x - 2y &= 6 \end{aligned}$$

33.
$$\begin{aligned} 3x + 5y &= -1 \\ y &= 2x - 8 \end{aligned}$$

36.
$$\begin{aligned} x &= 2y + 4 \\ 4x + 3y &= -17 \end{aligned}$$

39.
$$\begin{aligned} 3x + 2y &= 4 \\ y &= 1 - 2x \end{aligned}$$

42.
$$\begin{aligned} 7x - 3y &= 3 \\ x &= 2y + 2 \end{aligned}$$

45.
$$\begin{aligned} 3x + 7y &= -5 \\ y &= 6x - 5 \end{aligned}$$

48.
$$\begin{aligned} 6x - 4y &= 3 \\ 3x - 2y &= 9 \end{aligned}$$

51.
$$\begin{aligned} 3x + 5y &= 0 \\ x - 4y &= 0 \end{aligned}$$

54.
$$\begin{aligned} 3x - 12y &= -24 \\ -x + 4y &= 8 \end{aligned}$$

57.
$$\begin{aligned} y &= 3x + 1 \\ y &= 6x - 1 \end{aligned}$$

31.
$$\begin{aligned} x &= 2y - 3 \\ 3x + y &= 5 \end{aligned}$$

34.
$$\begin{aligned} 5x - 2y &= 9 \\ y &= 3x - 4 \end{aligned}$$

37.
$$\begin{aligned} 3x - 2y &= -11 \\ x &= 2y - 9 \end{aligned}$$

40.
$$\begin{aligned} 2x - 5y &= -9 \\ y &= 9 - 2x \end{aligned}$$

43.
$$\begin{aligned} 3x - 4y &= 6 \\ x &= 3y + 2 \end{aligned}$$

46.
$$\begin{aligned} 3x + y &= 5 \\ 2x + 3y &= 8 \end{aligned}$$

49.
$$\begin{aligned} 3x + 4y &= 14 \\ 2x + y &= 1 \end{aligned}$$

52.
$$\begin{aligned} 2x - 7y &= 0 \\ 3x + y &= 0 \end{aligned}$$

55.
$$\begin{aligned} y &= 3x + 2 \\ y &= 2x + 3 \end{aligned}$$

58.
$$\begin{aligned} y &= 2x - 3 \\ y &= 4x - 4 \end{aligned}$$



59. The system of equations at the right is inconsistent. What is the value of $\frac{a}{b}$?

$$\begin{aligned} 4x - 6y &= 7 \\ ax - by &= 9 \end{aligned}$$



60. Give an example of a system of linear equations in two variables that has $(0, 0)$ as its only solution.

OBJECTIVE C**To solve investment problems**

For Exercises 61 and 62, use the system of equations shown at the right. The system models the investment of x dollars in one simple interest account and y dollars in a second simple interest account.

$$\begin{aligned} x + y &= 6000 \\ 0.055x + 0.072y &= 391.20 \end{aligned}$$

61. What are the interest rates on the two accounts?
62. What is the total amount of money invested?

63. The Community Relief Charity Group is earning 3.5% simple interest on the \$2800 it invested in a savings account. It also earns 4.2% simple interest on an insured bond fund. The annual interest earned from both accounts is \$329. How much is invested in the insured bond fund?
64. Two investments earn an annual income of \$575. One investment earns an annual simple interest rate of 8.5%, and the other investment earns an annual simple interest rate of 6.4%. The total amount invested is \$8000. How much is invested in each account?
65. An investment club invested \$6000 at an annual simple interest rate of 4.0%. How much additional money must be invested at an annual simple interest rate of 6.5% so that the total annual interest earned will be 5% of the total investment?
66. A company invested \$30,000, putting part of it into a savings account that earned 3.2% annual simple interest and the remainder in a stock fund that earned 12.6% annual simple interest. If the investments earned \$1665 annually, how much was in each account?
67. An account executive divided \$42,000 between two simple interest accounts. On the tax-free account the annual simple interest rate is 3.5%, and on the money market fund the annual simple interest rate is 4.5%. How much should be invested in each account so that both accounts earn the same annual interest?
68. An investment club placed \$33,000 into two simple interest accounts. On one account, the annual simple interest rate is 6.5%. On the other, the annual simple interest rate is 4.5%. How much should be invested in each account so that both accounts earn the same annual interest?
69. The Cross Creek Investment Club decided to invest \$16,000 in two bond funds. The first, a mutual bond fund, earns 4.5% annual simple interest. The second, a corporate bond fund, earns 8% annual simple interest. If the club earned \$1070 from these two accounts, how much was invested in the mutual bond fund?
70. Cabin Financial Service Group recommends that a client purchase for \$10,000 a corporate bond that earns 5% annual simple interest. How much additional money must be placed in U.S. government securities that earn a simple interest rate of 3.5% so that the total annual interest earned from the two investments is 4% of the total investment?

Applying the Concepts



For Exercises 71 to 73, use a graphing calculator to estimate the solution to the system of equations. Round coordinates to the nearest hundredth. See the Projects and Group Activities at the end of this chapter for assistance.

$$\begin{aligned} 71. \quad y &= -\frac{1}{2}x + 2 \\ y &= 2x - 1 \end{aligned}$$

$$\begin{aligned} 72. \quad y &= \sqrt{2}x - 1 \\ y &= -\sqrt{3}x + 1 \end{aligned}$$

$$\begin{aligned} 73. \quad y &= \pi x - \frac{2}{3} \\ y &= -x + \frac{\pi}{2} \end{aligned}$$

SECTION

4.2

Solving Systems of Linear Equations by the Addition Method

OBJECTIVE A

To solve a system of two linear equations in two variables by the addition method



The **addition method** is an alternative method for solving a system of equations. This method is based on the Addition Property of Equations. Use the addition method when it is not convenient to solve one equation for one variable in terms of another variable.

Note for the system of equations at the right the effect of adding Equation (2) to Equation (1). Because $-3y$ and $3y$ are additive inverses, adding the equations results in an equation with only one variable.

$$\begin{array}{r} (1) \ 5x - 3y = 14 \\ (2) \ 2x + 3y = -7 \\ \hline 7x + 0y = 7 \\ 7x = 7 \end{array}$$

The solution of the resulting equation is the first coordinate of the ordered-pair solution of the system.

$$\begin{array}{r} 7x = 7 \\ x = 1 \end{array}$$

The second coordinate is found by substituting the value of x into Equation (1) or (2) and then solving for y . Equation (1) is used here.

$$\begin{array}{r} (1) \ 5x - 3y = 14 \\ 5(1) - 3y = 14 \\ 5 - 3y = 14 \\ -3y = 9 \\ y = -3 \end{array}$$

The solution is $(1, -3)$.

Sometimes each equation of the system of equations must be multiplied by a constant so that the coefficients of one of the variables are opposites.

HOW TO 1

Solve by the addition method:

$$\begin{array}{r} (1) \ 3x + 4y = 2 \\ (2) \ 2x + 5y = -1 \end{array}$$

To eliminate x , multiply Equation (1) by 2 and Equation (2) by -3 . Note at the right how the constants are chosen.

$$\begin{array}{r} 2(3x + 4y) = 2 \cdot 2 \\ \cancel{6x} + 8y = 4 \\ -3(2x + 5y) = -3(-1) \\ \hline -6x - 15y = 3 \end{array}$$

• The negative is used so that the coefficients will be opposites.

$$\begin{array}{r} 6x + 8y = 4 \\ -6x - 15y = 3 \\ \hline -7y = 7 \\ y = -1 \end{array}$$

- 2 times Equation (1).
- -3 times Equation (2).
- Add the equations.
- Solve for y .

Substitute the value of y into Equation (1) or Equation (2) and solve for x . Equation (1) will be used here.

$$\begin{array}{r} (1) \ 3x + 4y = 2 \\ 3x + 4(-1) = 2 \\ 3x - 4 = 2 \\ 3x = 6 \\ x = 2 \end{array}$$

- Substitute -1 for y .
- Solve for x .

The solution is $(2, -1)$.



Point of Interest

There are records of Babylonian mathematicians solving systems of equations 3600 years ago. Here is a system of equations from that time (in our modern notation):

$$\begin{array}{r} \frac{2}{3}x = \frac{1}{2}y + 500 \\ x + y = 1800 \end{array}$$

We say *modern notation* for many reasons. Foremost is the fact that the use of variables did not become widespread until the 17th century. There are many other reasons, however. The equals sign had not been invented, 2 and 3 did not look like they do today, and zero had not even been considered as a possible number.



Tips for Success

Always check the proposed solution of a system of equations. For the system at the right:

$$\begin{array}{r} 3x + 4y = 2 \\ 3(2) + 4(-1) \quad | \quad 2 \\ 6 - 4 \quad | \quad 2 \\ 2 = 2 \end{array}$$

$$\begin{array}{r} 2x + 5y = -1 \\ 2(2) + 5(-1) \quad | \quad -1 \\ 4 - 5 \quad | \quad -1 \\ -1 = -1 \end{array}$$

The solution checks.

**Take Note**

The result of adding Equations (3) and (2) is $0 = 0$. It is not $x = 0$ and it is not $y = 0$. There is no variable in the equation $0 = 0$. This result does not indicate that the solution is $(0, 0)$; rather, it indicates a dependent system of equations.

HOW TO 2

Solve by the addition method:

(1) $2x - y = 3$

(2) $4x - 2y = 6$

Eliminate y . Multiply Equation (1) by -2 .

(1) $-2(2x - y) = -2(3)$

• -2 times Equation (1).

(3) $-4x + 2y = -6$

• This is Equation (3).

Add Equation (3) to Equation (2).

(2) $4x - 2y = 6$

(3) $-4x + 2y = -6$

$0 = 0$

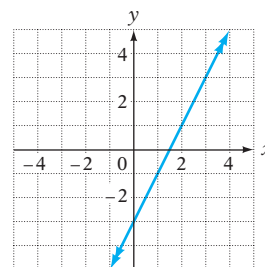
The equation $0 = 0$ indicates that the system of equations is dependent. This means that the graphs of the two lines are the same. Therefore, the solutions of the system of equations are the ordered-pair solutions of the equation of the line. Solve Equation (1) for y .

$2x - y = 3$

$-y = -2x + 3$

$y = 2x - 3$

The ordered-pair solutions are $(x, 2x - 3)$, where $2x - 3$ is substituted for y .

**HOW TO 3**

Solve by the addition method:

(1) $\frac{2}{3}x + \frac{1}{2}y = 4$

(2) $\frac{1}{4}x - \frac{3}{8}y = -\frac{3}{4}$

Clear fractions. Multiply each equation by the LCM of the denominators.

$6\left(\frac{2}{3}x + \frac{1}{2}y\right) = 6(4)$

• The LCM of 3 and 2 is 6.

$8\left(\frac{1}{4}x - \frac{3}{8}y\right) = 8\left(-\frac{3}{4}\right)$

• The LCM of 4 and 8 is 8.

$4x + 3y = 24$

$2x - 3y = -6$

$6x = 18$

$x = 3$

• Eliminate y . Add the equations. Then solve for x .

$\frac{2}{3}x + \frac{1}{2}y = 4$

• This is Equation (1).

$\frac{2}{3}(3) + \frac{1}{2}y = 4$

• Substitute $x = 3$ into Equation (1) and solve for y .

$2 + \frac{1}{2}y = 4$

$\frac{1}{2}y = 2$

$y = 4$

The solution is $(3, 4)$.

EXAMPLE • 1

Solve by the addition method:

$$\begin{aligned}(1) \quad & 3x - 2y = 2x + 5 \\(2) \quad & 2x + 3y = -4\end{aligned}$$

SolutionWrite Equation (1) in the form $Ax + By = C$.

$$\begin{aligned}3x - 2y &= 2x + 5 \\x - 2y &= 5\end{aligned}$$

Solve the system:

$$\begin{aligned}x - 2y &= 5 \\2x + 3y &= -4\end{aligned}$$

Eliminate x .

$$\begin{array}{r} -2(x - 2y) = -2(5) \\ 2x + 3y = -4 \\ \hline -2x + 4y = -10 \\ 2x + 3y = -4 \\ \hline 7y = -14 \\ y = -2 \end{array}$$

- Add the equations.
- Solve for y .

Replace y in Equation (2).

$$\begin{aligned}2x + 3y &= -4 \\ 2x + 3(-2) &= -4 \\ 2x - 6 &= -4 \\ 2x &= 2 \\ x &= 1\end{aligned}$$

The solution is $(1, -2)$.**EXAMPLE • 2**

Solve by the addition method:

$$\begin{aligned}(1) \quad & 4x - 8y = 36 \\(2) \quad & 3x - 6y = 27\end{aligned}$$

SolutionEliminate x .

$$\begin{array}{r} 3(4x - 8y) = 3(36) \\ -4(3x - 6y) = -4(27) \\ \hline 12x - 24y = 108 \\ -12x + 24y = -108 \\ \hline 0 = 0 \end{array}$$

- Add the equations.

The system of equations is dependent. Solve Equation (1) for y .

$$\begin{aligned}4x - 8y &= 36 \\ -8y &= -4x + 36 \\ y &= \frac{1}{2}x - \frac{9}{2}\end{aligned}$$

The solutions are the ordered pairs $\left(x, \frac{1}{2}x - \frac{9}{2}\right)$.**YOU TRY IT • 1**

Solve by the addition method:

$$\begin{aligned}2x + 5y &= 6 \\ 3x - 2y &= 6x + 2\end{aligned}$$

Your solution**YOU TRY IT • 2**

Solve by the addition method:

$$\begin{aligned}2x + y &= 5 \\ 4x + 2y &= 6\end{aligned}$$

Your solution

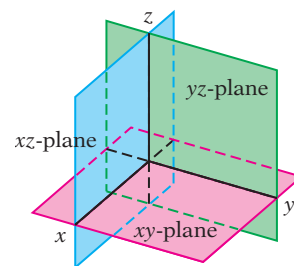
OBJECTIVE B**To solve a system of three linear equations in three variables by the addition method**

An equation of the form $Ax + By + Cz = D$, where A , B , and C are the coefficients of the variables and D is a constant, is a **linear equation in three variables**. Examples of this type of equation are shown at the right.

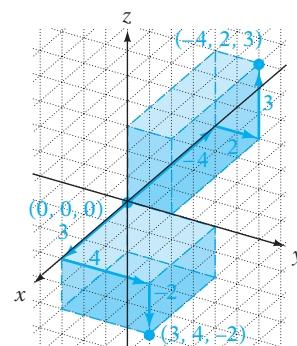
$$2x + 4y - 3z = 7$$

$$x - 6y + z = -3$$

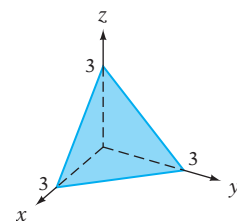
Graphing an equation in three variables requires a third coordinate axis perpendicular to the xy -plane. The third axis is commonly called the z -axis. The result is a three-dimensional coordinate system called the **xyz-coordinate system**. To help visualize a three-dimensional coordinate system, think of a corner of a room: The floor is the xy -plane, one wall is the yz -plane, and the other wall is the xz -plane. A three-dimensional coordinate system is shown at the right.



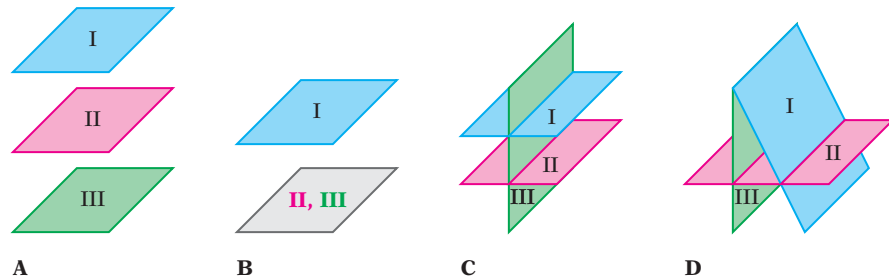
The graph of a point in an xyz -coordinate system is an **ordered triple** (x, y, z) . Graphing an ordered triple requires three moves, the first in the direction of the x -axis, the second in the direction of the y -axis, and the third in the direction of the z -axis. The graph of the points $(-4, 2, 3)$ and $(3, 4, -2)$ is shown at the right.



The graph of a linear equation in three variables is a plane. That is, if all the solutions of a linear equation in three variables were plotted in an xyz -coordinate system, the graph would look like a large piece of paper extending infinitely. The graph of $x + y + z = 3$ is shown at the right.



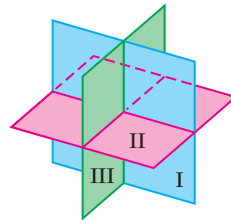
There are different ways in which three planes can be oriented in an xyz -coordinate system. The systems of equations represented by the planes below are inconsistent.



Graphs of Inconsistent Systems of Equations

For a system of three equations in three variables to have a solution, the graphs of the planes must intersect at a single point, they must intersect along a common line, or all equations must have a graph that is the same plane. These situations are shown in the figures below.

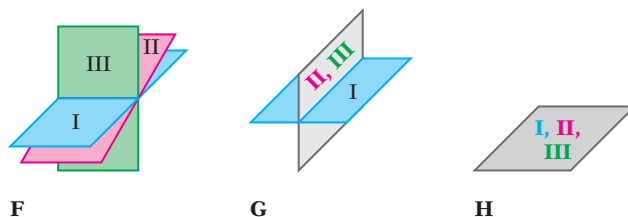
The three planes shown in Figure E intersect at a point. A system of equations represented by planes that intersect at a point is independent.



E

An Independent System of Equations

The planes shown in Figures F and G intersect along a common line. The system of equations represented by the planes in Figure H has a graph that is the same plane. The systems of equations represented by the graphs below are dependent.



Dependent Systems of Equations

**Point of Interest**

In the early 1980s, Stephen Hoppe became interested in winning Monopoly strategies. Finding these strategies required solving a system that contained 123 equations in 123 variables!

Just as a solution of an equation in two variables is an ordered pair (x, y) , a **solution of an equation in three variables** is an ordered triple (x, y, z) . For example, $(2, 1, -3)$ is a solution of the equation $2x - y - 2z = 9$. The ordered triple $(1, 3, 2)$ is not a solution.

A **system of linear equations in three variables** is shown at the right. A **solution of a system of equations in three variables** is an ordered triple that is a solution of each equation of the system.

$$\begin{aligned}x - 2y + z &= 6 \\3x + y - 2z &= 2 \\2x - 3y + 5z &= 1\end{aligned}$$

A system of linear equations in three variables can be solved by using the addition method. First, eliminate one variable from any two of the given equations. Then eliminate the same variable from any other two equations. The result will be a system of two equations in two variables. Solve this system by the addition method.

HOW TO 4

Solve:

$$\begin{aligned}(1) \quad & x + 4y - z = 10 \\(2) \quad & 3x + 2y + z = 4 \\(3) \quad & 2x - 3y + 2z = -7\end{aligned}$$

Eliminate z from Equations (1) and (2) by adding the two equations.

$$\begin{array}{r}x + 4y - z = 10 \\3x + 2y + z = 4 \\\hline(4) \quad 4x + 6y = 14\end{array} \quad \bullet \text{ Add the equations.}$$

Eliminate z from Equations (1) and (3). Multiply Equation (1) by 2 and add to Equation (3).

$$\begin{array}{r}2x + 8y - 2z = 20 \\2x - 3y + 2z = -7 \\\hline(5) \quad 4x + 5y = 13\end{array} \quad \begin{array}{l}\bullet \text{ 2 times Equation (1).} \\ \bullet \text{ This is Equation (3).} \\ \bullet \text{ Add the equations.}\end{array}$$

Using Equations (4) and (5), solve the system of two equations in two variables.

$$\begin{aligned}(4) \quad & 4x + 6y = 14 \\(5) \quad & 4x + 5y = 13\end{aligned}$$

Eliminate x . Multiply Equation (5) by -1 and add to Equation (4).

$$\begin{array}{r}4x + 6y = 14 \\-4x - 5y = -13 \\\hline y = 1\end{array} \quad \begin{array}{l}\bullet \text{ This is Equation (4).} \\ \bullet \text{ } -1 \text{ times Equation (5).} \\ \bullet \text{ Add the equations.}\end{array}$$

Substitute the value of y into Equation (4) or Equation (5) and solve for x . Equation (4) is used here.

$$\begin{array}{r}4x + 6y = 14 \\4x + 6(1) = 14 \\4x + 6 = 14 \\4x = 8 \\x = 2\end{array} \quad \begin{array}{l}\bullet \text{ This is Equation (4).} \\ \bullet y = 1 \\ \bullet \text{ Solve for } x.\end{array}$$

Substitute the value of y and the value of x into one of the equations in the original system. Equation (2) is used here.

$$\begin{array}{r}3x + 2y + z = 4 \\3(2) + 2(1) + z = 4 \\6 + 2 + z = 4 \\8 + z = 4 \\z = -4\end{array} \quad \bullet x = 2, y = 1$$

The solution is $(2, 1, -4)$.

**Tips for Success**

Always check the proposed solution of a system of equations. For the system at the right:

$$\begin{array}{r}x + 4y - z = 10 \\2 + 4(1) - (-4) \quad | \quad 10 \\10 = 10\end{array}$$

$$\begin{array}{r}3x + 2y + z = 4 \\3(2) + 2(1) + (-4) \quad | \quad 4 \\4 = 4\end{array}$$

$$\begin{array}{r}2x - 3y + 2z = -7 \\2(2) - 3(1) + 2(-4) \quad | \quad -7 \\-7 = -7\end{array}$$

The solution checks.

HOW TO • 5

Solve:

$$\begin{array}{ll} (1) & 2x - 3y - z = 1 \\ (2) & x + 4y + 3z = 2 \\ (3) & 4x - 6y - 2z = 5 \end{array}$$

Eliminate x from Equations (1) and (2).

$$\begin{array}{r} 2x - 3y - z = 1 \\ -2x - 8y - 6z = -4 \\ \hline -11y - 7z = -3 \end{array}$$

- This is Equation (1).
- -2 times Equation (2).
- Add the equations.

Eliminate x from Equations (1) and (3).

$$\begin{array}{r} -4x + 6y + 2z = -2 \\ 4x - 6y - 2z = 5 \\ \hline 0 = 3 \end{array}$$

- -2 times Equation (1).
- This is Equation (3).
- Add the equations.

The equation $0 = 3$ is not a true equation. The system of equations is inconsistent and therefore has no solution.

HOW TO • 6

Solve:

$$\begin{array}{ll} (1) & 3x - z = -1 \\ (2) & 2y - 3z = 10 \\ (3) & x + 3y - z = 7 \end{array}$$

Eliminate x from Equations (1) and (3). Multiply Equation (3) by -3 and add to Equation (1).

$$\begin{array}{r} 3x - z = -1 \\ -3x - 9y + 3z = -21 \\ \hline (4) \quad -9y + 2z = -22 \end{array}$$

- This is Equation (1).
- -3 times Equation (3).
- Add the equations.

Use Equations (2) and (4) to form a system of equations in two variables.

$$\begin{array}{ll} (2) & 2y - 3z = 10 \\ (4) & -9y + 2z = -22 \end{array}$$

Eliminate z . Multiply Equation (2) by 2 and Equation (4) by 3.

$$\begin{array}{r} 4y - 6z = 20 \\ -27y + 6z = -66 \\ \hline -23y = -46 \\ y = 2 \end{array}$$

- 2 times Equation (2).
- 3 times Equation (4).
- Add the equations.
- Solve for y .

Substitute the value of y into Equation (2) or Equation (4) and solve for z . Equation (2) is used here.

$$\begin{array}{ll} (2) & 2y - 3z = 10 \\ & 2(2) - 3z = 10 \\ & 4 - 3z = 10 \\ & -3z = 6 \\ & z = -2 \end{array}$$

- This is Equation (2).
- $y = 2$
- Solve for z .

Substitute the value of z into Equation (1) and solve for x .

$$\begin{array}{ll} (1) & 3x - z = -1 \\ & 3x - (-2) = -1 \\ & 3x + 2 = -1 \\ & 3x = -3 \\ & x = -1 \end{array}$$

- This is Equation (1).
- $z = -2$
- Solve for x .

The solution is $(-1, 2, -2)$.

EXAMPLE 3

Solve:

$$\begin{array}{l} (1) \quad 3x - y + 2z = 1 \\ (2) \quad 2x + 3y + 3z = 4 \\ (3) \quad x + y - 4z = -9 \end{array}$$

Solution

Eliminate y . Add Equations (1) and (3).

$$\begin{array}{r} 3x - y + 2z = 1 \\ x + y - 4z = -9 \\ \hline 4x - 2z = -8 \end{array}$$

Multiply each side of the equation by $\frac{1}{4}$.

$$2x - z = -4 \quad \bullet \text{ Equation (4)}$$

Multiply Equation (1) by 3 and add to Equation (2).

$$\begin{array}{r} 9x - 3y + 6z = 3 \\ 2x + 3y + 3z = 4 \\ \hline 11x + 9z = 7 \quad \bullet \text{ Equation (5)} \end{array}$$

Solve the system of two equations.

$$\begin{array}{l} (4) \quad 2x - z = -4 \\ (5) \quad 11x + 9z = 7 \end{array}$$

Multiply Equation (4) by 9 and add to Equation (5).

$$\begin{array}{r} 18x - 9z = -36 \\ 11x + 9z = 7 \\ \hline 29x = -29 \\ x = -1 \end{array}$$

Replace x by -1 in Equation (4).

$$\begin{array}{r} 2x - z = -4 \\ 2(-1) - z = -4 \\ -2 - z = -4 \\ -z = -2 \\ z = 2 \end{array}$$

Replace x by -1 and z by 2 in Equation (3).

$$\begin{array}{r} x + y - 4z = -9 \\ -1 + y - 4(2) = -9 \\ -9 + y = -9 \\ y = 0 \end{array}$$

The solution is $(-1, 0, 2)$.

YOU TRY IT 3

Solve:

$$\begin{array}{r} x - y + z = 6 \\ 2x + 3y - z = 1 \\ x + 2y + 2z = 5 \end{array}$$

Your solution

4.2 EXERCISES

OBJECTIVE A

To solve a system of two linear equations
in two variables by the addition method



For Exercises 1 and 2, use the system of equations at the right.

$$(1) 5x - 7y = 9$$

$$(2) 6x + 3y = 12$$

1. If you were using the addition method to eliminate x , you could multiply Equation (1) by _____ and Equation (2) by _____ and then add the resulting equations.
2. If you were using the addition method to eliminate y , you could multiply Equation (1) by _____ and Equation (2) by _____ and then add the resulting equations.

For Exercises 3 to 44, solve by the addition method.

$$3. \begin{cases} x - y = 5 \\ x + y = 7 \end{cases}$$

$$4. \begin{cases} x + y = 1 \\ 2x - y = 5 \end{cases}$$

$$5. \begin{cases} 3x + y = 4 \\ x + y = 2 \end{cases}$$

$$6. \begin{cases} x - 3y = 4 \\ x + 5y = -4 \end{cases}$$

$$7. \begin{cases} 3x + y = 7 \\ x + 2y = 4 \end{cases}$$

$$8. \begin{cases} x - 2y = 7 \\ 3x - 2y = 9 \end{cases}$$

$$9. \begin{cases} 2x + 3y = -1 \\ x + 5y = 3 \end{cases}$$

$$10. \begin{cases} x + 5y = 7 \\ 2x + 7y = 8 \end{cases}$$

$$11. \begin{cases} 3x - y = 4 \\ 6x - 2y = 8 \end{cases}$$

$$12. \begin{cases} x - 2y = -3 \\ -2x + 4y = 6 \end{cases}$$

$$13. \begin{cases} 2x + 5y = 9 \\ 4x - 7y = -16 \end{cases}$$

$$14. \begin{cases} 8x - 3y = 21 \\ 4x + 5y = -9 \end{cases}$$

$$15. \begin{cases} 4x - 6y = 5 \\ 2x - 3y = 7 \end{cases}$$

$$16. \begin{cases} 3x + 6y = 7 \\ 2x + 4y = 5 \end{cases}$$

$$17. \begin{cases} 3x - 5y = 7 \\ x - 2y = 3 \end{cases}$$

$$18. \begin{cases} 3x + 4y = 25 \\ 2x + y = 10 \end{cases}$$

$$19. \begin{cases} x + 3y = 7 \\ -2x + 3y = 22 \end{cases}$$

$$20. \begin{cases} 2x - 3y = 14 \\ 5x - 6y = 32 \end{cases}$$

$$\begin{aligned} 21. \quad & 3x + 2y = 16 \\ & 2x - 3y = -11 \end{aligned}$$

$$\begin{aligned} 22. \quad & 2x - 5y = 13 \\ & 5x + 3y = 17 \end{aligned}$$

$$\begin{aligned} 23. \quad & 4x + 4y = 5 \\ & 2x - 8y = -5 \end{aligned}$$

$$\begin{aligned} 24. \quad & 3x + 7y = 16 \\ & 4x - 3y = 9 \end{aligned}$$

$$\begin{aligned} 25. \quad & 5x + 4y = 0 \\ & 3x + 7y = 0 \end{aligned}$$

$$\begin{aligned} 26. \quad & 3x - 4y = 0 \\ & 4x - 7y = 0 \end{aligned}$$

$$\begin{aligned} 27. \quad & 5x + 2y = 1 \\ & 2x + 3y = 7 \end{aligned}$$

$$\begin{aligned} 28. \quad & 3x + 5y = 16 \\ & 5x - 7y = -4 \end{aligned}$$

$$\begin{aligned} 29. \quad & 3x - 6y = 6 \\ & 9x - 3y = 8 \end{aligned}$$

$$\begin{aligned} 30. \quad & \frac{2}{3}x - \frac{1}{2}y = 3 \\ & \frac{1}{3}x - \frac{1}{4}y = \frac{3}{2} \end{aligned}$$

$$\begin{aligned} 31. \quad & \frac{3}{4}x + \frac{1}{3}y = -\frac{1}{2} \\ & \frac{1}{2}x - \frac{5}{6}y = -\frac{7}{2} \end{aligned}$$

$$\begin{aligned} 32. \quad & \frac{2}{5}x - \frac{1}{3}y = 1 \\ & \frac{3}{5}x + \frac{2}{3}y = 5 \end{aligned}$$

$$\begin{aligned} 33. \quad & \frac{5x}{6} + \frac{y}{3} = \frac{4}{3} \\ & \frac{2x}{3} - \frac{y}{2} = \frac{11}{6} \end{aligned}$$

$$\begin{aligned} 34. \quad & \frac{3x}{4} + \frac{2y}{5} = -\frac{3}{20} \\ & \frac{3x}{2} - \frac{y}{4} = \frac{3}{4} \end{aligned}$$

$$\begin{aligned} 35. \quad & \frac{2x}{5} - \frac{y}{2} = \frac{13}{2} \\ & \frac{3x}{4} - \frac{y}{5} = \frac{17}{2} \end{aligned}$$

$$\begin{aligned} 36. \quad & \frac{x}{2} + \frac{y}{3} = \frac{5}{12} \\ & \frac{x}{2} - \frac{y}{3} = \frac{1}{12} \end{aligned}$$

$$\begin{aligned} 37. \quad & \frac{3x}{2} - \frac{y}{4} = -\frac{11}{12} \\ & \frac{x}{3} - y = -\frac{5}{6} \end{aligned}$$

$$\begin{aligned} 38. \quad & \frac{3x}{4} - \frac{2y}{3} = 0 \\ & \frac{5x}{4} - \frac{y}{3} = \frac{7}{12} \end{aligned}$$

$$\begin{aligned} 39. \quad & 4x - 5y = 3y + 4 \\ & 2x + 3y = 2x + 1 \end{aligned}$$

$$\begin{aligned} 40. \quad & 5x - 2y = 8x - 1 \\ & 2x + 7y = 4y + 9 \end{aligned}$$

$$\begin{aligned} 41. \quad & 2x + 5y = 5x + 1 \\ & 3x - 2y = 3y + 3 \end{aligned}$$

$$\begin{aligned} 42. \quad & 4x - 8y = 5 \\ & 8x + 2y = 1 \end{aligned}$$

$$\begin{aligned} 43. \quad & 5x + 2y = 2x + 1 \\ & 2x - 3y = 3x + 2 \end{aligned}$$

$$\begin{aligned} 44. \quad & 3x + 3y = y + 1 \\ & x + 3y = 9 - x \end{aligned}$$

OBJECTIVE B

To solve a system of three linear equations in three variables by the addition method

For Exercises 45 to 68, solve by the addition method.

$$\begin{aligned} 45. \quad & x + 2y - z = 1 \\ & 2x - y + z = 6 \\ & x + 3y - z = 2 \end{aligned}$$

$$\begin{aligned} 46. \quad & x + 3y + z = 6 \\ & 3x + y - z = -2 \\ & 2x + 2y - z = 1 \end{aligned}$$

$$\begin{aligned} 47. \quad & 2x - y + 2z = 7 \\ & x + y + z = 2 \\ & 3x - y + z = 6 \end{aligned}$$

$$\begin{aligned} 48. \quad & x - 2y + z = 6 \\ & x + 3y + z = 16 \\ & 3x - y - z = 12 \end{aligned}$$

$$\begin{aligned} 49. \quad & 3x + y = 5 \\ & 3y - z = 2 \\ & x + z = 5 \end{aligned}$$

$$\begin{aligned} 50. \quad & 2y + z = 7 \\ & 2x - z = 3 \\ & x - y = 3 \end{aligned}$$

$$\begin{aligned} 51. \quad & x - y + z = 1 \\ & 2x + 3y - z = 3 \\ & -x + 2y - 4z = 4 \end{aligned}$$

$$\begin{aligned} 52. \quad & 2x + y - 3z = 7 \\ & x - 2y + 3z = 1 \\ & 3x + 4y - 3z = 13 \end{aligned}$$

$$\begin{aligned} 53. \quad & 2x + 3z = 5 \\ & 3y + 2z = 3 \\ & 3x + 4y = -10 \end{aligned}$$

$$\begin{aligned} 54. \quad & 3x + 4z = 5 \\ & 2y + 3z = 2 \\ & 2x - 5y = 8 \end{aligned}$$

$$\begin{aligned} 55. \quad & 2x + 4y - 2z = 3 \\ & x + 3y + 4z = 1 \\ & x + 2y - z = 4 \end{aligned}$$

$$\begin{aligned} 56. \quad & x - 3y + 2z = 1 \\ & x - 2y + 3z = 5 \\ & 2x - 6y + 4z = 3 \end{aligned}$$

$$\begin{aligned} 57. \quad & 2x + y - z = 5 \\ & x + 3y + z = 14 \\ & 3x - y + 2z = 1 \end{aligned}$$

$$\begin{aligned} 58. \quad & 3x - y - 2z = 11 \\ & 2x + y - 2z = 11 \\ & x + 3y - z = 8 \end{aligned}$$

$$\begin{aligned} 59. \quad & 3x + y - 2z = 2 \\ & x + 2y + 3z = 13 \\ & 2x - 2y + 5z = 6 \end{aligned}$$

$$\begin{aligned} 60. \quad & 4x + 5y + z = 6 \\ & 2x - y + 2z = 11 \\ & x + 2y + 2z = 6 \end{aligned}$$

$$\begin{aligned} 61. \quad & 2x - y + z = 6 \\ & 3x + 2y + z = 4 \\ & x - 2y + 3z = 12 \end{aligned}$$

$$\begin{aligned} 62. \quad & 3x + 2y - 3z = 8 \\ & 2x + 3y + 2z = 10 \\ & x + y - z = 2 \end{aligned}$$

$$\begin{aligned} 63. \quad & 3x - 2y + 3z = -4 \\ & 2x + y - 3z = 2 \\ & 3x + 4y + 5z = 8 \end{aligned}$$

$$\begin{aligned} 64. \quad & 3x - 3y + 4z = 6 \\ & 4x - 5y + 2z = 10 \\ & x - 2y + 3z = 4 \end{aligned}$$

$$\begin{aligned} 65. \quad & 3x - y + 2z = 2 \\ & 4x + 2y - 7z = 0 \\ & 2x + 3y - 5z = 7 \end{aligned}$$

$$\begin{aligned} 66. \quad & 2x + 2y + 3z = 13 \\ & -3x + 4y - z = 5 \\ & 5x - 3y + z = 2 \end{aligned}$$

$$\begin{aligned} 67. \quad & 2x - 3y + 7z = 0 \\ & x + 4y - 4z = -2 \\ & 3x + 2y + 5z = 1 \end{aligned}$$

$$\begin{aligned} 68. \quad & 5x + 3y - z = 5 \\ & 3x - 2y + 4z = 13 \\ & 4x + 3y + 5z = 22 \end{aligned}$$



69. In the following sentences, fill in the blanks with one of the following phrases: (i) exactly one point, (ii) more than one point, or (iii) no points.
- For an inconsistent system of linear equations in three variables, the planes representing the equations intersect at _____.
 - For a dependent system of linear equations in three variables, the planes representing the equations intersect at _____.
 - For an independent system of linear equations in three variables, the planes representing the equations intersect at _____.

Applying the Concepts



70. Describe the graph of each of the following equations in an xyz -coordinate system.
- $x = 3$
 - $y = 4$
 - $z = 2$
 - $y = x$

In Exercises 71 to 74, the systems are not systems of linear equations. However, they can be solved by using a modification of the addition method. Solve each system of equations.

$$\begin{aligned} 71. \quad & \frac{1}{x} - \frac{2}{y} = 3 \\ & \frac{2}{x} + \frac{3}{y} = -1 \end{aligned}$$

$$\begin{aligned} 72. \quad & \frac{1}{x} + \frac{2}{y} = 3 \\ & \frac{1}{x} - \frac{3}{y} = -2 \end{aligned}$$

$$\begin{aligned} 73. \quad & \frac{3}{x} + \frac{2}{y} = 1 \\ & \frac{2}{x} + \frac{4}{y} = -2 \end{aligned}$$

$$\begin{aligned} 74. \quad & \frac{3}{x} - \frac{5}{y} = -\frac{3}{2} \\ & \frac{1}{x} - \frac{2}{y} = -\frac{2}{3} \end{aligned}$$

SECTION

4.3

Solving Systems of Equations by Using Determinants

OBJECTIVE A

To evaluate a determinant



Point of Interest

The word *matrix* was first used in a mathematical context in 1850. The root of this word is the Latin *mater*, meaning *mother*. A matrix was thought of as an object from which something else originates. The idea was that a determinant, discussed below, originated (was born) from a matrix. Today, matrices are one of the most widely used tools of applied mathematics.



Take Note

Note that vertical bars are used to represent the determinant and that brackets are used to represent the matrix.

A **matrix** is a rectangular array of numbers. Each number in a matrix is called an **element of the matrix**. The matrix at the right, with three rows and four columns, is called a 3×4 (read “3 by 4”) matrix.

$$A = \begin{bmatrix} 1 & -3 & 2 & 4 \\ 0 & 4 & -3 & 2 \\ 6 & -5 & 4 & -1 \end{bmatrix}$$

A matrix of m rows and n columns is said to be of **order** $m \times n$. The matrix above has order 3×4 . The notation a_{ij} refers to the element of a matrix in the i th row and the j th column. For matrix A , $a_{23} = -3$, $a_{31} = 6$, and $a_{13} = 2$.

A **square matrix** is one that has the same number of rows as columns. A 2×2 matrix and a 3×3 matrix are shown at the right. Associated with every square matrix is a number called its **determinant**.

$$\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 1 \\ 5 & -3 & 7 \\ 2 & 1 & 4 \end{bmatrix}$$

Determinant of a 2×2 Matrix

The **determinant** of a 2×2 matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is written $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$. The value of this determinant is given by the formula

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

HOW TO • 1

Find the value of the determinant $\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix}$.

$$\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix} = 3 \cdot 2 - 4(-1) = 6 - (-4) = 10$$

The value of the determinant is 10.

For a square matrix whose order is 3×3 or greater, the value of the determinant is found by using 2×2 determinants.

The **minor of an element** in a 3×3 determinant is the 2×2 determinant that is obtained by eliminating the row and column that contain that element.

HOW TO • 2

Find the minor of -3 for the determinant $\begin{vmatrix} 2 & -3 & 4 \\ 0 & 4 & 8 \\ -1 & 3 & 6 \end{vmatrix}$.

The minor of -3 is the 2×2 determinant created by eliminating the row and column that contain -3 .

Eliminate the row and column as shown:

$$\begin{vmatrix} \cancel{2} & \cancel{-3} & \cancel{4} \\ 0 & 4 & 8 \\ -1 & 3 & 6 \end{vmatrix}$$

The minor of -3 is $\begin{vmatrix} 0 & 8 \\ -1 & 6 \end{vmatrix}$.

**Take Note**

The only difference between the cofactor and the minor of an element is one of sign. The definition at the right can be stated with the use of symbols as follows: If C_{ij} is the cofactor and M_{ij} is the minor of the matrix element a_{ij} , then $C_{ij} = (-1)^{i+j}M_{ij}$. If $i + j$ is an even number, then $(-1)^{i+j} = 1$ and $C_{ij} = M_{ij}$. If $i + j$ is an odd number, then $(-1)^{i+j} = -1$ and $C_{ij} = -M_{ij}$.

Cofactor of an Element of a Matrix

The **cofactor of an element of a matrix** is $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is the column number of the element.

HOW TO • 3

For the determinant $\begin{vmatrix} 3 & -2 & 1 \\ 2 & -5 & -4 \\ 0 & 3 & 1 \end{vmatrix}$, find the cofactor of -2 and of -5 .

Because -2 is in the first row and the second column, $i = 1$ and $j = 2$. Thus

$$(-1)^{i+j} = (-1)^{1+2} = (-1)^3 = -1. \text{ The cofactor of } -2 \text{ is } (-1) \begin{vmatrix} 2 & -4 \\ 0 & 1 \end{vmatrix}.$$

Because -5 is in the second row and the second column, $i = 2$ and $j = 2$.

$$\text{Thus } (-1)^{i+j} = (-1)^{2+2} = (-1)^4 = 1. \text{ The cofactor of } -5 \text{ is } 1 \cdot \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix}.$$

Note from this example that the cofactor of an element is -1 times the minor of that element or 1 times the minor of that element, depending on whether the sum $i + j$ is an odd or an even integer.

The value of a 3×3 or larger determinant can be found by **expanding by cofactors** of any row or any column.

HOW TO • 4

Find the value of the determinant $\begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & -1 \\ 0 & -2 & 2 \end{vmatrix}$.

We will expand by cofactors of the first row. Any row or column would work.

$$\begin{aligned} \begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & -1 \\ 0 & -2 & 2 \end{vmatrix} &= 2(-1)^{1+1} \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix} + (-3)(-1)^{1+2} \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 2(-1)^{1+3} \begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix} \\ &= 2(1) \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix} + (-3)(-1) \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 2(1) \begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix} \\ &= 2(6 - 2) + 3(2 - 0) + 2(-2 - 0) \\ &= 2(4) + 3(2) + 2(-2) = 8 + 6 - 4 = 10 \end{aligned}$$

To illustrate that any row or column can be chosen when expanding by cofactors, we will now show the evaluation of the same determinant by expanding by cofactors of the second column.

$$\begin{aligned} \begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & -1 \\ 0 & -2 & 2 \end{vmatrix} &= -3(-1)^{1+2} \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 3(-1)^{2+2} \begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} + (-2)(-1)^{3+2} \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} \\ &= -3(-1) \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + 3(1) \begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} + (-2)(-1) \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} \\ &= 3(2 - 0) + 3(4 - 0) + 2(-2 - 2) \\ &= 3(2) + 3(4) + 2(-4) = 6 + 12 + (-8) = 10 \end{aligned}$$

Note that the value of the determinant is the same whether the first row or the second column is used to expand by cofactors. *Any row or column* can be used to evaluate a determinant by expanding by cofactors.

EXAMPLE • 1

Find the value of $\begin{vmatrix} 3 & -2 \\ 6 & -4 \end{vmatrix}$.

Solution

$$\begin{vmatrix} 3 & -2 \\ 6 & -4 \end{vmatrix} = 3(-4) - (-2)(6) = -12 + 12 = 0$$

The value of the determinant is 0.

YOU TRY IT • 1

Find the value of $\begin{vmatrix} -1 & -4 \\ 3 & -5 \end{vmatrix}$.

Your solution**EXAMPLE • 2**

Find the value of $\begin{vmatrix} -2 & 3 & 1 \\ 4 & -2 & 0 \\ 1 & -2 & 3 \end{vmatrix}$.

Solution

Expand by cofactors of the first row.

$$\begin{aligned} & \begin{vmatrix} -2 & 3 & 1 \\ 4 & -2 & 0 \\ 1 & -2 & 3 \end{vmatrix} \\ &= -2 \begin{vmatrix} -2 & 0 \\ -2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 4 & 0 \\ 1 & 3 \end{vmatrix} + 1 \begin{vmatrix} 4 & -2 \\ 1 & -2 \end{vmatrix} \\ &= -2(-6 - 0) - 3(12 - 0) + 1(-8 + 2) \\ &= -2(-6) - 3(12) + 1(-6) \\ &= 12 - 36 - 6 \\ &= -30 \end{aligned}$$

The value of the determinant is -30.

YOU TRY IT • 2

Find the value of $\begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix}$.

Your solution**EXAMPLE • 3**

Find the value of $\begin{vmatrix} 0 & -2 & 1 \\ 1 & 4 & 1 \\ 2 & -3 & 4 \end{vmatrix}$.

Solution

$$\begin{aligned} & \begin{vmatrix} 0 & -2 & 1 \\ 1 & 4 & 1 \\ 2 & -3 & 4 \end{vmatrix} \\ &= 0 \begin{vmatrix} 4 & 1 \\ -3 & 4 \end{vmatrix} - (-2) \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} + 1 \begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix} \\ &= 0 - (-2)(4 - 2) + 1(-3 - 8) \\ &= 2(2) + 1(-11) \\ &= 4 - 11 \\ &= -7 \end{aligned}$$

The value of the determinant is -7.

YOU TRY IT • 3

Find the value of $\begin{vmatrix} 3 & -2 & 0 \\ 1 & 4 & 2 \\ -2 & 1 & 3 \end{vmatrix}$.

Your solution

OBJECTIVE B

To solve a system of equations by using Cramer's Rule

The connection between determinants and systems of equations can be understood by solving a general system of linear equations.

Solve: (1) $a_1x + b_1y = c_1$
 (2) $a_2x + b_2y = c_2$

Eliminate y . Multiply Equation (1) by b_2 and Equation (2) by $-b_1$.

$$\begin{aligned} a_1b_2x + b_1b_2y &= c_1b_2 && \bullet \text{ } b_2 \text{ times Equation (1).} \\ -a_2b_1x - b_1b_2y &= -c_2b_1 && \bullet \text{ } -b_1 \text{ times Equation (2).} \end{aligned}$$

Add the equations.

$$\begin{aligned} a_1b_2x - a_2b_1x &= c_1b_2 - c_2b_1 \\ (a_1b_2 - a_2b_1)x &= c_1b_2 - c_2b_1 && \bullet \text{ Solve for } x, \text{ assuming } a_1b_2 - a_2b_1 \neq 0. \\ x &= \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \end{aligned}$$

The denominator $a_1b_2 - a_2b_1$ is the determinant of the coefficients of x and y . This is called the **coefficient determinant**.

$$a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

coefficients of x $\xrightarrow{\quad}$
 coefficients of y $\xrightarrow{\quad}$

The numerator $c_1b_2 - c_2b_1$ is the determinant obtained by replacing the first column in the coefficient determinant by the constants c_1 and c_2 . This is called a **numerator determinant**.

$$c_1b_2 - c_2b_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

constants of $\xrightarrow{\quad}$
 the equations

By following a similar procedure and eliminating x , it is also possible to express the y -component of the solution in determinant form. These results are summarized in Cramer's Rule.

 Point of Interest

Cramer's Rule is named after Gabriel Cramer, who used it in a book he published in 1750. However, this rule was also published in 1683 by the Japanese mathematician Seki Kowa. That publication occurred seven years before Cramer's birth.

Cramer's Rule

The solution of the system of equations $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ is given by $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$,

where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$, and $D \neq 0$.

HOW TO 5

Solve by using Cramer's Rule:

$$\begin{aligned} 3x - 2y &= 1 \\ 2x + 5y &= 3 \end{aligned}$$

$$D = \begin{vmatrix} 3 & -2 \\ 2 & 5 \end{vmatrix} = 19$$

• Find the value of the coefficient determinant.

$$D_x = \begin{vmatrix} 1 & -2 \\ 3 & 5 \end{vmatrix} = 11, D_y = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$

• Find the value of each of the numerator determinants.

$$x = \frac{D_x}{D} = \frac{11}{19}, y = \frac{D_y}{D} = \frac{7}{19}$$

• Use Cramer's Rule to write the solution.

The solution is $\left(\frac{11}{19}, \frac{7}{19}\right)$.

A procedure similar to that followed for two equations in two variables can be used to extend Cramer's Rule to three equations in three variables.

Cramer's Rule for a System of Three Equations in Three Variables

The solution of the system of equations

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$, where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \quad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}, \text{ and } D \neq 0.$$

HOW TO • 6

Solve by using Cramer's Rule:

$$\begin{aligned} 2x - y + z &= 1 \\ x + 3y - 2z &= -2 \\ 3x + y + 3z &= 4 \end{aligned}$$

Find the value of the coefficient determinant.

$$\begin{aligned} D &= \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 1 & 3 \end{vmatrix} = 2 \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} \\ &= 2(11) + 1(9) + 1(-8) \\ &= 23 \end{aligned}$$

Find the value of each of the numerator determinants.

$$\begin{aligned} D_x &= \begin{vmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 4 & 1 & 3 \end{vmatrix} = 1 \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} -2 & -2 \\ 4 & 3 \end{vmatrix} + 1 \begin{vmatrix} -2 & 3 \\ 4 & 1 \end{vmatrix} \\ &= 1(11) + 1(2) + 1(-14) \\ &= -1 \end{aligned}$$

$$\begin{aligned} D_y &= \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -2 \\ 3 & 4 & 3 \end{vmatrix} = 2 \begin{vmatrix} -2 & -2 \\ 4 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix} + 1 \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} \\ &= 2(2) - 1(9) + 1(10) \\ &= 5 \end{aligned}$$

$$\begin{aligned} D_z &= \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 1 & 4 \end{vmatrix} = 2 \begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} - (-1) \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} \\ &= 2(14) + 1(10) + 1(-8) \\ &= 30 \end{aligned}$$

Use Cramer's Rule to write the solution.

$$x = \frac{D_x}{D} = \frac{-1}{23}, \quad y = \frac{D_y}{D} = \frac{5}{23}, \quad z = \frac{D_z}{D} = \frac{30}{23}$$

The solution is $\left(-\frac{1}{23}, \frac{5}{23}, \frac{30}{23}\right)$.

EXAMPLE • 4

Solve by using Cramer's Rule.

$$6x - 9y = 5$$

$$4x - 6y = 4$$

Solution

$$D = \begin{vmatrix} 6 & -9 \\ 4 & -6 \end{vmatrix} = 0$$

Because $D = 0$, $\frac{D_x}{D}$ is undefined. Therefore, the system is dependent or inconsistent.

YOU TRY IT • 4

Solve by using Cramer's Rule.

$$3x - y = 4$$

$$6x - 2y = 5$$

Your solution**EXAMPLE • 5**

Solve by using Cramer's Rule.

$$3x - y + z = 5$$

$$x + 2y - 2z = -3$$

$$2x + 3y + z = 4$$

Solution

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 28,$$

$$D_x = \begin{vmatrix} 5 & -1 & 1 \\ -3 & 2 & -2 \\ 4 & 3 & 1 \end{vmatrix} = 28,$$

$$D_y = \begin{vmatrix} 3 & 5 & 1 \\ 1 & -3 & -2 \\ 2 & 4 & 1 \end{vmatrix} = 0,$$

$$D_z = \begin{vmatrix} 3 & -1 & 5 \\ 1 & 2 & -3 \\ 2 & 3 & 4 \end{vmatrix} = 56$$

$$x = \frac{D_x}{D} = \frac{28}{28} = 1$$

$$y = \frac{D_y}{D} = \frac{0}{28} = 0$$

$$z = \frac{D_z}{D} = \frac{56}{28} = 2$$

The solution is (1, 0, 2).

YOU TRY IT • 5

Solve by using Cramer's Rule.

$$2x - y + z = -1$$

$$3x + 2y - z = 3$$

$$x + 3y + z = -2$$

Your solution

4.3 EXERCISES

OBJECTIVE A

To evaluate a determinant



1. How do you find the value of the determinant associated with a 2×2 matrix?



2. What is the cofactor of a given element in a matrix?

For Exercises 3 to 14, evaluate the determinant.

3. $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$

4. $\begin{vmatrix} 5 & 1 \\ -1 & 2 \end{vmatrix}$

5. $\begin{vmatrix} 6 & -2 \\ -3 & 4 \end{vmatrix}$

6. $\begin{vmatrix} -3 & 5 \\ 1 & 7 \end{vmatrix}$

7. $\begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$

8. $\begin{vmatrix} 5 & -10 \\ 1 & -2 \end{vmatrix}$

9. $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 1 & 0 & 4 \end{vmatrix}$

10. $\begin{vmatrix} 4 & 1 & 3 \\ 2 & -2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

11. $\begin{vmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 3 & 2 & -2 \end{vmatrix}$

12. $\begin{vmatrix} 4 & 5 & -2 \\ 3 & -1 & 5 \\ 2 & 1 & 4 \end{vmatrix}$

13. $\begin{vmatrix} 4 & 2 & 6 \\ -2 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$

14. $\begin{vmatrix} 3 & 6 & -3 \\ 4 & -1 & 6 \\ -1 & -2 & 3 \end{vmatrix}$



15. What is the value of a determinant for which one row is all zeros?



16. What is the value of a determinant for which all the elements are the same number?

OBJECTIVE B

To solve a system of equations by using Cramer's Rule

For Exercises 17 to 34, solve by using Cramer's Rule.

17. $\begin{cases} 2x - 5y = 26 \\ 5x + 3y = 3 \end{cases}$

18. $\begin{cases} 3x + 7y = 15 \\ 2x + 5y = 11 \end{cases}$

19. $\begin{cases} x - 4y = 8 \\ 3x + 7y = 5 \end{cases}$

20. $\begin{cases} 5x + 2y = -5 \\ 3x + 4y = 11 \end{cases}$

21. $\begin{cases} 2x + 3y = 4 \\ 6x - 12y = -5 \end{cases}$

22. $\begin{cases} 5x + 4y = 3 \\ 15x - 8y = -21 \end{cases}$

23. $\begin{cases} 2x + 5y = 6 \\ 6x - 2y = 1 \end{cases}$

24. $\begin{cases} 7x + 3y = 4 \\ 5x - 4y = 9 \end{cases}$

25. $-2x + 3y = 7$
 $4x - 6y = 9$

26. $9x + 6y = 7$
 $3x + 2y = 4$

27. $2x - 5y = -2$
 $3x - 7y = -3$

28. $8x + 7y = -3$
 $2x + 2y = 5$

29. $2x - y + 3z = 9$
 $x + 4y + 4z = 5$
 $3x + 2y + 2z = 5$

30. $3x - 2y + z = 2$
 $2x + 3y + 2z = -6$
 $3x - y + z = 0$

31. $3x - y + z = 11$
 $x + 4y - 2z = -12$
 $2x + 2y - z = -3$

32. $x + 2y + 3z = 8$
 $2x - 3y + z = 5$
 $3x - 4y + 2z = 9$

33. $4x - 2y + 6z = 1$
 $3x + 4y + 2z = 1$
 $2x - y + 3z = 2$

34. $x - 3y + 2z = 1$
 $2x + y - 2z = 3$
 $3x - 9y + 6z = -3$



35. Can Cramer's Rule be used to solve a dependent system of equations?



36. Suppose a system of linear equations in two variables has $D_x \neq 0$, $D_y \neq 0$, and $D = 0$. Is the system of equations independent, dependent, or inconsistent?

Applying the Concepts

37. Determine whether the following statements are always true, sometimes true, or never true.
- The determinant of a matrix is a positive number.
 - A determinant can be evaluated by expanding about any row or column of the matrix.
 - Cramer's Rule can be used to solve a system of linear equations in three variables.

38. Show that $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = -\begin{vmatrix} c & d \\ a & b \end{vmatrix}$.

39. **Surveying** Surveyors use a formula to find the area of a plot of land. The *surveyor's area formula* states that if the vertices $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ of a simple polygon are listed counterclockwise around the perimeter, then the area of the polygon is

$$A = \frac{1}{2} \left\{ \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix} + \dots + \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix} \right\}$$

Use the surveyor's area formula to find the area of the polygon with vertices $(9, -3)$, $(26, 6)$, $(18, 21)$, $(16, 10)$, and $(1, 11)$. Measurements are given in feet.



SECTION

4.4 Application Problems

OBJECTIVE A

To solve rate-of-wind or rate-of-current problems



Solving motion problems that involve an object moving with or against a wind or current normally requires two variables.

HOW TO • 1

A motorboat traveling with the current can go 24 mi in 2 h. Against the current, it takes 3 h to go the same distance. Find the rate of the motorboat in calm water and the rate of the current.

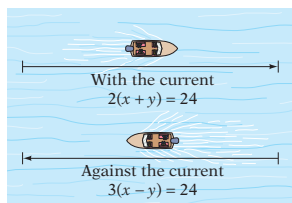
Strategy for Solving Rate-of-Wind or Rate-of-Current Problems

1. Choose one variable to represent the rate of the object in calm conditions and a second variable to represent the rate of the wind or current. Using these variables, express the rate of the object with and against the wind or current. Use the equation $rt = d$ to write expressions for the distance traveled by the object. The results can be recorded in a table.

Rate of the boat in calm water: x

Rate of the current: y

	Rate	•	Time	=	Distance
With the current	$x + y$	•	2	=	$2(x + y)$
Against the current	$x - y$	•	3	=	$3(x - y)$



2. Determine how the expressions for distance are related.

The distance traveled with the current is 24 mi: $2(x + y) = 24$

The distance traveled against the current is 24 mi: $3(x - y) = 24$

Solve the system of equations.

$$2(x + y) = 24 \xrightarrow{\text{Multiply by } \frac{1}{2}} \frac{1}{2} \cdot 2(x + y) = \frac{1}{2} \cdot 24 \longrightarrow x + y = 12$$

$$3(x - y) = 24 \xrightarrow{\text{Multiply by } \frac{1}{3}} \frac{1}{3} \cdot 3(x - y) = \frac{1}{3} \cdot 24 \longrightarrow x - y = 8$$

$$\begin{array}{rcl} 2x & = & 20 \\ x & = & 10 \end{array} \quad \bullet \text{ Add the equations.}$$

Replace x by 10 in the equation $x + y = 12$.
Solve for y .

$$\begin{array}{rcl} x + y & = & 12 \\ 10 + y & = & 12 \\ y & = & 2 \end{array}$$

The rate of the boat in calm water is 10 mph.
The rate of the current is 2 mph.

EXAMPLE 1

Flying with the wind, a plane flew 1000 mi in 5 h. Flying against the wind, the plane could fly only 500 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

Strategy

- Rate of the plane in still air: p
Rate of the wind: w

	<i>Rate</i>	<i>Time</i>	<i>Distance</i>
With wind	$p + w$	5	$5(p + w)$
Against wind	$p - w$	5	$5(p - w)$

- The distance traveled with the wind is 1000 mi.
The distance traveled against the wind is 500 mi.

$$5(p + w) = 1000$$

$$5(p - w) = 500$$

Solution

$$5(p + w) = 1000 \quad \frac{1}{5} \cdot 5(p + w) = \frac{1}{5} \cdot 1000$$

$$5(p - w) = 500 \quad \frac{1}{5} \cdot 5(p - w) = \frac{1}{5} \cdot 500$$

$$\begin{array}{r} p + w = 200 \\ p - w = 100 \\ \hline 2p = 300 \\ p = 150 \end{array}$$

$$p + w = 200$$

$$150 + w = 200$$

$$w = 50$$

- **Substitute 150 for p .**

The rate of the plane in calm air is 150 mph. The rate of the wind is 50 mph.

YOU TRY IT 1

A rowing team rowing with the current traveled 18 mi in 2 h. Against the current, the team rowed 10 mi in 2 h. Find the rate of the rowing team in calm water and the rate of the current.

Your strategy**Your solution**

Solution on p. S13

OBJECTIVE B**To solve application problems**

The application problems in this section are varieties of problems solved earlier in the text. Each of the strategies for the problems in this section will result in a system of equations.

HOW TO • 2

A store owner purchased twenty 60-watt light bulbs and 30 fluorescent bulbs for a total cost of \$80. A second purchase, at the same prices, included thirty 60-watt light bulbs and 10 fluorescent bulbs for a total cost of \$50. Find the cost of a 60-watt bulb and that of a fluorescent bulb.

Strategy for Solving Application Problems

1. Choose a variable to represent each of the unknown quantities. Write numerical or variable expressions for all the remaining quantities. These results may be recorded in tables, one for each condition.

Cost of 60-watt bulb: b

Cost of fluorescent bulb: f

First Purchase

	Amount	•	Unit Cost	=	Value
60-watt	20	•	b	=	$20b$
Fluorescent	30	•	f	=	$30f$

Second Purchase

	Amount	•	Unit Cost	=	Value
60-watt	30	•	b	=	$30b$
Fluorescent	10	•	f	=	$10f$

2. Determine a system of equations. The strategies presented in the chapter on First-Degree Equations and Inequalities can be used to determine the relationships among the expressions in the tables. Each table will give one equation of the system of equations.

The total of the first purchase was \$80: $20b + 30f = 80$

The total of the second purchase was \$50: $30b + 10f = 50$

Solve the system of equations: (1) $20b + 30f = 80$
 (2) $30b + 10f = 50$

$$\begin{array}{rcl}
 60b + 90f & = & 240 \\
 -60b - 20f & = & -100 \\
 \hline
 70f & = & 140 \\
 f & = & 2
 \end{array}
 \begin{array}{l}
 \bullet \text{ 3 times Equation (1).} \\
 \bullet \text{ -2 times Equation (2).}
 \end{array}$$

Replace f by 2 in Equation (1) and solve for b .

$$\begin{array}{rcl}
 20b + 30f & = & 80 \\
 20b + 30(2) & = & 80 \\
 20b + 60 & = & 80 \\
 20b & = & 20 \\
 b & = & 1
 \end{array}$$

The cost of a 60-watt bulb was \$1.00.

The cost of a fluorescent bulb was \$2.00.

Some application problems may require more than two variables, as shown in Example 2 and You Try It 2 on the next page.

EXAMPLE • 2

An investor has a total of \$20,000 deposited in three different accounts, which earn annual interest rates of 9%, 7%, and 5%. The amount deposited in the 9% account is twice the amount in the 7% account. If the total annual interest earned for the three accounts is \$1300, how much is invested in each account?

Strategy

- Amount invested at 9%: x
Amount invested at 7%: y
Amount invested at 5%: z

	<i>Principal</i>	<i>Rate</i>	<i>Interest</i>
Amount at 9%	x	0.09	$0.09x$
Amount at 7%	y	0.07	$0.07y$
Amount at 5%	z	0.05	$0.05z$

- The amount invested at 9% (x) is twice the amount invested at 7% (y): $x = 2y$
The sum of the interest earned for all three accounts is \$1300:
 $0.09x + 0.07y + 0.05z = 1300$
The total amount invested is \$20,000:
 $x + y + z = 20,000$

Solution

- (1) $x = 2y$
 - (2) $0.09x + 0.07y + 0.05z = 1300$
 - (3) $x + y + z = 20,000$
- Solve the system of equations. Substitute $2y$ for x in Equation (2) and Equation (3).
- $$0.09(2y) + 0.07y + 0.05z = 1300$$
- $$2y + y + z = 20,000$$
- (4) $0.25y + 0.05z = 1300$ • $0.09(2y) + 0.07y = 0.25y$
 - (5) $3y + z = 20,000$ • $2y + y = 3y$

Solve the system of equations in two variables by multiplying Equation (5) by -0.05 and adding to Equation (4).

$$\begin{array}{r}
 0.25y + 0.05z = 1300 \\
 -0.15y - 0.05z = -1000 \\
 \hline
 0.10y = 300 \\
 y = 3000
 \end{array}$$

Substituting the value of y into Equation (1),
 $x = 6000$.

Substituting the values of x and y into Equation (3),
 $z = 11,000$.

The investor placed \$6000 in the 9% account, \$3000 in the 7% account, and \$11,000 in the 5% account.

YOU TRY IT • 2

A coin bank contains only nickels, dimes, and quarters. The value of the 19 coins in the bank is \$2. If there are twice as many nickels as dimes, find the number of each type of coin in the bank.

Your strategy**Your solution**

4.4 EXERCISES

OBJECTIVE A

To solve rate-of-wind or rate-of-current problems



1. Traveling with the wind, a plane flies m miles in h hours. Traveling against the wind, the plane flies n miles in h hours. Is n less than, equal to, or greater than m ?



2. Traveling against the current, it takes a boat h hours to go m miles. Traveling with the current, the boat takes k hours to go m miles. Is h less than, equal to, or greater than k ?

3. A motorboat traveling with the current went 36 mi in 2 h. Against the current, it took 3 h to travel the same distance. Find the rate of the boat in calm water and the rate of the current.

4. A cabin cruiser traveling with the current went 45 mi in 3 h. Against the current, it took 5 h to travel the same distance. Find the rate of the cabin cruiser in calm water and the rate of the current.

5. A jet plane flying with the wind went 2200 mi in 4 h. Against the wind, the plane could fly only 1820 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

6. Flying with the wind, a small plane flew 300 mi in 2 h. Against the wind, the plane could fly only 270 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

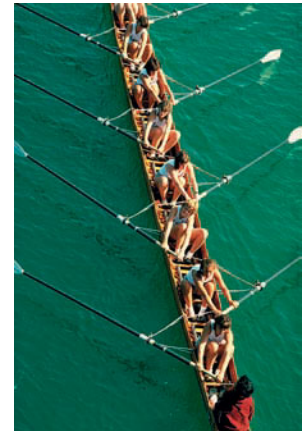
7. A rowing team rowing with the current traveled 20 km in 2 h. Rowing against the current, the team rowed 12 km in the same amount of time. Find the rate of the team in calm water and the rate of the current.

8. A motorboat traveling with the current went 72 km in 3 h. Against the current, the boat could go only 48 km in the same amount of time. Find the rate of the boat in calm water and the rate of the current.

9. A turboprop plane flying with the wind flew 800 mi in 4 h. Flying against the wind, the plane required 5 h to travel the same distance. Find the rate of the wind and the rate of the plane in calm air.

10. Flying with the wind, a pilot flew 600 mi between two cities in 4 h. The return trip against the wind took 5 h. Find the rate of the plane in calm air and the rate of the wind.

11. A plane flying with a tailwind flew 600 mi in 5 h. Against the wind, the plane required 6 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.



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12. Flying with the wind, a plane flew 720 mi in 3 h. Against the wind, the plane required 4 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

OBJECTIVE B To solve application problems



13. A coffee merchant's house blend contains 3 lb of dark roast coffee and 1 lb of light roast coffee. The merchant's breakfast blend contains 1 lb of the dark roast coffee and 3 lb of the light roast coffee. If the cost per pound of the house blend is greater than the cost per pound of the breakfast blend, is the cost per pound of the dark roast coffee less than, equal to, or greater than the cost per pound of the light roast coffee?



14. The total value of dimes and quarters in a bank is V dollars. If the dimes were quarters and the quarters were dimes, the total value would be more than V dollars. Is the number of quarters in the bank less than, equal to, or greater than the number of dimes in the bank?

15. **Coins** A coin bank contains only nickels and dimes. The total value of the coins in the bank is \$2.50. If the nickels were dimes and the dimes were nickels, the total value of the coins would be \$3.50. Find the number of nickels in the bank.

16. **Business** A merchant mixed 10 lb of cinnamon tea with 5 lb of spice tea. The 15-pound mixture cost \$40. A second mixture included 12 lb of the cinnamon tea and 8 lb of the spice tea. The 20-pound mixture cost \$54. Find the cost per pound of the cinnamon tea and of the spice tea.

17. **Purchasing** A carpenter purchased 60 ft of redwood and 80 ft of pine for a total cost of \$286. A second purchase, at the same prices, included 100 ft of redwood and 60 ft of pine for a total cost of \$396. Find the cost per foot of redwood and of pine.

18. **Coins** The total value of the quarters and dimes in a coin bank is \$5.75. If the quarters were dimes and the dimes were quarters, the total value of the coins would be \$6.50. Find the number of quarters in the bank.

19. **Purchasing** A contractor buys 16 yd of nylon carpet and 20 yd of wool carpet for \$1840. A second purchase, at the same prices, includes 18 yd of nylon carpet and 25 yd of wool carpet for \$2200. Find the cost per yard of the wool carpet.

20. **Finances** During one month, a homeowner used 500 units of electricity and 100 units of gas for a total cost of \$352. The next month, 400 units of electricity and 150 units of gas were used for a total cost of \$304. Find the cost per unit of gas.

21. **Manufacturing** A company manufactures both mountain bikes and trail bikes. The cost of materials for a mountain bike is \$70, and the cost of materials for a trail bike is \$50. The cost of labor to manufacture a mountain bike is \$80, and the cost of labor to manufacture a trail bike is \$40. During a week in which the company has budgeted \$2500 for materials and \$2600 for labor, how many mountain bikes does the company plan to manufacture?



- 22. Manufacturing** A company manufactures both LCD and plasma televisions. The cost for materials for an LCD television is \$125, and the cost of materials for a plasma TV is \$150. The cost of labor to manufacture one LCD television is \$80, and the cost of labor for one plasma television is \$85. How many of each television can a manufacturer produce during a week in which \$18,000 has been budgeted for materials and \$10,750 has been budgeted for labor?



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Fuel Economy Use the information in the article at the right for Exercises 23 and 24.

- 23.** One week, the owner of a hybrid car drove 394 mi and spent \$34.74 on gasoline. How many miles did the owner drive in the city? On the highway?
- 24.** Gasoline for one week of driving cost the owner of a hybrid car \$26.50. The owner would have spent \$51.50 for gasoline to drive the same number of miles in a traditional car. How many miles did the owner drive in the city? On the highway?
- 25. Chemistry** A chemist has two alloys, one of which is 10% gold and 15% lead, and the other of which is 30% gold and 40% lead. How many grams of each of the two alloys should be used to make an alloy that contains 60 g of gold and 88 g of lead?
- 26. Health Science** A pharmacist has two vitamin-supplement powders. The first powder is 20% vitamin B1 and 10% vitamin B2. The second is 15% vitamin B1 and 20% vitamin B2. How many milligrams of each powder should the pharmacist use to make a mixture that contains 130 mg of vitamin B1 and 80 mg of vitamin B2?
- 27. Business** On Monday, a computer manufacturing company sent out three shipments. The first order, which contained a bill for \$114,000, was for 4 Model II, 6 Model VI, and 10 Model IX computers. The second shipment, which contained a bill for \$72,000, was for 8 Model II, 3 Model VI, and 5 Model IX computers. The third shipment, which contained a bill for \$81,000, was for 2 Model II, 9 Model VI, and 5 Model IX computers. What does the manufacturer charge for each Model VI computer?
- 28. Purchasing** A relief organization supplies blankets, cots, and lanterns to victims of fires, floods, and other natural disasters. One week the organization purchased 15 blankets, 5 cots, and 10 lanterns for a total cost of \$1250. The next week, at the same prices, the organization purchased 20 blankets, 10 cots, and 15 lanterns for a total cost of \$2000. The next week, at the same prices, the organization purchased 10 blankets, 15 cots, and 5 lanterns for a total cost of \$1625. Find the cost of one blanket, the cost of one cot, and the cost of one lantern.

In the News

Hybrids Easier on the Pocketbook?

A hybrid car can make up for its high sticker price with savings at the pump. At current gas prices, here's a look at the cost per mile for one company's hybrid and traditional cars.

Gasoline Cost per Mile

Car Type	City (\$/mi)	Highway (\$/mi)
Hybrid	0.09	0.08
Traditional	0.18	0.13

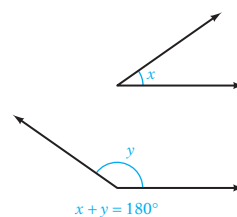
Source: www.fueleconomy.gov

29. **Investments** An investor has a total of \$25,000 deposited in three different accounts, which earn annual interest rates of 8%, 6%, and 4%. The amount deposited in the 8% account is twice the amount in the 6% account. If the three accounts earn total annual interest of \$1520, how much money is deposited in each account?

Applying the Concepts

30. **Geometry** Two angles are complementary. The measure of the larger angle is 9° more than eight times the measure of the smaller angle. Find the measures of the two angles. (Complementary angles are two angles whose sum is 90° .)

31. **Geometry** Two angles are supplementary. The measure of the larger angle is 40° more than three times the measure of the smaller angle. Find the measures of the two angles. (Supplementary angles are two angles whose sum is 180° .)



32. **Coins** The sum of the ages of a gold coin and a silver coin is 75 years. The age of the gold coin 10 years from now will be 5 years less than the age of the silver coin 10 years ago. Find the present ages of the two coins.
33. **Art** The difference between the ages of an oil painting and a watercolor is 35 years. The age of the oil painting 5 years from now will be twice the age of the watercolor 5 years ago. Find the present ages of each.
34. **Coins** A coin bank contains only dimes and quarters. The total value of all the coins is \$1. How many of each type of coin are in the bank? (*Hint:* There is more than one answer to this problem.)

SECTION

4.5

Solving Systems of Linear Inequalities

OBJECTIVE A

To graph the solution set of a system of linear inequalities



Point of Interest

Large systems of linear inequalities containing over 100 inequalities have been used to solve application problems in such diverse areas as providing health care and hardening a nuclear missile silo.



Take Note

You can use a test point to check that the correct region has been denoted as the solution set. We can see from the graph that the point $(2, 4)$ is in the solution set and, as shown below, it is a solution of each inequality in the system. This indicates that the solution set as graphed is correct.

$$\begin{aligned} 2x - y &\leq 3 \\ 2(2) - (4) &\leq 3 \\ 0 &\leq 3 \quad \text{True} \end{aligned}$$

$$\begin{aligned} 3x + 2y &> 8 \\ 3(2) + 2(4) &> 8 \\ 14 &> 8 \quad \text{True} \end{aligned}$$

Two or more inequalities considered together are called a **system of inequalities**. The **solution set of a system of inequalities** is the intersection of the solution sets of the individual inequalities. To graph the solution set of a system of inequalities, first graph the solution set of each inequality. The solution set of the system of inequalities is the region of the plane represented by the intersection of the shaded areas.

HOW TO • 1

Graph the solution set:

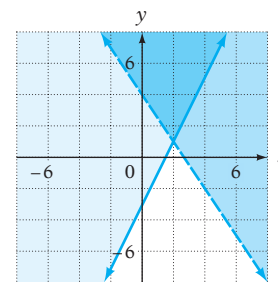
$$\begin{aligned} 2x - y &\leq 3 \\ 3x + 2y &> 8 \end{aligned}$$

Solve each inequality for y .

$$\begin{aligned} 2x - y &\leq 3 & 3x + 2y &> 8 \\ -y &\leq -2x + 3 & 2y &> -3x + 8 \\ y &\geq 2x - 3 & y &> -\frac{3}{2}x + 4 \end{aligned}$$

Graph $y = 2x - 3$ as a solid line. Because the inequality is $\geq$, shade above the line.

Graph $y = -\frac{3}{2}x + 4$ as a dashed line. Because the inequality is $>$, shade above the line.



The solution set of the system is the region of the plane represented by the intersection of the solution sets of the individual inequalities.

HOW TO • 2

Graph the solution set:

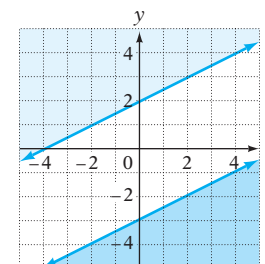
$$\begin{aligned} -x + 2y &\geq 4 \\ x - 2y &\geq 6 \end{aligned}$$

Solve each inequality for y .

$$\begin{aligned} -x + 2y &\geq 4 & x - 2y &\geq 6 \\ 2y &\geq x + 4 & -2y &\geq -x + 6 \\ y &\geq \frac{1}{2}x + 2 & y &\leq \frac{1}{2}x - 3 \end{aligned}$$

Shade above the solid line $y = \frac{1}{2}x + 2$.

Shade below the solid line $y = \frac{1}{2}x - 3$.



Because the solution sets of the two inequalities do not intersect, the solution of the system is the empty set.



Integrating Technology

See the Keystroke Guide: *Graphing Inequalities* for instructions on using a graphing calculator to graph the solution set of a system of inequalities.

EXAMPLE • 1

Graph the solution set:

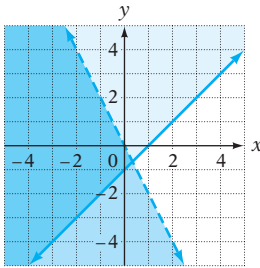
$$y \geq x - 1$$

$$y < -2x$$
Solution

Shade above the solid line $y = x - 1$.

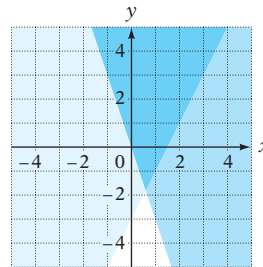
Shade below the dashed line $y = -2x$.

The solution of the system is the intersection of the solution sets of the individual inequalities.

**YOU TRY IT • 1**

Graph the solution set:

$$y \geq 2x - 3$$

$$y > -3x$$
Your solution**EXAMPLE • 2**

Graph the solution set:

$$2x + 3y > 9$$

$$y < -\frac{2}{3}x + 1$$
Solution

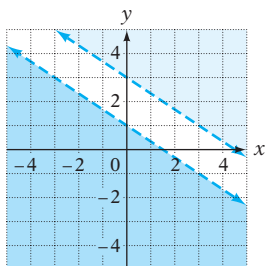
$$2x + 3y > 9$$

$$3y > -2x + 9$$

$$y > -\frac{2}{3}x + 3$$

Graph above the dashed line $y = -\frac{2}{3}x + 3$.

Graph below the dashed line $y = -\frac{2}{3}x + 1$.

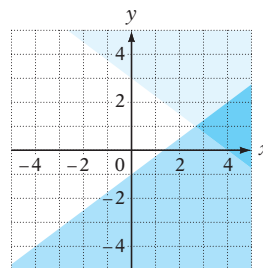


The intersection of the system is the empty set because the solution sets of the two inequalities do not intersect.

YOU TRY IT • 2

Graph the solution set:

$$3x + 4y > 12$$

$$y < \frac{3}{4}x - 1$$
Your solution

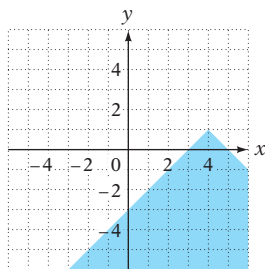
4.5 EXERCISES

OBJECTIVE A

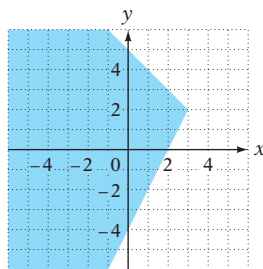
To graph the solution set of a system of linear inequalities

For Exercises 1 to 18, graph the solution set.

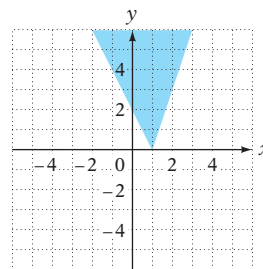
1. $x - y \geq 3$
 $x + y \leq 5$



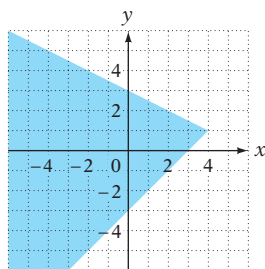
2. $2x - y < 4$
 $x + y < 5$



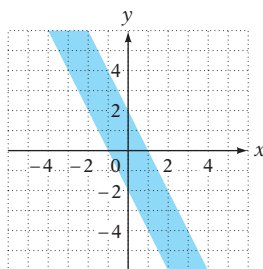
3. $3x - y < 3$
 $2x + y \geq 2$



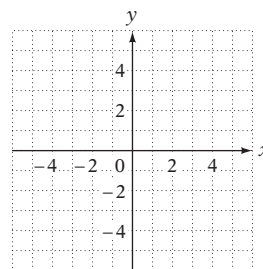
4. $x + 2y \leq 6$
 $x - y \leq 3$



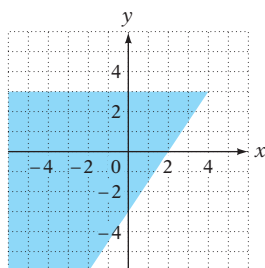
5. $2x + y \geq -2$
 $6x + 3y \leq 6$



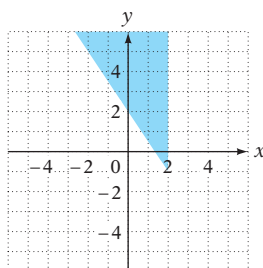
6. $x + y \geq 5$
 $3x + 3y \leq 6$



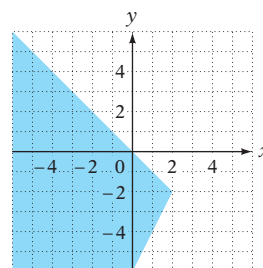
7. $3x - 2y < 6$
 $y \leq 3$



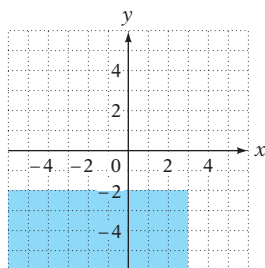
8. $x \leq 2$
 $3x + 2y > 4$



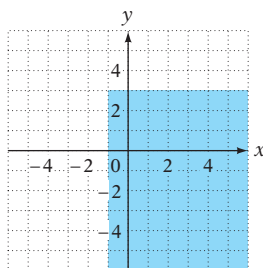
9. $y > 2x - 6$
 $x + y < 0$



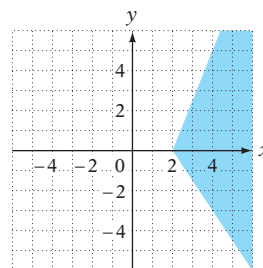
10. $x < 3$
 $y < -2$



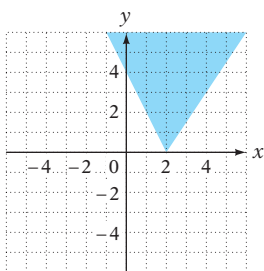
11. $x + 1 \geq 0$
 $y - 3 \leq 0$



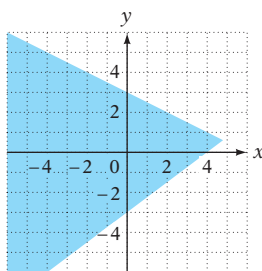
12. $5x - 2y \geq 10$
 $3x + 2y \geq 6$



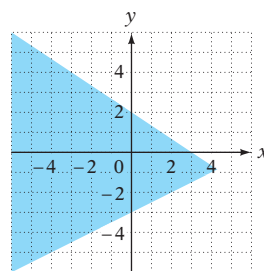
13. $2x + y \geq 4$
 $3x - 2y < 6$



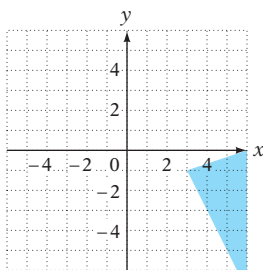
14. $3x - 4y < 12$
 $x + 2y < 6$



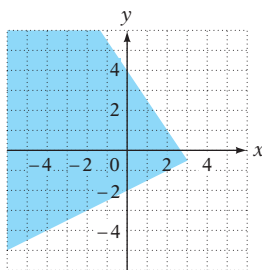
15. $x - 2y \leq 6$
 $2x + 3y \leq 6$



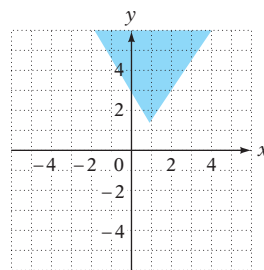
16. $x - 3y > 6$
 $2x + y > 5$



17. $x - 2y \leq 4$
 $3x + 2y \leq 8$



18. $3x - 2y < 0$
 $5x + 3y > 9$



For Exercises 19 to 22, assume that a and b are positive numbers such that $a > b$. Describe the solution set of each system of inequalities.

19. $x + y < a$
 $x + y < b$

20. $x + y > a$
 $x + y < b$

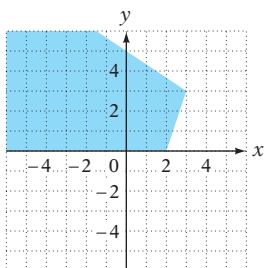
21. $x + y < a$
 $x + y > b$

22. $x + y > a$
 $x + y > b$

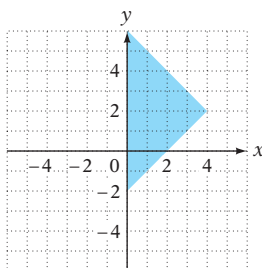
Applying the Concepts

For Exercises 23 to 25, graph the solution set.

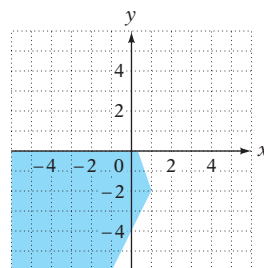
23. $2x + 3y \leq 15$
 $3x - y \leq 6$
 $y \geq 0$



24. $x + y \leq 6$
 $x - y \leq 2$
 $x \geq 0$



25. $2x - y \leq 4$
 $3x + y < 1$
 $y \leq 0$



FOCUS ON PROBLEM SOLVING

Solve an Easier Problem

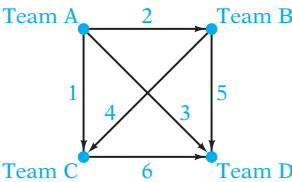
We begin this problem-solving feature with a restatement of the four steps of Polya’s recommended problem-solving model.

- 1. Understand the problem.
- 2. Devise a plan.
- 3. Carry out the plan.
- 4. Review the solution.



One of the several methods of devising a plan is to try to solve an easier problem. Suppose you are in charge of your softball league, which consists of 15 teams. You must devise a schedule in which each team plays every other team once. How many games must be scheduled?

To solve this problem, we will attempt an easier problem first. Suppose that your league contains only a small number of teams. For instance, if there were only 1 team, you would schedule 0 games. If there were 2 teams, you would schedule 1 game. If there were 3 teams, you would schedule 3 games. The diagram at the left shows that 6 games must be scheduled when there are 4 teams in the league.



Here is a table of our results so far. (Remember that making a table is another strategy to be used in problem solving.)

Number of Teams	Number of Games	Possible Pattern
1	0	0
2	1	1
3	3	1 + 2
4	6	1 + 2 + 3

- 1. Draw a diagram with 5 dots to represent the teams. Draw lines from each dot to a second dot, and determine the number of games required.
- 2. What is the apparent pattern for the number of games required?
- 3. Assuming the pattern continues, how many games must be scheduled for the 15 teams of the original problem?

After solving a problem, good problem solvers ask whether it is possible to solve the problem in a different manner. Here is a possible alternative method of solving the scheduling problem.

Begin with one of the 15 teams (say team A) and ask, “How many games must this team play?” Because there are 14 teams left to play, you must schedule 14 games. Now move to team B. It is already scheduled to play team A, and it does not play itself, so there are 13 teams left for it to play. Consequently, you must schedule 14 + 13 games.

- 4. Continue this reasoning for the remaining teams and determine the number of games that must be scheduled. Does this answer correspond to the answer you obtained using the first method?
- 5. Making connections to other problems you have solved is an important step toward becoming an excellent problem solver. How does the answer to the scheduling of teams relate to the triangular numbers discussed in the Focus on Problem Solving in the chapter titled Linear Functions and Inequalities in Two Variables?

PROJECTS AND GROUP ACTIVITIES

Using a Graphing Calculator to Solve a System of Equations



A graphing calculator can be used to solve a system of equations. For this procedure to work on most calculators, it is necessary that the point of intersection be on the screen. This means that you may have to experiment with Xmin, Xmax, Ymin, and Ymax values until the graphs intersect on the screen.

To solve a system of equations graphically, solve each equation for y . Then graph the equations of the system. Their point of intersection is the solution.

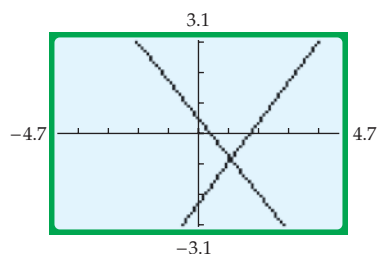
For instance, to solve the system of equations

$$\begin{aligned} 4x - 3y &= 7 \\ 5x + 4y &= 2 \end{aligned}$$

first solve each equation for y .

$$4x - 3y = 7 \Rightarrow y = \frac{4}{3}x - \frac{7}{3}$$

$$5x + 4y = 2 \Rightarrow y = -\frac{5}{4}x + \frac{1}{2}$$



The keystrokes needed to solve this system using a TI-84 are given below. We are using a viewing window of $[-4.7, 4.7]$ by $[-3.1, 3.1]$. The approximate solution is $(1.096774, -0.870968)$.

Y= CLEAR 4 X,T,θ,n ÷ 3 -
7 ÷ 3 ENTER CLEAR
(-) 5 X,T,θ,n ÷ 4 + 1 ÷
2 ZOOM 4

Once the calculator has drawn the graphs, use the TRACE feature and move the cursor to the approximate point of intersection. This will give you an approximate solution of the system of equations. A more accurate solution can be found by using the following keystrokes.

2ND CALC 5 ENTER ENTER ENTER

Some of the exercises in the first section of this chapter asked you to solve a system of equations by graphing. Try those exercises again, this time using your graphing calculator.

Here is an example of using a graphing calculator to solve an investment problem.

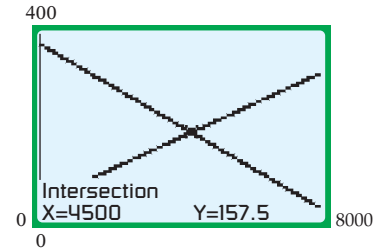
HOW TO 1

A marketing manager deposited \$8000 in two simple interest accounts, one with an interest rate of 3.5% and the other with an interest rate of 4.5%. How much is deposited in each account if both accounts earn the same interest?

Amount invested at 3.5%: x
 Amount invested at 4.5%: $8000 - x$

Interest earned on the 3.5% account: $0.035x$
 Interest earned on the 4.5% account: $0.045(8000 - x)$

Enter $0.035x$ into Y1. Enter $0.045(8000 - x)$ into Y2. Graph the equations. (We used a window of $X_{\min} = 0$, $X_{\max} = 8000$, $Y_{\min} = 0$, $Y_{\max} = 400$.) Use the intersect feature to find the point of intersection.



At the point of intersection,
 $x = 4500$. This is the amount in
 the 3.5% account.
 The amount in the 4.5% account is
 $8000 - x = 8000 - 4500 = 3500$.

$\$4500$ is invested at 3.5%. $\$3500$ is invested at 4.5%.

As shown on the graphing calculator screen in the example above, at the point of intersection, $y = 157.5$. This is the interest earned, \$157.50, on \$4500 invested at 3.5%. It is also the interest earned on \$3500 invested at 4.5%. In other words, it is the interest earned on each account when the interest earned on each account is the same.

For Exercises 1 to 4, solve by using a graphing calculator.



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1. **Finances** Suppose that a breadmaker costs \$190, and that the ingredients and electricity to make one loaf of bread cost \$.95. If a comparable loaf of bread at a grocery store costs \$1.98, how many loaves of bread must you make before the breadmaker pays for itself?
2. **Finances** Suppose a natural gas clothes washer costs \$590 and uses \$.50 of gas and water to wash a large load of clothes. If a laundromat charges \$3.50 to do a load of laundry, how many loads of clothes must you wash before the washing machine purchase becomes the more economical choice?
3. **Finances** Suppose a natural gas clothes dryer costs \$520 and uses \$.80 of gas to dry a load of clothes for 1 h. The laundromat charges \$3.50 to use a dryer for 1 h.
 - a. How many loads of clothes must you dry before the gas dryer purchase becomes the more economical choice?
 - b. What is the y-coordinate of the point of intersection? What does this represent in the context of the problem?
4. **Investments** When Mitch Deerfield changed jobs, he rolled over the \$7500 in his retirement account into two simple interest accounts. On one account, the annual simple interest rate is 6.25%; on the second account, the annual simple interest rate is 5.75%.
 - a. How much is invested in each account if the accounts earn the same amount of annual interest?
 - b. What is the y-coordinate of the point of intersection? What does this represent in the context of the problem?

CHAPTER 4

SUMMARY

KEY WORDS

A *system of equations* is two or more equations considered together. A *solution of a system of equations in two variables* is an ordered pair that is a solution of each equation of the system. [4.1A, p. 202]

When the graphs of a system of equations intersect at only one point, the system is called an *independent system of equations*. [4.1A, p. 202]

When the graphs of a system of equations do not intersect, the system has no solution and is called an *inconsistent system of equations*. [4.1A, p. 203]

When the graphs of a system of equations coincide, the system is called a *dependent system of equations*. [4.1A, p. 203]

An equation of the form $Ax + By + Cz = D$, where A , B , and C are the coefficients of the variables and D is a constant, is a *linear equation in three variables*. A *solution of an equation in three variables* is an *ordered triple* (x, y, z) . [4.2B, pp. 217, 219]

A *solution of a system of equations in three variables* is an ordered triple that is a solution of each equation of the system. [4.2B, p. 219]

A *matrix* is a rectangular array of numbers. Each number in the matrix is called an *element* of the matrix. A matrix of m rows and n columns is said to be of *order* $m \times n$. [4.3A, p. 226]

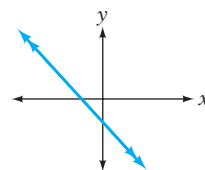
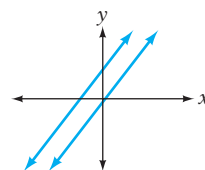
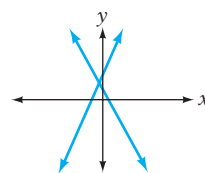
A *square matrix* has the same number of rows as columns. [4.3A, p. 226]

EXAMPLES

The solution of the system

$$\begin{aligned}x + y &= 2 \\x - y &= 4\end{aligned}$$

is the ordered pair $(3, -1)$. $(3, -1)$ is the only ordered pair that is a solution of both equations.



$3x + 2y - 5z = 12$ is a linear equation in three variables. One solution of this equation is the ordered triple $(0, 1, -2)$.

The solution of the system

$$\begin{aligned}3x + y - 3z &= 2 \\-x + 2y + 3z &= 6 \\2x + 2y - 2z &= 4\end{aligned}$$

is the ordered triple $(1, 2, 1)$. $(1, 2, 1)$ is the only ordered triple that is a solution of all three equations.

$\begin{bmatrix} 2 & 3 & 6 \\ -1 & 2 & 4 \end{bmatrix}$ is a 2×3 matrix. The elements in the first row are 2, 3, and 6.

$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ is a square matrix.

The *minor of an element* of a 3×3 determinant is the 2×2 determinant obtained by eliminating the row and column that contain the element. [4.3A, p. 226]

$$\begin{vmatrix} 2 & 1 & 3 \\ 4 & 6 & 2 \\ 1 & 8 & 5 \end{vmatrix} \quad \text{The minor of 4 is } \begin{vmatrix} 1 & 3 \\ 8 & 5 \end{vmatrix}.$$

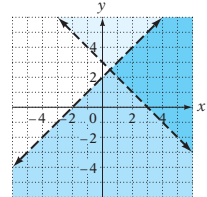
The *cofactor of an element* of a matrix is $(-1)^{i+j}$ times the minor of the element, where i is the row number of the element and j is its column number. [4.3A, p. 227]

In the determinant above, 4 is from the second row and the first column. The cofactor of 4 is

$$(-1)^{2+1} \begin{vmatrix} 1 & 3 \\ 8 & 5 \end{vmatrix} = - \begin{vmatrix} 1 & 3 \\ 8 & 5 \end{vmatrix}$$

Two or more inequalities considered together are called a *system of inequalities*. The *solution set of a system of inequalities* is the intersection of the solution sets of the individual inequalities. [4.5A, p. 242]

$$\begin{aligned} x + y &> 3 \\ x - y &> -2 \end{aligned}$$



ESSENTIAL RULES AND PROCEDURES

Solving Systems of Equations

A system of equations can be solved by:

a. *Graphing* [4.1A, p. 202]

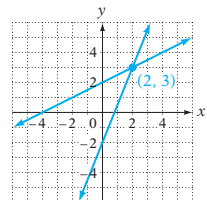
b. *The substitution method* [4.1B, p. 205]

c. *The addition method* [4.2A, p. 214]

EXAMPLES

$$y = \frac{1}{2}x + 2$$

$$y = \frac{5}{2}x - 2$$



$$(1) \quad 2x - 3y = 4$$

$$(2) \quad y = -x + 2$$

Substitute $-x + 2$ for y in equation (1).

$$2x - 3(-x + 2) = 4$$

$$-2x + 3y = 7$$

$$2x - 5y = 2$$

$$\hline -2y = 9$$

• Add the equations.

Annual Simple Interest Equation [4.1C, p. 208]

Principal $\cdot$ simple interest rate = simple interest

$$Pr = I$$

You have a total of \$10,000 to invest in two simple interest accounts, one earning 4.5% annual simple interest and the other earning 5% annual simple interest. If you earn \$485 per year in interest from these two investments, how much do you have invested in each account?

$$x + y = 10,000$$

$$0.045x + 0.05y = 485$$

Determinant of a 2×2 Matrix [4.3A, p. 226]

The *determinant* of a 2×2 matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is written

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

The value of this determinant is given by the formula

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

$$\begin{vmatrix} 2 & -3 \\ -4 & 5 \end{vmatrix} = 2(5) - (-3)(-4) \\ = 10 - 12 \\ = -2$$

Determinant of a 3×3 Matrix [4.3A, p. 227]

The determinant of a 3×3 matrix is found by *expanding by cofactors*.

Expand by cofactors of the first column.

$$\begin{vmatrix} 2 & 1 & 3 \\ 4 & 6 & 2 \\ 1 & 8 & 5 \end{vmatrix} = 2 \begin{vmatrix} 6 & 2 \\ 8 & 5 \end{vmatrix} - 4 \begin{vmatrix} 1 & 3 \\ 8 & 5 \end{vmatrix} \\ + 1 \begin{vmatrix} 1 & 3 \\ 6 & 2 \end{vmatrix}$$

Cramer's Rule [4.3B, pp. 229, 230]

For a system of two equations in two variables:

The solution of the system of equations

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}$$

is given by $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$, where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$,

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, \text{ and } D \neq 0.$$

$$\begin{aligned} 2x - y &= 6 \\ x + 3y &= 4 \end{aligned}$$

$$D = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7,$$

$$D_x = \begin{vmatrix} 6 & -1 \\ 4 & 3 \end{vmatrix} = 22,$$

$$D_y = \begin{vmatrix} 2 & 6 \\ 1 & 4 \end{vmatrix} = 2$$

$$x = \frac{D_x}{D} = \frac{22}{7}, y = \frac{D_y}{D} = \frac{2}{7}$$

For a system of three equations in three variables:

The solution of the system of equations

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$, where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$,

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}, \text{ and}$$

$D \neq 0$.

$$\begin{aligned} x - y + z &= 2 \\ 2x + y - 2z &= -2 \\ x - 2y + 3z &= 6 \end{aligned}$$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & -2 & 3 \end{vmatrix} = 2,$$

$$D_x = \begin{vmatrix} 2 & -1 & 1 \\ -2 & 1 & -2 \\ 6 & -2 & 3 \end{vmatrix} = 2,$$

$$D_y = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -2 & -2 \\ 1 & 6 & 3 \end{vmatrix} = 4,$$

$$D_z = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & -2 & 6 \end{vmatrix} = 6$$

$$x = \frac{D_x}{D} = \frac{2}{2} = 1, y = \frac{D_y}{D} = \frac{4}{2} = 2,$$

$$z = \frac{D_z}{D} = \frac{6}{2} = 3$$

CHAPTER 4**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. After graphing a system of linear equations, how is the solution determined?
2. What formula is used to solve a simple interest investment problem?
3. If a system of linear equations is dependent, how is the solution expressed?
4. How do you solve a system of two linear equations in two variables using the addition method?
5. What does it mean if, when solving a system of linear equations by the addition method, the result is $0 = 0$?
6. How do you check the solution to a system of three equations in three variables?
7. How do you evaluate the determinant of a 3×3 matrix?
8. When solving a system of equations using Cramer's Rule, what does it mean when the value of the coefficient matrix D is zero?
9. Using two variables, how do you represent the rate of a boat going against the current?
10. How is intersection used to solve a system of linear inequalities?
11. How do you find the minor of an element of a 3×3 determinant?
12. How do you determine the sign of the cofactor of an element of a matrix?

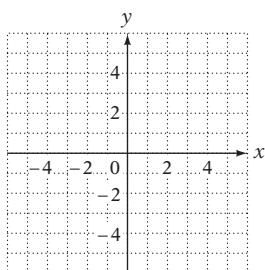
CHAPTER 4

REVIEW EXERCISES

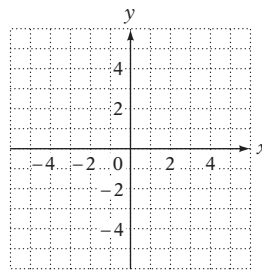
1. Solve by substitution: $2x - 6y = 15$
 $x = 4y + 8$

2. Solve by the addition method: $3x + 2y = 2$
 $x + y = 3$

3. Solve by graphing: $x + y = 3$
 $3x - 2y = -6$



4. Solve by graphing: $2x - y = 4$
 $y = 2x - 4$



5. Solve by substitution: $3x + 12y = 18$
 $x + 4y = 6$

6. Solve by the addition method: $5x - 15y = 30$
 $x - 3y = 6$

7. Solve by the addition method:
 $3x - 4y - 2z = 17$
 $4x - 3y + 5z = 5$
 $5x - 5y + 3z = 14$

8. Solve by the addition method:
 $3x + y = 13$
 $2y + 3z = 5$
 $x + 2z = 11$

9. Evaluate the determinant:

$$\begin{vmatrix} 6 & 1 \\ 2 & 5 \end{vmatrix}$$

10. Evaluate the determinant:

$$\begin{vmatrix} 1 & 5 & -2 \\ -2 & 1 & 4 \\ 4 & 3 & -8 \end{vmatrix}$$

11. Solve by using Cramer's Rule:

$$2x - y = 7$$

$$3x + 2y = 7$$

12. Solve by using Cramer's Rule:

$$3x - 4y = 10$$

$$2x + 5y = 15$$

13. Solve by using Cramer's Rule:

$$x + y + z = 0$$

$$x + 2y + 3z = 5$$

$$2x + y + 2z = 3$$

14. Solve by using Cramer's Rule:

$$x + 3y + z = 6$$

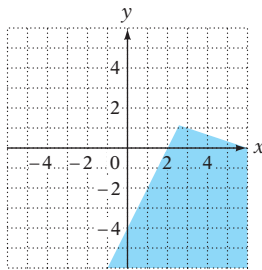
$$2x + y - z = 12$$

$$x + 2y - z = 13$$

15. Graph the solution set:

$$x + 3y \leq 6$$

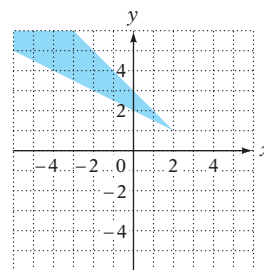
$$2x - y \geq 4$$



16. Graph the solution set:

$$2x + 4y \geq 8$$

$$x + y \leq 3$$

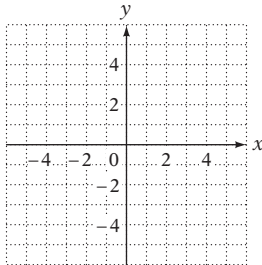


17. **Boating** A cabin cruiser traveling with the current went 60 mi in 3 h. Against the current, it took 5 h to travel the same distance. Find the rate of the cabin cruiser in calm water and the rate of the current.
18. **Aeronautics** A pilot flying with the wind flew 600 mi in 3 h. Flying against the wind, the pilot required 4 h to travel the same distance. Find the rate of the plane in calm air and the rate of the wind.
19. **Ticket Sales** At a movie theater, admission tickets are \$5 for children and \$8 for adults. The receipts for one Friday evening were \$2500. The next day there were three times as many children as the preceding evening and only half the number of adults as the night before, yet the receipts were still \$2500. Find the number of children who attended on Friday evening.
20. **Investments** A trust administrator divides \$20,000 between two accounts. One account earns an annual simple interest rate of 3%, and a second account earns an annual simple interest rate of 7%. The total annual income from the two accounts is \$1200. How much is invested in each account?

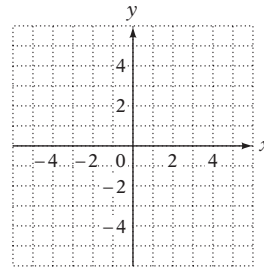
CHAPTER 4

TEST

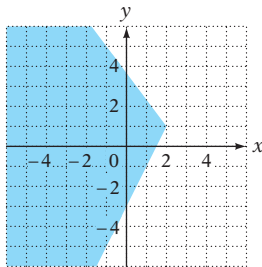
1. Solve by graphing: $2x - 3y = -6$
 $2x - y = 2$



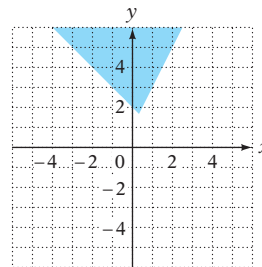
2. Solve by graphing: $x - 2y = -6$
 $y = \frac{1}{2}x - 4$



3. Graph the solution set: $2x - y < 3$
 $4x + 3y < 11$



4. Graph the solution set: $x + y > 2$
 $2x - y < -1$



5. Solve by substitution: $3x + 2y = 4$
 $x = 2y - 1$

6. Solve by substitution: $5x + 2y = -23$
 $2x + y = -10$

7. Solve by substitution: $y = 3x - 7$
 $y = -2x + 3$

8. Solve by the addition method:
 $3x + 4y = -2$
 $2x + 5y = 1$

9. Solve by the addition method:
 $4x - 6y = 5$
 $6x - 9y = 4$

10. Solve by the addition method:
 $3x - y = 2x + y - 1$
 $5x + 2y = y + 6$

11. Solve by the addition method:
 $2x + 4y - z = 3$
 $x + 2y + z = 5$
 $4x + 8y - 2z = 7$

12. Solve by the addition method:
 $x - y - z = 5$
 $2x + z = 2$
 $3y - 2z = 1$

13. Evaluate the determinant: $\begin{vmatrix} 3 & -1 \\ -2 & 4 \end{vmatrix}$

14. Evaluate the determinant: $\begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & 1 \\ 2 & -1 & -2 \end{vmatrix}$

15. Solve by using Cramer's Rule:
 $x - y = 3$
 $2x + y = -4$

16. Solve by using Cramer's Rule:
 $5x + 2y = 9$
 $3x + 5y = -7$

17. Solve by using Cramer's Rule:
 $x - y + z = 2$
 $2x - y - z = 1$
 $x + 2y - 3z = -4$

18. **Aeronautics** A plane flying with the wind went 350 mi in 2 h. The return trip, flying against the wind, took 2.8 h. Find the rate of the plane in calm air and the rate of the wind.

19. **Purchasing** A clothing manufacturer purchased 60 yd of cotton and 90 yd of wool for a total cost of \$1800. Another purchase, at the same prices, included 80 yd of cotton and 20 yd of wool for a total cost of \$1000. Find the cost per yard of the cotton and of the wool.

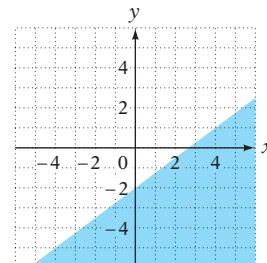
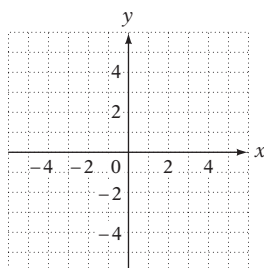


© Susan Van Etten/PhotoEdit, Inc.

20. **Investments** The annual interest earned on two investments is \$549. One investment is in a 2.7% tax-free annual simple interest account, and the other investment is in a 5.1% annual simple interest CD. The total amount invested is \$15,000. How much is invested in each account?

CUMULATIVE REVIEW EXERCISES

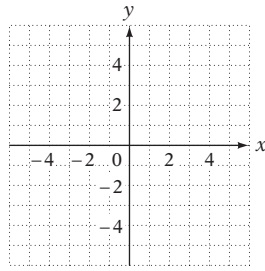
1. Solve: $\frac{3}{2}x - \frac{3}{8} + \frac{1}{4}x = \frac{7}{12}x - \frac{5}{6}$
2. Find the equation of the line that contains the points whose coordinates are $(2, -1)$ and $(3, 4)$.
3. Simplify: $3[x - 2(5 - 2x) - 4x] + 6$
4. Evaluate $a + bc \div 2$ when $a = 4$, $b = 8$, and $c = -2$.
5. Solve: $2x - 3 < 9$ or $5x - 1 < 4$
Write the solution set in interval notation.
6. Solve: $|x - 2| - 4 < 2$
7. Solve: $|2x - 3| > 5$
8. Given $f(x) = 3x^3 - 2x^2 + 1$, evaluate $f(-3)$.
9. Find the range of $f(x) = 3x^2 - 2x$ if the domain is $\{-2, -1, 0, 1, 2\}$.
10. Given $F(x) = x^2 - 3$, find $F(2)$.
11. Given $f(x) = 3x - 4$, write $f(2 + h) - f(2)$ in simplest form.
12. Graph the solution set of $\{x | x \leq 2\} \cap \{x | x > -3\}$.
13. Find the equation of the line that contains the point whose coordinates are $(-2, 3)$ and has slope $-\frac{2}{3}$.
14. Find the equation of the line that contains the point whose coordinates are $(-1, 2)$ and is perpendicular to the graph of $2x - 3y = 7$.
15. Find the distance, to the nearest hundredth, between the points whose coordinates are $(-4, 2)$ and $(2, 0)$.
16. Find the midpoint of the line segment connecting the points whose coordinates are $(-4, 3)$ and $(3, 5)$.
17. Graph $2x - 5y = 10$ by using the slope and y-intercept.
18. Graph the solution set of the inequality $3x - 4y \geq 8$.



19. Solve by graphing.

$$5x - 2y = 10$$

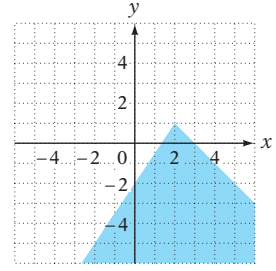
$$3x + 2y = 6$$



20. Graph the solution set.

$$3x - 2y \geq 4$$

$$x + y < 3$$



21. Solve by the addition method:

$$3x + 2z = 1$$

$$2y - z = 1$$

$$x + 2y = 1$$

22. Evaluate the determinant:

$$\begin{vmatrix} 2 & -5 & 1 \\ 3 & 1 & 2 \\ 6 & -1 & 4 \end{vmatrix}$$

23. Solve by using Cramer's Rule:

$$4x - 3y = 17$$

$$3x - 2y = 12$$

24. Solve by substitution:

$$3x - 2y = 7$$

$$y = 2x - 1$$

25. **Coins** A coin purse contains 40 coins in nickels, dimes, and quarters. There are three times as many dimes as quarters. The total value of the coins is \$4.10. How many nickels are in the coin purse?

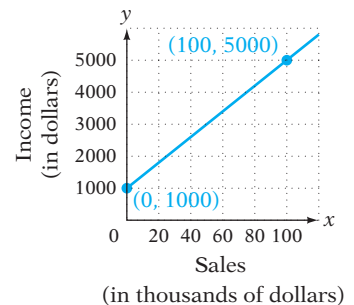
26. **Mixtures** How many milliliters of pure water must be added to 100 ml of a 4% salt solution to make a 2.5% salt solution?

27. **Aeronautics** Flying with the wind, a small plane required 2 h to fly 150 mi. Against the wind, it took 3 h to fly the same distance. Find the rate of the wind.

28. **Purchasing** A restaurant manager buys 100 lb of hamburger and 50 lb of steak for a total cost of \$540. A second purchase, at the same prices, includes 150 lb of hamburger and 100 lb of steak. The total cost is \$960. Find the cost of 1 lb of steak.

29. **Electronics** Find the lower and upper limits of a 12,000-ohm resistor with a 15% tolerance.

30. **Compensation** The graph shows the relationship between the monthly income and the sales of an account executive. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



Polynomials

CHAPTER

5

OBJECTIVES

SECTION 5.1

- A** To multiply monomials
- B** To divide monomials and simplify expressions with negative exponents
- C** To write a number using scientific notation
- D** To solve application problems

SECTION 5.2

- A** To evaluate polynomial functions
- B** To add or subtract polynomials

SECTION 5.3

- A** To multiply a polynomial by a monomial
- B** To multiply polynomials
- C** To multiply polynomials that have special products
- D** To solve application problems

SECTION 5.4

- A** To divide a polynomial by a monomial
- B** To divide polynomials
- C** To divide polynomials by using synthetic division
- D** To evaluate a polynomial function using synthetic division

SECTION 5.5

- A** To factor a monomial from a polynomial
- B** To factor by grouping
- C** To factor a trinomial of the form $x^2 + bx + c$
- D** To factor $ax^2 + bx + c$

SECTION 5.6

- A** To factor the difference of two perfect squares or a perfect-square trinomial
- B** To factor the sum or the difference of two perfect cubes
- C** To factor a trinomial that is quadratic in form
- D** To factor completely

SECTION 5.7

- A** To solve an equation by factoring
- B** To solve application problems

ARE YOU READY?

Take the Chapter 5 Prep Test to find out if you are ready to learn to:

- Add, subtract, multiply, and divide polynomials
- Simplify expressions with negative exponents
- Write a number in scientific notation
- Factor a polynomial completely
- Solve an equation by factoring

PREP TEST

Do these exercises to prepare for Chapter 5.

For Exercises 1 to 5, simplify.

1. $-4(3y)$

2. $(-2)^3$

3. $-4a - 8b + 7a$

4. $3x - 2[y - 4(x + 1) + 5]$

5. $-(x - y)$

6. Write 40 as a product of prime numbers.

7. Find the GCF of 16, 20, and 24.

8. Evaluate $x^3 - 2x^2 + x + 5$ for $x = -2$.

9. Solve: $3x + 1 = 0$

SECTION

5.1

Exponential Expressions

OBJECTIVE A

To multiply monomials



Point of Interest

Around A.D. 250, the monomial $3x^2$ shown at the right would have been written $\Delta^y 3$, or at least approximately like that. In A.D. 250, the symbol for 3 was not the one we use today.

A **monomial** is a number, a variable, or a product of a number and variables.

The examples at the right are monomials. The **degree of a monomial** is the sum of the exponents on the variables.

x	degree 1 ($x = x^1$)
$3x^2$	degree 2
$4x^2y$	degree 3
$6x^3y^4z^2$	degree 9

In this chapter, the variable n is considered a positive integer when used as an exponent.

x^n	degree n

The degree of a nonzero constant term is zero.

6	degree 0

The expression $5\sqrt{x}$ is not a monomial because $\sqrt{x}$ cannot be written as a product of variables. The expression $\frac{x}{y}$ is not a monomial because it is a quotient of variables.

The expression x^4 is an exponential expression. The exponent, 4, indicates the number of times the base, x , occurs as a factor.

The product of exponential expressions with the *same* base can be simplified by writing each expression in factored form and writing the result with an exponent.

$$x^3 \cdot x^4 = \overbrace{(x \cdot x \cdot x)}^{3 \text{ factors}} \cdot \overbrace{(x \cdot x \cdot x \cdot x)}^{4 \text{ factors}} = \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{7 \text{ factors}} = x^7$$

Note that adding the exponents results in the same product.

$$x^3 \cdot x^4 = x^{3+4} = x^7$$

Rule for Multiplying Exponential Expressions

If m and n are positive integers, then $x^m \cdot x^n = x^{m+n}$.

HOW TO • 1

Simplify: $(-4x^5y^3)(3xy^2)$

$$(-4x^5y^3)(3xy^2) = (-4 \cdot 3)(x^5 \cdot x)(y^3 \cdot y^2)$$

$$= -12(x^{5+1})(y^{3+2})$$

$$= -12x^6y^5$$

- Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.
- Multiply variables with the same base by adding their exponents.
- Simplify.

As shown below, the power of a monomial can be simplified by writing the power in factored form and then using the Rule for Multiplying Exponential Expressions. It can also be simplified by multiplying each exponent inside the parentheses by the exponent outside the parentheses.

$$\begin{aligned}(a^2)^3 &= a^2 \cdot a^2 \cdot a^2 \\ &= a^{2+2+2} \\ &= a^6\end{aligned}$$

$$(a^2)^3 = a^{2 \cdot 3} = a^6$$

$$\begin{aligned}(x^3y^4)^2 &= (x^3y^4)(x^3y^4) \\ &= x^{3+3}y^{4+4} \\ &= x^6y^8\end{aligned}$$

$$(x^3y^4)^2 = x^{3 \cdot 2}y^{4 \cdot 2} = x^6y^8$$

- Write in factored form. Then use the Rule for Multiplying Exponential Expressions.
- Multiply each exponent inside the parentheses by the exponent outside the parentheses.

Rule for Simplifying the Power of an Exponential Expression

If m and n are positive integers, then $(x^m)^n = x^{mn}$.

Rule for Simplifying Powers of Products

If m , n , and p are positive integers, then $(x^m y^n)^p = x^{mp} y^{np}$.

HOW TO • 2 Simplify: $(x^4)^5$

$$\begin{aligned}(x^4)^5 &= x^{4 \cdot 5} \\ &= x^{20}\end{aligned}$$

- Use the Rule for Simplifying Powers of Exponential Expressions to multiply the exponents.

HOW TO • 3 Simplify: $(2a^3b^4)^3$

$$\begin{aligned}(2a^3b^4)^3 &= 2^{1 \cdot 3} a^{3 \cdot 3} b^{4 \cdot 3} \\ &= 2^3 a^9 b^{12} \\ &= 8a^9b^{12}\end{aligned}$$

- Use the Rule for Simplifying Powers of Products to multiply each exponent inside the parentheses by the exponent outside the parentheses.

EXAMPLE • 1

Simplify: $(2xy^2)(-3xy^4)^3$

$$\begin{aligned}\text{Solution} \quad (2xy^2)(-3xy^4)^3 &= (2xy^2)[(-3)^3 x^3 y^{12}] \\ &= (2xy^2)(-27x^3 y^{12}) \\ &= -54x^4 y^{14}\end{aligned}$$

YOU TRY IT • 1

Simplify: $(-3a^2b^4)(-2ab^3)^4$

Your solution

EXAMPLE • 2

Simplify: $(-3x^3y)^2(-2x^3y^5)^3$

$$\begin{aligned}\text{Solution} \quad (-3x^3y)^2(-2x^3y^5)^3 &= [(-3)^2 x^6 y^2][(-2)^3 x^9 y^{15}] \\ &= [9x^6 y^2][-8x^9 y^{15}] \\ &= -72x^{15} y^{17}\end{aligned}$$

YOU TRY IT • 2

Simplify: $(-4ab^4)^2(2a^4b^2)^4$

Your solution

EXAMPLE • 3

Simplify: $(x^{n+2})^5$

$$\text{Solution} \quad (x^{n+2})^5 = x^{5n+10} \quad \bullet \text{ Multiply the exponents.}$$

YOU TRY IT • 3

Simplify: $(y^{n-3})^2$

Your solution

OBJECTIVE B**To divide monomials and simplify expressions with negative exponents**

The quotient of two exponential expressions with the same base can be simplified by writing each expression in factored form, dividing by the common factors, and then writing the result with an exponent.

$$\frac{x^5}{x^2} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{x} \cdot \overset{1}{x} \cdot \overset{1}{x}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = x^3$$

Note that subtracting the exponents gives the same result.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

To divide two monomials with the same base, subtract the exponents of the like bases.

HOW TO • 4 Simplify: $\frac{z^8}{z^2}$

$$\begin{aligned}\frac{z^8}{z^2} &= z^{8-2} \\ &= z^6\end{aligned}$$

- The bases are the same. Subtract the exponents.

HOW TO • 5 Simplify: $\frac{a^5b^9}{a^4b}$

$$\begin{aligned}\frac{a^5b^9}{a^4b} &= a^{5-4}b^{9-1} \\ &= ab^8\end{aligned}$$

- Subtract the exponents of the like bases.

Consider the expression $\frac{x^4}{x^4}$, $x \neq 0$. This expression can be simplified, as shown below, by subtracting exponents or dividing by common factors.

$$\frac{x^4}{x^4} = x^{4-4} = x^0$$

$$\frac{x^4}{x^4} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = 1$$

The equations $\frac{x^4}{x^4} = x^0$ and $\frac{x^4}{x^4} = 1$ suggest the following definition of x^0 .

Definition of Zero as an Exponent

If $x \neq 0$, then $x^0 = 1$. The expression 0^0 is not defined.

 Take Note

In the example at the right, we indicated that $z \neq 0$. If we try to evaluate $(16z^5)^0$ when $z = 0$, we have $[16(0)^5]^0 = [16(0)]^0 = 0^0$. However, 0^0 is not defined, so we must assume that $z \neq 0$. To avoid stating this for every example or exercise, we will assume that variables cannot take on values that result in the expression 0^0 .

HOW TO • 6 Simplify: $(16z^5)^0$, $z \neq 0$

$$(16z^5)^0 = 1$$

- Any nonzero expression to the zero power is 1.

HOW TO • 7 Simplify: $-(7x^4y^3)^0$

$$-(7x^4y^3)^0 = -(1) = -1$$

- The negative sign outside the parentheses is not affected by the exponent.

**Point of Interest**

In the 15th century, the expression $12^{2\overline{m}}$ was used to mean $12x^{-2}$. The use of $\overline{m}$ reflects an Italian influence, where m was used for minus and p was used for plus. It was understood that $2\overline{m}$ referred to an unnamed variable. Isaac Newton, in the 17th century, advocated the use of a negative exponent, the symbol we use today.

Consider the expression $\frac{x^4}{x^6}$, $x \neq 0$. This expression can be simplified, as shown below, by subtracting exponents or by dividing by common factors.

$$\frac{x^4}{x^6} = x^{4-6} = x^{-2} \qquad \frac{x^4}{x^6} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot x} = \frac{1}{x^2}$$

The equations $\frac{x^4}{x^6} = x^{-2}$ and $\frac{x^4}{x^6} = \frac{1}{x^2}$ suggest that $x^{-2} = \frac{1}{x^2}$.

Definition of a Negative Exponent

If $x \neq 0$ and n is a positive integer, then

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

**Take Note**

Note from the example at the right that 2^{-4} is a *positive* number. A negative exponent does not change the sign of a number.

HOW TO 8

Evaluate: 2^{-4}

$$\begin{aligned} 2^{-4} &= \frac{1}{2^4} \\ &= \frac{1}{16} \end{aligned}$$

• Use the Definition of a Negative Exponent.

• Evaluate the expression.

The expression $\left(\frac{x^3}{y^4}\right)^2$, $y \neq 0$, can be simplified by squaring $\frac{x^3}{y^4}$ or by multiplying each exponent in the quotient by the exponent outside the parentheses.

$$\left(\frac{x^3}{y^4}\right)^2 = \left(\frac{x^3}{y^4}\right)\left(\frac{x^3}{y^4}\right) = \frac{x^3 \cdot x^3}{y^4 \cdot y^4} = \frac{x^{3+3}}{y^{4+4}} = \frac{x^6}{y^8} \qquad \left(\frac{x^3}{y^4}\right)^2 = \frac{x^{3 \cdot 2}}{y^{4 \cdot 2}} = \frac{x^6}{y^8}$$

Rule for Simplifying Powers of Quotients

If m , n , and p are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

HOW TO 9

Simplify: $\left(\frac{a^2}{b^3}\right)^{-2}$

$$\begin{aligned} \left(\frac{a^2}{b^3}\right)^{-2} &= \frac{a^{2(-2)}}{b^{3(-2)}} \\ &= \frac{a^{-4}}{b^{-6}} = \frac{b^6}{a^4} \end{aligned}$$

• Use the Rule for Simplifying Powers of Quotients.

• Use the Definition of a Negative Exponent to write the expression with positive exponents.

The preceding example suggests the following rule.

Rule for Negative Exponents on Fractional Expressions

If $a \neq 0$, $b \neq 0$, and n is a positive integer, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Take Note

The exponent on d is -5 (negative 5). The d^{-5} is written in the denominator as d^5 . The exponent on 7 is 1 (positive 1). The 7 remains in the numerator.

Also, note that we indicated $d \neq 0$. This is necessary because division by zero is not defined. In this textbook, we will assume that values of the variables are chosen such that division by zero does not occur.

An exponential expression is in simplest form when it is written with only positive exponents.

HOW TO • 10 Simplify: $7d^{-5}$, $d \neq 0$

$$7d^{-5} = 7 \cdot \frac{1}{d^5} = \frac{7}{d^5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

HOW TO • 11 Simplify: $\frac{2}{5a^{-4}}$

$$\frac{2}{5a^{-4}} = \frac{2}{5} \cdot \frac{1}{a^{-4}} = \frac{2}{5} \cdot a^4 = \frac{2a^4}{5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

Now that zero as an exponent and negative exponents have been defined, a rule for dividing exponential expressions can be stated.

Rule for Dividing Exponential Expressions

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

HOW TO • 12 Simplify: $\frac{x^4}{x^9}$

$$\begin{aligned} \frac{x^4}{x^9} &= x^{4-9} \\ &= x^{-5} \\ &= \frac{1}{x^5} \end{aligned}$$

- Use the Rule for Dividing Exponential Expressions.
- Subtract the exponents.
- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

The rules for simplifying exponential expressions and powers of exponential expressions are true for all integers. These rules are restated here for convenience.

Rules of Exponents

If m , n , and p are integers, then

$$x^m \cdot x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(x^m y^n)^p = x^{mp} y^{np}$$

$$\frac{x^m}{x^n} = x^{m-n}, x \neq 0$$

$$\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}, y \neq 0$$

$$x^{-n} = \frac{1}{x^n}, x \neq 0$$

$$x^0 = 1, x \neq 0$$

HOW TO • 13 Simplify: $(3ab^{-4})(-2a^{-3}b^7)$

$$\begin{aligned}
 (3ab^{-4})(-2a^{-3}b^7) &= [3 \cdot (-2)](a^{1+(-3)}b^{-4+7}) \\
 &= -6a^{-2}b^3 \\
 &= -\frac{6b^3}{a^2}
 \end{aligned}$$

- When multiplying expressions, add the exponents on like bases.

HOW TO • 14 Simplify: $\frac{4a^{-2}b^5}{6a^5b^2}$

$$\begin{aligned}
 \frac{4a^{-2}b^5}{6a^5b^2} &= \frac{2 \cdot 2a^{-2}b^5}{2 \cdot 3a^5b^2} = \frac{2a^{-2}b^5}{3a^5b^2} \\
 &= \frac{2a^{-2-5}b^{5-2}}{3} \\
 &= \frac{2a^{-7}b^3}{3} = \frac{2b^3}{3a^7}
 \end{aligned}$$

- Divide the coefficients by their common factor.
- Use the Rule for Dividing Exponential Expressions.
- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

HOW TO • 15 Simplify: $\left[\frac{6m^2n^3}{8m^7n^2}\right]^{-3}$

$$\begin{aligned}
 \left[\frac{6m^2n^3}{8m^7n^2}\right]^{-3} &= \left[\frac{3m^{2-7}n^{3-2}}{4}\right]^{-3} \\
 &= \left[\frac{3m^{-5}n}{4}\right]^{-3} \\
 &= \frac{3^{-3}m^{15}n^{-3}}{4^{-3}} \\
 &= \frac{4^3m^{15}}{3^3n^3} = \frac{64m^{15}}{27n^3}
 \end{aligned}$$

- Simplify inside the brackets.
- Subtract the exponents.
- Use the Rule for Simplifying Powers of Quotients.
- Use the Definition of a Negative Exponent to rewrite the expression with positive exponents. Then simplify.

EXAMPLE • 4

Simplify: $\frac{-28x^6z^{-3}}{42x^{-1}z^4}$

Solution

$$\begin{aligned}
 \frac{-28x^6z^{-3}}{42x^{-1}z^4} &= -\frac{14 \cdot 2x^{6-(-1)}z^{-3-4}}{14 \cdot 3} \\
 &= -\frac{2x^7z^{-7}}{3} = -\frac{2x^7}{3z^7}
 \end{aligned}$$

EXAMPLE • 5

Simplify: $\frac{(3a^{-1}b^4)^{-3}}{(6^{-1}a^{-3}b^{-4})^3}$

Solution

$$\begin{aligned}
 \frac{(3a^{-1}b^4)^{-3}}{(6^{-1}a^{-3}b^{-4})^3} &= \frac{3^{-3}a^3b^{-12}}{6^{-3}a^{-9}b^{-12}} = 3^{-3} \cdot 6^3 a^{12} b^0 \\
 &= \frac{6^3 a^{12}}{3^3} = \frac{216a^{12}}{27} = 8a^{12}
 \end{aligned}$$

YOU TRY IT • 4

Simplify: $\frac{20r^{-2}t^{-5}}{-16r^{-3}s^{-2}}$

Your solution**YOU TRY IT • 5**

Simplify: $\frac{(9u^{-6}v^4)^{-1}}{(6u^{-3}v^{-2})^{-2}}$

Your solution

EXAMPLE 6Simplify: $\frac{x^{4n-2}}{x^{2n-5}}$ **Solution**

$$\begin{aligned}\frac{x^{4n-2}}{x^{2n-5}} &= x^{4n-2-(2n-5)} \\ &= x^{4n-2-2n+5} \\ &= x^{2n+3}\end{aligned}$$

- Subtract the exponents.

YOU TRY IT 6Simplify: $\frac{a^{2n+1}}{a^{n+3}}$ **Your solution**

Solution on p. S14

OBJECTIVE C**To write a number using scientific notation****Point of Interest**

Astronomers measure the distance of some stars by using a unit called the parsec. One parsec is approximately 1.91×10^{13} mi.

Integer exponents are used to represent the very large and very small numbers encountered in the fields of science and engineering. For example, the mass of an electron is 0.000000000000000000000009 g. Numbers such as this are difficult to read and write, so a more convenient system for writing such numbers has been developed. It is called **scientific notation**.

To express a number in scientific notation, write the number as the product of a number between 1 and 10 and a power of 10. The form for scientific notation is $a \times 10^n$, where $1 \leq a < 10$.

For numbers greater than 10, move the decimal point to the right of the first digit. The exponent n is positive and equal to the number of places the decimal point has been moved.

For numbers less than 1, move the decimal point to the right of the first nonzero digit. The exponent n is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved.

$$\begin{aligned}965,000 &= 9.65 \times 10^5 \\ 3,600,000 &= 3.6 \times 10^6 \\ 92,000,000,000 &= 9.2 \times 10^{10} \\ 0.0002 &= 2 \times 10^{-4} \\ 0.0000000974 &= 9.74 \times 10^{-8} \\ 0.000000000086 &= 8.6 \times 10^{-11}\end{aligned}$$

**Take Note**

There are two steps involved in writing a number in scientific notation:

- (1) determine the number between 1 and 10, and
- (2) determine the exponent on 10.

Converting a number written in scientific notation to decimal notation requires moving the decimal point.

When the exponent is positive, move the decimal point to the right the same number of places as the exponent.

When the exponent is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

$$\begin{aligned}1.32 \times 10^4 &= 13,200 \\ 1.4 \times 10^8 &= 140,000,000 \\ 1.32 \times 10^{-2} &= 0.0132 \\ 1.4 \times 10^{-4} &= 0.00014\end{aligned}$$

Numerical calculations involving numbers that have more digits than a hand-held calculator is able to handle can be performed using scientific notation.

HOW TO 16Simplify: $\frac{220,000 \times 0.000000092}{0.0000011}$

$$\begin{aligned}\frac{220,000 \times 0.000000092}{0.0000011} &= \frac{2.2 \times 10^5 \times 9.2 \times 10^{-8}}{1.1 \times 10^{-6}} \\ &= \frac{(2.2)(9.2) \times 10^{5+(-8)-(-6)}}{1.1} \\ &= 18.4 \times 10^3 = 18,400\end{aligned}$$

- Write the numbers in scientific notation.
- Simplify.

**Integrating Technology**

See the Keystroke Guide: *Scientific Notation* for instructions on entering a number that is in scientific notation into a graphing calculator.

EXAMPLE • 7

Write 0.000041 in scientific notation.

Solution $0.000041 = 4.1 \times 10^{-5}$ **EXAMPLE • 8**Write 3.3×10^7 in decimal notation.**Solution** $3.3 \times 10^7 = 33,000,000$ **EXAMPLE • 9**

Simplify:

$$\frac{2,400,000,000 \times 0.0000063}{0.00009 \times 480}$$

Solution

$$\begin{aligned} & \frac{2,400,000,000 \times 0.0000063}{0.00009 \times 480} \\ &= \frac{2.4 \times 10^9 \times 6.3 \times 10^{-6}}{9 \times 10^{-5} \times 4.8 \times 10^2} \\ &= \frac{(2.4)(6.3) \times 10^{9+(-6)-(-5)-2}}{(9)(4.8)} \\ &= 0.35 \times 10^6 = 350,000 \end{aligned}$$
YOU TRY IT • 7

Write 942,000,000 in scientific notation.

Your solution**YOU TRY IT • 8**Write 2.7×10^{-5} in decimal notation.**Your solution****YOU TRY IT • 9**

Simplify:

$$\frac{5,600,000 \times 0.000000081}{900 \times 0.000000028}$$

Your solution*Solutions on p. S14***OBJECTIVE D****To solve application problems****EXAMPLE • 10**

How many miles does light travel in 1 day? The speed of light is 186,000 mi/s. Write the answer in scientific notation.

Strategy

To find the distance traveled:

- Write the speed of light in scientific notation.
- Write the number of seconds in 1 day in scientific notation.
- Use the equation $d = rt$, where r is the speed of light and t is the number of seconds in 1 day.

Solution

$$\begin{aligned} r &= 186,000 = 1.86 \times 10^5 \\ t &= 24 \cdot 60 \cdot 60 = 86,400 = 8.64 \times 10^4 \\ d &= rt \\ d &= (1.86 \times 10^5)(8.64 \times 10^4) \\ &= 1.86 \times 8.64 \times 10^9 \\ &= 16.0704 \times 10^9 \\ &= 1.60704 \times 10^{10} \end{aligned}$$

Light travels 1.60704×10^{10} mi in 1 day.**YOU TRY IT • 10**

The Roadrunner supercomputer from IBM can perform one arithmetic operation, called a FLOP (FLoating-point OPeration), in 9.74×10^{-16} s. In scientific notation, how many arithmetic operations can be performed in 1 min?

Your strategy**Your solution***Solution on p. S14*

5.1 EXERCISES

OBJECTIVE A

To multiply monomials

For Exercises 1 to 37, simplify.

1. $(ab^3)(a^3b)$

2. $(-2ab^4)(-3a^2b^4)$

3. $(9xy^2)(-2x^2y^2)$

4. $(x^2y)^2$

5. $(x^2y^4)^4$

6. $(-2ab^2)^3$

7. $(-3x^2y^3)^4$

8. $(4a^2b^3)^3$

9. $(27a^5b^3)^2$

10. $[(2ab)^3]^2$

11. $[(2a^4b^3)^3]^2$

12. $(xy)(x^2y)^4$

13. $(x^2y^2)(xy^3)^3$

14. $(-4x^3y)^2(2x^2y)$

15. $(-5ab)(3a^3b^2)^2$

16. $(-4r^2s^3)^3(2s^2)$

17. $(3x^5y)(-4x^3)^3$

18. $(4x^3z)(-3y^4z^5)$

19. $(-6a^4b^2)(-7a^2c^5)$

20. $(4ab)^2(-2ab^2c^3)^3$

21. $(-2ab^2)(-3a^4b^5)^3$

22. $(2a^2b)^3(-3ab^4)^2$

23. $(-3ab^3)^3(-2a^2b)^2$

24. $(-c^3)(-2a^2bc)(3a^2b)$

25. $(-2x^2y^3z)(3x^2yz^4)$

26. $(x^2z^4)(2xyz^4)(-3x^3y^2)$

27. $(2xy)(-3x^2yz)(x^2y^3z^3)$

28. $(2a^2b)(-3ab^2c)(-b^3c^5)$

29. $(3b^5)(2ab^2)(-2ab^2c^2)$

30. $y^n \cdot y^{2n}$

31. $x^n \cdot x^{n+1}$

32. $y^{2n} \cdot y^{4n+1}$

33. $y^{3n} \cdot y^{3n-2}$

34. $(a^n)^{2n}$

35. $(a^{n-3})^{2n}$

36. $(y^{2n-1})^3$

37. $(x^{3n+2})^5$



38. If $x^m \cdot x^n = x^9$ and $n = 3$, what is the value of m ?



39. What is the value of n in the equation $2^{32} + 2^{32} = 2^n$?

OBJECTIVE B

To divide monomials and simplify expressions with negative exponents

For Exercises 40 to 90, simplify.

40. 2^{-3}

41. $\frac{1}{3^{-5}}$

42. $\frac{1}{x^{-4}}$

43. $\frac{1}{y^{-3}}$

44. $\frac{2x^{-2}}{y^4}$ 45. $\frac{a^3}{4b^{-2}}$ 46. $x^{-3}y$ 47. xy^{-4}
48. $-5x^0$ 49. $\frac{1}{2x^0}$ 50. $\frac{(2x)^0}{-2^3}$ 51. $\frac{-3^{-2}}{(2y)^0}$
52. $(x^2y^{-4})^2$ 53. $(x^3y^5)^{-2}$ 54. $(2x^3y^{-2})(-3x^{-4}y^3)$ 55. $(-3a^{-4}b^{-5})(-5a^{-2}b^4)$
56. $(5m^{-3}n^2)^{-2}(10m^2n)$ 57. $(4y^{-3}z^{-4})(-3y^3z^{-3})^{-2}$ 58. $(-6mn^{-2})^3(-4m^{-3}n^{-1})^{-2}$ 59. $(4x^{-3}y^2)^{-3}(2xy^{-3})^4$
60. $\frac{6a^4}{4a^6}$ 61. $\frac{9x^5}{12x^8}$ 62. $\frac{x^{17}y^5}{-x^7y^{10}}$ 63. $\frac{-6x^2y}{12x^4y}$
64. $\frac{y^{-7}}{y^{-8}}$ 65. $\frac{y^{-2}}{y^6}$ 66. $\frac{x^{-2}y^{-11}}{xy^{-2}}$ 67. $\frac{x^4y^3}{x^{-1}y^{-2}}$
68. $\frac{a^{-1}b^{-3}}{a^4b^{-5}}$ 69. $\frac{a^6b^{-4}}{a^{-2}b^5}$ 70. $\frac{2x^2y^4}{(3xy^2)^3}$ 71. $\frac{-3ab^2}{(9a^2b^4)^3}$
72. $\frac{(4x^2y)^2}{(2xy^3)^3}$ 73. $\frac{(3a^2b)^3}{(-6ab^3)^2}$ 74. $\frac{(-4xy^3)^3}{(-2x^7y)^4}$ 75. $\frac{(-8x^2y^2)^4}{(16x^3y^7)^2}$
76. $\frac{(-3x^3y^{-2})^3}{(2xy^{-3})^{-2}}$ 77. $\frac{(3a^4b^{-2})^{-2}}{(2a^{-3}b)^3}$ 78. $\frac{(2m^{-2}n^3)^{-2}}{(6m^{-1}n^{-2})^{-3}}$ 79. $\frac{(-2x^{-5}y^2)^{-3}}{(4xy^{-2})^{-4}}$
80. $\frac{a^{5n}}{a^{3n}}$ 81. $\frac{b^{6n}}{b^{10n}}$ 82. $\frac{-x^{5n}}{x^{2n}}$ 83. $\frac{y^{2n}}{-y^{8n}}$
84. $\frac{x^{2n-1}}{x^{n-3}}$ 85. $\frac{y^{3n+2}}{y^{2n+4}}$ 86. $\frac{a^{3n-2}b^{n+1}}{a^{2n+1}b^{2n+2}}$ 87. $\frac{x^{2n-1}y^{n-3}}{x^{n+4}y^{n+3}}$
88. $\left(\frac{4^{-2}xy^{-3}}{x^{-3}y}\right)^3\left(\frac{8^{-1}x^{-2}y}{x^4y^{-1}}\right)^{-2}$ 89. $\left(\frac{9ab^{-2}}{8a^{-2}b}\right)^{-2}\left(\frac{3a^{-2}b}{2a^2b^{-2}}\right)^3$ 90. $\left(\frac{2ab^{-1}}{ab}\right)^{-1}\left(\frac{3a^{-2}b}{a^2b^2}\right)^{-2}$



91. If $\frac{x^p}{x^q} = 1$, what is the value of $p - q$?



92. If $m < n$, is the value of $\frac{2^m}{2^n}$ less than 1 or greater than 1?

OBJECTIVE C**To write a number using scientific notation**

For Exercises 93 to 98, write in scientific notation.

93. 0.00000467

94. 0.00000005

95. 0.00000000017

96. 4,300,000

97. 200,000,000,000

98. 9,800,000,000

For Exercises 99 to 104, write in decimal notation.

99. 1.23×10^{-7}

100. 6.2×10^{-12}

101. 8.2×10^{15}

102. 6.34×10^5

103. 3.9×10^{-2}

104. 4.35×10^9

For Exercises 105 to 116, simplify. Write the answer in decimal notation.

105. $(3 \times 10^{-12})(5 \times 10^{16})$

106. $(8.9 \times 10^{-5})(3.2 \times 10^{-6})$

107. $(0.0000065)(3,200,000,000,000)$

108. $(480,000)(0.0000000096)$

109. $\frac{9 \times 10^{-3}}{6 \times 10^5}$

110. $\frac{2.7 \times 10^4}{3 \times 10^{-6}}$

111. $\frac{0.0089}{500,000,000}$

112. $\frac{0.000000346}{0.0000005}$

113. $\frac{(3.3 \times 10^{-11})(2.7 \times 10^{15})}{8.1 \times 10^{-3}}$

114. $\frac{(6.9 \times 10^{27})(8.2 \times 10^{-13})}{4.1 \times 10^{15}}$

115. $\frac{(0.00000004)(84,000)}{(0.0003)(1,400,000)}$

116. $\frac{(720)(0.0000000039)}{(26,000,000,000)(0.018)}$



117. Is 5.27×10^{-6} less than zero or greater than zero?



118. Place the correct symbol, $<$, $=$, or $>$, between the two numbers:
 4.61×10^5 _____ 46.1×10^4 .

OBJECTIVE D**To solve application problems**

For Exercises 119 to 127, solve. Write the answer in scientific notation.



119. **Astronomy** Our galaxy is estimated to be 5.6×10^{19} mi across. How many years would it take a spaceship to cross the galaxy traveling at 25,000 mph?



120. **Astronomy** How long does it take light to travel to Earth from the sun? The sun is 9.3×10^7 mi from Earth, and light travels 1.86×10^5 mi/s.



The Milky Way

© Stocktrek/Spirit/Corbis



- 121. Physics** The mass of an electron is 9.109×10^{-31} kg. The mass of a proton is 1.673×10^{-27} kg. How many times larger is the mass of a proton than the mass of an electron?



- 122. The Federal Government** In 2010, the gross national debt was approximately 1.1×10^{13} dollars. How much would each American have to pay in order to pay off the debt? Use 3×10^8 as the number of citizens.



- 123. Geology** The mass of Earth is 5.9×10^{24} kg. The mass of the sun is 2×10^{30} kg. How many times larger is the mass of the sun than the mass of Earth?



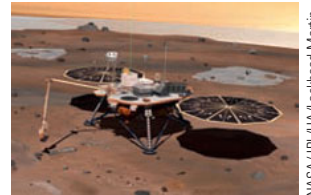
- 124. Astronomy** Use the information in the article below to determine the average number of miles traveled per day by *Phoenix* on its trip to Mars.

In the News

A Mars Landing for *Phoenix*

At 7:53 P.M., a safe landing on the surface of Mars brought an end to the *Phoenix* spacecraft's 296-day, 422-million-mile journey to the Red Planet.

Source: *The Los Angeles Times*



NASA/JPL/UA/Lockheed Martin



- 125. Astronomy** It took 11 min for the commands from a computer on Earth to travel to the *Phoenix* Mars Lander, a distance of 119 million miles. How fast did the signals from Earth to Mars travel?



- 126. Forestry** Use the information in the article at the right. If every burned acre of Yellowstone Park had 12,000 lodgepole pine seedlings growing on it 1 year after the fire, how many new seedlings would be growing?



- 127. Forestry** Use the information in the article at the right. Find the number of seeds released by the lodgepole pine trees for each surviving seedling.



- 128.** One light-year is approximately 5.9×10^{12} mi and is defined as the distance light can travel in a vacuum in 1 year. *Voyager 1* is approximately 15 light-hours away from Earth and took about 30 years to travel that distance. One light-hour is approximately 6.7×10^8 mi. True or false: $\frac{5.9 \times 10^{12}}{6.7 \times 10^8} \approx$ number of hours in 1 year.

In the News

Forest Fires Spread Seeds

Forest fires may be feared by humans, but not by the lodgepole pine, a tree that uses the intense heat of a fire to release its seeds from their cones. After a blaze that burned 12,000,000 acres of Yellowstone National Park, scientists counted 2 million lodgepole pine seeds on a single acre of the park. One year later, they returned to find 12,000 lodgepole pine seedlings growing.

Source: National Public Radio

Applying the Concepts



- 129.** Correct the error in each of the following expressions. Explain which rule or property was used incorrectly.

a. $x^0 = 0$ b. $(x^4)^5 = x^9$ c. $x^2 \cdot x^3 = x^6$

- 130.** Simplify. a. $1 + [1 + (1 + 2^{-1})^{-1}]^{-1}$ b. $2 - [2 - (2 - 2^{-1})^{-1}]^{-1}$

SECTION

5.2

Introduction to Polynomial Functions

OBJECTIVE A

To evaluate polynomial functions



Tips for Success

A great many new vocabulary words are introduced in this chapter. All of these terms are in **boldface type**. The bold type indicates that these are concepts you must know to learn the material. Be sure to study each new term as it is presented.

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of one term is a **monomial**.

$$5x$$

A polynomial of two terms is a **binomial**.

$$5x^2y + 6x$$

A polynomial of three terms is a **trinomial**.

$$3x^2 + 9xy - 5y$$

Polynomials with more than three terms do not have special names.

The **degree of a polynomial** is the greatest of the degrees of any of its terms.

$3x + 2$	degree 1
$3x^2 + 2x - 4$	degree 2
$4x^3y^2 + 6x^4$	degree 5
$3x^{2n} - 5x^n - 2$	degree $2n$

The terms of a polynomial in one variable are usually arranged so that the exponents on the variable decrease from left to right. This is called **descending order**.

$2x^2 - x + 8$
$3y^3 - 3y^2 + y - 12$

For a polynomial in more than one variable, descending order may refer to any one of the variables.

The polynomial at the right is shown first in descending order of the x variable and then in descending order of the y variable.

$2x^2 + 3xy + 5y^2$
$5y^2 + 3xy + 2x^2$

Polynomial functions have many applications in mathematics. In general, a **polynomial function** is an expression whose terms are monomials. The **linear function** given by $f(x) = mx + b$ is an example of a polynomial function. It is a polynomial function of degree 1. A second-degree polynomial function, called a **quadratic function**, is given by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. A third-degree polynomial function is called a **cubic function**.

To **evaluate a polynomial function**, replace the variable by its value and simplify.

HOW TO 1

Given $P(x) = x^3 - 3x^2 + 4$, evaluate $P(-3)$.

$$\begin{aligned} P(x) &= x^3 - 3x^2 + 4 \\ P(-3) &= (-3)^3 - 3(-3)^2 + 4 \\ &= -27 - 27 + 4 \\ &= -50 \end{aligned}$$

• Substitute -3 for x and simplify.



Integrating Technology

See the Keystroke Guide: *Evaluating Functions* for instructions on using a graphing calculator to evaluate a function.

The **leading coefficient** of a polynomial function is the coefficient of the variable with the largest exponent. The **constant term** is the term without a variable.

HOW TO 2

Find the leading coefficient, the constant term, and the degree of the polynomial function $P(x) = 7x^4 - 3x^2 + 2x - 4$.

The leading coefficient is 7, the constant term is -4, and the degree is 4.

The three equations below do not represent polynomial functions.

$$f(x) = 3x^2 + 2x^{-1}$$

A polynomial function does not have a variable raised to a negative power.

$$g(x) = 2\sqrt{x} - 3$$

A polynomial function does not have a variable expression within a radical.

$$h(x) = \frac{x}{x-1}$$

A polynomial function does not have a variable in the denominator of a fraction.

The graph of a linear function is a straight line and can be found by plotting just two points. The graph of a polynomial function of degree greater than 1 is a curve. Consequently, many points may have to be found before an accurate graph can be drawn.

Evaluating the quadratic function given by the equation $f(x) = x^2 - x - 6$ when $x = -3, -2, -1, 0, 1, 2, 3$, and 4 gives the points shown in Figure 1 below. For instance, $f(-3) = 6$, so $(-3, 6)$ is graphed; $f(2) = -4$, so $(2, -4)$ is graphed; and $f(4) = 6$, so $(4, 6)$ is graphed. Evaluating the function when x is not an integer, such as when $x = -\frac{3}{2}$ and $x = \frac{5}{2}$, produces more points to graph, as shown in Figure 2. Connecting the points with a smooth curve results in Figure 3, which is the graph of f .

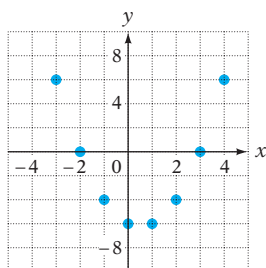


FIGURE 1

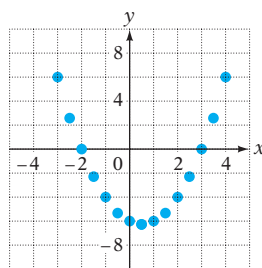


FIGURE 2

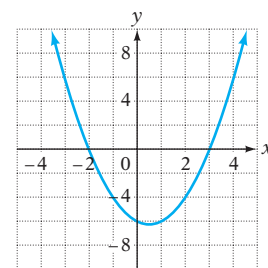


FIGURE 3

Here is an example of graphing a cubic function, $P(x) = x^3 - 2x^2 - 5x + 6$. Evaluating the function when $x = -2, -1, 0, 1, 2, 3$, and 4 gives the graph in Figure 4 below. Evaluating for some noninteger values gives the graph in Figure 5. Finally, connecting the dots with a smooth curve gives the graph in Figure 6.



Integrating Technology

You can verify the graphs of these polynomial functions by using a graphing calculator. See the Keystroke Guide: *Graph* for instructions on using a graphing calculator to graph a function.

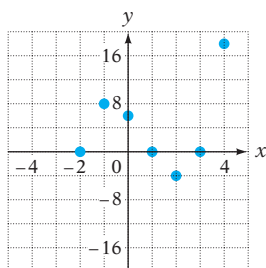


FIGURE 4

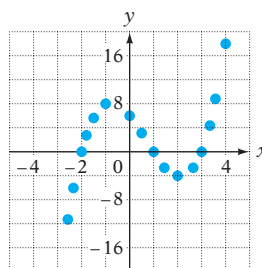


FIGURE 5

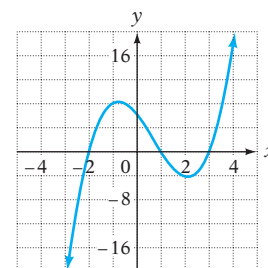


FIGURE 6

EXAMPLE • 1

Given $P(x) = x^3 + 3x^2 - 2x + 8$, evaluate $P(-2)$.

Solution

$$\begin{aligned}
 P(x) &= x^3 + 3x^2 - 2x + 8 \\
 P(-2) &= (-2)^3 + 3(-2)^2 - 2(-2) + 8 && \bullet \text{ Replace } x \\
 &= (-8) + 3(4) + 4 + 8 && \text{by } -2. \\
 &= -8 + 12 + 4 + 8 && \text{Simplify.} \\
 &= 16
 \end{aligned}$$

YOU TRY IT • 1

Given $R(x) = -2x^4 - 5x^3 + 2x - 8$, evaluate $R(2)$.

Your solution**EXAMPLE • 2**

Find the leading coefficient, the constant term, and the degree of the polynomial.

$$P(x) = 5x^6 - 4x^5 - 3x^2 + 7$$

Solution

The leading coefficient is 5, the constant term is 7, and the degree is 6.

YOU TRY IT • 2

Find the leading coefficient, the constant term, and the degree of the polynomial.

$$r(x) = -3x^4 + 3x^3 + 3x^2 - 2x - 12$$

Your solution**EXAMPLE • 3**

Which of the following is a polynomial function?

- $P(x) = 3x^{\frac{1}{3}} + 2x^2 - 3$
- $T(x) = 3\sqrt{x} - 2x^2 - 3x + 2$
- $R(x) = 14x^3 - \pi x^2 + 3x + 2$

Solution

- This is not a polynomial function. A polynomial function does not have a variable raised to a fractional power.
- This is not a polynomial function. A polynomial function does not have a variable expression within a radical.
- This is a polynomial function.

YOU TRY IT • 3

Which of the following is a polynomial function?

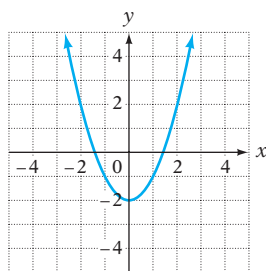
- $R(x) = 5x^{14} - 5$
- $V(x) = -x^{-1} + 2x - 7$
- $P(x) = 2x^4 - 3\sqrt{x} - 3$

Your solution**EXAMPLE • 4**

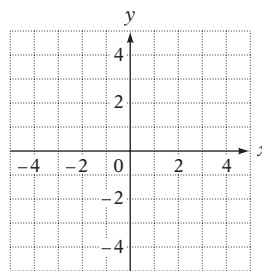
Graph $f(x) = x^2 - 2$.

Solution

x	$y = f(x)$
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

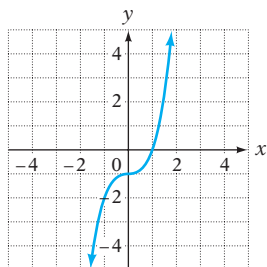
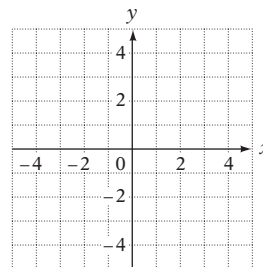
**YOU TRY IT • 4**

Graph $f(x) = x^2 + 2x - 3$.

Your solution

EXAMPLE 5Graph $f(x) = x^3 - 1$.**Solution**

x	$y = f(x)$
-2	-9
-1	-2
0	-1
1	0
2	7

**YOU TRY IT 5**Graph $f(x) = -x^3 + 1$.**Your solution**

Solution on p. S15

OBJECTIVE B**To add or subtract polynomials**

Polynomials can be added by combining like terms. Either a vertical or a horizontal format can be used.

HOW TO 3Add $(3x^2 + 2x - 7) + (7x^3 - 3 + 4x^2)$. Use a horizontal format.

Use the Commutative and Associative Properties of Addition to rearrange and group like terms.

$$\begin{aligned}
 (3x^2 + 2x - 7) + (7x^3 - 3 + 4x^2) \\
 &= 7x^3 + (3x^2 + 4x^2) + 2x + (-7 - 3) \\
 &= 7x^3 + 7x^2 + 2x - 10
 \end{aligned}$$

• **Combine like terms.****HOW TO 4**Add $(4x^2 + 5x - 3) + (7x^3 - 7x + 1) + (2x - 3x^2 + 4x^3 + 1)$.

Use a vertical format.

Arrange the terms of each polynomial in descending order, with like terms in the same column.

$$\begin{array}{r}
 4x^2 + 5x - 3 \\
 7x^3 \quad - 7x + 1 \\
 4x^3 - 3x^2 + 2x + 1 \\
 \hline
 11x^3 + x^2 \quad - 1
 \end{array}$$

• **Add the terms in each column.****Take Note**

The additive inverse of a polynomial is that polynomial with the sign of every term changed.

The **additive inverse of the polynomial** $x^2 + 5x - 4$ is $-(x^2 + 5x - 4)$.

To simplify the additive inverse of a polynomial, change the sign of every term inside the parentheses.

$$-(x^2 + 5x - 4) = -x^2 - 5x + 4$$

**Take Note**

This is the same definition used for subtraction of integers: subtraction is addition of the opposite.

To subtract two polynomials, add the additive inverse of the second polynomial to the first.

HOW TO • 5

Subtract $(3x^2 - 7xy + y^2) - (-4x^2 + 7xy - 3y^2)$. Use a horizontal format.

Rewrite the subtraction as addition of the additive inverse.

$$\begin{aligned}(3x^2 - 7xy + y^2) - (-4x^2 + 7xy - 3y^2) \\&= (3x^2 - 7xy + y^2) + (4x^2 - 7xy + 3y^2) \\&= 7x^2 - 14xy + 4y^2\end{aligned}$$

• Combine like terms.

HOW TO • 6

Subtract $(6x^3 - 3x + 7) - (3x^2 - 5x + 12)$. Use a vertical format.

Rewrite the subtraction as addition of the additive inverse.

$$(6x^3 - 3x + 7) - (3x^2 - 5x + 12) = (6x^3 - 3x + 7) + (-3x^2 + 5x - 12)$$

Arrange the terms of each polynomial in descending order, with like terms in the same column.

$$\begin{array}{r} 6x^3 \qquad - 3x + 7 \\ - 3x^2 + 5x - 12 \\ \hline 6x^3 - 3x^2 + 2x - 5 \end{array}$$

• Combine the terms in each column.

Function notation can be used when adding or subtracting polynomials.

HOW TO • 7

Given $P(x) = 3x^2 - 2x + 4$ and $R(x) = -5x^3 + 4x + 7$, find

$$P(x) + R(x).$$

$$\begin{aligned}P(x) + R(x) &= (3x^2 - 2x + 4) + (-5x^3 + 4x + 7) \\&= -5x^3 + 3x^2 + 2x + 11\end{aligned}$$

HOW TO • 8

Given $P(x) = -5x^2 + 8x - 4$ and $R(x) = -3x^2 - 5x + 9$, find

$$P(x) - R(x).$$

$$\begin{aligned}P(x) - R(x) &= (-5x^2 + 8x - 4) - (-3x^2 - 5x + 9) \\&= (-5x^2 + 8x - 4) + (3x^2 + 5x - 9) \\&= -2x^2 + 13x - 13\end{aligned}$$

HOW TO • 9

Given $P(x) = 3x^2 - 5x + 6$ and $R(x) = 2x^2 - 5x - 7$, find $S(x)$, the sum of the two polynomials.

$$\begin{aligned}S(x) = P(x) + R(x) &= (3x^2 - 5x + 6) + (2x^2 - 5x - 7) \\&= 5x^2 - 10x - 1\end{aligned}$$

Note from the preceding example that evaluating $P(x) = 3x^2 - 5x + 6$ and $R(x) = 2x^2 - 5x - 7$ at, for example, $x = 3$ and then adding the values is the same as evaluating $S(x) = 5x^2 - 10x - 1$ at 3.

$$P(3) = 3(3)^2 - 5(3) + 6 = 27 - 15 + 6 = 18$$

$$R(3) = 2(3)^2 - 5(3) - 7 = 18 - 15 - 7 = -4$$

$$P(3) + R(3) = 18 + (-4) = 14$$

$$S(3) = 5(3)^2 - 10(3) - 1 = 45 - 30 - 1 = 14$$

EXAMPLE • 6

Add:

$$(4x^2 - 3xy + 7y^2) + (-3x^2 + 7xy + y^2)$$

Use a vertical format.

Solution

$$\begin{array}{r} 4x^2 - 3xy + 7y^2 \\ -3x^2 + 7xy + y^2 \\ \hline x^2 + 4xy + 8y^2 \end{array}$$

YOU TRY IT • 6

Add:

$$(-3x^2 - 4x + 9) + (-5x^2 - 7x + 1)$$

Use a vertical format.

Your solution**EXAMPLE • 7**

Subtract:

$$(3x^2 - 2x + 4) - (7x^2 + 3x - 12)$$

Use a vertical format.

SolutionAdd the additive inverse of $7x^2 + 3x - 12$ to $3x^2 - 2x + 4$.

$$\begin{array}{r} 3x^2 - 2x + 4 \\ -7x^2 - 3x + 12 \\ \hline -4x^2 - 5x + 16 \end{array}$$

YOU TRY IT • 7

Subtract:

$$(-5x^2 + 2x - 3) - (6x^2 + 3x - 7)$$

Use a vertical format.

Your solution**EXAMPLE • 8**Given $P(x) = -3x^2 + 2x - 6$ and $R(x) = 4x^3 - 3x + 4$, find $S(x) = P(x) + R(x)$.Evaluate $S(-2)$.**Solution**

$$S(x) = P(x) + R(x)$$

$$\begin{aligned} S(x) &= (-3x^2 + 2x - 6) + (4x^3 - 3x + 4) \\ &= 4x^3 - 3x^2 - x - 2 \end{aligned}$$

$$\begin{aligned} S(-2) &= 4(-2)^3 - 3(-2)^2 - (-2) - 2 \\ &= 4(-8) - 3(4) + 2 - 2 \\ &= -44 \end{aligned}$$

YOU TRY IT • 8Given $P(x) = 4x^3 - 3x^2 + 2$ and $R(x) = -2x^2 + 2x - 3$, find $S(x) = P(x) + R(x)$.Evaluate $S(-1)$.**Your solution****EXAMPLE • 9**Given $P(x) = (2x^{2n} - 3x^n + 7)$ and $R(x) = (3x^{2n} + 3x^n + 5)$, find $D(x) = P(x) - R(x)$.**Solution**

$$D(x) = P(x) - R(x)$$

$$\begin{aligned} D(x) &= (2x^{2n} - 3x^n + 7) - (3x^{2n} + 3x^n + 5) \\ &= (2x^{2n} - 3x^n + 7) + (-3x^{2n} - 3x^n - 5) \\ &= -x^{2n} - 6x^n + 2 \end{aligned}$$

YOU TRY IT • 9Given $P(x) = (5x^{2n} - 3x^n - 7)$ and $R(x) = (-2x^{2n} - 5x^n + 8)$, find $D(x) = P(x) - R(x)$.**Your solution**

5.2 EXERCISES

OBJECTIVE A

To evaluate polynomial functions

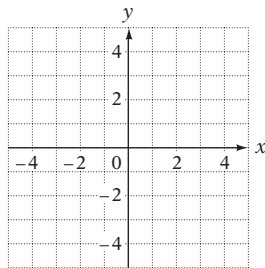
- Given $P(x) = 3x^2 - 2x - 8$, evaluate $P(3)$.
- Given $P(x) = -3x^2 - 5x + 8$, evaluate $P(-5)$.
- Given $R(x) = 2x^3 - 3x^2 + 4x - 2$, evaluate $R(2)$.
- Given $R(x) = -x^3 + 2x^2 - 3x + 4$, evaluate $R(-1)$.
- Given $f(x) = x^4 - 2x^2 - 10$, evaluate $f(-1)$.
- Given $f(x) = x^5 - 2x^3 + 4x$, evaluate $f(2)$.

In Exercises 7 to 18, indicate whether the expression defines a polynomial function. For those that are polynomial functions: **a.** Identify the leading coefficient. **b.** Identify the constant term. **c.** State the degree.

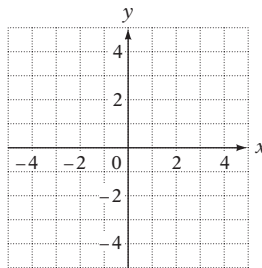
- $P(x) = -x^2 + 3x + 8$
- $P(x) = 3x^4 - 3x - 7$
- $f(x) = \sqrt{x} - x^2 + 2$
- $f(x) = x^2 - \sqrt{x+2} - 8$
- $R(x) = \frac{x}{x+1}$
- $R(x) = \frac{3x^2 - 2x + 1}{x}$
- $g(x) = 3x^5 - 2x^2 + \pi$
- $g(x) = -4x^5 - 3x^2 + x - \sqrt{7}$
- $P(x) = 3x^2 - 5x^3 + 2$
- $P(x) = x^2 - 5x^4 - x^6$
- $R(x) = 14$
- $R(x) = \frac{1}{x} + 2$

For Exercises 19 to 24, graph.

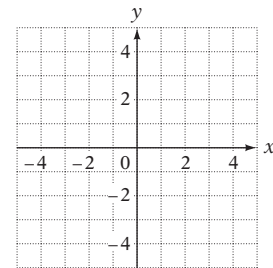
19. $P(x) = x^2 - 1$



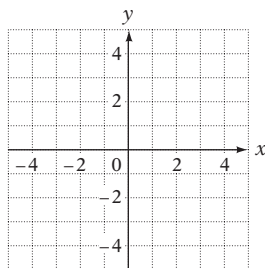
20. $P(x) = 2x^2 + 3$



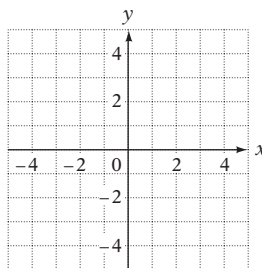
21. $R(x) = x^3 + 2$



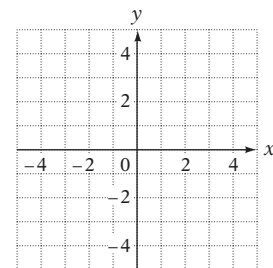
22. $R(x) = x^4 + 1$



23. $f(x) = x^3 - 2x$



24. $f(x) = x^2 - x - 2$



-  25. Suppose $f(x) = x^2$ and $g(x) = x^3$. For any number c between 0 and 1, is $f(c) - g(c) > 0$ or is $f(c) - g(c) < 0$? Suppose c is between -1 and 0. Is $f(c) - g(c) > 0$ or is $f(c) - g(c) < 0$?

OBJECTIVE B To add or subtract polynomials

26. If $P(x)$ and $Q(x)$ are polynomials of degree 3, is it possible for the sum of the two polynomials to have degree 2? If so, give an example. If not, explain why.

For Exercises 27 to 30, simplify. Use a vertical format.

27. $(5x^2 + 2x - 7) + (x^2 - 8x + 12)$ 28. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$
29. $(x^2 - 3x + 8) - (2x^2 - 3x + 7)$ 30. $(2x^2 + 3x - 7) - (5x^2 - 8x - 1)$

For Exercises 31 to 34, simplify. Use a horizontal format.

31. $(3y^2 - 7y) + (2y^2 - 8y + 2)$ 32. $(-2y^2 - 4y - 12) + (5y^2 - 5y)$
33. $(2a^2 - 3a - 7) - (-5a^2 - 2a - 9)$ 34. $(3a^2 - 9a) - (-5a^2 + 7a - 6)$
35. Given $P(x) = x^2 - 3xy + y^2$ and $R(x) = 2x^2 - 3y^2$, find $P(x) + R(x)$.
36. Given $P(x) = x^{2n} + 7x^n - 3$ and $R(x) = -x^{2n} + 2x^n + 8$, find $P(x) + R(x)$.
37. Given $P(x) = 3x^2 + 2y^2$ and $R(x) = -5x^2 + 2xy - 3y^2$, find $P(x) - R(x)$.
38. Given $P(x) = 2x^{2n} - x^n - 1$ and $R(x) = 5x^{2n} + 7x^n + 1$, find $P(x) - R(x)$.
39. Given $P(x) = 3x^4 - 3x^3 - x^2$ and $R(x) = 3x^3 - 7x^2 + 2x$, find $S(x) = P(x) + R(x)$. Evaluate $S(2)$.
40. Given $P(x) = 3x^4 - 2x + 1$ and $R(x) = 3x^5 - 5x - 8$, find $S(x) = P(x) + R(x)$. Evaluate $S(-1)$.

Applying the Concepts

41. For what value of k is the given equation an identity?
- a. $(2x^3 + 3x^2 + kx + 5) - (x^3 + 2x^2 + 3x + 7) = x^3 + x^2 + 5x - 2$
- b. $(6x^3 + kx^2 - 2x - 1) - (4x^3 - 3x^2 + 1) = 2x^3 - x^2 - 2x - 2$

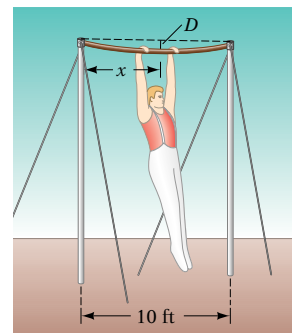


42. If $P(x)$ is a third-degree polynomial and $Q(x)$ is a fourth-degree polynomial, what can be said about the degree of $P(x) + Q(x)$? Give some examples of polynomials that support your answer.



43. If $P(x)$ is a fifth-degree polynomial and $Q(x)$ is a fourth-degree polynomial, what can be said about the degree of $P(x) - Q(x)$? Give some examples of polynomials that support your answer.

44. **Sports** The deflection D (in inches) of a beam that is uniformly loaded is given by the polynomial function $D(x) = 0.005x^4 - 0.1x^3 + 0.5x^2$, where x is the distance from one end of the beam. See the figure at the right. The maximum deflection occurs when x is the midpoint of the beam. Determine the maximum deflection for the beam in the diagram.



SECTION

5.3

Multiplication of Polynomials

OBJECTIVE A

To multiply a polynomial by a monomial 

To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions.

HOW TO • 1 Multiply: $-3x^2(2x^2 - 5x + 3)$

$$-3x^2(2x^2 - 5x + 3)$$

$$= -3x^2(2x^2) - (-3x^2)(5x) + (-3x^2)(3)$$

$$= -6x^4 + 15x^3 - 9x^2$$

- Use the Distributive Property.
- Use the Rule for Multiplying Exponential Expressions.

HOW TO • 2 Simplify: $5x(3x - 6) + 3(4x - 2)$

$$5x(3x - 6) + 3(4x - 2)$$

$$= 5x(3x) - 5x(6) + 3(4x) - 3(2)$$

$$= 15x^2 - 30x + 12x - 6$$

$$= 15x^2 - 18x - 6$$

- Use the Distributive Property.
- Simplify.

HOW TO • 3 Simplify: $2x^2 - 3x[2 - x(4x + 1) + 2]$

$$2x^2 - 3x[2 - x(4x + 1) + 2]$$

$$= 2x^2 - 3x[2 - 4x^2 - x + 2]$$

$$= 2x^2 - 3x[-4x^2 - x + 4]$$

$$= 2x^2 + 12x^3 + 3x^2 - 12x$$

$$= 12x^3 + 5x^2 - 12x$$

- Use the Distributive Property to remove the parentheses.
- Simplify.
- Use the Distributive Property to remove the brackets.
- Simplify.

EXAMPLE • 1Multiply: $(3a^2 - 2a + 4)(-3a)$ **Solution**

$$(3a^2 - 2a + 4)(-3a)$$

$$= 3a^2(-3a) - 2a(-3a) + 4(-3a)$$

$$= -9a^3 + 6a^2 - 12a$$

- Use the Distributive Property.

YOU TRY IT • 1Multiply: $(2b^2 - 7b - 8)(-5b)$ **Your solution**

EXAMPLE • 2Simplify: $y - 3y[y - 2(3y - 6) + 2]$ **Solution**

$$\begin{aligned}
 y - 3y[y - 2(3y - 6) + 2] \\
 &= y - 3y[y - 6y + 12 + 2] \\
 &= y - 3y[-5y + 14] \\
 &= y + 15y^2 - 42y \\
 &= 15y^2 - 41y
 \end{aligned}$$

YOU TRY IT • 2Simplify: $x^2 - 2x[x - x(4x - 5) + x^2]$ **Your solution**

Solution on p. S15

OBJECTIVE B**To multiply polynomials**

Multiplication of polynomials requires repeated application of the Distributive Property.

HOW TO • 4 Multiply: $(2x^2 - 5x + 1)(3x + 2)$

Use the Distributive Property to multiply the trinomial by each term of the binomial.

$$\begin{aligned}
 (2x^2 - 5x + 1)(3x + 2) &= (2x^2 - 5x + 1)3x + (2x^2 - 5x + 1)2 \\
 &= (6x^3 - 15x^2 + 3x) + (4x^2 - 10x + 2) \\
 &= 6x^3 - 11x^2 - 7x + 2
 \end{aligned}$$

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

HOW TO • 5 Multiply: $(2x^2 - 5x + 1)(3x + 2)$

$$\begin{array}{r}
 2x^2 - 5x + 1 \\
 \underline{3x + 2} \\
 4x^2 - 10x + 2 = (2x^2 - 5x + 1)2 \\
 6x^3 - 15x^2 + 3x = (2x^2 - 5x + 1)3x \\
 \hline
 6x^3 - 11x^2 - 7x + 2
 \end{array}$$

 Take Note

FOIL is not really a different way of multiplying. It is based on the Distributive Property.

$$\begin{aligned}
 (3x - 2)(2x + 5) \\
 &= 3x(2x + 5) - 2(2x + 5) \\
 &= 6x^2 + 15x - 4x - 10 \\
 &= 6x^2 + 11x - 10
 \end{aligned}$$

FOIL is an efficient way of remembering how to do binomial multiplication.

It is frequently necessary to find the product of two binomials. The product can be found by using a method called **FOIL**, which is based on the Distributive Property. The letters of FOIL stand for **F**irst, **O**uter, **I**nnner, and **L**ast.Multiply: $(3x - 2)(2x + 5)$ Multiply the **F**irst terms.

$(3x - 2)(2x + 5)$

$3x \cdot 2x = 6x^2$

Multiply the **O**uter terms.

$(3x - 2)(2x + 5)$

$3x \cdot 5 = 15x$

Multiply the **I**nnner terms.

$(3x - 2)(2x + 5)$

$-2 \cdot 2x = -4x$

Multiply the **L**ast terms.

$(3x - 2)(2x + 5)$

$-2 \cdot 5 = -10$

F O I L

Add the products.

$(3x - 2)(2x + 5) = 6x^2 + 15x - 4x - 10$

Combine like terms.

$= 6x^2 + 11x - 10$

HOW TO • 6 Multiply: $(6x - 5)(3x - 4)$

$$\begin{aligned}
 (6x - 5)(3x - 4) &= 6x(3x) + 6x(-4) + (-5)(3x) + (-5)(-4) \\
 &= 18x^2 - 24x - 15x + 20 \\
 &= 18x^2 - 39x + 20
 \end{aligned}$$

 Take Note

The product of $x^2 - x - 12$ and $x + 2$ could have been found using a vertical format.

$$\begin{array}{r}
 x^2 - x - 12 \\
 x + 2 \\
 \hline
 2x^2 - 2x - 24 \\
 x^3 - x^2 - 12x \\
 \hline
 x^3 + x^2 - 14x - 24
 \end{array}$$

HOW TO • 7 Multiply: $(x + 3)(x - 4)(x + 2)$

$$\begin{aligned}
 (x + 3)(x - 4)(x + 2) &= (x^2 - x - 12)(x + 2) \\
 &= x^2(x + 2) - x(x + 2) - 12(x + 2) \\
 &= x^3 + 2x^2 - x^2 - 2x - 12x - 24 \\
 &= x^3 + x^2 - 14x - 24
 \end{aligned}$$

- **Multiply $(x + 3)(x - 4)$.**
- **Use the Distributive Property.**
- **Simplify.**

EXAMPLE • 3

Multiply: $(4a^3 - 3a + 7)(a - 5)$

Solution Keep like terms in the same columns.

$$\begin{array}{r}
 4a^3 \\
 - 3a + 7 \\
 \hline
 -20a^3 \\
 4a^4 \\
 \hline
 4a^4 - 20a^3 - 3a^2 + 22a - 35
 \end{array}$$

- $-5(4a^3 - 3a + 7)$
- $a(4a^3 - 3a + 7)$

YOU TRY IT • 3

Multiply: $(-2b^2 + 5b - 4)(-3b + 2)$

Your solution

EXAMPLE • 4

Multiply: $(5a - 3b)(2a + 7b)$

Solution

$$\begin{aligned}
 (5a - 3b)(2a + 7b) &= 10a^2 + 35ab - 6ab - 21b^2 \\
 &= 10a^2 + 29ab - 21b^2
 \end{aligned}$$

- **FOIL**

YOU TRY IT • 4

Multiply: $(3x - 4)(2x - 3)$

Your solution

EXAMPLE • 5

Multiply: $(2x^2 - 3)(4x^2 + 1)$

Solution

$$\begin{aligned}
 (2x^2 - 3)(4x^2 + 1) &= 8x^4 + 2x^2 - 12x^2 - 3 \\
 &= 8x^4 - 10x^2 - 3
 \end{aligned}$$

YOU TRY IT • 5

Multiply: $(3ab + 4)(5ab - 3)$

Your solution

OBJECTIVE C**To multiply polynomials that have special products**

Using FOIL, a pattern can be found for the **product of the sum and difference of two terms** [that is, a polynomial that can be expressed in the form $(a + b)(a - b)$] and for the **square of a binomial** [that is, a polynomial that can be expressed in the form $(a + b)^2$].

The Sum and Difference of Two Terms

$$(a + b)(a - b) = a^2 - ab + ab - b^2 \\ = a^2 - b^2$$

Square of the first term

Square of the second term

The Square of a Binomial

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 \\ = a^2 + 2ab + b^2$$

Square of the first term

Twice the product of the two terms

Square of the second term

HOW TO • 8Multiply: $(4x + 3)(4x - 3)$

$(4x + 3)(4x - 3)$ is the sum and difference of the same two terms. The product is the difference of the squares of the terms.

$$(4x + 3)(4x - 3) = (4x)^2 - 3^2 \\ = 16x^2 - 9$$

**Take Note**

The word *expand* is sometimes used to mean “multiply,” especially when referring to a power of a binomial.

HOW TO • 9Expand: $(2x - 3y)^2$

$(2x - 3y)^2$ is the square of a binomial.

$$(2x - 3y)^2 = (2x)^2 + 2(2x)(-3y) + (-3y)^2 \\ = 4x^2 - 12xy + 9y^2$$

EXAMPLE • 6Multiply: $(2a - 3)(2a + 3)$ **Solution**

$$(2a - 3)(2a + 3) = 4a^2 - 9$$

- The sum and difference of two terms

YOU TRY IT • 6Multiply: $(3x - 7)(3x + 7)$ **Your solution****EXAMPLE • 7**Multiply: $(5x + y)(5x - y)$ **Solution**

$$(5x + y)(5x - y) = 25x^2 - y^2$$

- The sum and difference of two terms

YOU TRY IT • 7Multiply: $(2ab + 7)(2ab - 7)$ **Your solution**

EXAMPLE • 8Expand: $(2x + 7y)^2$ **Solution**

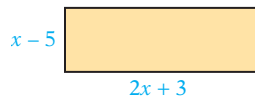
$$(2x + 7y)^2 = 4x^2 + 28xy + 49y^2 \quad \bullet \text{ The square of a binomial}$$

YOU TRY IT • 8Expand: $(3x - 4y)^2$ **Your solution****EXAMPLE • 9**Expand: $(3a^2 - b)^2$ **Solution**

$$(3a^2 - b)^2 = 9a^4 - 6a^2b + b^2 \quad \bullet \text{ The square of a binomial}$$

YOU TRY IT • 9Expand: $(5xy + 4)^2$ **Your solution***Solutions on p. S15***OBJECTIVE D****To solve application problems****EXAMPLE • 10**

The length of a rectangle is $(2x + 3)$ ft. The width is $(x - 5)$ ft. Find the area of the rectangle in terms of the variable x .

**Strategy**

To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values, and solve for A .

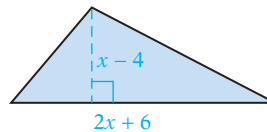
Solution

$$\begin{aligned} A &= L \cdot W \\ A &= (2x + 3)(x - 5) \\ &= 2x^2 - 10x + 3x - 15 \\ &= 2x^2 - 7x - 15 \end{aligned} \quad \bullet \text{ FOIL}$$

The area is $(2x^2 - 7x - 15)$ ft².

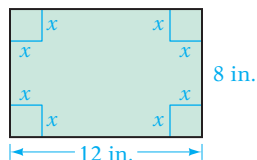
YOU TRY IT • 10

The base of a triangle is $(2x + 6)$ ft. The height is $(x - 4)$ ft. Find the area of the triangle in terms of the variable x .

**Your strategy****Your solution***Solution on p. S15*

EXAMPLE • 11

The corners are cut from a rectangular piece of cardboard measuring 8 in. by 12 in. The sides are folded up to make a box. Find the volume of the box in terms of the variable x , where x is the length of a side of the square cut from each corner of the rectangle.

**Strategy**

Length of the box: $12 - 2x$

Width of the box: $8 - 2x$

Height of the box: x

To find the volume, replace the variables L , W , and H in the equation $V = L \cdot W \cdot H$, and solve for V .

Solution

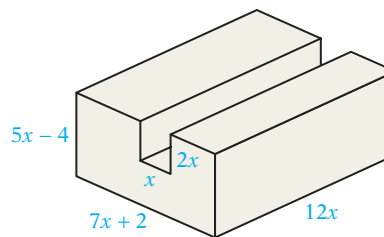
$$V = L \cdot W \cdot H$$

$$\begin{aligned} V &= (12 - 2x)(8 - 2x)x \\ &= (96 - 24x - 16x + 4x^2)x && \bullet \text{ FOIL} \\ &= (96 - 40x + 4x^2)x \\ &= 96x - 40x^2 + 4x^3 \\ &= 4x^3 - 40x^2 + 96x \end{aligned}$$

The volume is $(4x^3 - 40x^2 + 96x) \text{ in}^3$.

YOU TRY IT • 11

Find the volume of the solid shown in the diagram below. All dimensions are in feet.

**Your strategy****Your solution****EXAMPLE • 12**

The radius of a circle is $(3x - 2) \text{ cm}$.

Find the area of the circle in terms of the variable x .

Use 3.14 for π .

Strategy

To find the area, replace the variable r in the equation $A = \pi r^2$ by the given value, and solve for A .

Solution

$$\begin{aligned} A &= \pi r^2 \\ A &\approx 3.14(3x - 2)^2 \\ &= 3.14(9x^2 - 12x + 4) \\ &= 28.26x^2 - 37.68x + 12.56 \end{aligned}$$

The area is $(28.26x^2 - 37.68x + 12.56) \text{ cm}^2$.

YOU TRY IT • 12

The radius of a circle is $(2x + 3) \text{ cm}$.

Find the area of the circle in terms of the variable x .

Use 3.14 for π .

Your strategy**Your solution**

5.3 EXERCISES

OBJECTIVE A

To multiply a polynomial by a monomial



1. What is the name of the property that is used to remove parentheses from the expression $3x(4x^2 - 5x - 7)$?



2. What is the first step when simplifying $5 + 2(2x + 1)$? Why?

For Exercises 3 to 28, simplify.

3. $2x(x - 3)$

4. $2a(2a + 4)$

5. $3x^2(2x^2 - x)$

6. $-4y^2(4y - 6y^2)$

7. $3xy(2x - 3y)$

8. $-4ab(5a - 3b)$

9. $x^n(x + 1)$

10. $y^n(y^{2n} - 3)$

11. $x^n(x^n + y^n)$

12. $x - 2x(x - 2)$

13. $2b + 4b(2 - b)$

14. $-2y(3 - y) + 2y^2$

15. $-2a^2(3a^2 - 2a + 3)$

16. $4b(3b^3 - 12b^2 - 6)$

17. $(-3y^2 - 4y + 2)(y^2)$

18. $(6b^4 - 5b^2 - 3)(-2b^3)$

19. $-5x^2(4 - 3x + 3x^2 + 4x^3)$

20. $-2y^2(3 - 2y - 3y^2 + 2y^3)$

21. $-2x^2y(x^2 - 3xy + 2y^2)$

22. $3ab^2(3a^2 - 2ab + 4b^2)$

23. $5x^3 - 4x(2x^2 + 3x - 7)$

24. $7a^3 - 2a(6a^2 - 5a - 3)$

25. $2y^2 - y[3 - 2(y - 4) - y]$

26. $3x^2 - x[x - 2(3x - 4)]$

27. $2y - 3[y - 2y(y - 3) + 4y]$

28. $4a^2 - 2a[3 - a(2 - a + a^2)]$

29. Given $P(b) = 3b$ and $Q(b) = 3b^4 - 3b^2 + 8$, find $P(b) \cdot Q(b)$.

30. Given $P(x) = -2x^2$ and $Q(x) = 2x^2 - 3x - 7$, find $P(x) \cdot Q(x)$.

OBJECTIVE B To multiply polynomials

For Exercises 31 to 60, multiply.

31. $(x - 2)(x + 7)$

32. $(y + 8)(y + 3)$

33. $(2y - 3)(4y + 7)$

34. $(5x - 7)(3x - 8)$

35. $(a + 3c)(4a - 5c)$

36. $(2m - 3n)(5m + 4n)$

37. $(5x - 7)(5x - 7)$

38. $(5r + 2t)(5r - 2t)$

39. $2(2x - 3y)(2x + 5y)$

40. $-3(7x - 3y)(2x - 9y)$

41. $(xy + 4)(xy - 3)$

42. $(xy - 5)(2xy + 7)$

43. $(2x^2 - 5)(x^2 - 5)$

44. $(x^2 - 4)(x^2 - 6)$

45. $(5x^2 - 5y)(2x^2 - y)$

46. $(x^2 - 2y^2)(x^2 + 4y^2)$

47. $(x + 5)(x^2 - 3x + 4)$

48. $(a + 2)(a^2 - 3a + 7)$

49. $(2a - 3b)(5a^2 - 6ab + 4b^2)$

50. $(3a + b)(2a^2 - 5ab - 3b^2)$

51. $(2x^3 + 3x^2 - 2x + 5)(2x - 3)$

52. $(3a^3 + 4a - 7)(4a - 2)$

53. $(2x - 5)(2x^4 - 3x^3 - 2x + 9)$

54. $(2a - 5)(3a^4 - 3a^2 + 2a - 5)$

55. $(x^2 + 2x - 3)(x^2 - 5x + 7)$

56. $(x^2 - 3x + 1)(x^2 - 2x + 7)$

57. $(a - 2)(2a - 3)(a + 7)$

58. $(b - 3)(3b - 2)(b - 1)$

59. $(2x + 3)(x - 4)(3x + 5)$

60. $(3a - 5)(2a + 1)(a - 3)$

61. Given $P(y) = 2y^2 - 1$ and $Q(y) = y^3 - 5y^2 - 3$, find $P(y) \cdot Q(y)$.

62. Given $P(b) = 2b^2 - 3$ and $Q(b) = 3b^2 - 3b + 6$, find $P(b) \cdot Q(b)$.



63. If $P(x)$ is a polynomial of degree 3 and $Q(x)$ is a polynomial of degree 2, what is the degree of the product of the two polynomials?



64. Do all polynomials of degree 2 factor over the integers? If not, give an example of a polynomial of degree 2 that does not factor over the integers.

OBJECTIVE C**To multiply polynomials that have special products**

For Exercises 65 to 88, simplify or expand.

65. $(3x - 2)(3x + 2)$

66. $(4y + 1)(4y - 1)$

67. $(6 - x)(6 + x)$

68. $(10 + b)(10 - b)$

69. $(2a - 3b)(2a + 3b)$

70. $(5x - 7y)(5x + 7y)$

71. $(3ab + 4)(3ab - 4)$

72. $(5xy - 8)(5xy + 8)$

73. $(x^2 + 1)(x^2 - 1)$

74. $(x^2 + y^2)(x^2 - y^2)$

75. $(x - 5)^2$

76. $(y + 2)^2$

77. $(3a + 5b)^2$

78. $(5x - 4y)^2$

79. $(x^2 - 3)^2$

80. $(x^2 + y^2)^2$

81. $(2x^2 - 3y^2)^2$

82. $(2xy + 3)^2$

83. $(3mn - 5)^2$

84. $(2 - 7xy)^2$

85. $y^2 - (x - y)^2$

86. $a^2 + (a + b)^2$

87. $(x - y)^2 - (x + y)^2$

88. $(a + b)^2 + (a - b)^2$



89. True or false? $a^2 + b^2 = (a + b)(a + b)$



90. If $P(x)$ is a polynomial of degree 2 that factors as the difference of squares, is the coefficient of x in $P(x)$ (i) less than zero, (ii) equal to zero, (iii) greater than zero, or (iv) either less than or greater than zero?

OBJECTIVE D**To solve application problems**

91. If the measures of the width and length of the floor of a room are given in feet, what is the unit of measure of the area?

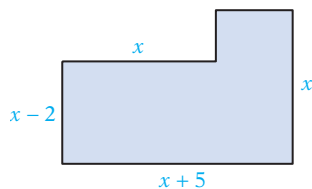


92. If the measure of the width, length, and height of a box are given in meters, what is the unit of measure of the volume of the box?

93. **Geometry** The length of a rectangle is $(3x - 2)$ ft. The width is $(x + 4)$ ft. Find the area of the rectangle in terms of the variable x .

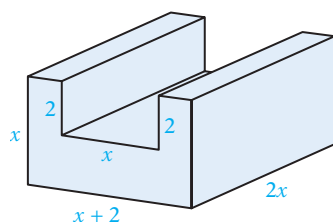
94. **Geometry** The base of a triangle is $(x - 4)$ ft. The height is $(3x + 2)$ ft. Find the area of the triangle in terms of the variable x .

95. **Geometry** Find the area of the figure shown below. All dimensions given are in meters.



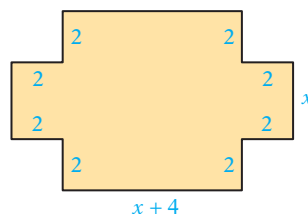
97. **Geometry** The length of the side of a cube is $(x + 3)$ cm. Find the volume of the cube in terms of the variable x .

99. **Geometry** Find the volume of the figure shown below. All dimensions given are in inches.



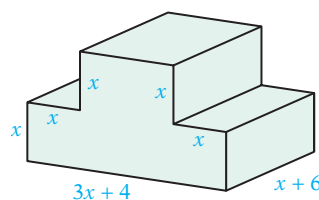
101. **Geometry** The radius of a circle is $(5x + 4)$ in. Find the area of the circle in terms of the variable x . Use 3.14 for π .

96. **Geometry** Find the area of the figure shown below. All dimensions given are in feet.



98. **Geometry** The length of a box is $(3x + 2)$ cm, the width is $(x - 4)$ cm, and the height is x cm. Find the volume of the box in terms of the variable x .

100. **Geometry** Find the volume of the figure shown below. All dimensions given are in centimeters.



102. **Geometry** The radius of a circle is $(x - 2)$ in. Find the area of the circle in terms of the variable x . Use 3.14 for π .

Applying the Concepts

103. Find the product.

a. $(a - b)(a^2 + ab + b^2)$

b. $(x + y)(x^2 - xy + y^2)$

104. Correct the error in each of the following.

a. $(x + 3)^2 = x^2 + 9$

b. $(a - b)^2 = a^2 - b^2$

105. For what value of k is the given equation an identity?

a. $(3x - k)(2x + k) = 6x^2 + 5x - k^2$

b. $(4x + k)^2 = 16x^2 + 8x + k^2$

106. Complete.

a. If $m = n + 1$, then $\frac{a^m}{a^n} = \underline{\hspace{2cm}}$.

b. If $m = n + 2$, then $\frac{a^m}{a^n} = \underline{\hspace{2cm}}$.

107. Subtract the product of $4a + b$ and $2a - b$ from $9a^2 - 2ab$.

108. Subtract the product of $5x - y$ and $x + 3y$ from $6x^2 + 12xy - 2y^2$.

SECTION

5.4

Division of Polynomials

OBJECTIVE A

To divide a polynomial by a monomial

As shown below, $\frac{6+4}{2}$ can be simplified by first adding the terms in the numerator and then dividing the result by the denominator. It can also be simplified by first dividing each term in the numerator by the denominator and then adding the results.

$$\frac{6+4}{2} = \frac{10}{2} = 5 \qquad \frac{6+4}{2} = \frac{6}{2} + \frac{4}{2} = 3 + 2 = 5$$

It is this second method that is used to divide a polynomial by a monomial: Divide each term in the numerator by the denominator, and then write the sum of the quotients.

To divide $\frac{6x^2 + 4x}{2x}$, divide each term of the polynomial $6x^2 + 4x$ by the monomial $2x$. Then simplify each quotient.

 Take Note

Recall that the fraction bar can be read “divided by.”

$$\begin{aligned} \frac{6x^2 + 4x}{2x} &= \frac{6x^2}{2x} + \frac{4x}{2x} \\ &= 3x + 2 \end{aligned}$$

- Divide each term in the numerator by the denominator.
- Simplify each quotient.

We can check this quotient by multiplying it by the divisor.

$$2x(3x + 2) = 6x^2 + 4x \quad \bullet \text{ The product is the dividend. The quotient checks.}$$

HOW TO • 1 Divide and check: $\frac{16x^5 - 8x^3 + 4x}{2x}$

$$\begin{aligned} \frac{16x^5 - 8x^3 + 4x}{2x} &= \frac{16x^5}{2x} - \frac{8x^3}{2x} + \frac{4x}{2x} \\ &= 8x^4 - 4x^2 + 2 \end{aligned}$$

- Divide each term in the numerator by the denominator.
- Simplify each quotient.

Check:

$$2x(8x^4 - 4x^2 + 2) = 16x^5 - 8x^3 + 4x \quad \bullet \text{ The quotient checks.}$$

EXAMPLE • 1

Divide and check: $\frac{6x^3 - 3x^2 + 9x}{3x}$

Solution

$$\begin{aligned} \frac{6x^3 - 3x^2 + 9x}{3x} &= \frac{6x^3}{3x} - \frac{3x^2}{3x} + \frac{9x}{3x} \\ &= 2x^2 - x + 3 \end{aligned}$$

- Divide each term in the numerator by the denominator.
- Simplify each quotient.

Check: $3x(2x^2 - x + 3) = 6x^3 - 3x^2 + 9x$

YOU TRY IT • 1

Divide and check: $\frac{4x^3y + 8x^2y^2 - 4xy^3}{2xy}$

Your solution

OBJECTIVE B**To divide polynomials**

The division method illustrated in Objective A is appropriate only when the divisor is a monomial. To divide two polynomials in which the divisor is not a monomial, use a method similar to that used for division of whole numbers.

To check division of polynomials, use

$$\text{Dividend} = (\text{quotient} \times \text{divisor}) + \text{remainder}$$

HOW TO 2Divide: $(x^2 + 5x - 7) \div (x + 3)$

Step 1

$$\begin{array}{r} x \\ x+3 \overline{)x^2+5x-7} \\ \underline{x^2+3x} \\ 2x-7 \end{array}$$

$$\text{Think: } x \overline{)x^2} = \frac{x^2}{x} = x$$

$$\text{Multiply: } x(x+3) = x^2 + 3x$$

$$\text{Subtract: } (x^2 + 5x) - (x^2 + 3x) = 2x$$

Step 2

$$\begin{array}{r} x+2 \\ x+3 \overline{)x^2+5x-7} \\ \underline{x^2+3x} \\ 2x-7 \\ \underline{2x+6} \\ -13 \end{array}$$

$$\text{Think: } x \overline{)2x} = \frac{2x}{x} = 2$$

$$\text{Multiply: } 2(x+3) = 2x + 6$$

$$\text{Subtract: } (2x - 7) - (2x + 6) = -13$$

The remainder is -13 .

$$\text{Check: } (x+2)(x+3) + (-13) = x^2 + 3x + 2x + 6 - 13 = x^2 + 5x - 7$$

$$(x^2 + 5x - 7) \div (x + 3) = x + 2 - \frac{13}{x+3}$$

HOW TO 3Divide: $\frac{6 - 6x^2 + 4x^3}{2x + 3}$

Arrange the terms in descending order. Note that there is no term containing x in $4x^3 - 6x^2 + 6$. Insert a zero for the missing term so that like terms will be in the same columns.

$$\begin{array}{r} 2x^2 - 6x + 9 \\ 2x+3 \overline{)4x^3-6x^2+0x+6} \\ \underline{4x^3+6x^2} \\ -12x^2+0x \\ \underline{-12x^2-18x} \\ 18x+6 \\ \underline{18x+27} \\ -21 \end{array}$$

$$\frac{4x^3 - 6x^2 + 6}{2x + 3} = 2x^2 - 6x + 9 - \frac{21}{2x + 3}$$

EXAMPLE • 2

Divide: $\frac{12x^2 - 11x + 10}{4x - 5}$

Solution

$$\begin{array}{r}
 3x + 1 \\
 4x - 5 \overline{) 12x^2 - 11x + 10} \\
 \underline{12x^2 - 15x} \\
 4x + 10 \\
 \underline{4x - 5} \\
 15
 \end{array}$$

$$\frac{12x^2 - 11x + 10}{4x - 5} = 3x + 1 + \frac{15}{4x - 5}$$

YOU TRY IT • 2

Divide: $\frac{15x^2 + 17x - 20}{3x + 4}$

Your solution**EXAMPLE • 3**

Divide: $\frac{x^3 + 1}{x + 1}$

Solution

$$\begin{array}{r}
 x^2 - x + 1 \\
 x + 1 \overline{) x^3 + 0x^2 + 0x + 1} \\
 \underline{x^3 + x^2} \\
 -x^2 + 0x \\
 \underline{-x^2 - x} \\
 x + 1 \\
 \underline{x + 1} \\
 0
 \end{array}$$

• **Insert zeros for the missing terms.**

$$\frac{x^3 + 1}{x + 1} = x^2 - x + 1$$

YOU TRY IT • 3

Divide: $\frac{3x^3 + 8x^2 - 6x + 2}{3x - 1}$

Your solution**EXAMPLE • 4**

Divide: $(2x^4 - 7x^3 + 3x^2 + 4x - 5) \div (x^2 - 2x - 2)$

Solution

$$\begin{array}{r}
 2x^2 - 3x + 1 \\
 x^2 - 2x - 2 \overline{) 2x^4 - 7x^3 + 3x^2 + 4x - 5} \\
 \underline{2x^4 - 4x^3 - 4x^2} \\
 -3x^3 + 7x^2 + 4x \\
 \underline{-3x^3 + 6x^2 + 6x} \\
 x^2 - 2x - 5 \\
 \underline{x^2 - 2x - 2} \\
 -3
 \end{array}$$

$$\begin{aligned}
 (2x^4 - 7x^3 + 3x^2 + 4x - 5) \div (x^2 - 2x - 2) \\
 = 2x^2 - 3x + 1 - \frac{3}{x^2 - 2x - 2}
 \end{aligned}$$

YOU TRY IT • 4

Divide: $(3x^4 - 11x^3 + 16x^2 - 16x + 8) \div (x^2 - 3x + 2)$

Your solution

OBJECTIVE C**To divide polynomials by using synthetic division****Tips for Success**

An important element of success is practice. We cannot do anything well if we do not practice it repeatedly. Practice is crucial to success in mathematics. In this objective you are learning a new procedure, synthetic division. You will need to practice this procedure in order to be successful at it.

Synthetic division is a shorter method of dividing a polynomial by a binomial of the form $x - a$.

Divide $(3x^2 - 4x + 6) \div (x - 2)$ by using long division.

$$\begin{array}{r} 3x + 2 \\ x - 2 \overline{) 3x^2 - 4x + 6} \\ \underline{3x^2 - 6x} \\ 2x + 6 \\ \underline{2x - 4} \\ 10 \end{array}$$

$$(3x^2 - 4x + 6) \div (x - 2) = 3x + 2 + \frac{10}{x - 2}$$

The variables can be omitted because the position of a term indicates the power of the term.

$$\begin{array}{r} 3 2 \\ -2 \overline{) 3 -4 6} \\ 3 -6 \\ 2 6 \\ 2 -4 \\ 10 \end{array}$$

Each number shown in color is exactly the same as the number above it. Removing the colored numbers condenses the vertical spacing.

$$\begin{array}{r} 3 2 \\ -2 \overline{) 3 -4 6} \\ -6 -4 \\ 2 10 \end{array}$$

The number in color in the top row is the same as the one in the bottom row. Writing the 3 from the top row in the bottom row allows the spacing to be condensed even further.

-2	3	-4	6
	-6	-4	
	3	2	10
	Terms of the quotient		Remainder

Because the degree of the dividend $(3x^2 - 4x + 6)$ is 2 and the degree of the divisor $(x - 2)$ is 1, the degree of the quotient is $2 - 1 = 1$. This means that, using the terms of the quotient given above, that quotient is $3x + 2$. The remainder is 10.

In general, the degree of the quotient of two polynomials is the difference between the degree of the dividend and the degree of the divisor.

By replacing the constant term in the divisor by its additive inverse, we may add rather than subtract terms. This is illustrated in the following example.

HOW TO 4 Divide: $(3x^3 + 6x^2 - x - 2) \div (x + 3)$

The additive inverse of the binomial constant

$$\begin{array}{r}
 \downarrow \qquad \qquad \text{Coefficients of the polynomial} \\
 \begin{array}{r|rrrr}
 -3 & 3 & 6 & -1 & -2 \\
 & \downarrow & & & \\
 & 3 & & & \\
 \hline
 & & & & \\
 & 3 & & & \\
 \hline
 & & & & \\
 & 3 & -9 & & \\
 \hline
 & & & & \\
 & 3 & -3 & & \\
 \hline
 & & & & \\
 & 3 & -9 & 9 & \\
 \hline
 & & & & \\
 & 3 & -3 & 8 & \\
 \hline
 & & & & \\
 & 3 & -9 & 9 & -24 \\
 \hline
 & & & & \\
 & 3 & -3 & 8 & -26
 \end{array}
 \end{array}$$

Terms of the quotient
Remainder

• Bring down the 3.

• Multiply $-3(3)$ and add the product to 6.• Multiply $-3(-3)$ and add the product to -1 .• Multiply $-3(8)$ and add the product to -2 .

The degree of the dividend is 3 and the degree of the divisor is 1. Therefore, the degree of the quotient is $3 - 1 = 2$.

$$(3x^3 + 6x^2 - x - 2) \div (x + 3) = 3x^2 - 3x + 8 - \frac{26}{x + 3}$$

HOW TO 5 Divide: $(2x^3 - x + 2) \div (x - 2)$

The additive inverse of the binomial constant

$$\begin{array}{r}
 \downarrow \qquad \qquad \text{Coefficients of the polynomial} \\
 \begin{array}{r|rrrr}
 2 & 2 & 0 & -1 & 2 \\
 & \downarrow & & & \\
 & 2 & & & \\
 \hline
 & & & & \\
 & 2 & & & \\
 \hline
 & & & & \\
 & 2 & 0 & -1 & 2 \\
 & & 4 & & \\
 \hline
 & & & & \\
 & 2 & 4 & & \\
 \hline
 & & & & \\
 & 2 & 4 & 8 & \\
 \hline
 & & & & \\
 & 2 & 4 & 7 & \\
 \hline
 & & & & \\
 & 2 & 4 & 7 & 14 \\
 \hline
 & & & & \\
 & 2 & 4 & 7 & 16
 \end{array}
 \end{array}$$

Terms of the quotient
Remainder

• Insert a 0 for the missing term and bring down the 2.

• Multiply $2(2)$ and add the product to 0.• Multiply $2(4)$ and add the product to -1 .• Multiply $2(7)$ and add the product to 2.

$$(2x^3 - x + 2) \div (x - 2) = 2x^2 + 4x + 7 + \frac{16}{x - 2}$$

EXAMPLE • 5Divide: $(7 - 3x + 5x^2) \div (x - 1)$ **Solution**Arrange the coefficients in decreasing powers of x .

$$\begin{array}{r|rrr} 1 & 5 & -3 & 7 \\ & & 5 & 2 \\ \hline & 5 & 2 & 9 \end{array}$$

$$(5x^2 - 3x + 7) \div (x - 1) = 5x + 2 + \frac{9}{x - 1}$$

YOU TRY IT • 5Divide: $(6x^2 + 8x - 5) \div (x + 2)$ **Your solution****EXAMPLE • 6**Divide: $(2x^3 + 4x^2 - 3x + 12) \div (x + 4)$ **Solution**

$$\begin{array}{r|rrrr} -4 & 2 & 4 & -3 & 12 \\ & & -8 & 16 & -52 \\ \hline & 2 & -4 & 13 & -40 \end{array}$$

$$\begin{aligned} (2x^3 + 4x^2 - 3x + 12) \div (x + 4) \\ = 2x^2 - 4x + 13 - \frac{40}{x + 4} \end{aligned}$$

YOU TRY IT • 6Divide: $(5x^3 - 12x^2 - 8x + 16) \div (x - 2)$ **Your solution****EXAMPLE • 7**Divide: $(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$ **Solution**

Insert a zero for the missing term.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -8 & 2 & 1 \\ & & -6 & 12 & -8 & 12 \\ \hline & 3 & -6 & 4 & -6 & 13 \end{array}$$

$$\begin{aligned} (3x^4 - 8x^2 + 2x + 1) \div (x + 2) \\ = 3x^3 - 6x^2 + 4x - 6 + \frac{13}{x + 2} \end{aligned}$$

YOU TRY IT • 7Divide: $(2x^4 - 3x^3 - 8x^2 - 2) \div (x - 3)$ **Your solution**

Solutions on p. S16

OBJECTIVE D**To evaluate a polynomial function using synthetic division**

A polynomial can be evaluated by using synthetic division. Consider the polynomial $P(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 1$. One way to evaluate the polynomial when $x = 2$ is to replace x by 2 and then simplify the numerical expression.

$$P(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 1$$

$$\begin{aligned} P(2) &= 2(2)^4 - 3(2)^3 + 4(2)^2 - 5(2) + 1 \\ &= 2(16) - 3(8) + 4(4) - 5(2) + 1 \\ &= 32 - 24 + 16 - 10 + 1 \\ &= 15 \end{aligned}$$

Now use synthetic division to divide $(2x^4 - 3x^3 + 4x^2 - 5x + 1) \div (x - 2)$.

2	2	-3	4	-5	1
		4	2	12	14
	2	1	6	7	15

Terms of the quotient
Remainder

Note that the remainder is 15, which is the same value as $P(2)$. This is not a coincidence. The following theorem states that this situation is always true.

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

HOW TO • 6

Evaluate $P(x) = x^4 - 3x^2 + 4x - 5$ when $x = -2$ by using the Remainder Theorem.

↓ The value at which the polynomial is evaluated

-2	1	0	-3	4	-5
		-2	4	-2	-4
	1	-2	1	2	-9

• A 0 is inserted for the x^3 term.

← The remainder

$$P(-2) = -9$$

EXAMPLE • 8

Evaluate $P(x) = x^2 - 6x + 4$ when $x = 3$ by using the Remainder Theorem.

Solution

3	1	-6	4
		3	-9
	1	-3	-5

$$P(3) = -5$$

EXAMPLE • 9

Evaluate $P(x) = -x^4 + 3x^3 + 2x^2 - x - 5$ when $x = -2$ by using the Remainder Theorem.

Solution

-2	-1	3	2	-1	-5
		2	-10	16	-30
	-1	5	-8	15	-35

$$P(-2) = -35$$

YOU TRY IT • 8

Evaluate $P(x) = 2x^2 - 3x - 5$ when $x = 2$ by using the Remainder Theorem.

Your solution

YOU TRY IT • 9

Evaluate $P(x) = 2x^3 - 5x^2 + 7$ when $x = -3$ by using the Remainder Theorem.

Your solution

5.4 EXERCISES**OBJECTIVE A****To divide a polynomial by a monomial**

For Exercises 1 to 12, divide and check.

1. $\frac{3x^2 - 6x}{3x}$

2. $\frac{10y^2 - 6y}{2y}$

3. $\frac{5x^2 - 10x}{-5x}$

4. $\frac{3y^2 - 27y}{-3y}$

5. $\frac{5x^2y^2 + 10xy}{5xy}$

6. $\frac{8x^2y^2 - 24xy}{8xy}$

7. $\frac{x^3 + 3x^2 - 5x}{x}$

8. $\frac{a^3 - 5a^2 + 7a}{a}$

9. $\frac{9b^5 + 12b^4 + 6b^3}{3b^2}$

10. $\frac{a^8 - 5a^5 - 3a^3}{a^2}$

11. $\frac{a^5b - 6a^3b + ab}{ab}$

12. $\frac{5c^3d + 10c^2d^2 - 15cd^3}{5cd}$



13. If $\frac{P(x)}{3x} = 2x^2 + 7x - 5$, what is $P(x)$?



14. If $\frac{6x^3 + 15x^2 - 24x}{ax} = 2x^2 + 5x - 8$, what is the value of a ?

OBJECTIVE B**To divide polynomials**

For Exercises 15 to 36, divide by using long division.

15. $(x^2 + 3x - 40) \div (x - 5)$

16. $(x^2 - 14x + 24) \div (x - 2)$

17. $(x^3 - 3x + 2) \div (x - 3)$

18. $(x^3 + 4x^2 - 8) \div (x + 4)$

19. $(6x^2 + 13x + 8) \div (2x + 1)$

20. $(12x^2 + 13x - 14) \div (3x - 2)$

21. $(10x^2 + 9x - 5) \div (2x - 1)$

22. $(18x^2 - 3x + 2) \div (3x + 2)$

23. $(8x^3 - 9) \div (2x - 3)$

24. $(64x^3 + 4) \div (4x + 2)$

25. $(6x^4 - 13x^2 - 4) \div (2x^2 - 5)$

26. $(12x^4 - 11x^2 + 10) \div (3x^2 + 1)$

27. $\frac{-10 - 33x + 3x^3 - 8x^2}{3x + 1}$

28. $\frac{10 - 49x + 38x^2 - 8x^3}{1 - 4x}$

29. $\frac{x^3 - 5x^2 + 7x - 4}{x - 3}$

30. $\frac{2x^3 - 3x^2 + 6x + 4}{2x + 1}$

31. $\frac{16x^2 - 13x^3 + 2x^4 + 20 - 9x}{x - 5}$

32. $\frac{x - x^2 + 5x^3 + 3x^4 - 2}{x + 2}$

33. $\frac{2x^3 + 4x^2 - x + 2}{x^2 + 2x - 1}$

34. $\frac{3x^3 - 2x^2 + 5x - 4}{x^2 - x + 3}$

35. $\frac{x^4 + 2x^3 - 3x^2 - 6x + 2}{x^2 - 2x - 1}$

36. $\frac{x^4 - 3x^3 + 4x^2 - x + 1}{x^2 + x - 3}$

37. Given $Q(x) = 2x + 1$ and $P(x) = 2x^3 + x^2 + 8x + 7$, find $\frac{P(x)}{Q(x)}$.

38. Given $Q(x) = 3x - 2$ and $P(x) = 3x^3 - 2x^2 + 3x - 5$, find $\frac{P(x)}{Q(x)}$.



39. True or false? When a tenth-degree polynomial is divided by a second-degree polynomial, the quotient is a fifth-degree polynomial.



40. Let $p(x)$ be a polynomial of degree 5, and let $q(x)$ be a polynomial of degree 2. If $r(x)$ is the remainder of $\frac{p(x)}{q(x)}$, is the degree of $r(x)$ (i) greater than 5, (ii) between 2 and 5, or (iii) less than 2?

OBJECTIVE C**To divide polynomials by using synthetic division**

41. The display below shows the beginning of a synthetic division. What is the degree of the dividend?

4	2	-5	4	1



42. For the synthetic division shown below, what is the quotient and what is the remainder?

-2	3	7	4	-3
	-6	-2	-4	
	3	1	2	-7

For Exercises 43 to 58, divide by using synthetic division.

43. $(2x^2 - 6x - 8) \div (x + 1)$

44. $(3x^2 + 19x + 20) \div (x + 5)$

45. $(3x^2 - 14x + 16) \div (x - 2)$

46. $(4x^2 - 23x + 28) \div (x - 4)$

47. $(3x^2 - 4) \div (x - 1)$

48. $(4x^2 - 8) \div (x - 2)$

49. $(2x^3 - x^2 + 6x + 9) \div (x + 1)$

50. $(3x^3 + 10x^2 + 6x - 4) \div (x + 2)$

51. $(18 + x - 4x^3) \div (2 - x)$

52. $(12 - 3x^2 + x^3) \div (x + 3)$

53. $(2x^3 + 5x^2 - 5x + 20) \div (x + 4)$

54. $(5x^3 + 3x^2 - 17x + 6) \div (x + 2)$

55.
$$\frac{5 + 5x - 8x^2 + 4x^3 - 3x^4}{2 - x}$$

56.
$$\frac{3 - 13x - 5x^2 + 9x^3 - 2x^4}{3 - x}$$

57.
$$\frac{3x^4 + 3x^3 - x^2 + 3x + 2}{x + 1}$$

58.
$$\frac{4x^4 + 12x^3 - x^2 - x + 2}{x + 3}$$

59. Given $Q(x) = x - 2$ and $P(x) = 3x^2 - 5x + 6$, find $\frac{P(x)}{Q(x)}$.

60. Given $Q(x) = x + 5$ and $P(x) = 2x^2 + 7x - 12$, find $\frac{P(x)}{Q(x)}$.

OBJECTIVE D**To evaluate a polynomial function using synthetic division**

61. The result of a synthetic division is shown below. What is a first-degree polynomial factor of the dividend $p(x)$?

3	1	1	-7	-15
		3	12	15
	1	4	5	0



62. The result of a synthetic division of $p(x)$ is shown below. What is $p(-3)$?

-3	1	2	-7	-4
		-3	3	12
	1	-1	-4	8

For Exercises 63 to 80, use the Remainder Theorem to evaluate the polynomial function.

63. $P(x) = 2x^2 - 3x - 1; P(3)$

64. $Q(x) = 3x^2 - 5x - 1; Q(2)$

65. $R(x) = x^3 - 2x^2 + 3x - 1; R(4)$

66. $F(x) = x^3 + 4x^2 - 3x + 2; F(3)$

67. $P(z) = 2z^3 - 4z^2 + 3z - 1; P(-2)$

68. $R(t) = 3t^3 + t^2 - 4t + 2; R(-3)$

69. $Z(p) = 2p^3 - p^2 + 3; Z(-3)$

70. $P(y) = 3y^3 + 2y^2 - 5; P(-2)$

71. $Q(x) = x^4 + 3x^3 - 2x^2 + 4x - 9; Q(2)$

72. $Y(z) = z^4 - 2z^3 - 3z^2 - z + 7; Y(3)$

73. $F(x) = 2x^4 - x^3 + 2x - 5; F(-3)$

74. $Q(x) = x^4 - 2x^3 + 4x - 2; Q(-2)$

75. $P(x) = x^3 - 3; P(5)$

76. $S(t) = 4t^3 + 5; S(-4)$

77. $R(t) = 4t^4 - 3t^2 + 5; R(-3)$

78. $P(z) = 2z^4 + z^2 - 3; P(-4)$

79. $Q(x) = x^5 - 4x^3 - 2x^2 + 5x - 2; Q(2)$

80. $R(x) = 2x^5 - x^3 + 4x - 1; R(-2)$

Applying the Concepts

81. Divide by using long division.

a. $\frac{a^3 + b^3}{a + b}$

b. $\frac{x^5 + y^5}{x + y}$

c. $\frac{x^6 - y^6}{x + y}$

82. For what value of k will the remainder be zero?

a. $(x^3 - x^2 - 3x + k) \div (x + 3)$

b. $(2x^3 - x + k) \div (x - 1)$



83. Show how synthetic division can be modified so that the divisor can be of the form $ax + b$.

SECTION

5.5

Factoring Polynomials

OBJECTIVE A

To factor a monomial from a polynomial



One number is a **factor** of another when it can be divided into that other number with a remainder of zero.

The greatest common factor (GCF) of two or more monomials is the product of the common factors with the smallest exponents.

$$16a^4b = 2^4a^4b$$

$$40a^2b^5 = 2^3 \cdot 5a^2b^5$$

$$\text{GCF} = 2^3a^2b = 8a^2b$$

Note that the exponent on each variable in the GCF is the same as the smallest exponent on the variable in either of the monomials.

Factoring a polynomial means writing the polynomial as a product of other polynomials.

In the example at the right, $3x$ is the GCF of the terms $3x^2$ and $6x$. $3x$ is a **common monomial factor** of the terms of the binomial. $x - 2$ is a **binomial factor** of $3x^2 - 6x$.



HOW TO • 1

Factor: $4x^3y^2 + 12x^3y + 20xy^3$

The GCF of $4x^3y^2$, $12x^3y$, and $20xy^3$ is $4xy$.

$$\begin{aligned} 4x^3y^2 + 12x^3y + 20xy^3 &= 4xy \left(\frac{4x^3y^2}{4xy} + \frac{12x^3y}{4xy} + \frac{20xy^3}{4xy} \right) \\ &= 4xy(x^2y + 3x^2 + 5y^2) \end{aligned}$$

- Find the GCF of the terms of the polynomial.
- Factor the GCF from each term of the polynomial. Think of this as dividing each term of the polynomial by the GCF.

EXAMPLE • 1

Factor: $4x^2y^2 - 6xy^2 + 12xy^3$

Solution

The GCF of $4x^2y^2$, $6xy^2$, and $12xy^3$ is $2xy^2$.

$$4x^2y^2 - 6xy^2 + 12xy^3 = 2xy^2(2x - 3 + 6y)$$

YOU TRY IT • 1

Factor: $3x^3y - 6x^2y^2 - 3xy^3$

Your solution

EXAMPLE • 2Factor: $x^{2n} + x^{n+1} + x^n$ **Solution**The GCF of x^{2n} , x^{n+1} , and x^n is x^n .

$$x^{2n} + x^{n+1} + x^n = x^n(x^n + x + 1)$$

YOU TRY IT • 2Factor: $6t^{2n} - 9t^n$ **Your solution**

Solution on p. S17

OBJECTIVE B**To factor by grouping**

In the examples at the right, the binomials in parentheses are binomial factors.

$$4x^4(2x - 3) \\ -2r^2s(5r + 2s)$$

The Distributive Property is used to factor a common binomial factor from an expression.

HOW TO • 2Factor: $4a(2b + 3) - 5(2b + 3)$ The common binomial factor is $(2b + 3)$. Use the Distributive Property to write the expression as a product of factors.

$$4a(2b + 3) - 5(2b + 3) = (2b + 3)(4a - 5)$$

Consider the binomial $y - x$. Factoring -1 from this binomial gives

$$y - x = -(x - y)$$

This equation is used to factor a common binomial from an expression.

 Take Note

For the simplification at the right,

$$\begin{aligned} 6r(r - s) - 7(s - r) \\ = 6r(r - s) - 7[(-1)(r - s)] \\ = 6r(r - s) + 7(r - s) \end{aligned}$$

HOW TO • 3Factor: $6r(r - s) - 7(s - r)$

$$\begin{aligned} 6r(r - s) - 7(s - r) &= 6r(r - s) + 7(r - s) & \bullet s - r &= -(r - s) \\ &= (r - s)(6r + 7) \end{aligned}$$

Some polynomials can be factored by grouping terms so that a common binomial factor is found. This is called **factoring by grouping**.**HOW TO • 4**Factor: $3xz - 4yz - 3xa + 4ya$

$$\begin{aligned} 3xz - 4yz - 3xa + 4ya \\ = (3xz - 4yz) - (3xa - 4ya) \end{aligned}$$

$$\begin{aligned} &= z(3x - 4y) - a(3x - 4y) \\ &= (3x - 4y)(z - a) \end{aligned}$$

- Group the first two terms and the last two terms. Note that $-3xa + 4ya = -(3xa - 4ya)$.
- Factor the GCF from each group.
- Write the expression as the product of factors.

HOW TO 5 Factor: $8y^2 + 4y - 6ay - 3a$

$$\begin{aligned} 8y^2 + 4y - 6ay - 3a \\ = (8y^2 + 4y) - (6ay + 3a) \end{aligned}$$

$$\begin{aligned} &= 4y(2y + 1) - 3a(2y + 1) \\ &= (2y + 1)(4y - 3a) \end{aligned}$$

- Group the first two terms and the last two terms. Note that $-6ay - 3a = -(6ay + 3a)$.
- Factor the GCF from each group.
- Write the expression as a product of factors.

EXAMPLE 3

Factor: $x^2(5y - 2) - 7(5y - 2)$

Solution

$$\begin{aligned} x^2(5y - 2) - 7(5y - 2) \\ = (5y - 2)(x^2 - 7) \end{aligned}$$

- The common binomial factor is $(5y - 2)$.

YOU TRY IT 3

Factor: $3(6x - 7y) - 2x^2(6x - 7y)$

Your solution**EXAMPLE 4**

Factor: $15x^2 + 6x - 5xz - 2z$

Solution

$$\begin{aligned} 15x^2 + 6x - 5xz - 2z \\ = (15x^2 + 6x) - (5xz + 2z) \\ = 3x(5x + 2) - z(5x + 2) \\ = (5x + 2)(3x - z) \end{aligned}$$

YOU TRY IT 4

Factor: $4a^2 - 6a - 6ax + 9x$

Your solution

Solutions on p. S17

OBJECTIVE C**To factor a trinomial of the form $x^2 + bx + c$** 

A **quadratic trinomial** is a trinomial of the form $ax^2 + bx + c$, where a , b , and c are nonzero integers. The degree of a quadratic trinomial is 2. Here are examples of quadratic trinomials:

$$\begin{aligned} 4x^2 - 3x - 7 \\ (a = 4, b = -3, c = -7) \end{aligned}$$

$$\begin{aligned} z^2 + z + 10 \\ (a = 1, b = 1, c = 10) \end{aligned}$$

$$\begin{aligned} 2y^2 + 4y - 9 \\ (a = 2, b = 4, c = -9) \end{aligned}$$

Factoring a quadratic trinomial means expressing the trinomial as the product of two binomials. For example,

Trinomial		Factored Form
$2x^2 - x - 1$	=	$(2x + 1)(x - 1)$
$y^2 - 3y + 2$	=	$(y - 1)(y - 2)$

In this objective, trinomials of the form $x^2 + bx + c$ ($a = 1$) will be factored. The next objective deals with trinomials in which $a \neq 1$.

The method by which factors of a trinomial are found is based on FOIL. Consider the following binomial products, noting the relationship between the constant terms of the binomials and the terms of the trinomial.

sum of binomial constants	product of binomial constants
↓	↓

$$\begin{array}{rcl}
 & \text{F} & \text{O} & \text{I} & \text{L} & & \\
 (x + 4)(x + 5) & = & x \cdot x & + & 5x & + & 4x & + & 4 \cdot 5 & = & x^2 + 9x + 20 \\
 (x - 6)(x + 8) & = & x \cdot x & + & 8x & - & 6x & + & (-6) \cdot 8 & = & x^2 + 2x - 48 \\
 (x - 3)(x - 2) & = & x \cdot x & - & 2x & - & 3x & + & (-3)(-2) & = & x^2 - 5x + 6
 \end{array}$$

Observe two important points from these examples.

1. The constant term of the trinomial is the *product* of the constant terms of the binomials. The coefficient of x in the trinomial is the *sum* of the constant terms of the binomials.
2. When the constant term of the trinomial is positive, the constant terms of the binomials have the *same* sign. When the constant term of the trinomial is negative, the constant terms of the binomials have *opposite* signs.

HOW TO • 6 Factor: $x^2 - 7x + 12$

The constant term is positive. The signs of the binomial constants will be the same.

Find two negative factors of 12 whose sum is -7 .

Write the trinomial in factored form.

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

$$\begin{aligned}
 \text{Check: } (x - 3)(x - 4) &= x^2 - 4x - 3x + 12 \\
 &= x^2 - 7x + 12
 \end{aligned}$$

Negative Factors of 12	Sum
-1, -12	-13
-2, -6	-8
-3, -4	-7

Take Note

You can always check a proposed factorization by multiplying all the factors.

HOW TO • 7 Factor: $y^2 + 7y - 18$

The constant term is negative. The signs of the binomial constants will be opposite.

Find two factors of -18 that have opposite signs and whose sum is 7. All of the possible factors are shown at the right. In practice, once the correct pair is found, the remaining choices need not be checked.

Write the trinomial in factored form.

$$y^2 + 7y - 18 = (y - 2)(y + 9)$$

$$\begin{aligned}
 \text{Check: } (y - 2)(y + 9) &= y^2 + 9y - 2y - 18 \\
 &= y^2 + 7y - 18
 \end{aligned}$$

Factors of -18	Sum
-1, 18	17
1, -18	-17
-2, 9	7
2, -9	-7
-3, 6	3
3, -6	-3

Take Note

It is important to check proposed factorizations. For instance, we might have tried $(a + 2)(a - 5b^2)$. However,

$$(a + 2)(a - 5b^2) = a^2 - 5ab^2 + 2a - 10b^2$$

The first and last terms are correct but the middle term is not correct.

HOW TO • 8 Factor: $a^2 - 3ab - 10b^2$

The term $-10b^2$ is negative. Find two factors of -10 whose sum is -3 , the coefficient of ab . From the table, the numbers are 2 and -5 . Because the last term is $-10b^2$, we use $2b$ and $-5b$. The product of these terms is $2b(-5b) = -10b^2$.

$$a^2 - 3ab - 10b^2 = (a + 2b)(a - 5b)$$

$$\begin{aligned}
 \text{Check: } (a + 2b)(a - 5b) &= a^2 - 5ab + 2ab - 10b^2 \\
 &= a^2 - 3ab - 10b^2
 \end{aligned}$$

Factors of -10	Sum
-1, 10	9
1, -10	-9
-2, 5	3
2, -5	-3

When only integers are used, some trinomials do not factor. For example, to factor $x^2 + 11x + 5$, it would be necessary to find two positive integers whose product is 5 and whose sum is 11. This is not possible, because the only positive factors of 5 are 1 and 5, and the sum of 1 and 5 is 6. The polynomial $x^2 + 11x + 5$ is a **prime polynomial**. Such a polynomial is said to be **nonfactorable over the integers**. Binomials of the form $x + a$ or $x - a$ are also prime polynomials.

EXAMPLE • 5Factor: $x^2 - 5x - 6$ **Solution**The factors of -6 whose sum is -5 are -6 and 1 .

$$x^2 - 5x - 6 = (x - 6)(x + 1)$$

YOU TRY IT • 5Factor: $x^2 - x - 20$ **Your solution****EXAMPLE • 6**Factor: $10 - 3x - x^2$ **Solution**When the coefficient of x^2 is -1 , factoring -1 from the trinomial may make factoring the trinomial easier.

$$\begin{aligned} 10 - 3x - x^2 &= -x^2 - 3x + 10 \\ &= -(x^2 + 3x - 10) \\ &= -(x - 2)(x + 5) \end{aligned}$$

YOU TRY IT • 6Factor: $x^2 + 5xy + 6y^2$ **Your solution**

Solutions on p. S17

OBJECTIVE D**To factor $ax^2 + bx + c$**

There are various methods of factoring trinomials of the form $ax^2 + bx + c$, where $a \neq 1$. Factoring by using trial factors and factoring by grouping will be discussed in this objective. Factoring by using trial factors is illustrated first.

To use the trial factor method, use the factors of a and the factors of c to write all of the possible binomial factors of the trinomial. Then use FOIL to determine the correct factorization. To reduce the number of trial factors that must be considered, remember the following.

1. Use the signs of the constant term and the coefficient of x in the trinomial to determine the signs of the binomial factors. If the constant term is positive, the signs of the binomial factors will be the same as the sign of the coefficient of x in the trinomial. If the sign of the constant term is negative, the constant terms in the binomials will have opposite signs.
2. If the terms of the trinomial do not have a common factor, then the terms in either one of the binomial factors will not have a common factor.

 Take Note

Observe that when testing trial factors, it is important to test all possibilities. For the example at the right, note that we tried

$(x - 1)(3x - 4)$ and
 $(x - 4)(3x - 1)$.

HOW TO • 9 Factor: $3x^2 - 8x + 4$

The terms have no common factor.

The constant term is positive.

The coefficient of x is negative.

The binomial constants will be negative.

Write trial factors. Use the *Outer* and *Inner* products of FOIL to determine the middle term of the trinomial.

Write the trinomial in factored form.

<i>Positive Factors of 3 (coefficient of x^2)</i>	<i>Negative Factors of 4 (constant term)</i>
1, 3	-1, -4 -2, -2

<i>Trial Factors</i>	<i>Middle Term</i>
$(x - 1)(3x - 4)$	$-4x - 3x = -7x$
$(x - 4)(3x - 1)$	$-x - 12x = -13x$
$(x - 2)(3x - 2)$	$-2x - 6x = -8x$

$$3x^2 - 8x + 4 = (x - 2)(3x - 2)$$

HOW TO • 10 Factor: $5 - 3x - 2x^2$

Factor -1 from the trinomial:

$$5 - 3x - 2x^2 = -(2x^2 + 3x - 5).$$

The constant term, -5 , is negative; the signs of the binomial constants will be opposites.

Write trial factors. Use the *Outer* and *Inner* products of FOIL to determine the middle term of the trinomial.

Write the trinomial in factored form.

<i>Positive Factors of 2 (coefficient of x^2)</i>	<i>Factors of -5 (constant term)</i>
1, 2	1, -5 -1, 5

<i>Trial Factors</i>	<i>Middle Term</i>
$(x + 1)(2x - 5)$	$-5x + 2x = -3x$
$(x - 5)(2x + 1)$	$x - 10x = -9x$
$(x - 1)(2x + 5)$	$5x - 2x = 3x$
$(x + 5)(2x - 1)$	$-x + 10x = 9x$

$$5 - 3x - 2x^2 = -(x - 1)(2x + 5)$$

HOW TO • 11 Factor: $10y^3 + 44y^2 - 30y$

The GCF is $2y$. Factor the GCF from the terms.

Factor the trinomial
 $5y^2 + 22y - 15$.

The constant term is negative.

The binomial constants will have opposite signs.

Write trial factors. Use the *Outer* and *Inner* products of FOIL to determine the middle term of the trinomial.

It is not necessary to test trial factors that have a common factor.

Write the trinomial in factored form.

$$10y^3 + 44y^2 - 30y = 2y(5y^2 + 22y - 15)$$

<i>Positive Factors of 5 (coefficient of y^2)</i>	<i>Factors of -15 (constant term)</i>
1, 5	-1, 15 1, -15 -3, 5 3, -5

<i>Trial Factors</i>	<i>Middle Term</i>
$(y - 1)(5y + 15)$	common factor
$(y + 15)(5y - 1)$	$-y + 75y = 74y$
$(y + 1)(5y - 15)$	common factor
$(y - 15)(5y + 1)$	$y - 75y = -74y$
$(y - 3)(5y + 5)$	common factor
$(y + 5)(5y - 3)$	$-3y + 25y = 22y$
$(y + 3)(5y - 5)$	common factor
$(y - 5)(5y + 3)$	$3y - 25y = -22y$

$$10y^3 + 44y^2 - 30y = 2y(y + 5)(5y - 3)$$

**Take Note**

Either method of factoring discussed in this objective will always lead to a correct factorization of trinomials of the form $ax^2 + bx + c$ that are not prime polynomials.

For previous examples, all the trial factors were listed. Once the correct factors have been found, however, the remaining trial factors can be omitted.

Trinomials of the form $ax^2 + bx + c$ can also be factored by grouping. This method is an extension of the method discussed in the preceding objective.

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b . Then use factoring by grouping to write the factorization of the trinomial.

For the trinomial $3x^2 + 11x + 8$, $a = 3$, $b = 11$, and $c = 8$. To find two factors of $a \cdot c$ whose sum is b , first find the product $a \cdot c$ ($a \cdot c = 3 \cdot 8 = 24$). Then find two factors of 24 whose sum is 11. (3 and 8 are two factors of 24 whose sum is 11.)

HOW TO • 12 Factor: $3x^2 + 11x + 8$

Find two positive factors of 24 ($ac = 3 \cdot 8$) whose sum is 11, the coefficient of x .

Positive Factors of 24	Sum
1, 24	25
2, 12	14
3, 8	11

The required sum has been found. The remaining factors need not be checked.

Use the factors of 24 whose sum is 11 to write $11x$ as $3x + 8x$.

Factor by grouping.

$$\begin{aligned}
 3x^2 + 11x + 8 &= 3x^2 + 3x + 8x + 8 \\
 &= (3x^2 + 3x) + (8x + 8) \\
 &= 3x(x + 1) + 8(x + 1) \\
 &= (x + 1)(3x + 8)
 \end{aligned}$$

Check: $(x + 1)(3x + 8) = 3x^2 + 8x + 3x + 8 = 3x^2 + 11x + 8$

HOW TO • 13 Factor: $4z^2 - 17z - 21$

Find two factors of -84 [$ac = 4 \cdot (-21)$] whose sum is -17 , the coefficient of z .

Once the required sum is found, the remaining factors need not be checked.

Factors of -84	Sum
1, -84	-83
-1 , 84	83
2, -42	-40
-2 , 42	40
3, -28	-25
-3 , 28	25
4, -21	-17

$$4z^2 - 17z - 21 = 4z^2 + 4z - 21z - 21$$

Use the factors of -84 whose sum is -17 to write $-17z$ as $4z - 21z$.

Factor by grouping. Recall that $-21z - 21 = -(21z + 21)$.

$$\begin{aligned}
 &= (4z^2 + 4z) - (21z + 21) \\
 &= 4z(z + 1) - 21(z + 1) \\
 &= (z + 1)(4z - 21)
 \end{aligned}$$

Check: $(z + 1)(4z - 21) = 4z^2 - 21z + 4z - 21 = 4z^2 - 17z - 21$

HOW TO • 14 Factor: $3x^2 - 11x + 4$

Find two negative factors of 12 ($3 \cdot 4$) whose sum is -11 .

Negative Factors of 12	Sum
$-1, -12$	-13
$-2, -6$	-8
$-3, -4$	-7

Because no integer factors of 12 have a sum of -11 , $3x^2 - 11x + 4$ is **nonfactorable over the integers**. $3x^2 - 11x + 4$ is a prime polynomial over the integers.

EXAMPLE • 7

Factor: $6x^2 + 11x - 10$

Solution

$$6x^2 + 11x - 10 = (2x + 5)(3x - 2)$$

YOU TRY IT • 7

Factor: $4x^2 + 15x - 4$

Your solution**EXAMPLE • 8**

Factor: $12x^2 - 32x + 5$

Solution

$$12x^2 - 32x + 5 = (6x - 1)(2x - 5)$$

YOU TRY IT • 8

Factor: $10x^2 + 39x + 14$

Your solution**EXAMPLE • 9**

Factor: $30y + 2xy - 4x^2y$

Solution

The GCF of $30y$, $2xy$, and $4x^2y$ is $2y$.

$$\begin{aligned} 30y + 2xy - 4x^2y &= 2y(15 + x - 2x^2) \\ &= -2y(2x^2 - x - 15) \\ &= -2y(2x + 5)(x - 3) \end{aligned}$$

YOU TRY IT • 9

Factor: $3a^3b^3 + 3a^2b^2 - 60ab$

Your solution**EXAMPLE • 10**

Find two linear functions f and g such that $f(x) \cdot g(x) = 2x^2 + 9x - 5$.

Solution

We are looking for two linear functions that, when multiplied, equal $2x^2 + 9x - 5$. To find them, factor $2x^2 + 9x - 5$.

$$2x^2 + 9x - 5 = (2x - 1)(x + 5)$$

The two functions are $f(x) = 2x - 1$ and $g(x) = x + 5$.

YOU TRY IT • 10

Find two linear functions f and g such that $f(x) \cdot g(x) = 3x^2 - 17x - 6$.

Your solution

5.5 EXERCISES**OBJECTIVE A****To factor a monomial from a polynomial**

For Exercises 1 to 22, factor.

1. $6a^2 - 15a$

2. $32b^2 + 12b$

3. $4x^3 - 3x^2$

4. $12a^5b^2 + 16a^4b$

5. $3a^2 - 10b^3$

6. $9x^2 + 14y^4$

7. $x^5 - x^3 - x$

8. $y^4 - 3y^2 - 2y$

9. $16x^2 - 12x + 24$

10. $2x^5 + 3x^4 - 4x^2$

11. $5b^2 - 10b^3 + 25b^4$

12. $x^2y^4 - x^2y - 4x^2$

13. $x^{2n} - x^n$

14. $2a^{5n} + a^{2n}$

15. $x^{3n} - x^{2n}$

16. $y^{4n} + y^{2n}$

17. $a^{2n+2} + a^2$

18. $b^{n+5} - b^5$

19. $12x^2y^2 - 18x^3y + 24x^2y$

20. $14a^4b^4 - 42a^3b^3 + 28a^3b^2$

21. $24a^3b^2 - 4a^2b^2 - 16a^2b^4$

22. $10x^2y + 20x^2y^2 + 30x^2y^3$



23. If $\frac{p(x)}{3x} = 2x + 1$, what are the factors of $p(x)$?



24. If $m < n$, what is the GCF of $x^m + x^n$?

OBJECTIVE B**To factor by grouping**

For Exercises 25 to 45, factor.

25. $x(a + 2) - 2(a + 2)$

26. $3(x + y) + a(x + y)$

27. $a(x - 2) - b(2 - x)$

28. $3(a - 7) - b(7 - a)$

29. $x(a - 2b) + y(2b - a)$

30. $b(3 - 2c) - 5(2c - 3)$

31. $xy + 4y - 2x - 8$

32. $ab + 7b - 3a - 21$

33. $ax + bx - ay - by$

34. $2ax - 3ay - 2bx + 3by$

35. $x^2y - 3x^2 - 2y + 6$

36. $a^2b + 3a^2 + 2b + 6$

37. $6 + 2y + 3x^2 + x^2y$

38. $15 + 3b - 5a^2 - a^2b$

39. $2ax^2 + bx^2 - 4ay - 2by$

40. $4a^2x + 2a^2y - 6bx - 3by$

41. $6xb + 3ax - 4by - 2ay$

42. $a^2x - 3a^2y + 2x - 6y$

43. $x^n y - 5x^n + y - 5$

44. $a^n x^n + 2a^n + x^n + 2$

45. $2x^3 - x^2 + 4x - 2$

 46. Not all four-term expressions can be factored by grouping. Which expression(s) below can be factored by grouping?

(i) $xy + 6y + 3x - 18$ (ii) $xy - 6y - 3x - 18$ (iii) $xy + 6y - 3x - 18$

 47. a. Which of the following expressions are equivalent to $x^2 - x - 6$?
(i) $x^2 - 5x + 4x - 6$ (ii) $x^2 - 3x + 2x - 6$ (iii) $x^2 - 9x + 8x - 6$

b. Which expression in part (a) can be factored by grouping?

OBJECTIVE C**To factor a trinomial of the form $x^2 + bx + c$**

 48. If $x^2 + 3x - 18 = (x + a)(x + 6)$, what is the value of a ?

 49. If $x^2 + bx - 12$ factors over the integers, what are the possible values of b ?

For Exercises 50 to 76, factor.

50. $x^2 - 8x + 15$

51. $x^2 + 12x + 20$

52. $a^2 + 12a + 11$

53. $a^2 + a - 72$

54. $b^2 + 2b - 35$

55. $a^2 + 7a + 6$

56. $y^2 - 16y + 39$

57. $y^2 - 18y + 72$

58. $b^2 + 4b - 32$

59. $x^2 + x - 132$

60. $a^2 - 15a + 56$

61. $x^2 + 15x + 50$

62. $y^2 + 13y + 12$

63. $b^2 - 6b - 16$

64. $x^2 + 4x - 5$

65. $a^2 - 3ab + 2b^2$

66. $a^2 + 11ab + 30b^2$

67. $a^2 + 8ab - 33b^2$

68. $x^2 - 14xy + 24y^2$

69. $x^2 + 5xy + 6y^2$

70. $y^2 + 2xy - 63x^2$

71. $2 + x - x^2$

72. $21 - 4x - x^2$

73. $5 + 4x - x^2$

74. $50 + 5a - a^2$

75. $x^2 - 5x + 6$

76. $x^2 - 7x - 12$

OBJECTIVE D**To factor $ax^2 + bx + c$**

For Exercises 77 to 109, factor.

77. $2x^2 + 7x + 3$

78. $2x^2 - 11x - 40$

79. $6y^2 + 5y - 6$

80. $4y^2 - 15y + 9$

81. $6b^2 - b - 35$

82. $2a^2 + 13a + 6$

83. $3y^2 - 22y + 39$

84. $12y^2 - 13y - 72$

85. $6a^2 - 26a + 15$

86. $5x^2 + 26x + 5$

87. $4a^2 - a - 5$

88. $11x^2 - 122x + 11$

89. $10x^2 - 29x + 10$

90. $2x^2 + 5x + 12$

91. $4x^2 - 6x + 1$

92. $6x^2 + 5xy - 21y^2$

93. $6x^2 + 41xy - 7y^2$

94. $4a^2 + 43ab + 63b^2$

95. $7a^2 + 46ab - 21b^2$

96. $10x^2 - 23xy + 12y^2$

97. $18x^2 + 27xy + 10y^2$

98. $24 + 13x - 2x^2$

99. $6 - 7x - 5x^2$

100. $8 - 13x + 6x^2$

101. $30 + 17a - 20a^2$

102. $15 - 14a - 8a^2$

103. $35 - 6b - 8b^2$

104. $12y^3 + 22y^2 - 70y$

105. $5y^4 - 29y^3 + 20y^2$

106. $30a^2 + 85ab + 60b^2$

107. $20x^2 - 38x^3 - 30x^4$

108. $12x + x^2 - 6x^3$

109. $3y - 16y^2 + 16y^3$

 110. If $ax^2 + bx + c$ has no monomial factor, can either of the possible binomial factors have a monomial factor?

 111. Give an example of a polynomial of the form $ax^2 + bx + c$ that does not factor over the integers.

112. Find two linear functions f and g such that $f(x) \cdot g(x) = 2x^2 + 9x - 18$.

113. Find two linear functions f and h such that $f(x) \cdot h(x) = 2x^2 - 5x + 2$.

114. Find two linear functions F and G such that $F(x) \cdot G(x) = 2x^2 - 9x - 5$.

115. Find two linear functions f and g such that $f(a) \cdot g(a) = 3a^2 + 11a - 4$.

116. Find two linear functions f and g such that $f(b) \cdot g(b) = 4b^2 - 17b + 15$.

117. Find two linear functions f and g such that $f(x) \cdot g(x) = 2x^2 + 13x - 24$.

118. Find two linear functions f and h such that $f(x) \cdot h(x) = 4x^2 - 12x - 7$.

119. Find two linear functions g and h such that $g(x) \cdot h(x) = 6x^2 + 7x - 5$.

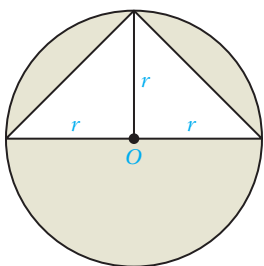
120. Find two linear functions f and g such that $f(x) \cdot g(x) = 4x^2 + 23x + 15$.

121. Find two linear functions f and g such that $f(t) \cdot g(t) = 6t^2 - 17t - 3$.

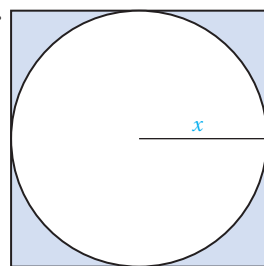
Applying the Concepts

122. **Geometry** Write the area of each shaded region in factored form.

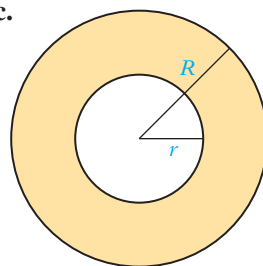
a.



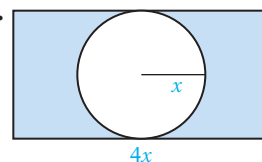
b.



c.



d.



123. Find all integers k such that the trinomial can be factored.

a. $x^2 + kx + 8$

b. $x^2 + kx - 6$

c. $2x^2 + kx + 3$

d. $2x^2 + kx - 5$

e. $3x^2 + kx + 5$

f. $2x^2 + kx - 3$

SECTION

5.6 Special Factoring

OBJECTIVE A

To factor the difference of two perfect squares or a perfect-square trinomial



The product of a term and itself is called a **perfect square**. The exponents on variables of perfect squares are always even numbers.

Term		Perfect Square
5	$5 \cdot 5 =$	25
x	$x \cdot x =$	x^2
$3y^4$	$3y^4 \cdot 3y^4 =$	$9y^8$
x^n	$x^n \cdot x^n =$	x^{2n}

The **square root of a perfect square** is one of the two equal factors of the perfect square. $\sqrt{\quad}$ is the symbol for square root. To find the exponent on the square root of a variable term, multiply the exponent by $\frac{1}{2}$.

$$\begin{aligned}\sqrt{25} &= 5 \\ \sqrt{x^2} &= x \\ \sqrt{9y^8} &= 3y^4 \\ \sqrt{x^{2n}} &= x^n\end{aligned}$$

The **difference of two perfect squares** is the **product of the sum and difference of two terms**. The factors of the difference of two perfect squares are the sum and difference of the square roots of the perfect squares.

Factors of the Difference of Two Perfect Squares

$$a^2 - b^2 = (a + b)(a - b)$$

The **sum of two perfect squares**, $a^2 + b^2$, is nonfactorable over the integers.

HOW TO 1 Factor: $4x^2 - 81y^2$

Write the binomial as the difference of two perfect squares.

The factors are the sum and difference of the square roots of the perfect squares.

$$4x^2 - 81y^2 = (2x)^2 - (9y)^2$$

$$= (2x + 9y)(2x - 9y)$$

A **perfect-square trinomial** is the square of a binomial.

Factors of a Perfect-Square Trinomial

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

In factoring a perfect-square trinomial, remember that the terms of the binomial are the square roots of the perfect squares of the trinomial. The sign of the binomial is the sign of the middle term of the trinomial.

HOW TO • 2 Factor: $4x^2 + 12x + 9$

Because $4x^2$ is a perfect square [$4x^2 = (2x)^2$] and 9 is a perfect square ($9 = 3^2$), try factoring $4x^2 + 12x + 9$ as the square of a binomial.

$$4x^2 + 12x + 9 \stackrel{?}{=} (2x + 3)^2$$

$$\text{Check: } (2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$$

The check verifies that $4x^2 + 12x + 9 = (2x + 3)^2$.

It is important to check a proposed factorization as we did above. The next example illustrates the importance of this check.

HOW TO • 3 Factor: $x^2 + 13x + 36$

Because x^2 is a perfect square and 36 is a perfect square, try factoring $x^2 + 13x + 36$ as the square of a binomial.

$$x^2 + 13x + 36 \stackrel{?}{=} (x + 6)^2$$

$$\text{Check: } (x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$$

In this case, the proposed factorization of $x^2 + 13x + 36$ does *not* check. Try another factorization. The numbers 4 and 9 are factors of 36 whose sum is 13.

$$x^2 + 13x + 36 = (x + 4)(x + 9)$$

EXAMPLE • 1

Factor: $25x^2 - 1$

Solution

$$\begin{aligned} 25x^2 - 1 &= (5x)^2 - (1)^2 \\ &= (5x + 1)(5x - 1) \end{aligned}$$

• Difference of
two squares

YOU TRY IT • 1

Factor: $x^2 - 36y^4$

Your solution**EXAMPLE • 2**

Factor: $4x^2 - 20x + 25$

Solution

$$4x^2 - 20x + 25 = (2x - 5)^2$$

• Perfect-square
trinomial

YOU TRY IT • 2

Factor: $9x^2 + 12x + 4$

Your solution**EXAMPLE • 3**

Factor: $(x + y)^2 - 4$

Solution

$$\begin{aligned} (x + y)^2 - 4 &= (x + y)^2 - (2)^2 \\ &= (x + y)^2 - (2)^2 \\ &= (x + y + 2)(x + y - 2) \end{aligned}$$

• Difference of
two squares

YOU TRY IT • 3

Factor: $(a + b)^2 - (a - b)^2$

Your solution

OBJECTIVE B**To factor the sum or the difference of two perfect cubes**

The product of the same three factors is called a **perfect cube**. The first seven perfect cube integers are:

$$1 = 1^3, 8 = 2^3, 27 = 3^3, 64 = 4^3, 125 = 5^3, 216 = 6^3, 343 = 7^3$$

A variable term is a perfect cube if the coefficient is a perfect cube and the exponent on each variable is divisible by 3. The table at the right shows some perfect-cube variable terms. Note that each exponent of the perfect cube is divisible by 3.

Term		Perfect Cube
x	$x \cdot x \cdot x = x^3 =$	x^3
$2y$	$2y \cdot 2y \cdot 2y = (2y)^3 =$	$8y^3$
$4x^2$	$4x^2 \cdot 4x^2 \cdot 4x^2 = (4x^2)^3 =$	$64x^6$
$3x^4y^3$	$3x^4y^3 \cdot 3x^4y^3 \cdot 3x^4y^3 = (3x^4y^3)^3 =$	$27x^{12}y^9$

The **cube root** of a perfect cube is one of the three equal factors of the perfect cube. $\sqrt[3]{}$ is the symbol for cube root. To find the exponents on the cube root of a perfect-cube variable expression, multiply the exponents on the variables by $\frac{1}{3}$.

$$\begin{aligned}\sqrt[3]{x^3} &= x \\ \sqrt[3]{8y^3} &= 2y \\ \sqrt[3]{64x^6} &= 4x^2 \\ \sqrt[3]{27x^{12}y^9} &= 3x^4y^3\end{aligned}$$

**Take Note**

The factoring formulas at the right are the result of finding, for instance, the quotient

$$\frac{a^3 + b^3}{a + b} = a^2 - ab + b^2$$

See Exercise 81(a) in Section 5.4.

Similarly,

$$\frac{a^3 - b^3}{a - b} = a^2 + ab + b^2.$$

The following rules are used to factor the sum or difference of two perfect cubes.

Factoring the Sum or Difference of Two Perfect Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

To factor $27x^3 - 1$:

$$27x^3 - 1 = (3x)^3 - 1^3$$

Write the binomial as the difference of two perfect cubes.

The terms of the binomial factor are the cube roots of the perfect cubes. The sign of the binomial factor is the same as the sign of the given binomial. The trinomial factor is obtained from the binomial factor.

$$\begin{aligned}27x^3 - 1 &= (3x - 1)(9x^2 + 3x + 1) \\ \text{Square of the first term} &\quad \uparrow \\ \text{Opposite of the product of the two terms} &\quad \uparrow \\ \text{Square of the last term} &\quad \uparrow\end{aligned}$$

**Take Note**

Note the placement of the signs. The sign of the binomial factor is the same as the sign of the sum or difference of the perfect cubes. The first sign of the trinomial factor is the opposite of the sign of the binomial factor.

HOW TO 4

Factor: $m^3 + 64n^3$

$$m^3 + 64n^3 = (m)^3 + (4n)^3$$

$$\begin{aligned}(m)^2 &\quad \quad \quad m(4n) & \quad \quad \quad (4n)^2 \\ &\quad \quad \quad \downarrow & \quad \quad \quad \downarrow \\ &= (m + 4n)(m^2 - 4mn + 16n^2)\end{aligned}$$

• Write as the sum of two perfect cubes.

• Use $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

HOW TO • 5 Factor: $8x^3 - 27$

$$8x^3 - 27 = (2x)^3 - 3^3$$

- Write as the difference of two perfect cubes.

$$\begin{array}{ccc} (2x)^2 & \xrightarrow{\quad} & 2x(3) \\ \downarrow & & \downarrow \\ = (2x - 3)(4x^2 + 6x + 9) & & 3^2 \end{array}$$

- Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

HOW TO • 6 Factor: $64y^4 - 125y$

$$\begin{aligned} 64y^4 - 125y &= y(64y^3 - 125) \\ &= y[(4y)^3 - 5^3] \end{aligned}$$

- Factor out y , the GCF.
- Write the binomial as the difference of two perfect cubes.

$$= y(4y - 5)(16y^2 + 20y + 25)$$

- Factor.

EXAMPLE • 4

Factor: $x^3y^3 - 1$

Solution

$$\begin{aligned} x^3y^3 - 1 &= (xy)^3 - 1^3 \\ &= (xy - 1)(x^2y^2 + xy + 1) \end{aligned}$$

- Difference of two cubes

YOU TRY IT • 4

Factor: $a^3b^3 - 27$

Your solution**EXAMPLE • 5**

Factor: $64c^3 + 8d^3$

Solution

$$\begin{aligned} 64c^3 + 8d^3 &= 8(8c^3 + d^3) \\ &= 8[(2c)^3 + d^3] \\ &= 8(2c + d)(4c^2 - 2cd + d^2) \end{aligned}$$

- GCF
- Sum of two cubes

YOU TRY IT • 5

Factor: $8x^3 + y^3z^3$

Your solution**EXAMPLE • 6**

Factor: $(x + y)^3 - x^3$

Solution

$$\begin{aligned} (x + y)^3 - x^3 &= [(x + y) - x][(x + y)^2 + x(x + y) + x^2] \\ &= y(x^2 + 2xy + y^2 + x^2 + xy + x^2) \\ &= y(3x^2 + 3xy + y^2) \end{aligned}$$

- Difference of two cubes

YOU TRY IT • 6

Factor: $(x - y)^3 + (x + y)^3$

Your solution

OBJECTIVE C**To factor a trinomial that is quadratic in form**

Certain trinomials that are not quadratic can be expressed as quadratic trinomials by making suitable variable substitutions. A trinomial is **quadratic in form** if it can be written as $au^2 + bu + c$.

As shown below, the trinomials $x^4 + 5x^2 + 6$ and $2x^2y^2 + 3xy - 9$ are quadratic in form.

$x^4 + 5x^2 + 6$	$2x^2y^2 + 3xy - 9$
$(x^2)^2 + 5(x^2) + 6$	$2(xy)^2 + 3(xy) - 9$
Let $u = x^2$. $u^2 + 5u + 6$	Let $u = xy$. $2u^2 + 3u - 9$

When we use this method to factor a trinomial that is quadratic in form, the variable part of the first term in each binomial will be u .

**Take Note**

The trinomial $x^4 + 5x^2 + 6$ was shown above to be quadratic in form.

HOW TO • 7 Factor: $x^4 + 5x^2 + 6$

$$\begin{aligned} x^4 + 5x^2 + 6 &= u^2 + 5u + 6 \\ &= (u + 3)(u + 2) \\ &= (x^2 + 3)(x^2 + 2) \end{aligned}$$

- Let $u = x^2$.
- Factor.
- Replace u by x^2 .

Here is an example in which $u = \sqrt{x}$.

HOW TO • 8 Factor: $x - 2\sqrt{x} - 15$

$$\begin{aligned} x - 2\sqrt{x} - 15 &= u^2 - 2u - 15 \\ &= (u - 5)(u + 3) \\ &= (\sqrt{x} - 5)(\sqrt{x} + 3) \end{aligned}$$

- Let $u = \sqrt{x}$. Then $u^2 = x$.
- Factor.
- Replace u by $\sqrt{x}$.

EXAMPLE • 7

Factor: $6x^2y^2 - xy - 12$

Solution

Let $u = xy$.

$$\begin{aligned} 6x^2y^2 - xy - 12 &= 6u^2 - u - 12 \\ &= (3u + 4)(2u - 3) \\ &= (3xy + 4)(2xy - 3) \end{aligned}$$

- Replace u by xy .

YOU TRY IT • 7

Factor: $3x^4 + 4x^2 - 4$

Your solution

OBJECTIVE D**To factor completely****Tips for Success**

You now have completed all the lessons on factoring polynomials. You will need to be able to recognize all of the factoring patterns. To test yourself, try the Chapter 5 Review Exercises.

**Take Note**

Remember that you may have to factor more than once in order to write the polynomial as a product of *prime* factors.

General Factoring Strategy

1. Is there a common factor? If so, factor out the GCF.
2. If the polynomial is a binomial, is it the difference of two perfect squares, the sum of two perfect cubes, or the difference of two perfect cubes? If so, factor.
3. If the polynomial is a trinomial, is it a perfect-square trinomial or the product of two binomials? If so, factor.
4. Can the polynomial be factored by grouping? If so, factor.
5. Is each factor nonfactorable over the integers? If not, factor.

EXAMPLE • 8

Factor: $6a^3 + 15a^2 - 36a$

Solution

$$\begin{aligned} 6a^3 + 15a^2 - 36a &= 3a(2a^2 + 5a - 12) && \bullet \text{ GCF} \\ &= 3a(2a - 3)(a + 4) \end{aligned}$$

YOU TRY IT • 8

Factor: $18x^3 - 6x^2 - 60x$

Your solution**EXAMPLE • 9**

Factor: $x^2y + 2x^2 - y - 2$

Solution

$$\begin{aligned} x^2y + 2x^2 - y - 2 &= (x^2y + 2x^2) - (y + 2) \\ &= x^2(y + 2) - (y + 2) \\ &= (y + 2)(x^2 - 1) \\ &= (y + 2)(x + 1)(x - 1) \end{aligned} \quad \bullet \text{ Factor by grouping.}$$

YOU TRY IT • 9

Factor: $4x - 4y - x^3 + x^2y$

Your solution**EXAMPLE • 10**

Factor: $x^{4n} - y^{4n}$

Solution

$$\begin{aligned} x^{4n} - y^{4n} &= (x^{2n})^2 - (y^{2n})^2 \\ &= (x^{2n} + y^{2n})(x^{2n} - y^{2n}) \\ &= (x^{2n} + y^{2n})[(x^n)^2 - (y^n)^2] \\ &= (x^{2n} + y^{2n})(x^n + y^n)(x^n - y^n) \end{aligned} \quad \bullet \text{ Difference of two squares}$$

YOU TRY IT • 10

Factor: $x^{4n} - x^{2n}y^{2n}$

Your solution

5.6 EXERCISES

OBJECTIVE A

To factor the difference of two perfect squares or a perfect-square trinomial

For Exercises 1 and 2, determine which expressions are perfect squares.

1. $4; 8; 25x^6; 12y^{10}; 100x^4y^4$

2. $9; 18; 15a^8; 49b^{12}; 64a^{16}b^2$

For Exercises 3 to 6, find the square root of the expression.

3. $16z^8$

4. $36d^{10}$

5. $81a^4b^6$

6. $25m^2n^{12}$



7. Which of the following polynomials, if any, is a difference of squares?

(i) $x^2 - 12$ (ii) $a^2 + 25$ (iii) $x^3 - 4x$ (iv) $m^4 - n^2$



8. Determine if each statement is true or false. Explain.

a. $a^2 + b^2 = (a + b)^2$

b. $a^2 - b^2 = (a - b)(a - b)$

c. $a^2 + 10a + 9$ is a perfect-square trinomial.

d. $a^2 + 3a - 4$ is a perfect-square trinomial.

For Exercises 9 to 44, factor.

9. $x^2 - 16$

10. $y^2 - 49$

11. $4x^2 - 1$

12. $81x^2 - 4$

13. $16x^2 - 121$

14. $49y^2 - 36$

15. $1 - 9a^2$

16. $16 - 81y^2$

17. $x^2y^2 - 100$

18. $a^2b^2 - 25$

19. $x^2 + 4$

20. $a^2 + 16$

21. $25 - a^2b^2$

22. $64 - x^2y^2$

23. $a^{2n} - 1$

24. $b^{2n} - 16$

25. $x^2 - 12x + 36$

26. $y^2 - 6y + 9$

27. $b^2 - 2b + 1$

28. $a^2 + 14a + 49$

29. $16x^2 - 40x + 25$

30. $49x^2 + 28x + 4$

31. $4a^2 + 4a - 1$

32. $9x^2 + 12x - 4$

33. $b^2 + 7b + 14$

34. $y^2 - 5y + 25$

35. $x^2 + 6xy + 9y^2$

36. $4x^2y^2 + 12xy + 9$

37. $25a^2 - 40ab + 16b^2$

38. $4a^2 - 36ab + 81b^2$

39. $x^{2n} + 6x^n + 9$

40. $y^{2n} - 16y^n + 64$

41. $(x - 4)^2 - 9$

42. $16 - (a - 3)^2$

43. $(x - y)^2 - (a + b)^2$

44. $(x - 2y)^2 - (x + y)^2$

OBJECTIVE B**To factor the sum or the difference of two perfect cubes**

For Exercises 45 and 46, determine which expressions are perfect cubes.

45. 4; 8; x^9 ; a^8b^8 ; $27c^{15}d^{18}$

46. 9; 27; y^{12} ; m^3n^6 ; $64mn^9$

For Exercises 47 to 50, find the cube root of the expression.

47. $8x^9$

48. $27y^{15}$

49. $64a^6b^{18}$

50. $125c^{12}d^3$



For Exercises 51 to 54, state whether the polynomial can be written as either the sum or difference of two cubes.

51. $x^6 + 64$

52. $x^9 - 32$

53. $a^6x^3 - b^3y^6$

54. $a^3x^3 - b^3y^3$

For Exercises 55 to 78, factor.

55. $x^3 - 27$

56. $y^3 + 125$

57. $8x^3 - 1$

58. $64a^3 + 27$

59. $x^3 - y^3$

60. $x^3 - 8y^3$

61. $m^3 + n^3$

62. $27a^3 + b^3$

63. $64x^3 + 1$

64. $1 - 125b^3$

65. $27x^3 - 8y^3$

66. $64x^3 + 27y^3$

67. $x^3y^3 + 64$

68. $8x^3y^3 + 27$

69. $16x^3 - y^3$

70. $27x^3 - 8y^2$

71. $8x^3 - 9y^3$

72. $27a^3 - 16$

73. $(a - b)^3 - b^3$

74. $a^3 + (a + b)^3$

75. $x^{6n} + y^{3n}$

76. $x^{3n} + y^{3n}$

77. $x^{3n} + 8$

78. $a^{3n} + 64$

OBJECTIVE C**To factor a trinomial that is quadratic in form**

79. The expression $x - \sqrt{x} - 6$ is quadratic in form. Is the expression a polynomial? Explain.



80. The polynomial $x^4 - 2x^2 - 3$ is quadratic in form. Is the polynomial a quadratic polynomial? Explain.

For Exercises 81 to 101, factor.

81. $x^2y^2 - 8xy + 15$

82. $x^2y^2 - 8xy - 33$

83. $x^2y^2 - 17xy + 60$

84. $a^2b^2 + 10ab + 24$

85. $x^4 - 9x^2 + 18$

86. $y^4 - 6y^2 - 16$

87. $b^4 - 13b^2 - 90$

88. $a^4 + 14a^2 + 45$

89. $x^4y^4 - 8x^2y^2 + 12$

90. $a^4b^4 + 11a^2b^2 - 26$

91. $x^{2n} + 3x^n + 2$

92. $a^{2n} - a^n - 12$

93. $3x^2y^2 - 14xy + 15$

94. $5x^2y^2 - 59xy + 44$

95. $6a^2b^2 - 23ab + 21$

96. $10a^2b^2 + 3ab - 7$

97. $2x^4 - 13x^2 - 15$

98. $3x^4 + 20x^2 + 32$

99. $2x^{2n} - 7x^n + 3$

100. $4x^{2n} + 8x^n - 5$

101. $6a^{2n} + 19a^n + 10$

OBJECTIVE D**To factor completely**

For Exercises 102 to 135, factor.

102. $5x^2 + 10x + 5$

103. $12x^2 - 36x + 27$

104. $3x^4 - 81x$

105. $27a^4 - a$

106. $7x^2 - 28$

107. $20x^2 - 5$

108. $y^4 - 10y^3 + 21y^2$

109. $y^5 + 6y^4 - 55y^3$

110. $x^4 - 16$

111. $16x^4 - 81$

112. $8x^5 - 98x^3$

113. $16a - 2a^4$

114. $x^3y^3 - x^3$

115. $a^3b^6 - b^3$

116. $x^6y^6 - x^3y^3$

117. $8x^4 - 40x^3 + 50x^2$

118. $6x^5 + 74x^4 + 24x^3$

119. $x^4 - y^4$

120. $16a^4 - b^4$

121. $x^6 + y^6$

122. $x^4 - 5x^2 - 4$

123. $a^4 - 25a^2 - 144$

124. $3b^5 - 24b^2$

125. $16a^4 - 2a$

126. $x^4y^2 - 5x^3y^3 + 6x^2y^4$

127. $a^4b^2 - 8a^3b^3 - 48a^2b^4$

128. $x^3 - 2x^2 - x + 2$

129. $x^3 - 2x^2 - 4x + 8$

130. $4x^3 + 8x^2 - 9x - 18$

131. $2x^3 + x^2 - 32x - 16$

132. $4x^2y^2 - 4x^2 - 9y^2 + 9$

133. $4x^4 - x^2 - 4x^2y^2 + y^2$

134. $2x^{n+2} - 7x^{n+1} + 3x^n$

135. $3b^{n+2} + 4b^{n+1} - 4b^n$

 136. What is the degree of the polynomial whose factored form is $3x^3(x^2 + 4)(x - 2)(x + 3)$?

 137. What is the coefficient of x^6 when $4x^2(x^2 + 3)(x + 4)(2x - 5)$ is expanded and written as a polynomial?

Applying the Concepts

138. Factor: $x^2(x - 3) - 3x(x - 3) + 2(x - 3)$

 139. Given that $(x - 3)$ and $(x + 4)$ are factors of $x^3 + 6x^2 - 7x - 60$, explain how you can find a third *first-degree* factor of $x^3 + 6x^2 - 7x - 60$. Then find the factor.

SECTION

5.7

Solving Equations by Factoring

OBJECTIVE A

To solve an equation by factoring 

Consider the equation $ab = 0$. If a is not zero, then b must be zero. Conversely, if b is not zero, then a must be zero. This is summarized in the **Principle of Zero Products**.

Principle of Zero Products

If the product of two factors is zero, then at least one of the factors must be zero.

If $ab = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products is used to solve equations.

HOW TO • 1 Solve: $(x - 4)(x + 2) = 0$

By the Principle of Zero Products, if $(x - 4)(x + 2) = 0$, then $x - 4 = 0$ or $x + 2 = 0$.

$$\begin{aligned}x - 4 &= 0 \\x &= 4\end{aligned}$$

$$\begin{aligned}x + 2 &= 0 \\x &= -2\end{aligned}$$

Check:	$(x - 4)(x + 2) = 0$	$(x - 4)(x + 2) = 0$
	$(4 - 4)(4 + 2) \quad \quad 0$	$(-2 - 4)(-2 + 2) \quad \quad 0$
	$0 \cdot 6 \quad \quad 0$	$-6 \cdot 0 \quad \quad 0$
	$0 = 0$	$0 = 0$

-2 and 4 check as solutions. The solutions are -2 and 4 .

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a **quadratic equation**. In a **quadratic equation in standard form**, the polynomial is written in descending order and equal to zero.

Some quadratic equations can be solved by factoring and then using the Principle of Zero Products.

**Take Note**

Note the steps involved in solving a quadratic equation by factoring:

1. Write in standard form.
2. Factor.
3. Set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

HOW TO • 2 Solve: $(2x + 1)(x - 1) = 2x + 8$

$$\begin{aligned}(2x + 1)(x - 1) &= 2x + 8 \\2x^2 - x - 1 &= 2x + 8 \\2x^2 - 3x - 9 &= 0 \\(2x + 3)(x - 3) &= 0\end{aligned}$$

$$\begin{aligned}2x + 3 &= 0 \\2x &= -3 \\x &= -\frac{3}{2}\end{aligned}$$

$$\begin{aligned}x - 3 &= 0 \\x &= 3\end{aligned}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products.
- Solve each equation.

The solutions are $-\frac{3}{2}$ and 3 . You should check this solution.

 Take Note

The Principle of Zero Products can be extended to more than two factors. For example, if $abc = 0$, then $a = 0$, $b = 0$, or $c = 0$.

HOW TO 3Solve: $2x^3 + x^2 - 32x - 16 = 0$

$$2x^3 + x^2 - 32x - 16 = 0$$

$$(2x^3 + x^2) - (32x + 16) = 0$$

$$x^2(2x + 1) - 16(2x + 1) = 0$$

$$(2x + 1)(x^2 - 16) = 0$$

$$(2x + 1)(x - 4)(x + 4) = 0$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$x - 4 = 0$$

$$x = 4$$

$$x + 4 = 0$$

$$x = -4$$

- Factor by grouping.

- Use the Principle of Zero Products.
- Solve each equation.

The solutions are -4 , $-\frac{1}{2}$ and 4 .

EXAMPLE 1Solve: $x^2 + (x + 2)^2 = 100$ **Solution**

$$x^2 + (x + 2)^2 = 100$$

$$x^2 + x^2 + 4x + 4 = 100$$

$$2x^2 + 4x - 96 = 0$$

$$2(x^2 + 2x - 48) = 0$$

$$2(x - 6)(x + 8) = 0$$

$$x - 6 = 0$$

$$x = 6$$

$$x + 8 = 0$$

$$x = -8$$

- Square $(x + 2)$.
- Write in standard form.
- Factor the left side.

- Principle of Zero Products

The solutions are -8 and 6 .

YOU TRY IT 1Solve: $(x + 4)(x - 1) = 14$ **Your solution**

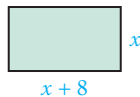
Solution on p. S17

OBJECTIVE B**To solve application problems****EXAMPLE 2**

The length of a rectangle is 8 in. more than the width. The area of the rectangle is 240 in^2 . Find the width of the rectangle.

Strategy

Draw a diagram. Then use the formula for the area of a rectangle.

**Solution**

$$A = LW$$

$$240 = (x + 8)x$$

$$240 = x^2 + 8x$$

$$0 = x^2 + 8x - 240$$

$$0 = (x + 20)(x - 12)$$

$$x + 20 = 0$$

$$x = -20$$

$$x - 12 = 0$$

$$x = 12$$

The width cannot be negative.

The width is 12 in.

YOU TRY IT 2

The height of a triangle is 3 cm more than the length of the base of the triangle. The area of the triangle is 54 cm^2 . Find the height of the triangle and the length of the base.

Your strategy**Your solution**

Solution on p. S18

5.7 EXERCISES

OBJECTIVE A

To solve an equation by factoring



1. If
- $ab = 0$
- , does this mean that
- $b = 0$
- ? Explain.



2. If
- $ab = 0$
- , then
- $a = 0$
- or
- $b = 0$
- . Suppose
- $ab = 6$
- . Does this mean that
- $a = 2$
- and
- $b = 3$
- ? Explain.

For Exercises 3 to 37, solve.

3. $(x - 5)(x + 3) = 0$	4. $(x - 2)(x + 6) = 0$	5. $(x + 7)(x - 8) = 0$
6. $x(2x + 5)(x - 6) = 0$	7. $2x(3x - 2)(x + 4) = 0$	8. $6x(3x - 7)(x + 7) = 0$
9. $x^2 + 2x - 15 = 0$	10. $t^2 + 3t - 10 = 0$	11. $z^2 - 4z + 3 = 0$
12. $6x^2 - 9x = 0$	13. $r^2 - 10 = 3r$	14. $t^2 - 12 = 4t$
15. $4t^2 = 4t + 3$	16. $5y^2 + 11y = 12$	17. $4v^2 - 4v + 1 = 0$
18. $9s^2 - 6s + 1 = 0$	19. $x^2 - 9 = 0$	20. $t^2 - 16 = 0$
21. $4y^2 - 1 = 0$	22. $9z^2 - 4 = 0$	23. $x(x - 1) = x + 15$
24. $x^2 + 2x - 6 = 3x$	25. $2x^2 - 3x - 8 = x^2 + 20$	
26. $(2x - 1)(x - 3) = x^2 - x - 2$	27. $(3v + 2)(v - 4) = v^2 + v + 5$	
28. $z^2 + 5z - 4 = (2z + 1)(z - 4)$	29. $4x^2 + x - 10 = (x - 2)(x + 1)$	
30. $x^3 + 2x^2 - 15x = 0$	31. $c^3 + 3c^2 - 10c = 0$	
32. $x^3 + x^2 - 4x - 4 = 0$	33. $a^3 + a^2 - 9a - 9 = 0$	
34. $2x^3 - x^2 - 2x + 1 = 0$	35. $3x^3 + 2x^2 - 12x - 8 = 0$	
36. $2x^3 + 3x^2 - 18x - 27 = 0$	37. $5x^3 + 2x^2 - 20x - 8 = 0$	

OBJECTIVE B To solve application problems

38. **Mathematics** The sum of a number and its square is 72. Find the number.
39. **Mathematics** The sum of a number and its square is 210. Find the number.
40. **Geometry** The length of a rectangle is 2 ft more than twice the width. The area of the rectangle is 84 ft^2 . Find the length and width of the rectangle.
41. **Geometry** The height of a triangle is 8 cm more than the length of the base. The area of the triangle is 64 cm^2 . Find the base and height of the triangle.
42. **Physics** An object is thrown downward, with an initial speed of 16 ft/s, from the top of a building 480 ft high. How many seconds later will the object hit the ground? Use the equation $d = vt + 16t^2$, where d is the distance in feet, v is the initial speed, and t is the time in seconds.
43. **Publishing** Read the article at the right. The length of the electronic paper rectangle was 3 cm less than three times the width. Find the length and width of the electronic paper rectangle.
44. **Publishing** The electronic paper rectangle described in the article in Exercise 43 was only a small portion of the magazine cover. The area of the magazine cover was about 560 cm^2 . If the length of the magazine cover was 8 cm more than the width, find the length and width of the magazine cover.
45. “Lucky Larry” is a feature in *The AMATYC Review*, a periodical published by the American Mathematical Association of Two-Year Colleges. This feature shows an incorrect procedure that yields the correct answer to a problem. Here is a problem and the solution by Larry. Explain why Larry was lucky.

The length of a rectangle is 2 ft longer than its width. The area of the rectangle is 15 ft^2 . Find the length and width of the rectangle.

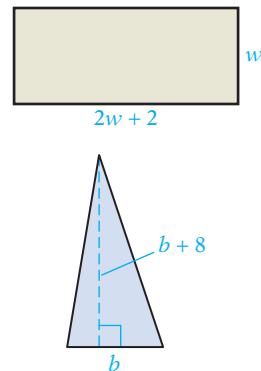
Strategy:Width of rectangle: x Length of rectangle: $x + 2$ Area = LW **Solution**

$$x(x + 2) = 15$$

$$x = 3 \text{ or } x + 2 = 5$$

$$x = 3$$

The width is 3 ft; the length is 5 ft. Because $3 \text{ ft} \cdot 5 \text{ ft} = 15 \text{ ft}^2$, the solution is correct.

**In the News****Esquire First with E-Ink**

This week, *Esquire* became the first magazine to employ the use of E-ink, the same technology used in electronic book devices. The magazine's cover featured an “electronic paper” rectangle of about 60 cm^2 .

Source: news.cnet.com

Applying the Concepts

46. Consider the equation $3x = x$. If we divide both sides of the equation by x , we have $3 = 1$. What went wrong? How should the equation be solved?

By using the Principle of Zero Products, an equation with specific solutions can be formed. For instance, to create an equation with solutions -3 and 5 , use the Principle of Zero Products and write $[x - (-3)][x - 5] = 0$. Expanding the left side, we have $x^2 - 2x - 15 = 0$. For Exercises 47 to 50, find an equation having the given solutions.

47. 3, 7

48. 4, -2

49. -1 , -3

50. -6 , 2

51. **Construction** A rectangular piece of cardboard is 10 in. longer than it is wide. Squares 2 in. on a side are to be cut from each corner, and then the sides are to be folded up to make an open box with a volume of 112 in^3 . Find the length and width of the piece of cardboard.

FOCUS ON PROBLEM SOLVING

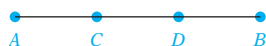
Find a Counterexample

When you are faced with an assertion, it may be that the assertion is false. For instance, consider the statement “Every prime number is an odd number.” This assertion is false because the prime number 2 is an even number.

Finding an example that illustrates that an assertion is false is called finding a *counterexample*. The number 2 is a counterexample to the assertion that every prime number is an odd number.

If you are given an unfamiliar problem, one strategy to consider as a means of solving the problem is to try to find a counterexample. For each of the following problems, answer *true* if the assertion is always true. If the assertion is not true, answer *false* and give a counterexample. If there are terms used that you do not understand, consult a reference to find the meaning of the term.

1. If x is a real number, then x^2 is always positive.
2. The product of an odd integer and an even integer is an even integer.
3. If m is a positive integer, then $2m + 1$ is always a positive odd integer.
4. If $x < y$, then $x^2 < y^2$.
5. Given any three positive numbers a , b , and c , it is possible to construct a triangle whose sides have lengths a , b , and c .
6. The product of two irrational numbers is an irrational number.
7. If n is a positive integer greater than 2, then $1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdots \cdot n + 1$ is a prime number.
8. Draw a polygon with more than three sides. Select two different points inside the polygon and join the points with a line segment. The line segment always lies completely inside the polygon.
9. Let A , B , and C be three points in the plane that are not collinear. Let d_1 be the distance from A to B , and let d_2 be the distance from A to C . Then the distance between B and C is less than $d_1 + d_2$.
10. Consider the line segment AB shown at the left. Two points, C and D , are randomly selected on the line segment and three new segments are formed: AC , CD , and DB . The three new line segments can always be connected to form a triangle.



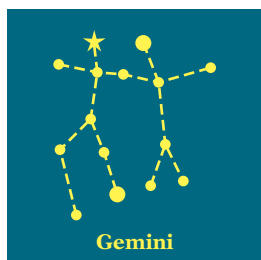
It may not be easy to establish that an assertion is true or to find a counterexample to the assertion. For instance, consider the assertion that every positive integer greater than 3 can be written as the sum of two primes. For example, $6 = 3 + 3$, $8 = 3 + 5$, $9 = 2 + 7$. Is this assertion always true? (Note: This assertion, called Goldbach's conjecture, has never been proved, nor has a counterexample been found!)

PROJECTS AND GROUP ACTIVITIES

Astronomical Distances and Scientific Notation



Astronomers have units of measurement that are useful for measuring vast distances in space. Two of these units are the astronomical unit and the light-year. An **astronomical unit** is the average distance between Earth and the sun. A **light-year** is the distance a ray of light travels in 1 year.



1. Light travels at a speed of 1.86×10^5 mi/s. Find the measure of 1 light-year in miles. Use a 365-day year.
2. The distance between Earth and the star Alpha Centauri is approximately 25 trillion miles. Find the distance between Earth and Alpha Centauri in light-years. Round to the nearest hundredth.
3. The Coma cluster of galaxies is approximately 2.8×10^8 light-years from Earth. Find the distance, in miles, from the Coma cluster to Earth. Write the answer in scientific notation.
4. One astronomical unit (A.U.) is 9.3×10^7 mi. The star Pollux in the constellation Gemini is 1.8228×10^{12} mi from Earth. Find the distance from Pollux to Earth in astronomical units.
5. One light-year is equal to approximately how many astronomical units? Round to the nearest thousand.

Shown below are data on the planets in our solar system. The planets are listed in alphabetical order.

Point of Interest

In November 2001, the Hubble Space Telescope took photos of the atmosphere of a planet orbiting a star 150 light-years from Earth in the constellation Pegasus. The planet is about the size of Jupiter and orbits close to the star HD209458. It was the first discovery of an atmosphere around a planet outside our solar system.



NASA/Roger Ressmeyer/Corbis

Jupiter

<i>Planet</i>	<i>Distance from the Sun (in kilometers)</i>	<i>Mass (in kilograms)</i>
Earth	1.50×10^8	5.97×10^{24}
Jupiter	7.79×10^8	1.90×10^{27}
Mars	2.28×10^8	6.42×10^{23}
Mercury	5.79×10^7	3.30×10^{23}
Neptune	4.50×10^9	1.02×10^{26}
Saturn	1.43×10^9	5.68×10^{26}
Uranus	2.87×10^9	8.68×10^{25}
Venus	1.08×10^8	4.87×10^{24}

6. Arrange the planets in order from closest to the sun to farthest from the sun.
7. Arrange the planets in order from the one with the greatest mass to the one with the least mass.
8. Write a rule for ordering numbers written in scientific notation.

CHAPTER 5

SUMMARY

KEY WORDS

A *monomial* is a number, a variable, or a product of numbers and variables. [5.1A, p. 260]

The *degree of a monomial* is the sum of the exponents on the variables. [5.1A, p. 260]

A *polynomial* is a variable expression in which the terms are monomials. [5.2A, p. 272]

A polynomial of one term is a *monomial*, a polynomial of two terms is a *binomial*, and a polynomial of three terms is a *trinomial*. [5.2A, p. 272]

The *degree of a polynomial* is the greatest of the degrees of any of its terms. [5.2A, p. 272]

The terms of a polynomial in one variable are usually arranged so that the exponents on the variable decrease from left to right. This is called *descending order*. [5.2A, p. 272]

A *polynomial function* is an expression whose terms are monomials. Polynomial functions include the *linear function* given by $f(x) = mx + b$; the *quadratic function* given by $f(x) = ax^2 + bx + c$, $a \neq 0$; and the *cubic function*, which is a third-degree polynomial function. The *leading coefficient* of a polynomial function is the coefficient of the variable with the largest exponent. The *constant term* is the term without a variable. [5.2A, p. 272]

To *factor a polynomial* means to write the polynomial as the product of other polynomials. [5.5A, p. 301]

A *quadratic trinomial* is a polynomial of the form $ax^2 + bx + c$, where a and b are nonzero coefficients and c is a nonzero constant. To *factor a quadratic trinomial* means to express the trinomial as the product of two binomials. [5.5C, p. 303]

EXAMPLES

5 is a number; y is a variable. $8a^2b^2$ is a product of a number and variables. 5, y , and $8a^2b^2$ are monomials.

The degree of $8x^4y^5z$ is 10.

$x^4 - 2xy - 32x + 8$ is a polynomial. The terms are x^4 , $-2xy$, $-32x$, and 8.

$5x^4$ is a monomial.
 $= 6y^3 - 2y$ is a binomial.
 $= 2x^2 - 5x + 3$ is a trinomial.

The degree of the polynomial $x^3 + 3x^2y^2 - 4xy - 3$ is 4.

The polynomial $4x^3 + 5x^2 - x + 7$ is written in descending order.

$f(x) = 5x - 4$ is a linear function.
 $f(x) = 3x^2 - 2x + 1$ is a quadratic function. 3 is the leading coefficient, and 1 is the constant term.
 $f(x) = x^3 - 1$ is a cubic function.

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$3x^2 + 10x - 8$ is a quadratic trinomial in which $a = 3$, $b = 10$, and $c = -8$.
 $3x^2 + 10x - 8 = (3x - 2)(x + 4)$

A polynomial is *nonfactorable over the integers* if it does not factor using only integers. [5.5C, p. 305]

$x^2 + x + 1$ is nonfactorable over the integers.

The product of a term and itself is called a *perfect square*. The *square root* of a perfect square is one of the two equal factors of the perfect square. [5.6A, p. 313]

$(5x)(5x) = 25x^2$; $25x^2$ is a perfect square.
 $\sqrt{25x^2} = 5x$

The product of the same three factors is called a *perfect cube*. The *cube root* of a perfect cube is one of the three equal factors of the perfect cube. [5.6B, p. 315]

$(2x)(2x)(2x) = 8x^3$; $8x^3$ is a perfect cube.
 $\sqrt[3]{8x^3} = 2x$

A trinomial is *quadratic in form* if it can be written as $au^2 + bu + c$. [5.6C, p. 317]

$$\begin{aligned} 6x^4 - 5x^2 - 4 \\ &= 6(x^2)^2 - 5(x^2) - 4 \\ &= 6u^2 - 5u - 4 \end{aligned}$$

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a *quadratic equation*. A quadratic equation is in *standard form* when the polynomial is written in descending order and is equal to zero. [5.7A, p. 323]

$3x^2 + 3x + 8 = 0$ is a quadratic equation in standard form.

ESSENTIAL RULES AND PROCEDURES

Rule for Multiplying Exponential Expressions [5.1A, p. 260]

$$x^m \cdot x^n = x^{m+n}$$

EXAMPLES

$$b^5 \cdot b^4 = b^{5+4} = b^9$$

Rule for Simplifying the Power of an Exponential Expression [5.1A, p. 261]

$$(x^m)^n = x^{mn}$$

$$(y^3)^7 = y^{3(7)} = y^{21}$$

Rule for Simplifying Powers of Products [5.1A, p. 261]

$$(x^m y^n)^p = x^{mp} y^{np}$$

$$(x^6 y^4 z^5)^2 = x^{6(2)} y^{4(2)} z^{5(2)} = x^{12} y^8 z^{10}$$

Definition of Zero as an Exponent [5.1B, p. 262]

For $x \neq 0$, $x^0 = 1$. The expression 0^0 is not defined.

$$\begin{aligned} 17^0 &= 1 \\ (5y)^0 &= 1, y \neq 0 \end{aligned}$$

Definition of a Negative Exponent [5.1B, p. 263]

For $x \neq 0$, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

$$x^{-6} = \frac{1}{x^6} \text{ and } \frac{1}{x^{-6}} = x^6$$

Rule for Simplifying Powers of Quotients [5.1B, p. 263]

For $y \neq 0$, $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

$$\left(\frac{x^2}{y^4}\right)^5 = \frac{x^{2 \cdot 5}}{y^{4 \cdot 5}} = \frac{x^{10}}{y^{20}}$$

Rule for Negative Exponents on Fractional Expressions [5.1B, p. 264]

For $a \neq 0$, $b \neq 0$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

$$\left(\frac{3}{8}\right)^{-4} = \left(\frac{8}{3}\right)^4$$

Rule for Dividing Exponential Expressions [5.1B, p. 264]

For $x \neq 0$, $\frac{x^m}{x^n} = x^{m-n}$.

$$\frac{y^8}{y^3} = y^{8-3} = y^5$$

Scientific Notation [5.1C, p. 266]

To express a number in scientific notation, write it in the form $a \times 10^n$, where a is a number between 1 and 10 and n is an integer.

If the number is greater than 10, the exponent on 10 will be positive.

If the number is less than 1, the exponent on 10 will be negative.

$$367,000,000 = 3.67 \times 10^8$$

$$0.0000059 = 5.9 \times 10^{-6}$$

To change a number written in scientific notation to decimal notation, move the decimal point to the right if the exponent on 10 is positive and to the left if the exponent on 10 is negative. Move the decimal point the same number of places as the absolute value of the exponent on 10.

$$2.418 \times 10^7 = 24,180,000$$

$$9.06 \times 10^{-5} = 0.0000906$$

To write the additive inverse of a polynomial, change the sign of every term of the polynomial. [5.2B, p. 275]

The additive inverse of $-y^2 + 4y - 5$ is $y^2 - 4y + 5$.

To add polynomials, combine like terms, which means to add the coefficients of the like terms. [5.2B, p. 275]

$$\begin{aligned} (8x^2 + 2x - 9) + (-3x^2 + 5x - 7) \\ = (8x^2 - 3x^2) + (2x + 5x) + (-9 - 7) \\ = 5x^2 + 7x - 16 \end{aligned}$$

To subtract two polynomials, add the additive inverse of the second polynomial to the first polynomial. [5.2B, p. 276]

$$\begin{aligned} (3y^2 - 8y + 6) - (-y^2 + 4y - 5) \\ = (3y^2 - 8y + 6) + (y^2 - 4y + 5) \\ = 4y^2 - 12y + 11 \end{aligned}$$

To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions. [5.3A, p. 280]

$$\begin{aligned} -2x^3(4x^2 + 5x - 1) \\ = -8x^5 - 10x^4 + 2x^3 \end{aligned}$$

The FOIL Method [5.3B, p. 281]

The product of two binomials can be found by adding the products of the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms.

$$\begin{aligned} (4x + 3)(2x - 5) \\ = (4x)(2x) + (4x)(-5) \\ \quad + (3)(2x) + (3)(-5) \\ = 8x^2 - 20x + 6x - 15 \\ = 8x^2 - 14x - 15 \end{aligned}$$

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. [5.4A, p. 290]

$$\frac{12x^5 + 8x^3 - 6x}{4x^2} = 3x^3 + 2x - \frac{3}{2x}$$

Synthetic Division [5.4C, p. 293]

Synthetic division is a shorter method of dividing a polynomial by a binomial of the form $x - a$. This method uses only the coefficients of the variable terms.

$$(3x^3 - 9x - 5) \div (x - 2)$$

2	3	0	-9	-5
		6	12	6
		3	6	1

$$(3x^3 - 9x - 5) \div (x - 2)$$

$$= 3x^2 + 6x + 3 + \frac{1}{x - 2}$$

Remainder Theorem [5.4D, p. 296]

If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

$$P(x) = x^3 - x^2 + x - 1$$

-2	1	-1	1	-1
		-2	6	-14
		1	-3	7
			7	-15

$$P(-2) = -15$$

Factoring Patterns [5.6A, p. 313]

The difference of two perfect squares equals the sum and difference of two terms: $a^2 - b^2 = (a + b)(a - b)$.

A perfect-square trinomial equals the square of a binomial:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$4x^2 - 9 = (2x + 3)(2x - 3)$$

$$4x^2 + 12x + 9 = (2x + 3)^2$$

Factoring the Sum or Difference of Two Cubes

[5.6B, p. 315]

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$x^3 + 64 = (x + 4)(x^2 - 4x + 16)$$

$$8b^3 - 1 = (2b - 1)(4b^2 + 2b + 1)$$

To Factor Completely [5.6D, p. 318]

When factoring a polynomial completely, ask the following questions about the polynomial.

1. Is there a common factor? If so, factor out the GCF.
2. If the polynomial is a binomial, is it the difference of two perfect squares, the sum of two perfect cubes, or the difference of two perfect cubes? If so, factor.
3. If the polynomial is a trinomial, is it a perfect-square trinomial or the product of two binomials? If so, factor.
4. Can the polynomial be factored by grouping? If so, factor.
5. Is each factor nonfactorable over the integers? If not, factor.

$$\begin{aligned} 54x^3 - 6x &= 6x(9x^2 - 1) \\ &= 6x(3x + 1)(3x - 1) \end{aligned}$$

Principle of Zero Products [5.7A, p. 323]

If the product of two factors is zero, then at least one of the factors must be zero.

If $ab = 0$, then $a = 0$ or $b = 0$.

$$\begin{aligned} (x - 4)(x + 2) &= 0 \\ x - 4 = 0 \quad x + 2 = 0 \end{aligned}$$

CHAPTER 5**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How do you determine the degree of a monomial with several variables?
2. How do you write a very small number in scientific notation?
3. How do you multiply two binomials?
4. How do you square a binomial?
5. After you divide a polynomial by a binomial, how do you check your answer?
6. How do you know if a binomial is a factor of a polynomial?
7. How do you use synthetic division to evaluate a polynomial function?
8. What type of divisor is necessary to use synthetic division for division of a polynomial?
9. How do you write a polynomial with a missing term when using synthetic division?
10. How is GCF used in factoring by grouping?
11. What are the binomial factors of the difference of two perfect squares?
12. To solve a quadratic equation by factoring, why must the equation be set equal to zero?
13. What does it mean to factor a polynomial completely?

CHAPTER 5

REVIEW EXERCISES

1. Factor: $18a^5b^2 - 12a^3b^3 + 30a^2b$
2. Divide: $\frac{15x^2 + 2x - 2}{3x - 2}$
3. Multiply: $(2x^{-1}y^2z^5)^4(-3x^3yz^{-3})^2$
4. Factor: $2ax + 4bx - 3ay - 6by$
5. Factor: $12 + x - x^2$
6. Use the Remainder Theorem to evaluate $P(x) = x^3 - 2x^2 + 3x - 5$ when $x = 2$.
7. Subtract: $(5x^2 - 8xy + 2y^2) - (x^2 - 3y^2)$
8. Factor: $24x^2 + 38x + 15$
9. Factor: $4x^2 + 12xy + 9y^2$
10. Multiply: $(-2a^2b^4)(3ab^2)$
11. Factor: $64a^3 - 27b^3$
12. Divide: $\frac{4x^3 + 27x^2 + 10x + 2}{x + 6}$
13. Given $P(x) = 2x^3 - x + 7$, evaluate $P(-2)$.
14. Factor: $x^2 - 3x - 40$
15. Factor: $x^2y^2 - 9$
16. Multiply: $4x^2y(3x^3y^2 + 2xy - 7y^3)$
17. Factor: $x^{2n} - 12x^n + 36$
18. Solve: $6x^2 + 60 = 39x$
19. Simplify: $5x^2 - 4x[x - 3(3x + 2) + x]$
20. Factor: $3a^6 - 15a^4 - 18a^2$
21. Expand: $(4x - 3y)^2$
22. Divide: $\frac{x^4 - 4}{x - 4}$

23. Add: $(3x^2 - 2x - 6) + (-x^2 - 3x + 4)$

24. Multiply: $(5x^2yz^4)(2xy^3z^{-1})(7x^{-2}y^{-2}z^3)$

25. Divide: $\frac{3x^4yz^{-1}}{-12xy^3z^2}$

26. Write 948,000,000 in scientific notation.

27. Simplify: $\frac{3 \times 10^{-3}}{15 \times 10^2}$

28. Use the Remainder Theorem to evaluate $P(x) = -2x^3 + 2x^2 - 4$ when $x = -3$.

29. Divide: $\frac{16x^5 - 8x^3 + 20x}{4x}$

30. Divide: $\frac{12x^2 - 16x - 7}{6x + 1}$

31. Multiply: $a^3(a^4 - 5a + 2)$

32. Multiply: $(x + 6)(x^3 - 3x^2 - 5x + 1)$

33. Factor: $10a^3b^3 - 20a^2b^4 + 35ab^2$

34. Factor: $5x^5 + x^3 + 4x^2$

35. Factor: $x(y - 3) + 4(3 - y)$

36. Factor: $x^2 - 16x + 63$

37. Factor: $24x^2 + 61x - 8$

38. Find two linear functions f and g such that $f(x) \cdot g(x) = 5x^2 + 3x - 2$.

39. Factor: $36 - a^{2n}$

40. Factor: $8 - y^{3n}$

41. Factor: $36x^8 - 36x^4 + 5$

42. Factor: $3a^4b - 3ab^4$

43. Factor: $x^4 - 8x^2 + 16$

44. Solve: $x^3 - x^2 - 6x = 0$

45. Solve: $x^3 - 16x = 0$

46. Solve: $y^3 + y^2 - 36y - 36 = 0$

47. Factor: $15x^4 + x^2 - 6$

48. Simplify: $\frac{(2a^4b^{-3}c^2)^3}{(2a^3b^2c^{-1})^4}$

49. Multiply: $(x - 4)(3x + 2)(2x - 3)$

50. Factor: $21x^4y^4 + 23x^2y^2 + 6$

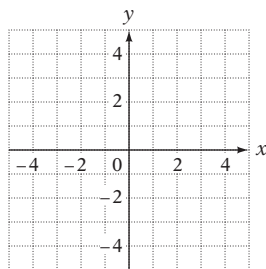
51. Solve: $x^3 + 16 = x(x + 16)$

52. Multiply: $(5a + 2b)(5a - 2b)$

53. Write 2.54×10^{-3} in decimal notation.

54. Factor: $6x^2 - 31x + 18$

55. Graph $y = x^2 + 1$.



56. For the polynomial $P(x) = 3x^5 - 6x^2 + 7x + 8$:

- Identify the leading coefficient.
- Identify the constant term.
- State the degree.



57. **Physics** The mass of the moon is 3.7×10^{-8} times the mass of the sun. The mass of the sun is 2.19×10^{27} tons. Find the mass of the moon. Write the answer in scientific notation.

58. **Mathematics** The sum of a number and its square is 56. Find the number.



59. **Astronomy** The most distant object visible from Earth without the aid of a telescope is the Great Galaxy of Andromeda. It takes light from the Great Galaxy of Andromeda 2.2×10^6 years to travel to Earth. Light travels about 6.7×10^8 mph. How far from Earth is the Great Galaxy of Andromeda? Use a 365-day year.

60. **Geometry** The length of a rectangle is $(5x + 3)$ cm. The width is $(2x - 7)$ cm. Find the area of the rectangle in terms of the variable x .



The Moon

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CHAPTER 5**TEST**

1. Factor: $16t^2 + 24t + 9$
2. Multiply: $-6rs^2(3r - 2s - 3)$
3. Given $P(x) = 3x^2 - 8x + 1$, evaluate $P(2)$.
4. Factor: $27x^3 - 8$
5. Factor: $16x^2 - 25$
6. Multiply: $(3t^3 - 4t^2 + 1)(2t^2 - 5)$
7. Simplify: $-5x[3 - 2(2x - 4) - 3x]$
8. Factor: $12x^3 + 12x^2 - 45x$
9. Solve: $6x^3 + x^2 - 6x - 1 = 0$
10. Subtract:
 $(6x^3 - 7x^2 + 6x - 7) - (4x^3 - 3x^2 + 7)$
11. Write the number 0.000000501 in scientific notation.
12. Divide: $\frac{14x^2 + x + 1}{7x - 3}$
13. Multiply: $(7 - 5x)(7 + 5x)$
14. Factor: $6a^4 - 13a^2 - 5$

15. Multiply: $(3a + 4b)(2a - 7b)$

16. Factor: $3x^4 - 23x^2 - 36$

17. Multiply: $(-4a^2b)^3(-ab^4)$

18. Solve: $6x^2 = x + 1$

19. Use the Remainder Theorem to evaluate $P(x) = -x^3 + 4x - 8$ when $x = -2$.

20. Simplify: $\frac{(2a^{-4}b^2)^3}{4a^{-2}b^{-1}}$

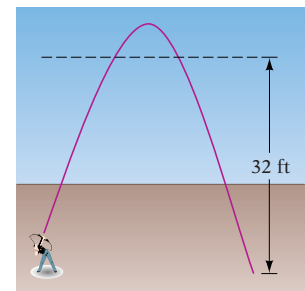
21. Divide: $\frac{x^3 - 2x^2 - 5x + 7}{x + 3}$

22. Factor: $12 - 17x + 6x^2$

23. Factor: $6x^2 - 4x - 3xa + 2a$

24. Write the number of seconds in 1 week in scientific notation.

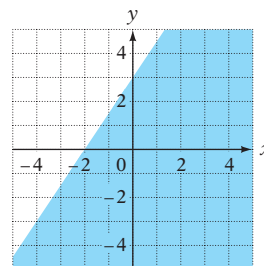
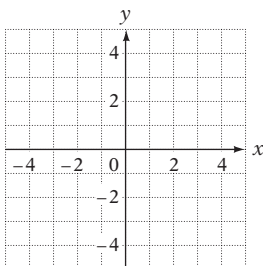
25. **Sports** An arrow is shot into the air with an upward velocity of 48 ft/s from a hill 32 ft high. How many seconds later will the arrow be 64 ft above the ground? Use the equation $h = 32 + 48t - 16t^2$, where h is the height in feet and t is the time in seconds.



26. **Geometry** The length of a rectangle is $(5x + 1)$ ft. The width is $(2x - 1)$ ft. Find the area of the rectangle in terms of the variable x .

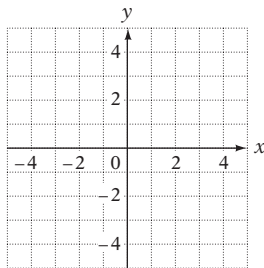
CUMULATIVE REVIEW EXERCISES

1. Simplify: $8 - 2[-3 - (-1)]^2 + 4$
2. Evaluate $\frac{2a - b}{b - c}$ when $a = 4$, $b = -2$, and $c = 6$.
3. Identify the property that justifies the statement $2x + (-2x) = 0$.
4. Simplify: $2x - 4[x - 2(3 - 2x) + 4]$
5. Solve: $\frac{2}{3} - y = \frac{5}{6}$
6. Solve: $8x - 3 - x = -6 + 3x - 8$
7. Divide: $\frac{x^3 - 3}{x - 3}$
8. Solve: $3 - |2 - 3x| = -2$
9. Given $P(x) = 3x^2 - 2x + 2$, evaluate $P(-2)$.
10. What values of x are excluded from the domain of the function $f(x) = \frac{x + 1}{x + 2}$?
11. Find the range of the function given by $F(x) = 3x^2 - 4$ if the domain is $\{-2, -1, 0, 1, 2\}$.
12. Find the slope of the line containing the points $(-2, 3)$ and $(4, 2)$.
13. Find the equation of the line that contains the point $(-1, 2)$ and has slope $-\frac{3}{2}$.
14. Find the equation of the line that contains the point $(-2, 4)$ and is perpendicular to the line $3x + 2y = 4$.
15. Solve by using Cramer's Rule:
 $2x - 3y = 2$
 $x + y = -3$
16. Solve by the addition method:
 $x - y + z = 0$
 $2x + y - 3z = -7$
 $-x + 2y + 2z = 5$
17. Graph $3x - 4y = 12$ by using the x - and y -intercepts.
18. Graph the solution set: $-3x + 2y < 6$



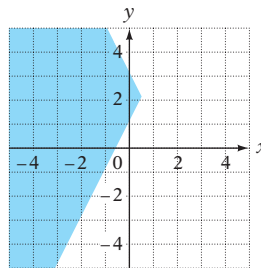
19. Solve by graphing:

$$\begin{aligned}x - 2y &= 3 \\ -2x + y &= -3\end{aligned}$$



20. Graph the solution set:

$$\begin{aligned}2x + y &< 3 \\ -2x + y &\geq 1\end{aligned}$$



21. Simplify:
- $(4a^{-2}b^3)(2a^3b^{-1})^{-2}$

22. Simplify:
- $\frac{(5x^3y^{-3}z)^{-2}}{y^4z^{-2}}$

23. Simplify:
- $3 - (3 - 3^{-1})^{-1}$

24. Multiply:
- $(2x + 3)(2x^2 - 3x + 1)$

25. Factor:
- $-4x^3 + 14x^2 - 12x$

26. Factor:
- $a(x - y) - b(y - x)$

27. Factor:
- $x^4 - 16$

28. Factor:
- $2x^3 - 16$

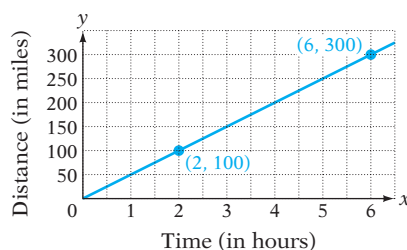
29. **Mathematics** The sum of two integers is twenty-four. The difference between four times the smaller integer and nine is three less than twice the larger integer. Find the integers.

30. **Mixtures** How many ounces of pure gold that costs \$360 per ounce must be mixed with 80 oz of an alloy that costs \$120 per ounce to make a mixture that costs \$200 per ounce?

31. **Uniform Motion** Two bicycles are 25 mi apart and are traveling toward each other. One cyclist is traveling at $\frac{2}{3}$ the rate of the other cyclist. They pass in 2 h. Find the rate of each cyclist.

32. **Astronomy** A space vehicle travels 2.4×10^5 mi from Earth to the moon at an average velocity of 2×10^4 mph. How long does it take the vehicle to reach the moon?

33. **Uniform Motion** The graph shows the relationship between the distance traveled and the time of travel. Find the slope of the line between the two points on the graph. Write a sentence that states the meaning of the slope.



Rational Expressions

OBJECTIVES

SECTION 6.1

- A** To find the domain of a rational function
- B** To simplify a rational expression
- C** To multiply rational expressions
- D** To divide rational expressions

SECTION 6.2

- A** To rewrite rational expressions in terms of a common denominator
- B** To add or subtract rational expressions

SECTION 6.3

- A** To simplify a complex fraction

SECTION 6.4

- A** To solve a proportion
- B** To solve application problems

SECTION 6.5

- A** To solve a rational equation
- B** To solve work problems
- C** To solve uniform motion problems

SECTION 6.6

- A** To solve variation problems

ARE YOU READY?

Take the Chapter 6 Prep Test to find out if you are ready to learn to:

- Add, subtract, multiply, and divide rational expressions
- Simplify a complex fraction
- Solve a proportion
- Solve a rational equation
- Solve work and uniform motion problems
- Solve variation problems

PREP TEST

Do these exercises to prepare for Chapter 6.

1. Find the LCM of 10 and 25.

For Exercises 2 to 5, add, subtract, multiply, or divide.

$$2. -\frac{3}{8} \cdot \frac{4}{9}$$

$$3. -\frac{4}{5} \div \frac{8}{15}$$

$$4. -\frac{5}{6} + \frac{7}{8}$$

$$5. -\frac{3}{8} - \left(-\frac{7}{12}\right)$$

$$6. \text{Simplify: } \frac{\frac{2}{3} - \frac{1}{4}}{\frac{1}{8} - 2}$$

$$7. \text{Evaluate } \frac{2x-3}{x^2-x+1} \text{ for } x=2.$$

$$8. \text{Solve: } 4(2x+1) = 3(x-2) \quad 9. \text{Solve: } 10\left(\frac{t}{2} + \frac{t}{5}\right) = 10(1)$$

10. Two planes start from the same point and fly in opposite directions. The first plane is flying 20 mph slower than the second plane. In 2 h, the planes are 480 mi apart. Find the rate of each plane.

SECTION

6.1

Multiplication and Division of Rational Expressions

OBJECTIVE A

To find the domain of a rational function



An expression in which the numerator and denominator are polynomials is called a **rational expression**. Examples of rational expressions are shown at the right. Both the numerator and denominator are polynomials.

$$\frac{9}{z} \quad \frac{3x+4}{2x^2+1} \quad \frac{x^3-x+1}{x^2-3x-5}$$

The expression $\frac{\sqrt{x}+3}{x}$ is not a rational expression because $\sqrt{x}+3$ is not a polynomial.

A function that is written in terms of a rational expression is a **rational function**. Each of the following equations represents a rational function.

$$f(x) = \frac{x^2+3}{2x-1} \quad g(t) = \frac{3}{t^2-4} \quad R(z) = \frac{z^2+3z-1}{z^2+z-12}$$

To evaluate a rational function, replace the variable by its value. Then simplify.



Integrating Technology

See the Keystroke Guide: *Evaluating Functions* for instructions on using a graphing calculator to evaluate a function.

HOW TO 1

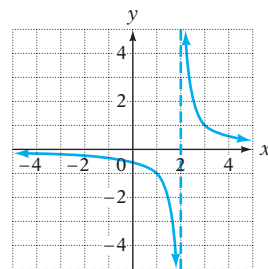
Evaluate $f(-2)$ given $f(x) = \frac{x^2}{3x^2-x-9}$.

$$\begin{aligned} f(x) &= \frac{x^2}{3x^2-x-9} \\ f(-2) &= \frac{(-2)^2}{3(-2)^2 - (-2) - 9} = \frac{4}{12 + 2 - 9} = \frac{4}{5} \end{aligned}$$

- Replace x by -2 . Then simplify.

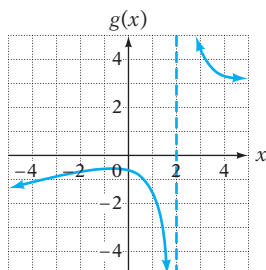
Because division by zero is not defined, the domain of a rational function must exclude those numbers for which the value of the polynomial in the denominator is zero.

The graph of $f(x) = \frac{1}{x-2}$ is shown at the right. Note that the graph never intersects the graph of $x = 2$ (shown as a dashed line). The value 2 is excluded from the domain of $f(x) = \frac{1}{x-2}$.



Integrating Technology

See the Keystroke Guide: *Graph* for instructions on using a graphing calculator to graph a function.



HOW TO 2

Determine the domain of $g(x) = \frac{x^2+4}{3x-6}$.

The domain of g must exclude values of x for which the denominator is zero. To find these values, set the denominator equal to zero and solve for x .

$$\begin{aligned} 3x - 6 &= 0 && \bullet \text{ Set the denominator equal to zero.} \\ 3x &= 6 && \bullet \text{ Solve for } x. \\ x &= 2 && \bullet \text{ This value must be excluded from the domain.} \end{aligned}$$

The domain of g is $\{x \mid x \neq 2\}$.

EXAMPLE • 1

Given $f(x) = \frac{3x - 4}{x^2 - 2x + 1}$, find $f(-2)$.

Solution

$$\begin{aligned}
 f(x) &= \frac{3x - 4}{x^2 - 2x + 1} \\
 f(-2) &= \frac{3(-2) - 4}{(-2)^2 - 2(-2) + 1} && \bullet \text{ Replace } x \text{ by } -2. \\
 &= \frac{-6 - 4}{4 + 4 + 1} \\
 &= \frac{-10}{9} = -\frac{10}{9}
 \end{aligned}$$

YOU TRY IT • 1

Given $f(x) = \frac{3 - 5x}{x^2 + 5x + 6}$, find $f(2)$.

Your solution**EXAMPLE • 2**

Find the domain of $f(x) = \frac{x^2 - 1}{x^2 - 2x - 15}$.

Solution

Set the denominator equal to zero. Then solve for x .

$$\begin{aligned}
 x^2 - 2x - 15 &= 0 && \bullet \text{ Solve the quadratic equation by factoring.} \\
 (x - 5)(x + 3) &= 0 \\
 x - 5 = 0 & \quad x + 3 = 0 \\
 x = 5 & \quad x = -3
 \end{aligned}$$

The domain is $\{x \mid x \neq -3, 5\}$.

YOU TRY IT • 2

Find the domain of $f(x) = \frac{2x + 1}{2x^2 - 7x + 3}$.

Your solution

Solutions on p. S18

OBJECTIVE B**To simplify a rational expression**

The Multiplication Property of One is used to write the **simplest form of a rational expression**, which means that the numerator and denominator of the rational expression have no common factors.

HOW TO • 3

Simplify: $\frac{x^2 - 25}{x^2 + 13x + 40}$

$$\frac{x^2 - 25}{x^2 + 13x + 40} = \frac{(x - 5)}{(x + 8)} \cdot \frac{\boxed{(x + 5)}}{\boxed{(x + 5)}} = \frac{x - 5}{x + 8} \cdot 1 = \frac{x - 5}{x + 8}, \quad x \neq -8, -5$$

The requirement $x \neq -8, -5$ is necessary because division by 0 is not allowed.

The simplification above is usually shown with slashes to indicate that a common factor has been removed:

$$\frac{x^2 - 25}{x^2 + 13x + 40} = \frac{(x - 5)\cancel{(x + 5)}}{(x + 8)\cancel{(x + 5)}} = \frac{x - 5}{x + 8}, \quad x \neq -8, -5$$

We will show a simplification with slashes. We will not show the restrictions that prevent division by zero. Nonetheless, those restrictions *always* are implied.

Take Note

Recall that

$$b - a = -(a - b).$$

Therefore,

$$4 - x = -(x - 4).$$

In general,

$$\frac{b - a}{a - b} = \frac{-\cancel{a}^1 \cancel{b}^1}{\cancel{a}^1 \cancel{b}^1} = \frac{-1}{1} = -1.$$

HOW TO • 4Simplify: $\frac{12 + 5x - 2x^2}{2x^2 - 3x - 20}$

$$\begin{aligned}\frac{12 + 5x - 2x^2}{2x^2 - 3x - 20} &= \frac{(4 - x)(3 + 2x)}{(x - 4)(2x + 5)} \\ &= \frac{\overset{-1}{\cancel{(4 - x)}}(3 + 2x)}{\cancel{(x - 4)}^1(2x + 5)} \\ &= -\frac{2x + 3}{2x + 5}\end{aligned}$$

• Factor the numerator and denominator.

$$\bullet \frac{4 - x}{x - 4} = \frac{-\overset{1}{\cancel{(x - 4)}}}{\cancel{x - 4}^1} = \frac{-1}{1} = -1$$

• Write the answer in simplest form.

EXAMPLE • 3Simplify: $\frac{12x^3y^2 + 6x^3y^3}{6x^2y^2}$ **Solution**

$$\begin{aligned}\frac{12x^3y^2 + 6x^3y^3}{6x^2y^2} &= \frac{6x^3y^2(2 + y)}{6x^2y^2} \\ &= x(2 + y)\end{aligned}$$

• Factor. Then divide by the common factors.

YOU TRY IT • 3Simplify: $\frac{21a^3b - 14a^3b^2}{7a^2b}$ **Your solution****EXAMPLE • 4**Simplify: $\frac{6x^3 - 9x^2}{12x^2 - 18x}$ **Solution**

$$\begin{aligned}\frac{6x^3 - 9x^2}{12x^2 - 18x} &= \frac{3x^2(2x - 3)}{6x(2x - 3)} \\ &= \frac{\overset{1}{\cancel{3x^2(2x - 3)}}}{\cancel{6x(2x - 3)}^1} = \frac{x}{2}\end{aligned}$$

• Factor. Then divide by the common factors.

YOU TRY IT • 4Simplify: $\frac{6x^4 - 24x^3}{12x^3 - 48x^2}$ **Your solution****EXAMPLE • 5**Simplify: $\frac{2x^2 - 8x^3}{16x^3 - 28x^2 + 6x}$ **Solution**

$$\begin{aligned}\frac{2x^2 - 8x^3}{16x^3 - 28x^2 + 6x} &= \frac{2x^2(1 - 4x)}{2x(8x^2 - 14x + 3)} \\ &= \frac{2x^2(1 - 4x)}{2x(4x - 1)(2x - 3)} \\ &= \frac{\overset{-1}{\cancel{2x^2(1 - 4x)}}}{\cancel{2x(4x - 1)}^1(2x - 3)} \\ &= -\frac{x}{2x - 3}\end{aligned}$$

YOU TRY IT • 5Simplify: $\frac{20x - 15x^2}{15x^3 - 5x^2 - 20x}$ **Your solution**

EXAMPLE • 6

Simplify: $\frac{x^{2n} + x^n - 2}{x^{2n} - 1}$

Solution

$$\begin{aligned}\frac{x^{2n} + x^n - 2}{x^{2n} - 1} &= \frac{(x^n - 1)(x^n + 2)}{(x^n - 1)(x^n + 1)} \\ &= \frac{\overbrace{(x^n - 1)}^1 \overbrace{(x^n + 2)}^1}{\overbrace{(x^n - 1)}^1 \overbrace{(x^n + 1)}^1} = \frac{x^n + 2}{x^n + 1}\end{aligned}$$

YOU TRY IT • 6

Simplify: $\frac{x^{2n} + x^n - 12}{x^{2n} - 3x^n}$

Your solution

Solution on p. S18

OBJECTIVE C**To multiply rational expressions**

The product of two fractions is a fraction whose numerator is the product of the numerators of the two fractions and whose denominator is the product of the denominators of the two fractions.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \frac{5}{a+2} \cdot \frac{b-3}{3} = \frac{5(b-3)}{(a+2)3} = \frac{5b-15}{3a+6}$$

HOW TO • 5

Simplify: $\frac{12x^4y^5}{25a^3b^4} \cdot \frac{15a^5b^4}{16x^7y^2}$

$$\begin{aligned}\frac{12x^4y^5}{25a^3b^4} \cdot \frac{15a^5b^4}{16x^7y^2} &= \frac{12x^4y^5 \cdot 15a^5b^4}{25a^3b^4 \cdot 16x^7y^2} \\ &= \frac{9a^2y^3}{20x^3}\end{aligned}$$

- **Multiply numerators and denominators. Then simplify using the Rules of Exponents.**

The product of two rational expressions can often be simplified by factoring the numerator and the denominator.

HOW TO • 6

Simplify: $\frac{x^2 - 2x}{2x^2 + x - 15} \cdot \frac{2x^2 - x - 10}{x^2 - 4}$

$$\begin{aligned}\frac{x^2 - 2x}{2x^2 + x - 15} \cdot \frac{2x^2 - x - 10}{x^2 - 4} &= \frac{x(x-2)}{(x+3)(2x-5)} \cdot \frac{(x+2)(2x-5)}{(x+2)(x-2)} \\ &= \frac{x(x-2)(x+2)(2x-5)}{(x+3)(2x-5)(x+2)(x-2)} \\ &= \frac{\overbrace{x}^1 \overbrace{(x-2)}^1 \overbrace{(x+2)}^1 \overbrace{(2x-5)}^1}{(x+3) \overbrace{(2x-5)}^1 \overbrace{(x+2)}^1 \overbrace{(x-2)}^1} \\ &= \frac{x}{x+3}\end{aligned}$$

- **Factor the numerator and the denominator of each fraction.**
- **Multiply.**
- **Simplify.**
- **Write the answer in simplest form.**

EXAMPLE • 7

Simplify: $\frac{2x^2 - 6x}{3x - 6} \cdot \frac{6x - 12}{8x^3 - 12x^2}$

Solution

$$\begin{aligned} & \frac{2x^2 - 6x}{3x - 6} \cdot \frac{6x - 12}{8x^3 - 12x^2} \\ &= \frac{2x(x - 3)}{3(x - 2)} \cdot \frac{6(x - 2)}{4x^2(2x - 3)} \\ &= \frac{2x(x - 3) \cdot 6(x - 2)}{3(x - 2) \cdot 4x^2(2x - 3)} \\ &= \frac{2x(x - 3) \cdot \cancel{6(x - 2)}}{3\cancel{(x - 2)} \cdot 4x^2(2x - 3)} \\ &= \frac{x - 3}{x(2x - 3)} \end{aligned}$$

EXAMPLE • 8

Simplify: $\frac{6x^2 + x - 2}{6x^2 + 7x + 2} \cdot \frac{2x^2 + 9x + 4}{4 - 7x - 2x^2}$

Solution

$$\begin{aligned} & \frac{6x^2 + x - 2}{6x^2 + 7x + 2} \cdot \frac{2x^2 + 9x + 4}{4 - 7x - 2x^2} \\ &= \frac{(2x - 1)(3x + 2)}{(3x + 2)(2x + 1)} \cdot \frac{(2x + 1)(x + 4)}{(1 - 2x)(4 + x)} \\ &= \frac{(2x - 1)(3x + 2) \cdot (2x + 1)(x + 4)}{(3x + 2)(2x + 1) \cdot (1 - 2x)(4 + x)} \\ &= \frac{\cancel{(2x - 1)}\cancel{(3x + 2)}\cancel{(2x + 1)}\cancel{(x + 4)}}{\cancel{(3x + 2)}\cancel{(2x + 1)}\cancel{(1 - 2x)}\cancel{(4 + x)}} \\ &= -1 \end{aligned}$$

YOU TRY IT • 7

Simplify: $\frac{12 + 5x - 3x^2}{x^2 + 2x - 15} \cdot \frac{2x^2 + x - 45}{3x^2 + 4x}$

Your solution**YOU TRY IT • 8**

Simplify: $\frac{2x^2 - 13x + 20}{x^2 - 16} \cdot \frac{2x^2 + 9x + 4}{6x^2 - 7x - 5}$

Your solution

Solutions on p. S18

OBJECTIVE D**To divide rational expressions**

The **reciprocal of a rational expression** is the rational expression with the numerator and denominator interchanged.

$$\text{Rational Expression } \left\{ \begin{array}{l} \frac{a}{b} \\ \frac{a^2 - 2y}{4} \end{array} \quad \begin{array}{l} \frac{b}{a} \\ \frac{4}{a^2 - 2y} \end{array} \right\} \text{ Reciprocal}$$

To divide two rational expressions, multiply by the reciprocal of the divisor.

$$\begin{aligned} \frac{a}{b} \div \frac{c}{d} &= \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \\ \frac{2}{a} \div \frac{5}{b} &= \frac{2}{a} \cdot \frac{b}{5} = \frac{2b}{5a} \\ \frac{x+y}{2} \div \frac{x-y}{5} &= \frac{x+y}{2} \cdot \frac{5}{x-y} = \frac{(x+y)5}{2(x-y)} = \frac{5x+5y}{2x-2y} \end{aligned}$$

EXAMPLE • 9

Simplify: $\frac{12x^2y}{25a^2b^5} \div \frac{9x^3y^4}{10a^3b^4}$

Solution

$$\begin{aligned}\frac{12x^2y}{25a^2b^5} \div \frac{9x^3y^4}{10a^3b^4} &= \frac{12x^2y}{25a^2b^5} \cdot \frac{10a^3b^4}{9x^3y^4} && \bullet \text{ Rewrite division as multiplication by the reciprocal.} \\ &= \frac{12x^2y \cdot 10a^3b^4}{25a^2b^5 \cdot 9x^3y^4} \\ &= \frac{8a}{15xy^3b}\end{aligned}$$

YOU TRY IT • 9

Simplify: $\frac{7a^3b^7}{15x^2y^3} \div \frac{21a^5b^2}{20x^4y}$

Your solution**EXAMPLE • 10**

Simplify: $\frac{12x^2y^2 - 24xy^2}{5z^2} \div \frac{4x^3y - 8x^2y}{3z^4}$

Solution

$$\begin{aligned}\frac{12x^2y^2 - 24xy^2}{5z^2} \div \frac{4x^3y - 8x^2y}{3z^4} &= \frac{12x^2y^2 - 24xy^2}{5z^2} \cdot \frac{3z^4}{4x^3y - 8x^2y} && \bullet \text{ Rewrite division as multiplication by the reciprocal.} \\ &= \frac{12xy^2(x - 2)}{5z^2} \cdot \frac{3z^4}{4x^2y(x - 2)} \\ &= \frac{12xy^2(x - 2)3z^4}{5z^2 \cdot 4x^2y(x - 2)} \\ &= \frac{12xy^2(\cancel{x - 2})3z^4}{5z^2 \cdot 4x^2y(\cancel{x - 2})} \\ &= \frac{9yz^2}{5x}\end{aligned}$$

YOU TRY IT • 10

Simplify: $\frac{6x^2 - 3xy}{10ab^4} \div \frac{16x^2y^2 - 8xy^3}{15a^2b^2}$

Your solution**EXAMPLE • 11**

Simplify: $\frac{3y^2 - 10y + 8}{3y^2 + 8y - 16} \div \frac{2y^2 - 7y + 6}{2y^2 + 5y - 12}$

Solution

$$\begin{aligned}\frac{3y^2 - 10y + 8}{3y^2 + 8y - 16} \div \frac{2y^2 - 7y + 6}{2y^2 + 5y - 12} &= \frac{3y^2 - 10y + 8}{3y^2 + 8y - 16} \cdot \frac{2y^2 + 5y - 12}{2y^2 - 7y + 6} \\ &= \frac{(y - 2)(3y - 4)}{(3y - 4)(y + 4)} \cdot \frac{(y + 4)(2y - 3)}{(y - 2)(2y - 3)} \\ &= \frac{(y - 2)(3y - 4)(y + 4)(2y - 3)}{(3y - 4)(y + 4)(y - 2)(2y - 3)} \\ &= \frac{(\cancel{y - 2})(\cancel{3y - 4})(\cancel{y + 4})(\cancel{2y - 3})}{(\cancel{3y - 4})(\cancel{y + 4})(\cancel{y - 2})(\cancel{2y - 3})} \\ &= 1\end{aligned}$$

YOU TRY IT • 11

Simplify: $\frac{6x^2 - 7x + 2}{3x^2 + x - 2} \div \frac{4x^2 - 8x + 3}{5x^2 + x - 4}$

Your solution

6.1 EXERCISES

OBJECTIVE A

To find the domain of a rational function



1. What is a rational function? Give an example of a rational function.



2. What values are excluded from the domain of a rational function?

3. Given $f(x) = \frac{2}{x-3}$, find $f(4)$.

4. Given $f(x) = \frac{-7}{5-x}$, find $f(-2)$.

5. Given $f(x) = \frac{x-2}{x+4}$, find $f(-2)$.

6. Given $f(x) = \frac{x-3}{2x-1}$, find $f(3)$.

7. Given $f(x) = \frac{1}{x^2-2x+1}$, find $f(-2)$.

8. Given $f(x) = \frac{-3}{x^2-4x+2}$, find $f(-1)$.

9. Given $f(x) = \frac{x-2}{2x^2+3x+8}$, find $f(3)$.

10. Given $f(x) = \frac{x^2}{3x^2-3x+5}$, find $f(4)$.

11. Given $f(x) = \frac{x^2-2x}{x^3-x+4}$, find $f(-1)$.

12. Given $f(x) = \frac{8-x^2}{x^3-x^2+4}$, find $f(-3)$.

For Exercises 13 to 24, find the domain of the function.

13. $f(x) = \frac{4}{x-3}$

14. $G(x) = \frac{-2}{x+2}$

15. $H(x) = \frac{x}{x+4}$

16. $F(x) = \frac{3x}{x-5}$

17. $h(x) = \frac{5x}{3x+9}$

18. $f(x) = \frac{-2x}{6-2x}$

19. $q(x) = \frac{4-x}{(x-4)(3x-2)}$

20. $p(x) = \frac{2x+1}{(2x+5)(3x-6)}$

21. $f(x) = \frac{2x-1}{x^2+x-6}$

22. $G(x) = \frac{3-4x}{x^2+4x-5}$

23. $f(x) = \frac{x+1}{x^2+1}$

24. $g(x) = \frac{x^2+1}{x^2}$



25. Which of the following represent(s) a rational function?

(i) $f(x) = \frac{1}{x}$ (ii) $g(x) = \frac{\pi}{x^2-4}$ (iii) $h(x) = \frac{\sqrt{3}}{x-3}$ (iv) $r(x) = \frac{4}{\sqrt{x}-4}$



26. Give an example of a rational function for which 2 is not in the domain of the function.

OBJECTIVE B**To simplify a rational expression**

27. When is a rational expression in simplest form?



28. Are the rational expressions $\frac{x(x-2)}{2(x-2)}$ and $\frac{x}{2}$ equal for all values of x ? Why or why not?

For Exercises 29 to 61, simplify.

29. $\frac{4 - 8x}{4}$

30. $\frac{8y + 2}{2}$

31. $\frac{6x^2 - 2x}{2x}$

32. $\frac{3y - 12y^2}{3y}$

33. $\frac{3x^3y^3 - 12x^2y^2 + 15xy}{3xy}$

34. $\frac{-10a^4 - 20a^3 + 30a^2}{-10a^2}$

35. $\frac{8x^2(x - 3)}{4x(x - 3)}$

36. $\frac{16y^4(y + 8)}{12y^3(y + 8)}$

37. $\frac{-36a^2 - 48a}{18a^3 + 24a^2}$

38. $\frac{24r^3t^2 + 36rt^4}{12r^3t + 18rt^3}$

39. $\frac{3x - 6}{x^2 + 2x}$

40. $\frac{a^2 + 4a}{4a - 16}$

41. $\frac{x^2 - 7x + 12}{x^2 - 9x + 20}$

42. $\frac{x^2 - 2x - 24}{x^2 - 10x + 24}$

43. $\frac{2x^2 - 5x - 3}{2x^2 - 3x - 9}$

44. $\frac{6 - x - x^2}{3x^2 - 10x + 8}$

45. $\frac{3x^2 + 10x - 8}{8 - 14x + 3x^2}$

46. $\frac{14 - 19x - 3x^2}{3x^2 - 23x + 14}$

47. $\frac{a^2 - b^2}{a^3 + b^3}$

48. $\frac{x^4 - y^4}{x^2 + y^2}$

49. $\frac{8x^3 - y^3}{4x^2 - y^2}$

50. $\frac{x^2 - 4}{a(x + 2) - b(x + 2)}$

51. $\frac{x^2(a - 2) - a + 2}{ax^2 - ax}$

52. $\frac{x^4 + 3x^2 + 2}{x^4 - 1}$

53. $\frac{x^4 - 2x^2 - 3}{x^4 + 2x^2 + 1}$

54. $\frac{x^2y^2 + 4xy - 21}{x^2y^2 - 10xy + 21}$

55. $\frac{6x^2y^2 + 11xy + 4}{9x^2y^2 + 9xy - 4}$

56. $\frac{a^{2n} - a^n - 2}{a^{2n} + 3a^n + 2}$

57. $\frac{a^{2n} + a^n - 12}{a^{2n} - 2a^n - 3}$

58. $\frac{a^{2n} - 1}{a^{2n} - 2a^n + 1}$

59. $\frac{a^{2n} + 2a^n b^n + b^{2n}}{a^{2n} - b^{2n}}$

60. $\frac{(x-3) - b(x-3)}{b(x+3) - x-3}$

61. $\frac{x^2(a+b) + a+b}{x^4 - 1}$



62. If $\frac{x^2 - x - 6}{x^2 - n} = \frac{x+2}{x+3}$, what is the value of n ?



63. If $\frac{x^2 + kx - 5}{x^2 + 7x + 10} = \frac{x-1}{x+2}$, what is the value of k ?

OBJECTIVE C**To multiply rational expressions**

For Exercises 64 to 77, simplify.

64. $\frac{27a^2b^5}{16xy^2} \cdot \frac{20x^2y^3}{9a^2b}$

65. $\frac{15x^2y^4}{24ab^3} \cdot \frac{28a^2b^4}{35xy^4}$

66. $\frac{3x-15}{4x^2-2x} \cdot \frac{20x^2-10x}{15x-75}$

67. $\frac{2x^2+4x}{8x^2-40x} \cdot \frac{6x^3-30x^2}{3x^2+6x}$

68. $\frac{x^2y^3}{x^2-4x-5} \cdot \frac{2x^2-13x+15}{x^4y^3}$

69. $\frac{2x^2-5x+3}{x^6y^3} \cdot \frac{x^4y^4}{2x^2-x-3}$

70. $\frac{x^2-3x+2}{x^2-8x+15} \cdot \frac{x^2+x-12}{8-2x-x^2}$

71. $\frac{x^2+x-6}{12+x-x^2} \cdot \frac{x^2+x-20}{x^2-4x+4}$

72. $\frac{x^3-y^3}{2x^2+xy-3y^2} \cdot \frac{2x^2+5xy+3y^2}{x^2+xy+y^2}$

73. $\frac{x^4-5x^2+4}{3x^2-4x-4} \cdot \frac{3x^2-10x-8}{x^2-4}$

74. $\frac{x^2+x-6}{3x^2+5x-12} \cdot \frac{2x^2-14x}{16x^2-4x} \cdot \frac{12x^2-19x+4}{x^2+5x-14}$

75. $\frac{x^2-y^2}{x^2+xy+y^2} \cdot \frac{x^2-xy}{3x^2-3xy} \cdot \frac{x^3-y^3}{x^2-2xy+y^2}$

76. $\frac{x^{2n}-x^n-6}{x^{2n}+x^n-2} \cdot \frac{x^{2n}-5x^n-6}{x^{2n}-2x^n-3}$

77. $\frac{x^{2n}+3x^n+2}{x^{2n}-x^n-6} \cdot \frac{x^{2n}+x^n-12}{x^{2n}-1}$



78. If $\frac{x^2-x-6}{x^2-5x+6} \cdot \frac{x^2+x-6}{x^2+6x+n} = \frac{x+3}{x+4}$, what is the value of n ?



79. If $\frac{p(x)}{x^2+7x+12} \cdot \frac{x+4}{x+5} = \frac{x-3}{x+3}$, what is $p(x)$?

OBJECTIVE D**To divide rational expressions**

For Exercises 80 to 95, simplify.

80. $\frac{6x^2y^4}{35a^2b^5} \div \frac{12x^3y^3}{7a^4b^5}$

81. $\frac{12a^4b^7}{13x^2y^2} \div \frac{18a^5b^6}{26xy^3}$

82. $\frac{2x - 6}{6x^2 - 15x} \div \frac{4x^2 - 12x}{18x^3 - 45x^2}$

83. $\frac{4x^2 - 4y^2}{6x^2y^2} \div \frac{3x^2 + 3xy}{2x^2y - 2xy^2}$

84. $\frac{2x^2 - 2y^2}{14x^2y^4} \div \frac{x^2 + 2xy + y^2}{35xy^3}$

85. $\frac{8x^3 + 12x^2y}{4x^2 - 9y^2} \div \frac{16x^2y^2}{4x^2 - 12xy + 9y^2}$

86. $\frac{x^2 - 8x + 15}{x^2 + 2x - 35} \div \frac{15 - 2x - x^2}{x^2 + 9x + 14}$

87. $\frac{2x^2 + 13x + 20}{8 - 10x - 3x^2} \div \frac{6x^2 - 13x - 5}{9x^2 - 3x - 2}$

88. $\frac{14 + 17x - 6x^2}{3x^2 + 14x + 8} \div \frac{4x^2 - 49}{2x^2 + 15x + 28}$

89. $\frac{16x^2 - 9}{6 - 5x - 4x^2} \div \frac{16x^2 + 24x + 9}{4x^2 + 11x + 6}$

90. $\frac{6x^2 + 6x}{3x + 6x^2 + 3x^3} \div \frac{x^2 - 1}{1 - x^3}$

91. $\frac{x^3 + y^3}{2x^3 + 2x^2y} \div \frac{3x^3 - 3x^2y + 3xy^2}{6x^2 - 6y^2}$

92. $\frac{x^2 - 9}{x^2 - x - 6} \div \frac{2x^2 + x - 15}{x^2 + 7x + 10} \cdot \frac{2x^2 - x - 10}{x^2 - 25}$

93. $\frac{3x^2 + 10x - 8}{x^2 + 4x + 3} \cdot \frac{x^2 + 6x + 9}{3x^2 - 5x + 2} \div \frac{x^2 + x - 12}{x^2 - 1}$

94. $\frac{2x^{2n} - x^n - 6}{x^{2n} - x^n - 2} \div \frac{2x^{2n} + x^n - 3}{x^{2n} - 1}$

95. $\frac{x^{4n} - 1}{x^{2n} + x^n - 2} \div \frac{x^{2n} + 1}{x^{2n} + 3x^n + 2}$



96. If $\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$, what is $\frac{c}{d} \cdot \frac{e}{f}$?



97. If $\frac{x^2 + x - 6}{x^2 - x - 20} \div \frac{p(x)}{x^2 + 6x + 8} = \frac{x + 3}{x - 5}$, what is $p(x)$?

Applying the Concepts

For Exercises 98 and 99, simplify.

98. $\frac{3x^2 + 6x}{4x^2 - 16} \cdot \frac{2x + 8}{x^2 + 2x} \div \frac{3x - 9}{5x - 20}$

99. $\frac{5y^2 - 20}{3y^2 - 12y} \cdot \frac{9y^3 + 6y}{2y^2 - 4y} \div \frac{y^3 + 2y^2}{2y^2 - 8y}$

SECTION

6.2

Addition and Subtraction of Rational Expressions

OBJECTIVE A

To rewrite rational expressions in terms of a common denominator



Tips for Success

As you know, often in mathematics you learn one skill in order to perform another. This is true of this objective. You are learning to rewrite rational expressions in terms of a common denominator in order to add and subtract rational expressions in the next objective. To ensure success, be certain you understand this lesson before studying the next.

In adding or subtracting rational expressions, it is frequently necessary to express the rational expressions in terms of a common denominator. This common denominator is the least common multiple (LCM) of the denominators.

The **least common multiple (LCM) of two or more polynomials** is the simplest polynomial that contains the factors of each polynomial. To find the LCM, first factor each polynomial completely. The LCM is the product of each factor the greatest number of times it occurs in any one factorization.

HOW TO 1

Find the LCM of $3x^2 + 15x$ and $6x^4 + 24x^3 - 30x^2$.

Factor each polynomial.

$$\begin{aligned} 3x^2 + 15x &= 3x(x + 5) \\ 6x^4 + 24x^3 - 30x^2 &= 6x^2(x^2 + 4x - 5) \\ &= 6x^2(x - 1)(x + 5) \end{aligned}$$

The LCM is the product of the LCM of the numerical coefficients and each variable factor the greatest number of times it occurs in any one factorization.

$$\text{LCM} = 6x^2(x - 1)(x + 5)$$

HOW TO 2

Write the fractions $\frac{5}{6x^2y}$ and $\frac{a}{9xy^3}$ in terms of the LCM of the denominators.The LCM is $18x^2y^3$.

$$\begin{aligned} \frac{5}{6x^2y} &= \frac{5}{6x^2y} \cdot \frac{3y^2}{3y^2} = \frac{15y^2}{18x^2y^3} \\ \frac{a}{9xy^3} &= \frac{a}{9xy^3} \cdot \frac{2x}{2x} = \frac{2ax}{18x^2y^3} \end{aligned}$$

- Find the LCM of the denominators.
- For each fraction, multiply the numerator and denominator by the factor whose product with the denominator is the LCM.

HOW TO 3

Write the fractions $\frac{x+2}{x^2-2x}$ and $\frac{5x}{3x-6}$ in terms of the LCM of the denominators.The LCM is $3x(x-2)$.

$$\begin{aligned} \frac{x+2}{x^2-2x} &= \frac{x+2}{x(x-2)} \cdot \frac{3}{3} = \frac{3x+6}{3x(x-2)} \\ \frac{5x}{3x-6} &= \frac{5x}{3(x-2)} \cdot \frac{x}{x} = \frac{5x^2}{3x(x-2)} \end{aligned}$$

- Find the LCM of the denominators.
- For each fraction, multiply the numerator and denominator by the factor whose product with the denominator is the LCM.



Take Note

$x^2 - 2x = x(x - 2)$
 $3x - 6 = 3(x - 2)$
 The LCM of $x(x - 2)$ and $3(x - 2)$ is $3x(x - 2)$.

EXAMPLE • 1

Write the fractions $\frac{3x}{x-1}$ and $\frac{4}{2x+5}$ in terms of the LCM of the denominators.

Solution

The LCM of $x-1$ and $2x+5$ is $(x-1)(2x+5)$.

$$\begin{aligned}\frac{3x}{x-1} &= \frac{3x}{x-1} \cdot \frac{2x+5}{2x+5} = \frac{6x^2 + 15x}{(x-1)(2x+5)} \\ \frac{4}{2x+5} &= \frac{4}{2x+5} \cdot \frac{x-1}{x-1} = \frac{4x-4}{(x-1)(2x+5)}\end{aligned}$$

EXAMPLE • 2

Write the fractions $\frac{2a-3}{a^2-2a}$ and $\frac{a+1}{2a^2-a-6}$ in terms of the LCM of the denominators.

Solution

$$a^2 - 2a = a(a-2);$$

$$2a^2 - a - 6 = (2a+3)(a-2)$$

The LCM is $a(a-2)(2a+3)$.

$$\begin{aligned}\frac{2a-3}{a^2-2a} &= \frac{2a-3}{a(a-2)} \cdot \frac{2a+3}{2a+3} \\ &= \frac{4a^2-9}{a(a-2)(2a+3)} \\ \frac{a+1}{2a^2-a-6} &= \frac{a+1}{(a-2)(2a+3)} \cdot \frac{a}{a} \\ &= \frac{a^2+a}{a(a-2)(2a+3)}\end{aligned}$$

EXAMPLE • 3

Write the fractions $\frac{2x-3}{3x-x^2}$ and $\frac{3x}{x^2-4x+3}$ in terms of the LCM of the denominators.

Solution

$$3x - x^2 = x(3-x) = -x(x-3);$$

$$x^2 - 4x + 3 = (x-3)(x-1)$$

The LCM is $x(x-3)(x-1)$.

$$\begin{aligned}\frac{2x-3}{3x-x^2} &= -\frac{2x-3}{x(x-3)} \cdot \frac{x-1}{x-1} \\ &= -\frac{2x^2-5x+3}{x(x-3)(x-1)} \\ \frac{3x}{x^2-4x+3} &= \frac{3x}{(x-3)(x-1)} \cdot \frac{x}{x} \\ &= \frac{3x^2}{x(x-3)(x-1)}\end{aligned}$$

YOU TRY IT • 1

Write the fractions $\frac{2x}{2x-5}$ and $\frac{3}{x+4}$ in terms of the LCM of the denominators.

Your solution**YOU TRY IT • 2**

Write the fractions $\frac{3x}{2x^2-11x+15}$ and $\frac{x-2}{x^2-3x}$ in terms of the LCM of the denominators.

Your solution**YOU TRY IT • 3**

Write the fractions $\frac{2x-7}{2x-x^2}$ and $\frac{3x-2}{3x^2-5x-2}$ in terms of the LCM of the denominators.

Your solution

OBJECTIVE B**To add or subtract rational expressions**

When adding rational expressions in which the denominators are the same, add the numerators. The denominator of the sum is the common denominator.

$$\frac{4x}{15} + \frac{8x}{15} = \frac{4x + 8x}{15} = \frac{12x}{15} = \frac{4x}{5}$$

• Note that the sum is written in simplest form.

$$\frac{a}{a^2 - b^2} + \frac{b}{a^2 - b^2} = \frac{a + b}{a^2 - b^2} = \frac{a + b}{(a - b)(a + b)} = \frac{\cancel{a + b}}{(a - b)\cancel{(a + b)}} = \frac{1}{a - b}$$

When subtracting rational expressions in which the denominators are the same, subtract the numerators. The denominator of the difference is the common denominator. Write the answer in simplest form.

$$\begin{aligned} \frac{7x - 12}{2x^2 + 5x - 12} - \frac{3x - 6}{2x^2 + 5x - 12} &= \frac{(7x - 12) - (3x - 6)}{2x^2 + 5x - 12} = \frac{4x - 6}{2x^2 + 5x - 12} \\ &= \frac{2(\cancel{2x} - 3)}{(\cancel{2x} - 3)(x + 4)} = \frac{2}{x + 4} \end{aligned}$$

Before two rational expressions with *different* denominators can be added or subtracted, each rational expression must be expressed in terms of a common denominator. This common denominator is the LCM of the denominators of the rational expressions.

Take Note

Note the steps involved in adding or subtracting rational expressions with different denominators:

1. Find the LCM of the denominators.
2. Rewrite each fraction in terms of the common denominator.
3. Add or subtract the rational expressions.
4. Simplify the resulting sum or difference.

HOW TO 4

Simplify: $\frac{x}{x - 3} - \frac{x + 1}{x - 2}$

The LCM is $(x - 3)(x - 2)$.

$$\begin{aligned} \frac{x}{x - 3} - \frac{x + 1}{x - 2} &= \frac{x}{x - 3} \cdot \frac{x - 2}{x - 2} - \frac{x + 1}{x - 2} \cdot \frac{x - 3}{x - 3} \\ &= \frac{x(x - 2) - (x + 1)(x - 3)}{(x - 3)(x - 2)} \\ &= \frac{(x^2 - 2x) - (x^2 - 2x - 3)}{(x - 3)(x - 2)} \\ &= \frac{3}{(x - 3)(x - 2)} \end{aligned}$$

- Find the LCM of the denominators.
- Express each fraction in terms of the LCM.
- Subtract the fractions.
- Simplify.

HOW TO 5

Simplify: $\frac{3x}{2x - 3} + \frac{3x + 6}{2x^2 + x - 6}$

The LCM of $2x - 3$ and $2x^2 + x - 6$ is $(2x - 3)(x + 2)$.

$$\begin{aligned} \frac{3x}{2x - 3} + \frac{3x + 6}{2x^2 + x - 6} &= \frac{3x}{2x - 3} \cdot \frac{x + 2}{x + 2} + \frac{3x + 6}{(2x - 3)(x + 2)} \\ &= \frac{3x(x + 2) + (3x + 6)}{(2x - 3)(x + 2)} \\ &= \frac{(3x^2 + 6x) + (3x + 6)}{(2x - 3)(x + 2)} \\ &= \frac{3x^2 + 9x + 6}{(2x - 3)(x + 2)} = \frac{3(\cancel{x + 2})(x + 1)}{(2x - 3)\cancel{(x + 2)}} \\ &= \frac{3(x + 1)}{2x - 3} \end{aligned}$$

- Find the LCM of the denominators.
- Express each fraction in terms of the LCM.
- Add the fractions.
- Simplify.

EXAMPLE • 4Simplify: $\frac{2}{x} - \frac{3}{x^2} + \frac{1}{xy}$ **Solution**The LCM is x^2y .

$$\begin{aligned}
 \frac{2}{x} - \frac{3}{x^2} + \frac{1}{xy} &= \frac{2}{x} \cdot \frac{xy}{xy} - \frac{3}{x^2} \cdot \frac{y}{y} + \frac{1}{xy} \cdot \frac{x}{x} \\
 &= \frac{2xy}{x^2y} - \frac{3y}{x^2y} + \frac{x}{x^2y} \\
 &= \frac{2xy - 3y + x}{x^2y}
 \end{aligned}$$

YOU TRY IT • 4Simplify: $\frac{2}{b} - \frac{1}{a} + \frac{4}{ab}$ **Your solution****EXAMPLE • 5**Simplify: $\frac{x}{2x-4} - \frac{4-x}{x^2-2x}$ **Solution** $2x - 4 = 2(x - 2)$; $x^2 - 2x = x(x - 2)$ The LCM is $2x(x - 2)$.

$$\begin{aligned}
 \frac{x}{2x-4} - \frac{4-x}{x^2-2x} &= \frac{x}{2(x-2)} \cdot \frac{x}{x} - \frac{4-x}{x(x-2)} \cdot \frac{2}{2} \\
 &= \frac{x^2 - (4-x)2}{2x(x-2)} \\
 &= \frac{x^2 - (8-2x)}{2x(x-2)} = \frac{x^2 + 2x - 8}{2x(x-2)} \\
 &= \frac{(x+4)(x-2)}{2x(x-2)} = \frac{(x+4)\cancel{(x-2)}}{2x\cancel{(x-2)}} \\
 &= \frac{x+4}{2x}
 \end{aligned}$$

YOU TRY IT • 5Simplify: $\frac{a-3}{a^2-5a} + \frac{a-9}{a^2-25}$ **Your solution****EXAMPLE • 6**Simplify: $\frac{x}{x+1} - \frac{2}{x-2} - \frac{3}{x^2-x-2}$ **Solution**The LCM is $(x+1)(x-2)$.

$$\begin{aligned}
 \frac{x}{x+1} - \frac{2}{x-2} - \frac{3}{x^2-x-2} \\
 &= \frac{x}{x+1} \cdot \frac{x-2}{x-2} - \frac{2}{x-2} \cdot \frac{x+1}{x+1} \\
 &\quad - \frac{3}{(x+1)(x-2)} \\
 &= \frac{x(x-2) - 2(x+1) - 3}{(x+1)(x-2)} \\
 &= \frac{x^2 - 2x - 2x - 2 - 3}{(x+1)(x-2)} \\
 &= \frac{x^2 - 4x - 5}{(x+1)(x-2)} = \frac{\cancel{(x+1)}(x-5)}{\cancel{(x+1)}(x-2)} = \frac{x-5}{x-2}
 \end{aligned}$$

YOU TRY IT • 6Simplify: $\frac{2x}{x-4} - \frac{x-1}{x+1} + \frac{2}{x^2-3x-4}$ **Your solution**

6.2 EXERCISES

OBJECTIVE A

To rewrite rational expressions in terms of a common denominator



1. a. How many factors of a are in the LCM of $(a^2b)^3$ and a^4b^4 ?
 b. How many factors of b are in the LCM of $(a^2b)^3$ and a^4b^4 ?



2. a. How many factors of $x - 4$ are in the LCM of $x^2 - x - 12$ and $x^2 - 8x + 16$?
 b. How many factors of $x - 4$ are in the LCM of $x^2 - x - 12$ and $x^2 - 16$?

For Exercises 3 to 25, write each fraction in terms of the LCM of the denominators.

3. $\frac{3}{4x^2y}, \frac{17}{12xy^4}$

4. $\frac{5}{16a^3b^3}, \frac{7}{30a^5b}$

5. $\frac{x-2}{3x(x-2)}, \frac{3}{6x^2}$

6. $\frac{5x-1}{4x(2x+1)}, \frac{2}{5x^3}$

7. $\frac{3x-1}{2x(x-5)}, -3x$

8. $\frac{4x-3}{3x(x-2)}, 2x$

9. $\frac{3x}{2x-3}, \frac{5x}{2x+3}$

10. $\frac{2}{7y-3}, \frac{-3}{7y+3}$

11. $\frac{2x}{x^2-9}, \frac{x+1}{x-3}$

12. $\frac{3x}{16-x^2}, \frac{2x}{16-4x}$

13. $\frac{3}{3x^2-12y^2}, \frac{5}{6x-12y}$

14. $\frac{2x}{x^2-36}, \frac{x-1}{6x-36}$

15. $\frac{3x}{x^2-1}, \frac{5x}{x^2-2x+1}$

16. $\frac{x^2+2}{x^3-1}, \frac{3}{x^2+x+1}$

17. $\frac{x-3}{8-x^3}, \frac{2}{4+2x+x^2}$

18. $\frac{2x}{x^2+x-6}, \frac{-4x}{x^2+5x+6}$

19. $\frac{2x}{x^2+2x-3}, \frac{-x}{x^2+6x+9}$

$$20. \frac{3x}{2x^2 - x - 3}, \frac{-2x}{2x^2 - 11x + 12}$$

$$21. \frac{-4x}{4x^2 - 16x + 15}, \frac{3x}{6x^2 - 19x + 10}$$

$$22. \frac{3}{2x^2 + 5x - 12}, \frac{2x}{3 - 2x}, \frac{3x - 1}{x + 4}$$

$$23. \frac{5}{6x^2 - 17x + 12}, \frac{2x}{4 - 3x}, \frac{x + 1}{2x - 3}$$

$$24. \frac{3x}{x - 4}, \frac{4}{x + 5}, \frac{x + 2}{20 - x - x^2}$$

$$25. \frac{2x}{x - 3}, \frac{-2}{x + 5}, \frac{x - 1}{15 - 2x - x^2}$$

OBJECTIVE B**To add or subtract rational expressions**

$$26. \text{ True or false? } \frac{1}{2x} + \frac{1}{3x} = \frac{1}{5x}$$



$$27. \text{ True or false? } \frac{1}{x - 3} + \frac{1}{3 - x} = 0$$

For Exercises 28 to 83, simplify.

$$28. \frac{3}{2xy} - \frac{7}{2xy} - \frac{9}{2xy}$$

$$29. -\frac{3}{4x^2} + \frac{8}{4x^2} - \frac{3}{4x^2}$$

$$30. \frac{x}{x^2 - 3x + 2} - \frac{2}{x^2 - 3x + 2}$$

$$31. \frac{3x}{3x^2 + x - 10} - \frac{5}{3x^2 + x - 10}$$

$$32. \frac{3}{2x^2y} - \frac{8}{5x} - \frac{9}{10xy}$$

$$33. \frac{2}{5ab} - \frac{3}{10a^2b} + \frac{4}{15ab^2}$$

$$34. \frac{2}{3x} - \frac{3}{2xy} + \frac{4}{5xy} - \frac{5}{6x}$$

$$35. \frac{3}{4ab} - \frac{2}{5a} + \frac{3}{10b} - \frac{5}{8ab}$$

$$36. \frac{2x - 1}{12x} - \frac{3x + 4}{9x}$$

$$37. \frac{3x - 4}{6x} - \frac{2x - 5}{4x}$$

$$38. \frac{3x + 2}{4x^2y} - \frac{y - 5}{6xy^2}$$

$$39. \frac{2y - 4}{5xy^2} + \frac{3 - 2x}{10x^2y}$$

$$40. \frac{2x}{x - 3} - \frac{3x}{x - 5}$$

$$41. \frac{3a}{a - 2} - \frac{5a}{a + 1}$$

$$42. \frac{3}{2a - 3} + \frac{2a}{3 - 2a}$$

43. $\frac{x}{2x-5} - \frac{2}{5x-2}$

44. $\frac{1}{x+h} - \frac{1}{h}$

45. $\frac{1}{a-b} + \frac{1}{b}$

46. $\frac{2}{x} - 3 - \frac{10}{x-4}$

47. $\frac{6a}{a-3} - 5 + \frac{3}{a}$

48. $\frac{1}{2x-3} - \frac{5}{2x} + 1$

49. $\frac{5}{x} - \frac{5x}{5-6x} + 2$

50. $\frac{3}{x^2-1} + \frac{2x}{x^2+2x+1}$

51. $\frac{1}{x^2-6x+9} - \frac{1}{x^2-9}$

52. $\frac{x}{x+3} - \frac{3-x}{x^2-9}$

53. $\frac{1}{x+2} - \frac{3x}{x^2+4x+4}$

54. $\frac{2x-3}{x+5} - \frac{x^2-4x-19}{x^2+8x+15}$

55. $\frac{-3x^2+8x+2}{x^2+2x-8} - \frac{2x-5}{x+4}$

56. $\frac{x^n}{x^{2n}-1} - \frac{2}{x^n+1}$

57. $\frac{2}{x^n-1} + \frac{x^n}{x^{2n}-1}$

58. $\frac{2}{x^n-1} - \frac{6}{x^{2n}+x^n-2}$

59. $\frac{2x^n-6}{x^{2n}-x^n-6} + \frac{x^n}{x^n+2}$

60. $\frac{2x-2}{4x^2-9} - \frac{5}{3-2x}$

61. $\frac{x^2+4}{4x^2-36} - \frac{13}{x+3}$

62. $\frac{x-2}{x+1} - \frac{3-12x}{2x^2-x-3}$

63. $\frac{3x-4}{4x+1} + \frac{3x+6}{4x^2+9x+2}$

64. $\frac{x+1}{x^2+x-6} - \frac{x+2}{x^2+4x+3}$

65. $\frac{x+1}{x^2+x-12} - \frac{x-3}{x^2+7x+12}$

66. $\frac{x^2+6x}{x^2+3x-18} - \frac{2x-1}{x+6} + \frac{x-2}{3-x}$

67. $\frac{2x^2-2x}{x^2-2x-15} - \frac{2}{x+3} + \frac{x}{5-x}$

$$68. \frac{7-4x}{2x^2-9x+10} + \frac{x-3}{x-2} - \frac{x+1}{2x-5}$$

$$69. \frac{x}{3x+4} + \frac{3x+2}{x-5} - \frac{7x^2+24x+28}{3x^2-11x-20}$$

$$70. \frac{32x-9}{2x^2+7x-15} + \frac{x-2}{3-2x} + \frac{3x+2}{x+5}$$

$$71. \frac{x+1}{1-2x} - \frac{x+3}{4x-3} + \frac{10x^2+7x-9}{8x^2-10x+3}$$

$$72. \frac{x^2}{x^3-8} - \frac{x+2}{x^2+2x+4}$$

$$73. \frac{2x}{4x^2+2x+1} + \frac{4x+1}{8x^3-1}$$

$$74. \frac{2x^2}{x^4-1} - \frac{1}{x^2-1} + \frac{1}{x^2+1}$$

$$75. \frac{x^2-12}{x^4-16} + \frac{1}{x^2-4} - \frac{1}{x^2+4}$$

$$76. \left(\frac{x+8}{4} + \frac{4}{x} \right) \div \frac{x+4}{16x^2}$$

$$77. \left(\frac{a-3}{a^2} - \frac{a-3}{9} \right) \div \frac{a^2-9}{3a}$$

$$78. \frac{3}{x-2} - \frac{x^2+x}{2x^3+3x^2} \cdot \frac{2x^2+x-3}{x^2+3x+2}$$

$$79. \frac{x^2-4x+4}{2x+1} \cdot \frac{2x^2+x}{x^3-4x} - \frac{3x-2}{x+1}$$

$$80. \left(\frac{x-y}{x^2} - \frac{x-y}{y^2} \right) \div \frac{x^2-y^2}{xy}$$

$$81. \left(\frac{a-2b}{b} + \frac{b}{a} \right) \div \left(\frac{b+a}{a} - \frac{2a}{b} \right)$$

$$82. \frac{2}{x-3} - \frac{x}{x^2-x-6} \cdot \frac{x^2-2x-3}{x^2-x}$$

$$83. \frac{2x}{x^2-x-6} - \frac{6x-6}{2x^2-9x+9} \div \frac{x^2+x-2}{2x-3}$$

Applying the Concepts

84. Correct the right hand side of the following equations.

a. $\frac{3}{4} + \frac{x}{5} = \frac{3+x}{4+5}$

b. $\frac{4x+5}{4} = x+5$

c. $\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y}$

85. Let $f(x) = \frac{x}{x+2}$, $g(x) = \frac{4}{x-3}$, and $S(x) = \frac{x^2+x+8}{x^2-x-6}$.

a. Evaluate $f(4)$, $g(4)$, and $S(4)$. b. Does $f(4) + g(4) = S(4)$?

SECTION

6.3 Complex Fractions

OBJECTIVE A

To simplify a complex fraction 

A **complex fraction** is a fraction in which the numerator or denominator contains one or more fractions. Examples of complex fractions are shown at the right.

$$\frac{5}{2 + \frac{1}{2}} \quad \frac{5 + \frac{1}{y}}{5 - \frac{1}{y}} \quad \frac{x + 4 + \frac{1}{x+2}}{x - 2 + \frac{1}{x+2}}$$

If the LCMs of the denominators in the numerator and denominator of a complex fraction are the same, the complex fraction can be simplified by multiplying the numerator and denominator by that expression.

 Take Note

Begin with the numerator of the complex fraction. The LCM of the denominators is x^2 . Now consider the denominator of the complex fraction. The LCM of these denominators is also x^2 .

HOW TO • 1

Simplify: $\frac{1 + \frac{5}{x} - \frac{6}{x^2}}{1 + \frac{8}{x} + \frac{12}{x^2}}$

$$\begin{aligned} \frac{1 + \frac{5}{x} - \frac{6}{x^2}}{1 + \frac{8}{x} + \frac{12}{x^2}} &= \frac{1 + \frac{5}{x} - \frac{6}{x^2}}{1 + \frac{8}{x} + \frac{12}{x^2}} \cdot \frac{x^2}{x^2} \\ &= \frac{1 \cdot x^2 + \frac{5}{x} \cdot x^2 - \frac{6}{x^2} \cdot x^2}{1 \cdot x^2 + \frac{8}{x} \cdot x^2 + \frac{12}{x^2} \cdot x^2} \\ &= \frac{x^2 + 5x - 6}{x^2 + 8x + 12} = \frac{(x+6)(x-1)}{(x+6)(x+2)} = \frac{x-1}{x+2} \end{aligned}$$

• Multiply the numerator and denominator of the complex fraction by x^2 .

• Simplify.

If the LCMs of the denominators in the numerator and denominator of a complex fraction are different, it may be easier to simplify the complex fraction by using a different approach.

 Take Note

Begin with the numerator of the complex fraction. The LCM of the denominators is $x-3$. Now consider the denominator of the complex fraction. The LCM of the denominators is $x+1$. These expressions are different.

 Take Note

Either method of simplifying a complex fraction will always work. With experience, you will be able to decide which method works best for a particular complex fraction.

HOW TO • 2

Simplify: $\frac{3 + \frac{10}{x-3}}{3 - \frac{2}{x+1}}$

$$\begin{aligned} \frac{3 + \frac{10}{x-3}}{3 - \frac{2}{x+1}} &= \frac{\frac{3(x-3)}{x-3} + \frac{10}{x-3}}{\frac{3(x+1)}{x+1} - \frac{2}{x+1}} \\ &= \frac{\frac{3x-9}{x-3} + \frac{10}{x-3}}{\frac{3x+3}{x+1} - \frac{2}{x+1}} = \frac{\frac{3x+1}{x-3}}{\frac{3x+1}{x+1}} \\ &= \frac{3x+1}{x-3} \div \frac{3x+1}{x+1} = \frac{3x+1}{x-3} \cdot \frac{x+1}{3x+1} = \frac{x+1}{x-3} \end{aligned}$$

• Simplify the numerator and denominator of the complex fraction by rewriting each as a single fraction.

• Both the numerator and denominator of the complex fraction are now written as single fractions.

• Divide the numerator by the denominator.

EXAMPLE • 1

Simplify: $\frac{2x - 1 + \frac{7}{x + 4}}{3x - 8 + \frac{17}{x + 4}}$

Solution

The LCM of $x + 4$ and $x + 4$ is $x + 4$.

$$\begin{aligned} \frac{2x - 1 + \frac{7}{x + 4}}{3x - 8 + \frac{17}{x + 4}} &= \frac{2x - 1 + \frac{7}{x + 4}}{3x - 8 + \frac{17}{x + 4}} \cdot \frac{x + 4}{x + 4} \\ &= \frac{(2x - 1)(x + 4) + \frac{7}{x + 4}(x + 4)}{(3x - 8)(x + 4) + \frac{17}{x + 4}(x + 4)} \\ &= \frac{2x^2 + 7x - 4 + 7}{3x^2 + 4x - 32 + 17} \\ &= \frac{2x^2 + 7x + 3}{3x^2 + 4x - 15} = \frac{(2x + 1)(x + 3)}{(3x - 5)(x + 3)} \\ &= \frac{(2x + 1)\cancel{(x + 3)}}{(3x - 5)\cancel{(x + 3)}} = \frac{2x + 1}{3x - 5} \end{aligned}$$

YOU TRY IT • 1

Simplify: $\frac{2x + 5 + \frac{14}{x - 3}}{4x + 16 + \frac{49}{x - 3}}$

Your solution**EXAMPLE • 2**

Simplify: $1 + \frac{a}{2 + \frac{1}{a}}$

Solution

The LCM of the denominators of 2 and $\frac{1}{a}$ is a .

$$\begin{aligned} 1 + \frac{a}{2 + \frac{1}{a}} &= 1 + \frac{a}{2 + \frac{1}{a}} \cdot \frac{a}{a} \\ &= 1 + \frac{a \cdot a}{2 \cdot a + \frac{1}{a} \cdot a} = 1 + \frac{a^2}{2a + 1} \end{aligned}$$

The LCM of the denominators of 1 and $\frac{a^2}{2a + 1}$ is $2a + 1$.

$$\begin{aligned} 1 + \frac{a^2}{2a + 1} &= 1 \cdot \frac{2a + 1}{2a + 1} + \frac{a^2}{2a + 1} \\ &= \frac{2a + 1}{2a + 1} + \frac{a^2}{2a + 1} = \frac{2a + 1 + a^2}{2a + 1} \\ &= \frac{a^2 + 2a + 1}{2a + 1} = \frac{(a + 1)^2}{2a + 1} \end{aligned}$$

YOU TRY IT • 2

Simplify: $2 - \frac{1}{2 - \frac{1}{x}}$

Your solution

6.3 EXERCISES

OBJECTIVE A

To simplify a complex fraction



1. What is a complex fraction?



2. What is the general goal of simplifying a complex fraction?

For Exercises 3 to 46, simplify.

3.
$$\frac{2 - \frac{1}{3}}{4 + \frac{11}{3}}$$

4.
$$\frac{3 + \frac{5}{2}}{8 - \frac{3}{2}}$$

5.
$$\frac{3 - \frac{2}{3}}{5 + \frac{5}{6}}$$

6.
$$\frac{5 - \frac{3}{4}}{2 + \frac{1}{2}}$$

7.
$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x^2}}$$

8.
$$\frac{\frac{1}{y^2} - 1}{1 + \frac{1}{y}}$$

9.
$$\frac{\frac{a-2}{4} - a}{a}$$

10.
$$\frac{\frac{25}{a} - a}{5 + a}$$

11.
$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

12.
$$\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

13.
$$\frac{x - \frac{1}{x}}{x + \frac{1}{x}}$$

14.
$$\frac{a - \frac{1}{a}}{\frac{1}{a} + a}$$

15.
$$\frac{\frac{1}{a^2} - \frac{1}{a}}{\frac{1}{a^2} + \frac{1}{a}}$$

16.
$$\frac{\frac{1}{b} + \frac{1}{2}}{\frac{4}{b^2} - 1}$$

17.
$$\frac{2 - \frac{4}{x+2}}{5 - \frac{10}{x+2}}$$

18.
$$\frac{4 + \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}$$

19.
$$\frac{\frac{3}{2a-3} + 2}{\frac{-6}{2a-3} - 4}$$

20.
$$\frac{\frac{-5}{b-5} - 3}{\frac{10}{b-5} + 6}$$

21.
$$\frac{1 - \frac{1}{x-4}}{1 - \frac{6}{x+1}}$$

22.
$$\frac{1 + \frac{3}{x+2}}{1 + \frac{6}{x-1}}$$

23.
$$\frac{1 - \frac{2}{x-3}}{1 + \frac{3}{2-x}}$$

24.
$$\frac{1 + \frac{x}{x+1}}{1 + \frac{x-1}{x+2}}$$

25.
$$\frac{x-4 + \frac{9}{2x+3}}{x+3 - \frac{5}{2x+3}}$$

26.
$$\frac{2x-3 - \frac{10}{4x-5}}{3x+2 + \frac{11}{4x-5}}$$

$$27. \frac{x - 3 + \frac{10}{x+4}}{x + 7 + \frac{16}{x-3}}$$

$$28. \frac{x + 9 + \frac{30}{x-2}}{x - 1 + \frac{8}{x+5}}$$

$$29. \frac{1 - \frac{1}{x} - \frac{6}{x^2}}{1 - \frac{4}{x} + \frac{3}{x^2}}$$

$$30. \frac{1 - \frac{3}{x} - \frac{10}{x^2}}{1 + \frac{11}{x} + \frac{18}{x^2}}$$

$$31. \frac{1 + \frac{1}{x} - \frac{12}{x^2}}{\frac{9}{x^2} + \frac{3}{x} - 2}$$

$$32. \frac{\frac{15}{x^2} - \frac{2}{x} - 1}{\frac{4}{x^2} - \frac{5}{x} + 4}$$

$$33. \frac{\frac{1}{y^2} - \frac{1}{xy} - \frac{2}{x^2}}{\frac{1}{y^2} - \frac{3}{xy} + \frac{2}{x^2}}$$

$$34. \frac{\frac{2}{b^2} - \frac{5}{ab} - \frac{3}{a^2}}{\frac{2}{b^2} + \frac{7}{ab} + \frac{3}{a^2}}$$

$$35. \frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$$

$$36. \frac{\frac{2a}{a-1} - \frac{3}{a}}{\frac{1}{a-1} + \frac{2}{a}}$$

$$37. \frac{\frac{1}{a} - \frac{3}{a-2}}{\frac{2}{a} + \frac{5}{a-2}}$$

$$38. \frac{\frac{2}{b} - \frac{5}{b+3}}{\frac{3}{b} + \frac{3}{b+3}}$$

$$39. \frac{\frac{x-1}{x+1} - \frac{x+1}{x-1}}{\frac{x-1}{x+1} + \frac{x+1}{x-1}}$$

$$40. \frac{\frac{y}{y+2} - \frac{y}{y-2}}{\frac{y}{y+2} + \frac{y}{y-2}}$$

$$41. a + \frac{a}{a + \frac{1}{a}}$$

$$42. 4 - \frac{2}{2 - \frac{3}{x}}$$

$$43. a - \frac{a}{1 - \frac{a}{1-a}}$$

$$44. 3 - \frac{3}{3 - \frac{3}{3-x}}$$

$$45. 3 - \frac{2}{1 - \frac{2}{3 - \frac{2}{x}}}$$

$$46. a + \frac{a}{2 + \frac{1}{1 - \frac{2}{a}}}$$



47. In simplest form, what is the reciprocal of the complex fraction $\frac{1}{1 - \frac{1}{a}}$?



48. The denominator of a complex fraction is the reciprocal of its numerator. Which of the following is the simplified form of the complex fraction?
 (i) 1 (ii) the square of the numerator of the complex fraction
 (iii) the square of the denominator of the complex fraction

Applying the Concepts

For Exercises 49 to 52, simplify.

$$49. (3 + 3^{-1})^{-1}$$

$$50. [x + (x + 1)^{-1}]^{-1}$$

$$51. \frac{x^{-1}}{x^{-1} + 2^{-1}}$$

$$52. \frac{1 + x^{-1}}{1 - x^{-1}}$$

SECTION

6.4 Ratio and Proportion

OBJECTIVE A

To solve a proportion

Quantities such as 3 feet, 5 liters, and 2 miles are number quantities written with units. In these examples, the units are feet, liters, and miles.

A **ratio** is the quotient of two quantities that have the same unit.

The weekly wages of a painter are \$800. The painter spends \$150 a week for food. The ratio of the wages spent for food to the total weekly wages is written

$$\frac{\$150}{\$800} = \frac{150}{800} = \frac{3}{16}$$

A ratio is in simplest form when the two numbers do not have a common factor. Note that the units are not written.

A **rate** is the quotient of two quantities that have different units.

A car travels 180 mi on 3 gal of gas. The miles-to-gallon rate is

$$\frac{180 \text{ mi}}{3 \text{ gal}} = \frac{60 \text{ mi}}{1 \text{ gal}}$$

A rate is in simplest form when the two numbers do not have a common factor. The units are written as part of the rate.

A **proportion** is an equation that states the equality of two ratios or rates. For example, $\frac{90 \text{ km}}{4 \text{ L}} = \frac{45 \text{ km}}{2 \text{ L}}$ and $\frac{3}{4} = \frac{x+2}{16}$ are proportions.



Tips for Success

Always check the proposed solution of an equation. For the equation at the right:

$$\begin{array}{r} \frac{2}{7} = \frac{x}{5} \\ \hline \frac{2}{7} \cdot \frac{10}{7} = \frac{x}{5} \cdot \frac{10}{7} \\ \frac{2}{7} \cdot \frac{10}{7} = \frac{10x}{35} \\ \frac{2}{7} = \frac{2}{7} \end{array}$$

The solution checks.

HOW TO • 1

Solve: $\frac{2}{7} = \frac{x}{5}$

$$\begin{array}{l} \frac{2}{7} = \frac{x}{5} \\ 35 \cdot \frac{2}{7} = 35 \cdot \frac{x}{5} \\ 10 = 7x \\ \frac{10}{7} = x \end{array}$$

The solution is $\frac{10}{7}$.

- Multiply each side of the proportion by the LCM of the denominators.
- Solve the equation.

EXAMPLE • 1

Solve: $\frac{3}{12} = \frac{5}{x+5}$

Solution

$$\begin{array}{l} \frac{3}{12} = \frac{5}{x+5} \\ 12(x+5) \cdot \frac{3}{12} = 12(x+5) \cdot \frac{5}{x+5} \quad \bullet \text{ Multiply each side by the LCM of 12 and } x+5. \\ (x+5)3 = 12 \cdot 5 \\ 3x + 15 = 60 \\ 3x = 45 \\ x = 15 \end{array}$$

The solution is 15.

YOU TRY IT • 1

Solve: $\frac{5}{x-2} = \frac{3}{4}$

Your solution

Solution on p. S20

EXAMPLE • 2

Solve: $\frac{3}{x-2} = \frac{4}{2x+1}$

Solution

$$\begin{aligned}\frac{3}{x-2} &= \frac{4}{2x+1} \\ \frac{3}{x-2}(2x+1)(x-2) &= \frac{4}{2x+1}(2x+1)(x-2) \\ 3(2x+1) &= 4(x-2) \\ 6x+3 &= 4x-8 \\ 2x+3 &= -8 \\ 2x &= -11 \\ x &= -\frac{11}{2}\end{aligned}$$

The solution is $-\frac{11}{2}$.

YOU TRY IT • 2

Solve: $\frac{5}{2x-3} = \frac{-2}{x+1}$

Your solution

Solution on p. S20

OBJECTIVE B**To solve application problems****EXAMPLE • 3**

A stock investment of 50 shares pays a dividend of \$106. At this rate, how many additional shares are required to earn a dividend of \$424?

Strategy

To find the additional number of shares that are required, write and solve a proportion using x to represent the additional number of shares. Then $50 + x$ is the total number of shares of stock.

Solution

$$\begin{aligned}\frac{106}{50} &= \frac{424}{50+x} \\ \frac{53}{25} &= \frac{424}{50+x} && \bullet \text{ Simplify } \frac{106}{50}. \\ \frac{53}{25}(25)(50+x) &= \frac{424}{50+x}(25)(50+x) \\ 53(50+x) &= 424(25) \\ 2650 + 53x &= 10,600 \\ 53x &= 7950 \\ x &= 150\end{aligned}$$

An additional 150 shares of stock are required.

YOU TRY IT • 3

Two pounds of cashews cost \$12.40. At this rate, how much would 15 lb of cashews cost?

Your strategy**Your solution**

Solution on p. S20

6.4 EXERCISES

OBJECTIVE A

To solve a proportion



1. How does a ratio differ from a rate?



2. What is a proportion?

For Exercises 3 to 22, solve.

3. $\frac{x}{30} = \frac{3}{10}$

4. $\frac{5}{15} = \frac{x}{75}$

5. $\frac{2}{x} = \frac{8}{30}$

6. $\frac{80}{16} = \frac{15}{x}$

7. $\frac{x+1}{10} = \frac{2}{5}$

8. $\frac{5-x}{10} = \frac{3}{2}$

9. $\frac{4}{x+2} = \frac{3}{4}$

10. $\frac{8}{3} = \frac{24}{x+3}$

11. $\frac{x}{4} = \frac{x-2}{8}$

12. $\frac{8}{x-5} = \frac{3}{x}$

13. $\frac{16}{2-x} = \frac{4}{x}$

14. $\frac{6}{x-5} = \frac{1}{x}$

15. $\frac{8}{x-2} = \frac{4}{x+1}$

16. $\frac{4}{x-4} = \frac{2}{x-2}$

17. $\frac{x}{3} = \frac{x+1}{7}$

18. $\frac{x-3}{2} = \frac{3+x}{5}$

19. $\frac{8}{3x-2} = \frac{2}{2x+1}$

20. $\frac{3}{2x-4} = \frac{-5}{x+2}$

21. $\frac{3x+1}{3x-4} = \frac{x}{x-2}$

22. $\frac{x-2}{x-5} = \frac{2x}{2x+5}$



23. True or false? If
- $\frac{a}{b} = \frac{c}{d}$
- , then
- $\frac{b}{a} = \frac{d}{c}$
- .



24. True or false? If
- $\frac{a}{b} = \frac{c}{d}$
- , then
- $\frac{a+b}{b} = \frac{c+d}{d}$
- .

OBJECTIVE B

To solve application problems

25. **Nutrition** If a 56-gram serving of pasta contains 7 g of protein, how many grams of protein are in a 454-gram box of the pasta?
26. **Conservation** A team of biologists captured and tagged approximately 2400 northern squawfish at the Bonneville Dam on the Columbia River. Later, 225 squawfish were recaptured and 9 of them had tags. How many squawfish would you estimate are in the area?
27. **Computers** Of 300 people who purchased home computers from a national company, 15 received machines with defective USB ports. At this rate, how many of the 70,000 computers the company sold nationwide would you expect to have defective USB ports?



C. Allan Morgan



28. **Finance** The exchange rate gives the value of one country's money in terms of another country's money. Recently, 1.703 U.S. dollars would purchase 1 British pound. At this rate, what would be the cost in dollars of a waterproof jacket that cost 95 British pounds? Round to the nearest cent.



29. **Finance** The exchange rate gives the value of one country's money in terms of another country's money. Recently, 1 Argentine peso cost 0.346 U.S. dollars. At this rate, what would be the cost in dollars of a gallon of milk that cost 11 Argentine pesos? Round to the nearest cent.

30. **Interior Decorating** One hundred forty-four ceramic tiles are required to tile a 25-square-foot area. At this rate, how many tiles are required to tile 275 ft²?

31. **Architecture** On an architectural drawing, $\frac{1}{4}$ in. represents 1 ft. Using this scale, find the dimensions of a room that measures $4\frac{1}{2}$ in. by 6 in. on the drawing.



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32. **Fundraising** To raise money, a school is sponsoring a magazine subscription drive. By visiting 90 homes in the neighborhood, one student was able to sell \$375 worth of subscriptions. At this rate, how many additional homes will the student have to visit in order to meet a \$1000 goal?

33. **Exercise** Walking 4 mi in 2 h will use up 650 calories. Walking at the same rate, how many miles would a person need to walk to lose 1 lb? (Burning 3500 calories is equivalent to losing 1 pound.) Round to the nearest hundredth.

34. **Medicine** One and one-half ounces of a medication are required for a 140-pound adult. At the same rate, how many additional ounces of medication are required for a 210-pound adult?



35. **Electricity** Read the article at the right. Talking on a cell phone for 30 min requires approximately 13 watts of power. Write and solve a proportion to find the number of minutes of talk time the knee braces mentioned in the article would provide after walking for 1 min. Round to the nearest minute.



36. **Electricity** Read the article at the right. Suppose a walker could generate 7 watts of electricity per minute by walking a little faster. How many minutes of talk time (see Exercise 35) would a person generate by walking for 30 min? Round to the nearest minute.



37. If 1 U.S. dollar equals 0.59 British pound and 1 British pound equals 1.21 Euros, what is the value of 1 U.S. dollar in Euros? Round to the nearest cent.

In the News

Portable Power

American and Canadian scientists published results of tests of a knee brace device that walkers can use to generate their own electricity. The study showed that by walking at a leisurely pace, a person could generate 5 watts of electricity per minute with knee braces engaged on both legs.

Source: www.telegraph.co.uk

Applying the Concepts

38. **Lotteries** Three people put their money together to buy lottery tickets. The first person put in \$20, the second person put in \$25, and the third person put in \$30. One of the tickets was a winning ticket. If they won \$4.5 million, what was the first person's share of the winnings?

SECTION

6.5

Rational Equations

OBJECTIVE A

To solve a rational equation



To solve an equation containing fractions, begin by **clearing denominators**—that is, removing them by multiplying each side of the equation by the LCM of the denominators. Then solve for the variable. The solutions to the resulting equation must be checked because multiplying each side of an equation by a variable expression may produce a solution that is not a solution of the original equation.

HOW TO • 1

Solve: $\frac{4x}{x+3} + 3 = \frac{-5}{x+3}$

$$\frac{4x}{x+3} + 3 = \frac{-5}{x+3}$$

$$(x+3)\left(\frac{4x}{x+3} + 3\right) = (x+3)\left(\frac{-5}{x+3}\right)$$

$$(x+3)\left(\frac{4x}{x+3}\right) + (x+3)3 = (x+3)\left(\frac{-5}{x+3}\right)$$

$$4x + 3x + 9 = -5$$

$$7x + 9 = -5$$

$$7x = -14$$

$$x = -2$$

• Multiply each side by the LCM of the denominators.

• Simplify.

• Solve for x .

Check:

$$\begin{array}{r|l} \frac{4x}{x+3} + 3 = \frac{-5}{x+3} & \\ \hline \frac{4(-2)}{-2+3} + 3 & \frac{-5}{-2+3} \\ \frac{-8}{1} + 3 & \frac{-5}{1} \\ -8 + 3 & -5 \\ -5 & -5 \end{array}$$

The solution checks.

As shown at the left, -2 checks as a solution. The solution is -2 .

HOW TO • 2

Solve: $\frac{3x}{x-3} = 2 + \frac{9}{x-3}$

$$\frac{3x}{x-3} = 2 + \frac{9}{x-3}$$

$$(x-3)\left(\frac{3x}{x-3}\right) = (x-3)\left(2 + \frac{9}{x-3}\right)$$

$$(x-3)\left(\frac{3x}{x-3}\right) = (x-3)2 + (x-3)\left(\frac{9}{x-3}\right)$$

$$3x = 2x - 6 + 9$$

$$3x = 2x + 3$$

$$x = 3$$

• Multiply each side by the LCM of the denominators.

• Simplify.

• Solve for x .

Check:

$$\begin{array}{r|l} \frac{3x}{x-3} = 2 + \frac{9}{x-3} & \\ \hline \frac{3(3)}{3-3} & 2 + \frac{9}{3-3} \\ \frac{9}{0} & 2 + \frac{9}{0} \end{array}$$

Division by zero is not defined. The equation has no solution.

As shown at the left, 3 does not check as a solution. The equation has no solution.

**Take Note**

The domain of each of the rational expressions $\frac{3}{x+1}$ and $\frac{5x+8}{x+1}$ is all real numbers except -1 . Therefore, x cannot be -1 and the suggested solution of -1 is not a solution of the equation. This *extraneous* solution is the result of multiplying each side of the equation by $x+1$. A note of caution: As Example 2 shows, multiplying each side of an equation by a variable expression does not always result in extraneous solutions.

HOW TO 3

Solve: $\frac{3}{x+1} + 2x = \frac{5x+8}{x+1}$

$$\begin{aligned}\frac{3}{x+1} + 2x &= \frac{5x+8}{x+1} \\ (x+1)\left(\frac{3}{x+1} + 2x\right) &= (x+1)\left(\frac{5x+8}{x+1}\right) \\ 3 + 2x(x+1) &= 5x+8 \\ 3 + 2x^2 + 2x &= 5x+8 \\ 2x^2 - 3x - 5 &= 0 \\ (2x-5)(x+1) &= 0 \\ 2x-5=0 &\quad x+1=0 \\ 2x=5 &\quad x=-1 \\ x=\frac{5}{2} &\end{aligned}$$

- Multiply each side by $x+1$.
- Solve for x .
- This is a quadratic equation.
- Use the Principle of Zero Products.

$\frac{5}{2}$ checks as a solution. -1 does not check as a solution because substituting -1 into the original equation results in division by zero, which is not defined. -1 is called an **extraneous solution**. Extraneous solutions can arise when each side of an equation is multiplied by a variable expression. In this instance, you *must* check that the values of the variable are solutions of the *original* equation.

EXAMPLE 1

Solve: $\frac{2x}{x+2} - 3 = \frac{-2}{x+2}$

Solution

$$\begin{aligned}\frac{2x}{x+2} - 3 &= \frac{-2}{x+2} \\ (x+2)\left(\frac{2x}{x+2} - 3\right) &= (x+2)\left(\frac{-2}{x+2}\right) \\ 2x - 3(x+2) &= -2 \\ 2x - 3x - 6 &= -2 \\ -x &= 4 \\ x &= -4\end{aligned}$$

-4 checks as a solution. **The solution is -4 .**

YOU TRY IT 1

Solve: $\frac{3x}{x-3} = 2 + \frac{9}{x-3}$

Your solution

EXAMPLE 2

Solve: $\frac{1}{a} + \frac{1}{a+1} = \frac{3}{2}$

Solution

$$\begin{aligned}\frac{1}{a} + \frac{1}{a+1} &= \frac{3}{2} \\ 2a(a+1)\left(\frac{1}{a} + \frac{1}{a+1}\right) &= 2a(a+1)\frac{3}{2} \\ 2(a+1) + 2a &= a(a+1) \cdot 3 \\ 2a+2+2a &= 3a^2+3a \\ 0 &= 3a^2-a-2 \\ 0 &= (3a+2)(a-1)\end{aligned}$$

$$\begin{aligned}3a+2=0 &\quad a-1=0 \\ 3a=-2 &\quad a=1 \\ a=-\frac{2}{3} &\end{aligned}$$

$-\frac{2}{3}$ and 1 check as solutions. **The solutions are $-\frac{2}{3}$ and 1 .**

YOU TRY IT 2

Solve: $\frac{5}{x-2} = 2x + \frac{3x-1}{x-2}$

Your solution

OBJECTIVE B

To solve work problems



Point of Interest

The following problem was recorded in the *Jiuzhang*, a Chinese text that dates to the Han dynasty (about 200 B.C. to A.D. 200). “A reservoir has five channels bringing water to it. The first can fill the reservoir in $\frac{1}{3}$ day, the second in 1 day, the third in $2\frac{1}{2}$ days, the fourth in 3 days, and the fifth in 5 days. If all channels are open, how long does it take to fill the reservoir?” This problem is the earliest known work problem.

Rate of work is that part of a task that is completed in 1 unit of time. If a mason can build a retaining wall in 12 h, then in 1 h the mason can build $\frac{1}{12}$ of the wall. The mason’s rate of work is $\frac{1}{12}$ of the wall each hour. If an apprentice can build the wall in x hours, the rate of work for the apprentice is $\frac{1}{x}$ of the wall each hour.

In solving a work problem, the goal is to determine the time it takes to complete a task. The basic equation that is used to solve work problems is

$$\text{Rate of work} \times \text{time worked} = \text{part of task completed}$$

For example, if a pipe can fill a tank in 5 h, then in 2 h the pipe will fill $\frac{1}{5} \times 2 = \frac{2}{5}$ of the tank. In t hours, the pipe will fill $\frac{1}{5} \times t = \frac{t}{5}$ of the tank.



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HOW TO • 4

A mason can build a wall in 10 h. An apprentice can build a wall in 15 h. How long will it take them to build the wall if they work together?

Strategy for Solving a Work Problem

- For each person or machine, write a numerical or variable expression for the rate of work, the time worked, and the part of the task completed. The results can be recorded in a table.

Unknown time to build the wall working together: t

	Rate of Work	.	Time Worked	=	Part of Task Completed
Mason	$\frac{1}{10}$	.	t	=	$\frac{t}{10}$
Apprentice	$\frac{1}{15}$	.	t	=	$\frac{t}{15}$

- Determine how the parts of the task completed are related. Use the fact that the sum of the parts of the task completed must equal 1, the complete task.

The sum of the part of the task completed by the mason and the part of the task completed by the apprentice is 1.

$$\begin{aligned} \frac{t}{10} + \frac{t}{15} &= 1 \\ 30\left(\frac{t}{10} + \frac{t}{15}\right) &= 30(1) \\ 3t + 2t &= 30 \\ 5t &= 30 \\ t &= 6 \end{aligned}$$

Working together, they will build the wall in 6 h.



Tips for Success

Word problems are difficult because we must read the problem, determine the quantity we must find, think of a method to find it, actually solve the problem, and then check the answer. In short, we must devise a *strategy* and then use that strategy to find the *solution*. See *AIM for Success* in the Preface.

EXAMPLE • 3

An electrician requires 12 h to wire a house. The electrician's apprentice can wire a house in 16 h. After working alone on a job for 4 h, the electrician quits, and the apprentice completes the task. How long does it take the apprentice to finish wiring the house?

Strategy

- Time required for the apprentice to finish wiring the house: t

	<i>Rate</i>	<i>Time</i>	<i>Part</i>
Electrician	$\frac{1}{12}$	4	$\frac{4}{12}$
Apprentice	$\frac{1}{16}$	t	$\frac{t}{16}$

- The sum of the part of the task completed by the electrician and the part of the task completed by the apprentice is 1.

YOU TRY IT • 3

Two water pipes can fill a tank with water in 6 h. The larger pipe, working alone, can fill the tank in 9 h. How long will it take the smaller pipe, working alone, to fill the tank?

Your strategy**Solution**

$$\begin{aligned}\frac{4}{12} + \frac{t}{16} &= 1 \\ \frac{1}{3} + \frac{t}{16} &= 1 && \bullet \text{ Simplify } \frac{4}{12}. \\ 48\left(\frac{1}{3} + \frac{t}{16}\right) &= 48(1) && \bullet \text{ Multiply by the LCM of } 3 \text{ and } 16. \\ 16 + 3t &= 48 \\ 3t &= 32 \\ t &= \frac{32}{3} = 10\frac{2}{3}\end{aligned}$$

It will take the apprentice $10\frac{2}{3}$ h to finish wiring the house.

Your solution

OBJECTIVE C

To solve uniform motion problems

A car that travels constantly in a straight line at 55 mph is in uniform motion. **Uniform motion** means that the speed of an object does not change.

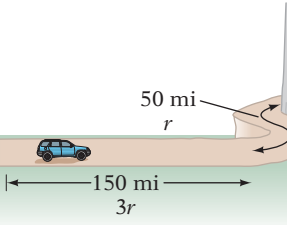
The basic equation used to solve uniform motion problems is

Distance = rate × time

An alternative form of this equation can be written by solving the equation for time.

$$\frac{\text{Distance}}{\text{Rate}} = \text{time}$$

This form of the equation is useful when the total time of travel for two objects or the time of travel between two points is known.



HOW TO • 5 A motorist drove 150 mi on country roads before driving 50 mi on mountain roads. The rate of speed on the country roads was three times the rate on the mountain roads. The time spent traveling the 200 mi was 5 h. Find the rate of the motorist on the country roads.

Strategy for Solving a Uniform Motion Problem

- 1. For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

Unknown rate of speed on the mountain roads: r
Rate of speed on the country roads: $3r$

	<i>Distance</i>	$\div$	<i>Rate</i>	=	<i>Time</i>
Country roads	150	$\div$	$3r$	=	$\frac{150}{3r}$
Mountain roads	50	$\div$	r	=	$\frac{50}{r}$

- 2. Determine how the times traveled by the different objects are related. For example, it may be known that the times are equal, or the total time may be known.

The total time of the trip is 5 h.

$$\begin{aligned} \frac{150}{3r} + \frac{50}{r} &= 5 \\ 3r\left(\frac{150}{3r} + \frac{50}{r}\right) &= 3r(5) \\ 150 + 150 &= 15r \\ 300 &= 15r \\ 20 &= r \end{aligned}$$

The rate of speed on the country roads was $3r$.
Replace r with 20 and evaluate.

$$3r = 3(20) = 60$$

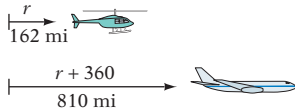
The rate of speed on the country roads was 60 mph.

EXAMPLE 4

A marketing executive traveled 810 mi on a corporate jet in the same amount of time it took to travel an additional 162 mi by helicopter. The rate of the jet was 360 mph greater than the rate of the helicopter. Find the rate of the jet.

Strategy

- Rate of the helicopter: r
Rate of the jet: $r + 360$



	<i>Distance</i>	<i>Rate</i>	<i>Time</i>
Jet	810	$r + 360$	$\frac{810}{r + 360}$
Helicopter	162	r	$\frac{162}{r}$

- The time traveled by jet is equal to the time traveled by helicopter.

Solution

$$\frac{810}{r + 360} = \frac{162}{r}$$

$$r(r + 360)\left(\frac{810}{r + 360}\right) = r(r + 360)\left(\frac{162}{r}\right) \quad \bullet \text{ Multiply by } r(r + 360).$$

$$810r = (r + 360)162$$

$$810r = 162r + 58,320$$

$$648r = 58,320$$

$$r = 90$$

$$r + 360 = 90 + 360 = 450$$

The rate of the jet was 450 mph.

YOU TRY IT 4

A plane can fly at a rate of 150 mph in calm air. Traveling with the wind, the plane flew 700 mi in the same amount of time it took to fly 500 mi against the wind. Find the rate of the wind.

Your strategy**Your solution**

6.5 EXERCISES

OBJECTIVE A

To solve a rational equation



A proposed solution of a rational equation must be an element of the domains of the rational expressions. For Exercises 1 and 2, state the values of x that are not in the domains of the rational expressions.

$$1. \frac{4x}{x+3} = 5x - \frac{12}{x-4}$$

$$2. \frac{1}{x-3} - 1 = \frac{2}{x^2 - 2x - 3}$$

For Exercises 3 to 29, solve.

$$3. \frac{x}{2} + \frac{5}{6} = \frac{x}{3}$$

$$4. \frac{x}{5} - \frac{2}{9} = \frac{x}{15}$$

$$5. \frac{8}{2x-1} = 2$$

$$6. 3 = \frac{18}{3x-4}$$

$$7. 1 - \frac{3}{y} = 4$$

$$8. 7 + \frac{6}{y} = 5$$

$$9. \frac{3}{x-2} = \frac{4}{x}$$

$$10. \frac{5}{x} = \frac{2}{x+3}$$

$$11. \frac{6}{2y+3} = \frac{6}{y}$$

$$12. \frac{3}{x-4} + 2 = \frac{5}{x-4}$$

$$13. \frac{5}{y+3} - 2 = \frac{7}{y+3}$$

$$14. 5 + \frac{8}{a-2} = \frac{4a}{a-2}$$

$$15. \frac{-4}{a-4} = 3 - \frac{a}{a-4}$$

$$16. \frac{5x}{x-4} + 3 = \frac{4}{x-4}$$

$$17. \frac{4}{x-5} = 9 - \frac{2x}{x-5}$$

$$18. \frac{2x}{x+3} - 2 = \frac{9}{x-2}$$

$$19. \frac{4}{x-2} - 4 = \frac{-12}{x+2}$$

$$20. 3x - \frac{8}{x+2} = \frac{4x}{x+2}$$

$$21. \frac{4}{x-4} = x + \frac{x}{x-4}$$

$$22. \frac{x}{x+1} - x = \frac{-4}{x+1}$$

$$23. \frac{2x}{x+2} + 3x = \frac{-5}{x+2}$$

$$24. \frac{x}{x-3} + x = \frac{3x-4}{x-3}$$

$$25. \frac{x}{2x-9} - 3x = \frac{10}{9-2x}$$

$$26. \frac{2}{4y^2-9} + \frac{1}{2y-3} = \frac{3}{2y+3}$$

$$27. \frac{5}{x-2} - \frac{2}{x+2} = \frac{3}{x^2-4}$$

$$28. \frac{5}{x^2-7x+12} = \frac{2}{x-3} + \frac{5}{x-4}$$

$$29. \frac{9}{x^2+7x+10} = \frac{5}{x+2} - \frac{3}{x+5}$$

OBJECTIVE B To solve work problems

30. A ski resort can manufacture enough machine-made snow in 12 h to open its steepest run, whereas it would take naturally falling snow 36 h to provide enough snow. If the resort makes snow at the same time that it is snowing naturally, how long will it take until the run can open?
31. An experienced bricklayer can work twice as fast as an apprentice bricklayer. After they work together on a job for 6 h, the experienced bricklayer quits. The apprentice requires 10 more hours to finish the job. How long would it take the experienced bricklayer, working alone, to do the job?
32. A roofer requires 8 h to shingle a roof. After the roofer and an apprentice work on a roof for 2 h, the roofer moves on to another job. The apprentice requires 10 more hours to finish the job. How long would it take the apprentice, working alone, to do the job?
33. A candy company knows that it will take one of its candy-wrapping machines 40 min to fill an order. After this machine has been working on an order for 15 min, the machine operator starts another candy-wrapping machine on the same order. With both machines running, the order is completed 15 min later. How long would it take the second machine, working alone, to complete the order?
34. The larger of two printers being used to print the payroll for a major corporation requires 30 min to print the payroll. After both printers have been operating for 10 min, the larger printer malfunctions. The smaller printer requires 40 more minutes to complete the payroll. How long would it take the smaller printer, working alone, to print the payroll?
35. Three machines are filling water bottles. The machines can fill the daily quota of water bottles in 10 h, 12 h, and 15 h, respectively. How long would it take to fill the daily quota of water bottles with all three machines working?
36. A goat can eat all the grass in a farmer's field in 12 days, whereas a cow can finish it in 15 days and a horse in 20 days. How long will it be before all the grass is eaten if all three animals graze in the field?
37. The inlet pipe can fill a water tank in 45 min. The outlet pipe can empty the tank in 30 min. How long would it take to empty a full tank with both pipes open?
38. Three computers can print out a task in 20 min, 30 min, and 60 min, respectively. How long would it take to complete the task with all three computers working?
39. Two circus clowns are blowing up balloons, but some of the balloons are popping before they can be sold. The first clown can blow up a balloon every 2 min, the second clown requires 3 min for each balloon, and every 5 min one balloon pops. How long will it take the clowns, working together, to have 76 balloons?



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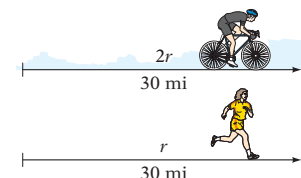
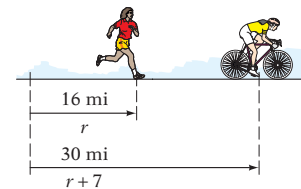


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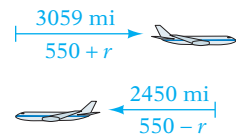
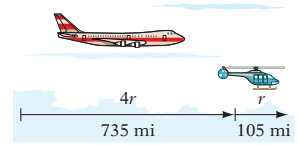
40. An oil tank has two inlet pipes and one outlet pipe. One inlet pipe can fill the tank in 12 h, and the other inlet pipe can fill the tank in 20 h. The outlet pipe can empty the tank in 10 h. How long would it take to fill the tank with all three pipes open?
41. Two clerks are addressing advertising envelopes for a company. One clerk can address one envelope every 30 s, whereas it takes 40 s for the second clerk to address one envelope. How long will it take them, working together, to address 140 envelopes?
42. A single-engine airplane carries enough fuel for an 8-hour flight. After the airplane has been flying for 1 h, the fuel tank begins to leak at a rate that would empty the tank in 12 h. How long after the leak begins does the plane have until it runs out of fuel?
-  43. It takes Katherine n minutes to weed a row of a garden, and it takes Rafael m minutes to weed a row of the garden, where $m > n$. Let t be the time it takes if they work together on the same row. Is t less than n , between n and m , or greater than m ?

OBJECTIVE C To solve uniform motion problems

44. A passenger train travels 240 mi in the same amount of time it takes a freight train to travel 168 mi. The rate of the passenger train is 18 mph greater than the rate of the freight train. Find the rate of each train.
45. The rate of a bicyclist is 7 mph faster than the rate of a long-distance runner. The bicyclist travels 30 mi in the same amount of time it takes the runner to travel 16 mi. Find the rate of the runner.
46. A cabin cruiser travels 20 mi in the same amount of time it takes a power boat to travel 45 mi. The rate of the cabin cruiser is 10 mph less than the rate of the power boat. Find the rate of the cabin cruiser.
47. A tortoise and a hare have joined a 360-foot race. Since the hare can run 180 times faster than the tortoise, it reaches the finish line 14 min and 55 s before the tortoise. How fast was each animal running?
48. A Porsche 911 Turbo has a top speed that is 20 mph faster than a Dodge Viper's top speed. At top speed, the Porsche can travel 630 mi in the same amount of time it takes the Viper to travel 560 mi. What is the top speed of each car?
49. A cyclist and a jogger start from a town at the same time and head for a destination 30 mi away. The rate of the cyclist is twice the rate of the jogger. The cyclist arrives 3 h ahead of the jogger. Find the rate of the cyclist.



50. A canoe can travel 8 mph in still water. Rowing with the current of a river, the canoe can travel 15 mi in the same amount of time it takes to travel 9 mi against the current. Find the rate of the current.
51. An insurance representative traveled 735 mi by commercial jet and then an additional 105 mi by helicopter. The rate of the jet was four times the rate of the helicopter. The entire trip took 2.2 h. Find the rate of the jet.
52. Some military fighter planes are capable of flying at a rate of 950 mph. One of these planes flew 6150 mi with the wind in the same amount of time it took to fly 5250 mi against the wind. Find the rate of the wind.
53. A tour boat used for river excursions can travel 7 mph in calm water. The amount of time it takes to travel 20 mi with the current is the same as the amount of time it takes to travel 8 mi against the current. Find the rate of the current.
54. A jet-ski can travel comfortably across calm water at 35 mph. If a rider traveled 8 mi down a river in the same amount of time it took to travel 6 mi back up the river, find the rate of the river's current.
55. A jet can travel 550 mph in calm air. Flying with the wind, the jet can travel 3059 mi in the same amount of time it takes to fly 2450 mi against the wind. Find the rate of the wind. Round to the nearest hundredth.
56. A pilot can fly a plane at 125 mph in calm air. A recent trip of 300 mi flying with the wind and 300 mi returning against the wind took 5 h. Find the rate of the wind.
57. A river excursion motorboat can travel 6 mph in calm water. A recent trip, going 16 mi down a river and then returning, took 6 h. Find the rate of the river's current.
58. Two people are d miles apart and are walking toward each other in a straight line along a beach. The rate of one person is r miles per hour, and the rate of the second person is $2r$ miles per hour. When they meet, what fraction of d has the slower person covered?



Applying the Concepts

59. Solve for x .
- a. $\frac{x-2}{y} = \frac{x+2}{5y}$ b. $\frac{x}{x+y} = \frac{2x}{4y}$ c. $\frac{x-y}{x} = \frac{2x}{9y}$
60. **Uniform Motion** Because of bad weather conditions, a bus driver reduced the usual speed along a 165-mile bus route by 5 mph. The bus arrived only 15 min later than its usual arrival time. How fast does the bus usually travel?
61. Henry and Hana painted a fence together in a hours. It would have taken Henry b hours to paint the fence working alone. What fraction of the fence did Hana paint?

SECTION

6.6 Variation

OBJECTIVE A

To solve variation problems

Direct variation is a special function that can be expressed as the equation $y = kx^n$, where k is a constant and n is a positive number. The equation $y = kx^n$ is read “ y varies directly as x to the n th” or “ y is directly proportional to x to the n th.” The constant k is called the **constant of variation** or the **constant of proportionality**.

A surveyor earns \$43 per hour. The total wage w of the surveyor varies directly as the number of hours worked h . The direct variation equation is $w = 43h$. The constant of variation is 43, and the value of n is 1.

The distance s , in feet, an object falls varies directly as the square of the time t , in seconds, that it falls. The direct variation equation is $s = kt^2$. The constant of variation is k , and the value of n is 2. If the object is dropped on Earth, $k = 16$; if the object is dropped on the moon, $k = 2.7$.

Many geometry formulas are expressed as direct variation. The circumference C of a circle is directly proportional to its diameter d . The direct variation equation is $C = \pi d$. The constant of proportionality is π , and the value of n is 1.

The area A of a circle varies directly as the square of its radius r . The direct variation equation is $A = \pi r^2$. The constant of proportionality is π , and the value of n is 2.

HOW TO • 1

Given that V varies directly as r and that $V = 20$ when $r = 4$, find the constant of variation and the variation equation.

$$V = kr$$

$$20 = k \cdot 4$$

$$5 = k$$

$$V = 5r$$

- Write the basic direct variation equation.
- Replace V and r by the given values. Then solve for k .
- This is the constant of variation.
- Write the direct variation equation by substituting the value of k into the basic direct variation equation.

HOW TO • 2

The tension T in a spring varies directly as the distance x it is stretched. If $T = 8$ lb when $x = 2$ in., find T when $x = 4$ in.

$$T = kx$$

$$8 = k \cdot 2$$

$$4 = k$$

$$T = 4x$$

- Write the basic direct variation equation.
- Replace T and x by the given values.
- Solve for the constant of variation.
- Write the direct variation equation.

To find T when $x = 4$ in., substitute 4 for x in the equation and solve for T .

$$T = 4x$$

$$T = 4 \cdot 4 = 16$$

The tension is 16 lb.

Inverse variation is a function that can be written as the equation $y = \frac{k}{x^n}$, where k is a constant and n is a positive number. The equation $y = \frac{k}{x^n}$ is read “ y varies inversely as x to the n th” or “ y is inversely proportional to x to the n th.”

The time t it takes a car to travel 100 mi varies inversely as the speed r of the car. The inverse variation equation is $t = \frac{100}{r}$. The variation constant is 100, and the value of n is 1.

The intensity I of a light source varies inversely as the square of the distance d from the source. The inverse variation equation is $I = \frac{k}{d^2}$. The constant of variation depends on the medium through which the light travels (air, water, glass), and the value of n is 2.



Point of Interest

This equation is important to concert sound engineers. Without additional speakers and reverberation, the sound intensity for someone about 20 rows back at this concert would be about 25 dB, the sound intensity of normal conversation.

HOW TO 3

The intensity I , in decibels (dB), of sound varies inversely as the square of the distance d from the source. If the intensity of an open-air concert is 110 dB in the front row, 10 ft from the band, find the variation equation.

$$I = \frac{k}{d^2}$$

- Write the basic inverse variation equation.

$$110 = \frac{k}{10^2}$$

- Replace I and d by the given values.

$$11,000 = k$$

- This is the constant of variation.

$$I = \frac{11,000}{d^2}$$

- This is the variation equation.

HOW TO 4

The length L of a rectangle with fixed area is inversely proportional to the width w . If $L = 6$ ft when $w = 2$ ft, find L when $w = 3$ ft.

$$L = \frac{k}{w}$$

- Write the basic inverse variation equation.

$$6 = \frac{k}{2}$$

- Replace L and w by the given values.

$$12 = k$$

- Solve for the constant of variation.

$$L = \frac{12}{w}$$

- Write the inverse variation equation.

To find L when $w = 3$ ft, substitute 3 for w in the equation and solve for L .

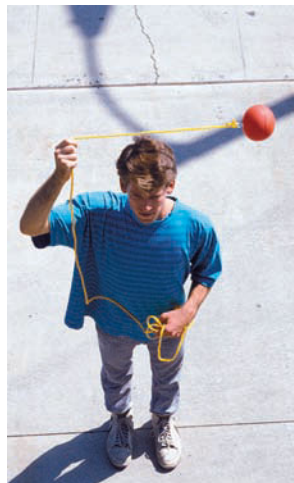
$$L = \frac{12}{w} = \frac{12}{3} = 4$$

The length is 4 ft.

Joint variation is variation in which a variable varies directly as the product of two or more other variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. The equation $z = kxy$ is read “ z varies jointly as x and y .”

The area A of a triangle varies jointly as the base b and the height h . The joint variation equation is written $A = \frac{1}{2}bh$. The constant of variation is $\frac{1}{2}$.

A **combined variation** is a variation in which two or more types of variation occur at the same time. For example, in physics, the volume V of a gas varies directly as the temperature T and inversely as the pressure P . This combined variation is written $V = \frac{kT}{P}$.



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HOW TO • 5

A ball is being twirled on the end of a string. The tension T in the string is directly proportional to the square of the speed v of the ball and inversely proportional to the length r of the string. If the tension is 96 lb when the length of the string is 0.5 ft and the speed is 4 ft/s, find the tension when the length of the string is 1 ft and the speed is 5 ft/s.

$$T = \frac{kv^2}{r}$$

- Write the basic combined variation equation.

$$96 = \frac{k \cdot 4^2}{0.5}$$

- Replace T , v , and r by the given values.

$$96 = \frac{k \cdot 16}{0.5}$$

$$96 = k \cdot 32$$

- Solve for the constant of variation.

$$3 = k$$

$$T = \frac{3v^2}{r}$$

- Write the combined variation equation.

To find T when $r = 1$ ft and $v = 5$ ft/s, substitute 1 for r and 5 for v , and solve for T .

$$T = \frac{3v^2}{r} = \frac{3 \cdot 5^2}{1} = 3 \cdot 25 = 75$$

The tension is 75 lb.

EXAMPLE • 1

The amount A of medication prescribed for a person is directly related to the person's weight W . For a 50-kilogram person, 2 ml of medication are prescribed. How many milliliters of medication are required for a person who weighs 75 kg?

Strategy

To find the required amount of medication:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k .
- Write the direct variation equation, replacing k by its value. Substitute 75 for W and solve for A .

Solution

$$A = kW$$

$$2 = k \cdot 50$$

$$\frac{1}{25} = k$$

$$A = \frac{1}{25}W = \frac{1}{25} \cdot 75 = 3$$

- Direct variation equation
- Replace A by 2 and W by 50.

- $k = \frac{1}{25}$, $W = 75$

The required amount of medication is 3 ml.

YOU TRY IT • 1

The distance s a body falls from rest varies directly as the square of the time t of the fall. An object falls 64 ft in 2 s. How far will it fall in 5 s?

Your strategy**Your solution**

EXAMPLE • 2

A company that produces personal computers has determined that the number of computers it can sell s is inversely proportional to the price P of the computer. Two thousand computers can be sold when the price is \$2500. How many computers can be sold when the price of a computer is \$2000?

Strategy

To find the number of computers:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k .
- Write the inverse variation equation, replacing k by its value. Substitute 2000 for P and solve for s .

Solution

$$s = \frac{k}{P} \quad \bullet \text{ Inverse variation equation}$$

$$2000 = \frac{k}{2500} \quad \bullet \text{ Replace } s \text{ by 2000 and } P \text{ by 2500.}$$

$$5,000,000 = k$$

$$s = \frac{5,000,000}{P} = \frac{5,000,000}{2000} = 2500 \quad \bullet \begin{matrix} k = 5,000,000, \\ P = 2000 \end{matrix}$$

At \$2000 each, 2500 computers can be sold.

EXAMPLE • 3

The pressure P of a gas varies directly as the temperature T and inversely as the volume V . When $T = 50^\circ$ and $V = 275 \text{ in}^3$, $P = 20 \text{ lb/in}^2$. Find the pressure of a gas when $T = 60^\circ$ and $V = 250 \text{ in}^3$.

Strategy

To find the pressure:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k .
- Write the combined variation equation, replacing k by its value. Substitute 60 for T and 250 for V , and solve for P .

Solution

$$P = \frac{kT}{V} \quad \bullet \text{ Combined variation equation}$$

$$20 = \frac{k \cdot 50}{275} \quad \bullet \text{ Replace } P \text{ by 20, } T \text{ by 50, and } V \text{ by 275.}$$

$$110 = k$$

$$P = \frac{110T}{V} = \frac{110 \cdot 60}{250} = 26.4 \quad \bullet \begin{matrix} k = 110, T = 60, \\ V = 250 \end{matrix}$$

The pressure is 26.4 lb/in².

YOU TRY IT • 2

The resistance R to the flow of electric current in a wire of fixed length is inversely proportional to the square of the diameter d of the wire. If a wire of diameter 0.01 cm has a resistance of 0.5 ohm, what is the resistance in a wire that is 0.02 cm in diameter?

Your strategy**Your solution****YOU TRY IT • 3**

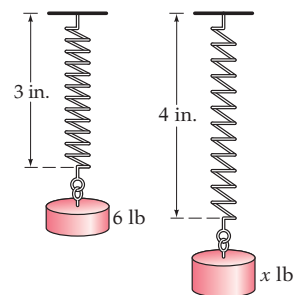
The strength s of a rectangular beam varies jointly as its width W and as the square of its depth d and inversely as its length L . If the strength of a beam 2 in. wide, 12 in. deep, and 12 ft long is 1200 lb, find the strength of a beam 4 in. wide, 8 in. deep, and 16 ft long.

Your strategy**Your solution**

6.6 EXERCISES

OBJECTIVE A To solve variation problems

- Business** The profit P realized by a company varies directly as the number of products it sells s . If a company makes a profit of \$4000 on the sale of 250 products, what is the profit when the company sells 5000 products?
- Compensation** The income I of a computer analyst varies directly as the number of hours h worked. If the analyst earns \$336 for working 8 h, how much will the analyst earn for working 36 h?
- Recreation** The pressure p on a diver in the water varies directly as the depth d . If the pressure is 4.5 lb/in^2 when the depth is 10 ft, what is the pressure when the depth is 15 ft?
- Physics** The distance d that a spring will stretch varies directly as the force f applied to the spring. If a force of 6 lb is required to stretch a spring 3 in., what force is required to stretch the spring 4 in.?
- Physics** The distance d an object will fall is directly proportional to the square of the time t of the fall. If an object falls 144 ft in 3 s, how far will the object fall in 10 s?
- Physics** The period p of a pendulum, or the time it takes the pendulum to make one complete swing, varies directly as the square root of the length L of the pendulum. If the period of a pendulum is 1.5 s when the length is 2 ft, find the period when the length is 5 ft. Round to the nearest hundredth.
- Computer Science** Parallel processing is the use of more than one computer to solve a particular problem. The time T it takes to solve a certain problem is inversely proportional to the number of computers n that are used. If it takes one computer 500 s to solve a problem, how long would it take five computers to solve the same problem?
- Safety** The stopping distance s of a car varies directly as the square of its speed v . If a car traveling 30 mph requires 63 ft to stop, find the stopping distance for a car traveling 55 mph.
- Sailing** The load L , in pounds, on a certain sail varies directly as the square of the wind speed v , in miles per hour. If the load on a sail is 640 lb when the wind speed is 20 mph, what is the load on the sail when the wind speed is 15 mph?
- Whirlpools** The speed v of the current in a whirlpool varies inversely as the distance d from the whirlpool's center. The Old Sow whirlpool, located off the coast of eastern Canada, is one of the most powerful whirlpools on Earth. At a distance of 10 ft from the center of the whirlpool, the speed of the current is about 2.5 ft/s. What is the speed of the current 2 ft from the center?



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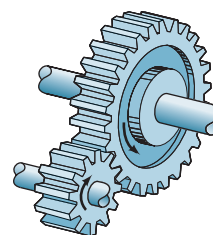
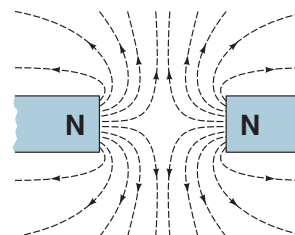
Image Courtesy of Jim Lowe of Eastport, Maine

11. **Real Estate** Real estate agents receive income in the form of a commission, which is a portion of the selling price of the house. The commission c on the sale of a house varies directly as the selling price p of the house. Read the article at the right. Find the commission the agency referred to in the article would receive for selling an average-priced home in January 2008.
12. **Architecture** The heat loss H , in watts, through a single-pane glass window varies jointly as the area A , in square meters, and the difference between the inside and outside temperatures T , in degrees Celsius. If the heat loss is 6 watts for a window with an area of 1.5 m^2 and a temperature difference of 2°C , what would be the heat loss for a window with an area of 2 m^2 and a temperature difference of 5°C ?
13. **Electronics** The current I in a wire varies directly as the voltage v and inversely as the resistance r . If the current is 10 amps when the voltage is 110 volts and the resistance is 11 ohms, find the current when the voltage is 180 volts and the resistance is 24 ohms.
14. **Magnetism** The repulsive force f between the north poles of two magnets is inversely proportional to the square of the distance d between them. If the repulsive force is 20 lb when the distance is 4 in., find the repulsive force when the distance is 2 in.
15. **Light** The intensity I of a light source is inversely proportional to the square of the distance d from the source. If the intensity is 12 foot-candles at a distance of 10 ft, what is the intensity when the distance is 5 ft?
16. **Mechanics** The speed v of a gear varies inversely as the number of teeth t . If a gear that has 45 teeth makes 24 revolutions per minute (rpm), how many revolutions per minute will a gear that has 36 teeth make?
-  17. In the direct variation equation $y = kx$, what is the effect on y when x is doubled?
-  18. In the inverse variation equation $y = \frac{k}{x}$, what is the effect on x when y is doubled?

In the News**Realtors' Incomes Hit Hard**

With lower house prices come lower commissions for real estate agents. One agent said, "In January 2007, my agency received a \$15,600 commission on the sale of an average-priced home (\$260,000). Now, in January 2008, that average-priced home is selling at \$230,000, and we will receive less than \$14,000 in commission."

Source: *The Arizona Republic*

**Applying the Concepts**

For Exercises 19 to 22, complete using the word *directly* or *inversely*.

19. If a varies _____ as b and c , then abc is constant.
20. If a varies directly as b and inversely as c , then c varies _____ as b and _____ as a .
21. If the area of a rectangle is held constant, the length of the rectangle varies _____ as the width.
22. If the length of a rectangle is held constant, the area of the rectangle varies _____ as the width.

FOCUS ON PROBLEM SOLVING

Implication Sentences that are constructed using “If . . . , then . . .” occur frequently in problem-solving situations. These sentences are called **implications**. The sentence “If it rains, then I will stay home” is an implication. The phrase *it rains* is called the **antecedent** of the implication. The phrase *I will stay home* is the **consequent** of the implication.

The sentence “If $x = 4$, then $x^2 = 16$ ” is a true sentence. The **contrapositive** of an implication is formed by switching the antecedent and the consequent and then negating each one. The contrapositive of “If $x = 4$, then $x^2 = 16$ ” is “If $x^2 \neq 16$, then $x \neq 4$.” This sentence is also true. It is a principle of logic that an implication and its contrapositive are either both true or both false.

The **converse** of an implication is formed by switching the antecedent and the consequent. The converse of “If $x = 4$, then $x^2 = 16$ ” is “If $x^2 = 16$, then $x = 4$.” Note that the converse is not a true statement because if $x^2 = 16$, then x could equal -4 . The converse of an implication may or may not be true.

Those statements for which the implication and the converse are both true are very important. They can be stated in the form “ x if and only if y .” For instance, a number is divisible by 5 if and only if the last digit of the number is 0 or 5. The implication “If a number is divisible by 5, then it ends in 0 or 5” and the converse “If a number ends in 0 or 5, then it is divisible by 5” are both true.

For Exercises 1 to 14, state the contrapositive and the converse of the implication. If the converse and the implication are both true statements, write a sentence using the phrase *if and only if*.

1. If I live in Chicago, then I live in Illinois.
2. If today is June 1, then yesterday was May 31.
3. If today is not Thursday, then tomorrow is not Friday.
4. If a number is divisible by 8, then it is divisible by 4.
5. If a number is an even number, then it is divisible by 2.
6. If a number is a multiple of 6, then it is a multiple of 3.
7. If $4z = 20$, then $z = 5$.
8. If an angle measures 90° , then it is a right angle.
9. If p is a prime number greater than 2, then p is an odd number.
10. If the equation of a graph is $y = mx + b$, then the graph of the equation is a straight line.
11. If $a = 0$ or $b = 0$, then $ab = 0$.
12. If the coordinates of a point are $(5, 0)$, then the point is on the x -axis.
13. If a quadrilateral is a square, then the quadrilateral has four sides of equal length.
14. If $x = y$, then $x^2 = y^2$.

PROJECTS AND GROUP ACTIVITIES

Graphing Variation Equations

1. Graph $y = kx$ when $k = 2$.
What kind of function does the graph represent?
2. Graph $y = kx$ when $k = \frac{1}{2}$.
What kind of function does the graph represent?
3. Graph $y = \frac{k}{x}$ when $k = 2$ and $x > 0$.
Is this the graph of a function?

Transformers



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Transformers are devices used to increase or decrease the voltage in an electrical system. For example, if the voltage in a wire is 120 volts, an electric current can pass through a transformer and “come out the other side” with a voltage of 60 volts.

Transformers have extensive applications; for instance, they are commonly used to decrease the voltage from a mainframe to individual houses (look for the big gray boxes on telephone poles). Transformers are also used in wall adapters to decrease wall voltage—for example, for an answering machine.

A transformer must have voltage and current entering and leaving. The voltage and current on both sides of a transformer can be determined by the equation

$$V_1 = \frac{V_2 I_2}{I_1}$$

where V_1 and I_1 are the voltage and current on one side of the transformer, and V_2 and I_2 are the voltage and current on the other side of the transformer.

HOW TO • 1

A transformer takes 1000 volts and reduces the voltage to 100 volts. If the current is 30 amperes when the voltage is 1000 volts, find the current after the voltage is reduced.

$$V_1 = \frac{V_2 I_2}{I_1}$$

$$1000 = \frac{100 I_2}{30}$$

$$30,000 = 100 I_2$$

$$300 = I_2$$

• The voltage is 1000 when the current is 30 amperes, so $V_1 = 1000$ and $I_1 = 30$. $V_2 = 100$. Find I_2 .

After the voltage is reduced, the current is 300 amperes.

HOW TO • 2

On one side of a transformer, the current is 40 amperes and the voltage is 500 volts. If the current on the other side is 400 amperes, what is the voltage on that side?

$$V_1 = \frac{V_2 I_2}{I_1}$$

$$500 = \frac{V_2 \cdot 400}{40}$$

$$20,000 = 400V_2$$

$$50 = V_2$$

• The voltage is 500 when the current is 40 amperes, so $V_1 = 500$ and $I_1 = 40$. $I_2 = 400$. Find V_2 .

After the current is increased, the voltage is 50 volts.

Solve Exercises 1 to 6.

1. A transformer takes 2000 volts and reduces the voltage to 1000 volts. If the current is 40 amperes when the voltage is 2000 volts, find the current after the voltage is reduced.
2. A wall adapter for a CD player reduces 180 volts to 9 volts. If the current was 15 amperes at 180 volts, find the current at 9 volts.
3. A wall adapter for a portable CD player reduces 120 volts to 9 volts. If the current was 12 amperes at 120 volts, find the current at 9 volts.
4. On one side of a transformer, the current is 12 amperes and the voltage is 120 volts. If the current on the other side is 40 amperes, what is the voltage on that side?
5. On one side of a transformer, the voltage is 12 volts and the current is 4.5 amperes. Find the voltage on the other side if the current on that side is 60 amperes.
6. On one side of a transformer, the current is 500 amperes and the voltage is 8 volts. If the current on the other side is 400 amperes, what is the voltage on that side?

CHAPTER 6**SUMMARY****KEY WORDS**

An expression in which the numerator and denominator are polynomials is called a *rational expression*. A function that is written in terms of a rational expression is a *rational function*. [6.1A, p. 342]

A rational expression is in *simplest form* when the numerator and denominator have no common factors. [6.1B, p. 343]

EXAMPLES

$\frac{x^2 + 2x + 1}{x - 3}$ is a rational expression.

$$\begin{aligned} \frac{x^2 + 3x + 2}{(x + 1)^2} &= \frac{(x + 1)(x + 2)}{(x + 1)(x + 1)} \\ &= \frac{x + 2}{x + 1} \end{aligned}$$

The *reciprocal of a rational expression* is the rational expression with the numerator and denominator interchanged. [6.1D, p. 346]

The reciprocal of $\frac{a^2 - b^2}{x + y}$ is $\frac{x + y}{a^2 - b^2}$.

The *least common multiple (LCM) of two or more polynomials* is the simplest polynomial that contains the factors of each polynomial. [6.2A, p. 352]

$$\begin{aligned} 4x^2 - 12x &= 4x(x - 3) \\ 3x^3 - 21x^2 + 36x &= 3x(x - 3)(x - 4) \\ \text{LCM} &= 12x(x - 3)(x - 4) \end{aligned}$$

A *complex fraction* is a fraction in which the numerator or denominator contains one or more fractions. [6.3A, p. 360]

$$\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x}}$$
 is a complex fraction.

A *ratio* is the quotient of two quantities that have the same unit. When a ratio is in simplest form, the units are not written. [6.4A, p. 364]

$$\frac{\$35}{\$100} \text{ written as a ratio in simplest form is } \frac{7}{20}.$$

A *rate* is the quotient of two quantities that have different units. The units are written as part of the rate. [6.4A, p. 364]

$$\frac{65 \text{ mi}}{2 \text{ gal}}$$
 is a rate.

A *proportion* is an equation that states the equality of two ratios or rates. [6.4A, p. 364]

$$\frac{75}{3} = \frac{x}{42}$$
 is a proportion.

Direct variation is a special function that can be expressed as the equation $y = kx^n$, where k is a constant called the *constant of variation* or the *constant of proportionality*, and n is a positive number. [6.6A, p. 378]

$E = mc^2$ is Einstein's formula relating energy and mass. c^2 is the constant of proportionality.

Inverse variation is a function that can be expressed as the equation $y = \frac{k}{x^n}$, where k is a constant and n is a positive number. [6.6A, p. 379]

$I = \frac{k}{d^2}$ gives the intensity of a light source at a distance d from the source.

Joint variation is a variation in which a variable varies directly as the product of two or more variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. [6.6A, p. 379]

$C = kAT$ is a formula for the cost of insulation, where A is the area to be insulated and T is the thickness of the insulation.

A *combined variation* is a variation in which two or more types of variation occur at the same time. [6.6A, p. 379]

$V = \frac{kT}{P}$ is a formula that states that the volume of a gas is directly proportional to the temperature and inversely proportional to the pressure.

ESSENTIAL RULES AND PROCEDURES

To Multiply Rational Expressions [6.1C, p. 345]

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To Divide Rational Expressions [6.1D, p. 346]

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

To Add Rational Expressions [6.2B, p. 354]

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To Subtract Rational Expressions [6.2B, p. 354]

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To Solve an Equation Containing Fractions [6.5A, p. 368]

Clear denominators by multiplying each side of the equation by the LCM of the denominators.

EXAMPLES

$$\begin{aligned} \frac{x^2 + x}{3} \cdot \frac{2}{5x} &= \frac{(x^2 + x) \cdot 2}{3(5x)} \\ &= \frac{2x(x+1)}{15x} = \frac{2(x+1)}{15} \end{aligned}$$

$$\begin{aligned} \frac{3x}{x-5} \div \frac{3}{x+4} &= \frac{3x}{x-5} \cdot \frac{x+4}{3} \\ &= \frac{3x(x+4)}{3(x-5)} = \frac{x(x+4)}{x-5} \end{aligned}$$

$$\begin{aligned} \frac{2x-7}{x^2+4} + \frac{x+2}{x^2+4} &= \frac{(2x-7) + (x+2)}{x^2+4} \\ &= \frac{3x-5}{x^2+4} \end{aligned}$$

$$\begin{aligned} \frac{5x-6}{x+2} - \frac{2x+4}{x+2} &= \frac{(5x-6) - (2x+4)}{x+2} \\ &= \frac{3x-10}{x+2} \end{aligned}$$

$$\begin{aligned} \frac{3}{x} + 5 &= \frac{7}{2x} \\ 2x\left(\frac{3}{x} + 5\right) &= 2x\left(\frac{7}{2x}\right) \\ 6 + 10x &= 7 \\ 10x &= 1 \\ x &= \frac{1}{10} \end{aligned}$$

Equation for Work Problems [6.5B, p. 370]

Rate of work $\times$ time worked = part of task completed

A roofer requires 24 h to shingle a roof. An apprentice can shingle the roof in 36 h. How long would it take to shingle the roof if both roofers worked together?

$$\frac{t}{24} + \frac{t}{36} = 1$$

Equation for Uniform Motion Problems [6.5C, p. 372]

Distance = rate $\times$ time or $\frac{\text{Distance}}{\text{Rate}} = \text{time}$

A motorcycle travels 195 mi in the same amount of time it takes a car to travel 159 mi. The rate of the motorcycle is 12 mph greater than the rate of the car. Find the rate of the car.

$$\frac{195}{r+12} = \frac{159}{r}$$

CHAPTER 6**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How are the excluded values of the domain determined in a rational function?
2. When is a rational expression in simplest form?
3. Do you need a common denominator when multiplying or dividing rational expressions?
4. After adding two rational expressions with the same denominator, how do you simplify the sum?
5. What is the first step in simplifying a complex fraction?
6. How are the units in a rate used to write a proportion?
7. How do you find the rate of work if a job is completed in a given number of hours?
8. Why do you have to check the solutions when solving a rational equation?
9. How do you find the value of the constant of proportionality?
10. How do you solve the formula for uniform motion for t ?
11. When simplifying a rational expression, why is it wrong to cross out common terms in the numerator and denominator?
12. In multiplying rational expressions, why do we begin by factoring the numerator and denominator of each fraction?

CHAPTER 6

REVIEW EXERCISES

1. Multiply: $\frac{a^6b^5 + a^5b^6}{a^5b^4 - a^4b^4} \cdot \frac{a - b}{a^2 - b^2}$
2. Simplify: $\frac{x}{x - 3} - 4 - \frac{2x - 5}{x + 2}$
3. Given $P(x) = \frac{x}{x - 3}$, find $P(4)$.
4. Solve: $\frac{3x - 2}{x + 6} = \frac{3x + 1}{x + 9}$
5. Simplify: $\frac{\frac{3x + 4}{3x - 4} + \frac{3x - 4}{3x + 4}}{\frac{3x - 4}{3x + 4} - \frac{3x + 4}{3x - 4}}$
6. Write each fraction in terms of the LCM of the denominators.

$$\frac{4x}{4x - 1}, \frac{3x - 1}{4x + 1}$$
7. Solve: $r = \frac{2}{3 - r}$
8. Given $P(x) = \frac{x^2 - 2}{3x^2 - 2x + 5}$, find $P(-2)$.
9. Solve: $\frac{10}{5x + 3} = \frac{2}{10x - 3}$
10. Determine the domain of $f(x) = \frac{2x - 7}{3x^2 + 3x - 18}$.
11. Simplify: $\frac{3x^4 + 11x^2 - 4}{3x^4 + 13x^2 + 4}$
12. Determine the domain of $g(x) = \frac{2x}{x - 3}$.
13. Multiply: $\frac{x^3 - 8}{x^3 + 2x^2 + 4x} \cdot \frac{x^3 + 2x^2}{x^2 - 4}$
14. Subtract: $\frac{3x^2 + 2}{x^2 - 4} - \frac{9x - x^2}{x^2 - 4}$
15. Solve: $\frac{4}{x - 3} = \frac{5}{x}$
16. Solve: $\frac{30}{x^2 + 5x + 4} + \frac{10}{x + 4} = \frac{4}{x + 1}$

17. Simplify: $\frac{x + 2 + \frac{3}{x-4}}{x-3 - \frac{2}{x-4}}$

18. Write each fraction in terms of the LCM of the denominators.

$$\frac{x-3}{x-5}, \frac{x}{x^2-9x+20}, \frac{1}{4-x}$$

19. Solve: $\frac{6}{2x-3} = \frac{5}{x+5} + \frac{5}{2x^2+7x-15}$

20. Subtract: $\frac{5}{x-4} - \frac{2x+1}{x^2-3x-4}$

21. Divide: $\frac{27x^3-8}{9x^3+6x^2+4x} \div \frac{9x^2-12x+4}{9x^2-4}$

22. Simplify: $\frac{6x}{3x^2-7x+2} - \frac{2}{3x-1} + \frac{3x}{x-2}$

23. Simplify: $\frac{x^3-27}{x^2-9}$

24. Subtract: $\frac{2x}{x-3} - \frac{5}{x+2}$

25. Find the domain of $F(x) = \frac{x^2-x}{3x^2+4}$.

26. Add: $\frac{x-4}{x+1} + \frac{x-3}{x-2}$

27. Simplify: $\frac{16-x^2}{x^3-2x^2-8x}$

28. Simplify: $\frac{8x^3-27}{4x^2-9}$

29. Multiply: $\frac{16-x^2}{6x+12} \cdot \frac{x^2+5x+6}{x^2-8x+16}$

30. Divide: $\frac{x^2-5x+4}{x^2-2x-8} \div \frac{x^2-4x+3}{x^2+8x+12}$

31. Divide: $\frac{8x^3-64}{4x^3+4x^2+x} \div \frac{x^2+2x+4}{4x^2-1}$

32. Divide: $\frac{3-x}{x^2+3x+9} \div \frac{x^2-9}{x^3-27}$

33. Add: $\frac{3}{2a^3b^2} + \frac{5}{6a^2b}$

34. Simplify: $\frac{8}{9x^2-4} + \frac{5}{3x-2} - \frac{4}{3x+2}$

35. Simplify: $\frac{x - 6 + \frac{6}{x-1}}{x + 3 - \frac{12}{x-1}}$

36. Simplify: $\frac{x + \frac{3}{x-4}}{3 + \frac{x}{x-4}}$

37. Solve: $\frac{x+2}{x-3} = \frac{2x-5}{x+1}$

38. Solve: $\frac{5x}{2x-3} + 4 = \frac{3}{2x-3}$

39. Solve $I = \frac{V}{R}$ for R .

40. Solve: $\frac{6}{x-3} - \frac{1}{x+3} = \frac{51}{x^2-9}$

41. **Work** The inlet pipe can fill a tub in 24 min. The drain pipe can empty the tub in 15 min. How long would it take to empty the tub with both pipes open?

42. **Uniform Motion** A bus and a cyclist leave a school at 8 A.M. and head for a stadium 90 mi away. The rate of the bus is three times the rate of the cyclist. The cyclist arrives 4 h after the bus. Find the rate of the bus.

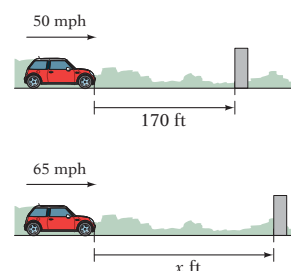
43. **Uniform Motion** A helicopter travels 9 mi in the same amount of time it takes an airplane to travel 10 mi. The rate of the airplane is 20 mph greater than the rate of the helicopter. Find the rate of the helicopter.

44. **Education** A student reads 2 pages of text in 5 min. At the same rate, how long will it take the student to read 150 pages?

45. **Electronics** The current I in an electric circuit varies inversely as the resistance R . If the current in the circuit is 4 amps when the resistance is 50 ohms, find the current in the circuit when the resistance is 100 ohms.

46. **Cartography** On a certain map, 2.5 in. represents 10 mi. How many miles would be represented by 12 in.?

47. **Safety** The stopping distance s of a car varies directly as the square of the speed v of the car. For a car traveling at 50 mph, the stopping distance is 170 ft. Find the stopping distance for a car that is traveling at 65 mph.



48. **Work** An electrician requires 65 min to install a ceiling fan. The electrician and an apprentice, working together, take 40 min to install the fan. How long would it take the apprentice, working alone, to install the ceiling fan?

CHAPTER 6

TEST

1. Solve: $\frac{3}{x+1} = \frac{2}{x}$

2. Divide: $\frac{x^2 + x - 6}{x^2 + 7x + 12} \div \frac{x^2 - 3x + 2}{x^2 + 6x + 8}$

3. Subtract: $\frac{2x-1}{x+2} - \frac{x}{x-3}$

4. Write each fraction in terms of the LCM of the denominators.

$$\frac{x+1}{x^2+x-6}, \frac{2x}{x^2-9}$$

5. Solve: $\frac{4x}{2x-1} = 2 - \frac{1}{2x-1}$

6. Add: $\frac{3}{x+5} + \frac{2x}{x^2+3x-10}$

7. Multiply: $\frac{3x^2-12}{5x-15} \cdot \frac{2x^2-18}{x^2+5x+6}$

8. Determine the domain of $f(x) = \frac{3x^2-x+1}{x^2-9}$.

9. Simplify: $\frac{1 - \frac{1}{x} - \frac{12}{x^2}}{1 + \frac{6}{x} + \frac{9}{x^2}}$

10. Simplify: $\frac{1 - \frac{1}{x+2}}{1 - \frac{3}{x+4}}$

11. Divide: $\frac{2x^2-x-3}{2x^2-5x+3} \div \frac{3x^2-x-4}{x^2-1}$

12. Solve: $\frac{4x}{x+1} - x = \frac{2}{x+1}$

13. Simplify: $\frac{2a^2-8a+8}{4+4a-3a^2}$

14. Solve: $x - \frac{12}{x-3} = \frac{x}{x-3}$

15. Given $f(x) = \frac{3 - x^2}{x^3 - 2x^2 + 4}$, find $f(-1)$.

16. Subtract: $\frac{x + 2}{x^2 + 3x - 4} - \frac{2x}{x^2 - 1}$

17. Solve: $\frac{x + 1}{2x + 5} = \frac{x - 3}{x}$

18. **Uniform Motion** A cyclist travels 20 mi in the same amount of time it takes a hiker to walk 6 mi. The rate of the cyclist is 7 mph greater than the rate of the hiker. Find the rate of the cyclist.

19. **Electronics** The electrical resistance r of a cable varies directly as its length l and inversely as the square of its diameter d . If a cable 16,000 ft long and $\frac{1}{4}$ in. in diameter has a resistance of 3.2 ohms, what is the resistance of a cable that is 8000 ft long and $\frac{1}{2}$ in. in diameter?

20. **Interior Design** An interior designer uses 2 rolls of wallpaper for every 45 ft² of wall space in an office. At this rate, how many rolls of wallpaper are needed for an office that has 315 ft² of wall space?

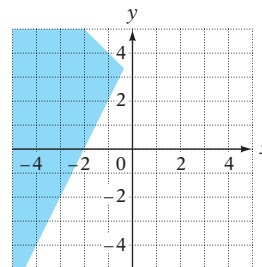
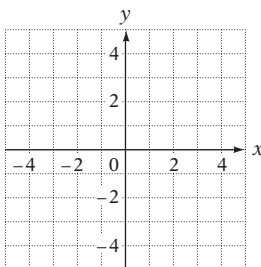


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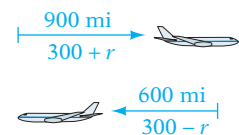
21. **Work** One landscaper can till the soil for a lawn in 30 min, whereas it takes a second landscaper 15 min to do the same job. How long would it take to till the soil for the lawn with both landscapers working together?
22. **Sound Intensity** The intensity I , in decibels, of a sound is inversely proportional to the square of the distance d , in meters, from the source. If the intensity is 50 decibels at a distance of 8 m from the source, what is the intensity at a distance of 5 m from the source?

CUMULATIVE REVIEW EXERCISES

1. Simplify: $8 - 4[-3 - (-2)]^2 \div 5$
2. Solve: $\frac{2x-3}{6} - \frac{x}{9} = \frac{x-4}{3}$
3. Solve: $5 - |x - 4| = 2$
4. Find the domain of $f(x) = \frac{x}{x-3}$.
5. Given $P(x) = \frac{x-1}{2x-3}$, find $P(-2)$.
6. Write 0.000000035 in scientific notation.
7. Simplify: $\frac{(2a^{-2}b^3)^{-2}}{(4a)^{-1}}$
8. Solve: $x - 3(1 - 2x) \geq 1 - 4(2 - 2x)$
Write the solution set in interval notation.
9. Simplify: $(2a^2 - 3a + 1)(-2a^2)$
10. Factor: $2x^2 + 3x - 2$
11. Factor: $x^3y^3 - 27$
12. Simplify: $\frac{x^4 + x^3y - 6x^2y^2}{x^3 - 2x^2y}$
13. Find the equation of the line that contains the point with coordinates $(-2, -1)$ and is parallel to the graph of $3x - 2y = 6$.
14. Solve: $8x^2 - 6x - 9 = 0$
15. Divide: $\frac{4x^3 + 2x^2 - 10x + 1}{x - 2}$
16. Divide: $\frac{16x^2 - 9y^2}{16x^2y - 12xy^2} \div \frac{4x^2 - xy - 3y^2}{12x^2y^2}$
17. Write each fraction in terms of the LCM of the denominators.
 $\frac{xy}{2x^2 + 2x}, \frac{2}{2x^4 - 2x^3 - 4x^2}$
18. Subtract: $\frac{5x}{3x^2 - x - 2} - \frac{2x}{x^2 - 1}$
19. Graph $-3x + 5y = -15$ by using the x - and y -intercepts.
20. Graph the solution set: $x + y \leq 3$
 $-2x + y > 4$



21. Evaluate the determinant: $\begin{vmatrix} 6 & 5 \\ 2 & -3 \end{vmatrix}$
22. Simplify: $\frac{x - 4 + \frac{5}{x+2}}{x + 2 - \frac{1}{x+2}}$
23. Solve: $\begin{cases} x + y + z = 3 \\ -2x + y + 3z = 2 \\ 2x + 4y + z = -1 \end{cases}$
24. Solve: $|3x - 2| > 4$
25. Solve: $\frac{2}{x-3} = \frac{5}{2x-3}$
26. Solve: $\frac{3}{x^2-36} = \frac{2}{x-6} - \frac{5}{x+6}$
27. Solve $I = \frac{E}{R+r}$ for r .
28. Simplify: $(1 - x^{-1})^{-1}$
29. **Integers** The sum of two integers is fifteen. Five times the smaller integer is five more than twice the larger integer. Find the integers.
30. **Mixtures** How many pounds of almonds that cost \$5.40 per pound must be mixed with 50 lb of peanuts that cost \$2.60 per pound to make a mixture that costs \$4.00 per pound?
31. **Elections** A pre-election survey showed that three out of five voters would vote in an election. At this rate, how many people would be expected to vote in a city of 125,000?
32. **Work** A new computer can work six times faster than an older computer. Working together, the computers can complete a job in 12 min. How long would it take the new computer, working alone, to complete the job?
33. **Uniform Motion** A plane can fly at a rate of 300 mph in calm air. Traveling with the wind, the plane flew 900 mi in the same amount of time it took to fly 600 mi against the wind. Find the rate of the wind.
34. **Uniform Motion** Two people start from the same point on a circular exercise track that is 0.25 mi in circumference. The first person walks at 3 mph, and the second person jogs at 5 mph. After 1 h, where along the track are the two people?



Exponents and Radicals

OBJECTIVES

SECTION 7.1

- A** To simplify expressions with rational exponents
- B** To write exponential expressions as radical expressions and to write radical expressions as exponential expressions
- C** To simplify radical expressions that are roots of perfect powers

SECTION 7.2

- A** To simplify radical expressions
- B** To add or subtract radical expressions
- C** To multiply radical expressions
- D** To divide radical expressions

SECTION 7.3

- A** To solve a radical equation
- B** To solve application problems

SECTION 7.4

- A** To simplify a complex number
- B** To add or subtract complex numbers
- C** To multiply complex numbers
- D** To divide complex numbers

ARE YOU READY?

Take the Chapter 7 Prep Test to find out if you are ready to learn to:

- Simplify expressions with rational exponents
- Simplify radical expressions
- Add, subtract, multiply, and divide radical expressions
- Solve a radical equation
- Add, subtract, multiply, and divide complex numbers

PREP TEST

Do these exercises to prepare for Chapter 7.

1. Complete: $48 = ? \cdot 3$

For Exercises 2 to 7, simplify.

2. 2^5

3. $6\left(\frac{3}{2}\right)$

4. $\frac{1}{2} - \frac{2}{3} + \frac{1}{4}$

5. $(3 - 7x) - (4 - 2x)$

6. $\frac{3x^5y^6}{12x^4y}$

7. $(3x - 2)^2$

For Exercises 8 and 9, multiply.

8. $(2 + 4x)(5 - 3x)$

9. $(6x - 1)(6x + 1)$

10. Solve: $x^2 - 14x - 5 = 10$

SECTION

7.1

Rational Exponents and Radical Expressions

OBJECTIVE A

To simplify expressions with rational exponents 

Point of Interest

Nicolas Chuquet (c. 1475), a French physician, wrote an algebra text in which he used a notation for expressions with fractional exponents. He wrote R^26 to mean $6^{1/2}$ and R^315 to mean $15^{1/3}$. This was an improvement over earlier notations that used words for these expressions.

In this section, the definition of an exponent is extended beyond integers so that any rational number can be used as an exponent. The definition is expressed in such a way that the Rules of Exponents hold true for rational exponents.

Consider the expression $(a^{1/n})^n$ for $a > 0$ and n a positive integer. Now simplify, assuming that the Rule for Simplifying the Power of an Exponential Expression is true.

$$(a^{1/n})^n = a^{\frac{1}{n} \cdot n} = a^1 = a$$

Because $(a^{1/n})^n = a$, the number $a^{1/n}$ is the number whose n th power is a .

If $a > 0$ and n is a positive number, then $a^{1/n}$ is called the n th root of a .

$$25^{1/2} = 5 \text{ because } (5)^2 = 25.$$

$$8^{1/3} = 2 \text{ because } (2)^3 = 8.$$

In the expression $a^{1/n}$, if a is a negative number and n is a positive even integer, then $a^{1/n}$ is not a real number.

$(-4)^{1/2}$ is not a real number, because there is no real number whose second power is -4 .

When n is a positive odd integer, a can be a positive or a negative number.

$$(-27)^{1/3} = -3 \text{ because } (-3)^3 = -27.$$

Using the definition of $a^{1/n}$ and the Rules of Exponents, it is possible to define any exponential expression that contains a rational exponent.



Integrating Technology

A calculator can be used to evaluate expressions with rational exponents. For example, to evaluate the expression at the right, press

$(-)$ 27 $\wedge$ $(\frac{\Box}{\Box})$ 1
 $\div$ 3 $)$ ENTER . The

display reads -3 .

Rule for Rational Exponents

If m and n are positive integers and $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^{1/n})^m$$

The expression $a^{m/n}$ can also be written $a^{m/n} = a^{m \cdot \frac{1}{n}} = (a^m)^{1/n}$. However, rewriting $a^{m/n}$ as $(a^m)^{1/n}$ is not as useful as rewriting it as $(a^{1/n})^m$. See the Take Note at the top of the next page.

As shown above, expressions that contain rational exponents do not always represent real numbers when the base of the exponential expression is a negative number. For this reason, **all variables in this chapter represent positive numbers unless otherwise stated.**

**Take Note**

Although we can simplify an expression by rewriting $a^{m/n}$ in the form $(a^m)^{1/n}$, it is usually easier to simplify the form $(a^{1/n})^m$. For instance, simplifying $(27^{1/3})^2$ is easier than simplifying $(27^2)^{1/3}$.

**Take Note**

Note that $32^{-2/5} = \frac{1}{4}$, a positive number. The negative exponent does not affect the sign of a number.

HOW TO • 1Simplify: $27^{2/3}$

$$\begin{aligned} 27^{2/3} &= (3^3)^{2/3} \\ &= 3^{3(2/3)} = 3^2 \\ &= 9 \end{aligned}$$

- Rewrite 27 as 3^3 .
- Multiply the exponents.

HOW TO • 2Simplify: $32^{-2/5}$

$$\begin{aligned} 32^{-2/5} &= (2^5)^{-2/5} \\ &= 2^{-2} = \frac{1}{2^2} \\ &= \frac{1}{4} \end{aligned}$$

- Rewrite 32 as 2^5 .
- Multiply the exponents. Then use the Rule of Negative Exponents.
- Simplify.

HOW TO • 3Simplify: $a^{1/2} \cdot a^{2/3} \cdot a^{-1/4}$

$$\begin{aligned} a^{1/2} \cdot a^{2/3} \cdot a^{-1/4} &= a^{1/2 + 2/3 - 1/4} \\ &= a^{6/12 + 8/12 - 3/12} \\ &= a^{11/12} \end{aligned}$$

- Use the Rule for Multiplying Exponential Expressions.
- Simplify.

HOW TO • 4Simplify: $(x^6y^4)^{3/2}$

$$\begin{aligned} (x^6y^4)^{3/2} &= x^{6(3/2)}y^{4(3/2)} \\ &= x^9y^6 \end{aligned}$$

- Use the Rule for Simplifying Powers of Products.
- Simplify.

HOW TO • 5Simplify: $3x^{-3/4}(2x^{11/4} - x^{-1/4})$

$$\begin{aligned} 3x^{-3/4}(2x^{11/4} - x^{-1/4}) &= 3x^{-3/4}(2x^{11/4}) - 3x^{-3/4}(x^{-1/4}) \\ &= 6x^{-3/4 + 11/4} - 3x^{-3/4 + (-1/4)} \\ &= 6x^{8/4} - 3x^{-4/4} = 6x^2 - 3x^{-1} \\ &= 6x^2 - \frac{3}{x} \end{aligned}$$

- Use the Distributive Property to remove parentheses.
- Use the Rule for Multiplying Exponential Expressions.
- Simplify.

**Tips for Success**

Remember that a HOW TO indicates a worked-out example. Using paper and pencil, work through the example. See *AIM for Success* in the Preface.

HOW TO • 6Simplify: $\left(\frac{8a^3b^{-4}}{64a^{-9}b^2}\right)^{2/3}$

$$\begin{aligned} \left(\frac{8a^3b^{-4}}{64a^{-9}b^2}\right)^{2/3} &= \left(\frac{2^3a^3b^{-4}}{2^6a^{-9}b^2}\right)^{2/3} \\ &= (2^{-3}a^{12}b^{-6})^{2/3} \\ &= 2^{-2}a^8b^{-4} \\ &= \frac{a^8}{2^2b^4} = \frac{a^8}{4b^4} \end{aligned}$$

- Rewrite 8 as 2^3 and 64 as 2^6 .
- Use the Rule for Dividing Exponential Expressions.
- Use the Rule for Simplifying Powers of Products.
- Use the Rule of Negative Exponents and simplify.

EXAMPLE • 1Simplify: $64^{-2/3}$

Solution $64^{-2/3} = (2^6)^{-2/3} = 2^{-4} \quad \bullet \quad 64 = 2^6$

$$= \frac{1}{2^4} = \frac{1}{16}$$

EXAMPLE • 2Simplify: $(-49)^{3/2}$ **Solution**

The base of the exponential expression is a negative number, while the denominator of the exponent is a positive even number.

Therefore, $(-49)^{3/2}$ is not a real number.

EXAMPLE • 3Simplify: $(x^{1/2}y^{-3/2}z^{1/4})^{-3/2}$

Solution $(x^{1/2}y^{-3/2}z^{1/4})^{-3/2}$ • Use the Rule for Simplifying Powers of Products.

$$= x^{-3/4}y^{9/4}z^{-3/8}$$

$$= \frac{y^{9/4}}{x^{3/4}z^{3/8}}$$

EXAMPLE • 4Simplify: $\frac{x^{1/2}y^{-5/4}}{x^{-4/3}y^{1/3}}$

Solution $\frac{x^{1/2}y^{-5/4}}{x^{-4/3}y^{1/3}}$ • Use the Rule for Dividing Exponential Expressions.

$$= x^{3/6 - (-8/6)}y^{-15/12 - 4/12}$$

$$= x^{11/6}y^{-19/12} = \frac{x^{11/6}}{y^{19/12}}$$

YOU TRY IT • 1Simplify: $16^{-3/4}$ **Your solution****YOU TRY IT • 2**Simplify: $(-81)^{3/4}$ **Your solution****YOU TRY IT • 3**Simplify: $(x^{3/4}y^{1/2}z^{-2/3})^{-4/3}$ **Your solution****YOU TRY IT • 4**Simplify: $\left(\frac{16a^{-2}b^{4/3}}{9a^4b^{-2/3}}\right)^{-1/2}$ **Your solution**

Solutions on pp. S21–S22

OBJECTIVE B

To write exponential expressions as radical expressions
and to write radical expressions as exponential expressions

**Point of Interest**

The radical sign was introduced in 1525 by Christoff Rudolff in a book called *Coss*. He modified the symbol to indicate square roots, cube roots, and fourth roots. The idea of using an index, as we do in our modern notation, did not occur until some years later.

Recall that $a^{1/n}$ is the n th root of a . The expression $\sqrt[n]{a}$ is another symbol for the n th root of a .

If a is a real number, then $a^{1/n} = \sqrt[n]{a}$.

In the expression $\sqrt[n]{a}$, the symbol $\sqrt[n]{}$ is called a **radical**, n is the **index** of the radical, and a is the **radicand**. When $n = 2$, the radical expression represents a square root and the index 2 is usually not written.

An exponential expression with a rational exponent can be written as a radical expression.

Writing Exponential Expressions as Radical Expressions

If $a^{1/n}$ is a real number, then $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = a^{m \cdot 1/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$.

The expression $a^{m/n}$ can also be written $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$.

The exponential expression at the right has been written as a radical expression.

$$y^{2/3} = (y^2)^{1/3} = \sqrt[3]{y^2}$$

The radical expressions at the right have been written as exponential expressions.

$$\sqrt[5]{x^6} = (x^6)^{1/5} = x^{6/5}$$

$$\sqrt{17} = 17^{1/2}$$

HOW TO • 7

Write $(5x)^{2/5}$ as a radical expression.

$$(5x)^{2/5} = \sqrt[5]{(5x)^2}$$

$$= \sqrt[5]{25x^2}$$

- The denominator of the rational exponent is the index of the radical. The numerator is the power of the radicand.
- Simplify.

HOW TO • 8

Write $\sqrt[3]{x^4}$ as an exponential expression with a rational exponent.

$$\sqrt[3]{x^4} = (x^4)^{1/3}$$

$$= x^{4/3}$$

- The index of the radical is the denominator of the rational exponent. The power of the radicand is the numerator of the rational exponent.
- Simplify.

HOW TO • 9

Write $\sqrt[3]{a^3 + b^3}$ as an exponential expression with a rational exponent.

$$\sqrt[3]{a^3 + b^3} = (a^3 + b^3)^{1/3}$$

Note that $(a^3 + b^3)^{1/3} \neq a + b$.

EXAMPLE • 5

Write $(3x)^{5/4}$ as a radical expression.

Solution $(3x)^{5/4} = \sqrt[4]{(3x)^5} = \sqrt[4]{243x^5}$

YOU TRY IT • 5

Write $(2x^3)^{3/4}$ as a radical expression.

Your solution

EXAMPLE • 6

Write $-2x^{2/3}$ as a radical expression.

Solution $-2x^{2/3} = -2(x^2)^{1/3} = -2\sqrt[3]{x^2}$

YOU TRY IT • 6

Write $-5a^{5/6}$ as a radical expression.

Your solution

EXAMPLE • 7

Write $\sqrt[4]{3a}$ as an exponential expression.

Solution $\sqrt[4]{3a} = (3a)^{1/4}$

YOU TRY IT • 7

Write $\sqrt[3]{3ab}$ as an exponential expression.

Your solution

EXAMPLE • 8

Write $\sqrt{a^2 - b^2}$ as an exponential expression.

Solution $\sqrt{a^2 - b^2} = (a^2 - b^2)^{1/2}$

YOU TRY IT • 8

Write $\sqrt[4]{x^4 + y^4}$ as an exponential expression.

Your solution

OBJECTIVE C**To simplify radical expressions that are roots of perfect powers**

Every positive number has two square roots, one a positive number and one a negative number. For example, because $(5)^2 = 25$ and $(-5)^2 = 25$, there are two square roots of 25: 5 and -5 .

The symbol $\sqrt{\quad}$ is used to indicate the positive square root, or **principal square root**. To indicate the negative square root of a number, a negative sign is placed in front of the radical.

$$\sqrt{25} = 5$$

$$-\sqrt{25} = -5$$

The square root of zero is zero.

$$\sqrt{0} = 0$$

The square root of a negative number is not a real number because the square of a real number must be positive.

$$\sqrt{-25} \text{ is not a real number.}$$

Note that

$$\sqrt{(-5)^2} = \sqrt{25} = 5 \text{ and } \sqrt{5^2} = \sqrt{25} = 5$$

This is true for all real numbers and is stated as the following result.

For any real number a , $\sqrt{a^2} = |a|$ and $-\sqrt{a^2} = -|a|$. If a is a positive real number, then $\sqrt{a^2} = a$ and $(\sqrt{a})^2 = a$.

Besides square roots, we can also determine cube roots, fourth roots, and so on.

$$\sqrt[3]{8} = 2, \text{ because } 2^3 = 8.$$

• The cube root of a positive number is positive.

$$\sqrt[3]{-8} = -2, \text{ because } (-2)^3 = -8.$$

• The cube root of a negative number is negative.

$$\sqrt[4]{625} = 5, \text{ because } 5^4 = 625.$$

$$\sqrt[5]{243} = 3, \text{ because } 3^5 = 243.$$

The following properties hold true for finding the n th root of a real number.

If n is an even integer, then $\sqrt[n]{a^n} = |a|$ and $-\sqrt[n]{a^n} = -|a|$. If n is an odd integer, then $\sqrt[n]{a^n} = a$.

For example,

$$\sqrt[6]{y^6} = |y| \quad -\sqrt[12]{x^{12}} = -|x| \quad \sqrt[5]{b^5} = b$$

Take Note

Note that when the index is an even natural number, the n th root requires absolute value symbols.

$$\sqrt[6]{y^6} = |y| \text{ but } \sqrt[5]{y^5} = y$$

Because we stated that variables within radicals represent *positive* numbers, we will omit the absolute value symbols when writing an answer.

HOW TO 10

Simplify: $\sqrt[4]{x^4 y^8}$

$$\sqrt[4]{x^4 y^8} = (x^4 y^8)^{1/4}$$

$$= xy^2$$

• The radicand is a perfect fourth power because the exponents on the variables are divisible by 4. Write the radical expression as an exponential expression.

• Use the Rule for Simplifying Powers of Products.

**Integrating Technology**

See the Keystroke Guide: *Radical Expressions* for instructions on using a graphing calculator to evaluate a numerical radical expression.

HOW TO • 11Simplify: $\sqrt[3]{125c^9d^6}$

$$\sqrt[3]{125c^9d^6} = (5^3c^9d^6)^{1/3}$$

$$= 5c^3d^2$$

- The radicand is a perfect cube because 125 is a perfect cube ($125 = 5^3$) and all the exponents on the variables are divisible by 3.

- Use the Rule for Simplifying Powers of Products.

Note that a variable expression is a perfect power if the exponents on the factors are evenly divisible by the index of the radical.

The chart below shows roots of perfect powers. Knowledge of these roots is very helpful when simplifying radical expressions.

Square Roots		Cube Roots	Fourth Roots	Fifth Roots
$\sqrt{1} = 1$	$\sqrt{36} = 6$	$\sqrt[3]{1} = 1$	$\sqrt[4]{1} = 1$	$\sqrt[5]{1} = 1$
$\sqrt{4} = 2$	$\sqrt{49} = 7$	$\sqrt[3]{8} = 2$	$\sqrt[4]{16} = 2$	$\sqrt[5]{32} = 2$
$\sqrt{9} = 3$	$\sqrt{64} = 8$	$\sqrt[3]{27} = 3$	$\sqrt[4]{81} = 3$	$\sqrt[5]{243} = 3$
$\sqrt{16} = 4$	$\sqrt{81} = 9$	$\sqrt[3]{64} = 4$	$\sqrt[4]{256} = 4$	
$\sqrt{25} = 5$	$\sqrt{100} = 10$	$\sqrt[3]{125} = 5$	$\sqrt[4]{625} = 5$	

**Take Note**

From the chart, $\sqrt[5]{243} = 3$, which means that $3^5 = 243$. From this we know that $(-3)^5 = -243$, which means $\sqrt[5]{-243} = -3$.

HOW TO • 12Simplify: $\sqrt[5]{-243x^5y^{15}}$

$$\sqrt[5]{-243x^5y^{15}} = -3xy^3$$

- From the chart, 243 is a perfect fifth power, and each exponent is divisible by 5. Therefore, the radicand is a perfect fifth power.

EXAMPLE • 9Simplify: $\sqrt[3]{-125a^6b^9}$ **Solution**

The radicand is a perfect cube.

$$\sqrt[3]{-125a^6b^9} = -5a^2b^3$$

- Divide each exponent by 3.

YOU TRY IT • 9Simplify: $\sqrt[3]{-8x^{12}y^3}$ **Your solution****EXAMPLE • 10**Simplify: $-\sqrt[4]{16a^4b^8}$ **Solution**

The radicand is a perfect fourth power.

$$-\sqrt[4]{16a^4b^8} = -2ab^2$$

- Divide each exponent by 4.

YOU TRY IT • 10Simplify: $-\sqrt[4]{81x^{12}y^8}$ **Your solution**

7.1 EXERCISES

OBJECTIVE A

To simplify expressions with rational exponents



1. Which of the following are not real numbers?

(i) $(-7)^{1/2}$ (ii) $(-7)^{1/3}$ (iii) $(-7)^{1/4}$ (iv) $(-7)^{1/5}$



2. If
- $x^{1/n} = a$
- , what is
- a^n
- ?

For Exercises 3 to 74, simplify.

3. $8^{1/3}$

4. $16^{1/2}$

5. $9^{3/2}$

6. $25^{3/2}$

7. $27^{-2/3}$

8. $64^{-1/3}$

9. $32^{2/5}$

10. $16^{3/4}$

11. $(-25)^{5/2}$

12. $(-36)^{1/4}$

13. $\left(\frac{25}{49}\right)^{-3/2}$

14. $\left(\frac{8}{27}\right)^{-2/3}$

15. $x^{1/2}x^{1/2}$

16. $a^{1/3}a^{5/3}$

17. $y^{-1/4}y^{3/4}$

18. $x^{2/5} \cdot x^{-4/5}$

19. $x^{-2/3} \cdot x^{3/4}$

20. $x \cdot x^{-1/2}$

21. $a^{1/3} \cdot a^{3/4} \cdot a^{-1/2}$

22. $y^{-1/6} \cdot y^{2/3} \cdot y^{1/2}$

23. $\frac{a^{1/2}}{a^{3/2}}$

24. $\frac{b^{1/3}}{b^{4/3}}$

25. $\frac{y^{-3/4}}{y^{1/4}}$

26. $\frac{x^{-3/5}}{x^{1/5}}$

27. $\frac{y^{2/3}}{y^{-5/6}}$

28. $\frac{b^{3/4}}{b^{-3/2}}$

29. $(x^2)^{-1/2}$

30. $(a^8)^{-3/4}$

31. $(x^{-2/3})^6$

32. $(y^{-5/6})^{12}$

33. $(a^{-1/2})^{-2}$

34. $(b^{-2/3})^{-6}$

35. $(x^{-3/8})^{-4/5}$

36. $(y^{-3/2})^{-2/9}$

37. $(a^{1/2} \cdot a)^2$

38. $(b^{2/3} \cdot b^{1/6})^6$

39. $(x^{-1/2} \cdot x^{3/4})^{-2}$

40. $(a^{1/2} \cdot a^{-2})^3$

41. $(y^{-1/2} \cdot y^{2/3})^{2/3}$

42. $(b^{-2/3} \cdot b^{1/4})^{-4/3}$

43. $(x^{-3}y^6)^{-1/3}$

44. $(a^2b^{-6})^{-1/2}$

45. $(x^{-2}y^{1/3})^{-3/4}$

46. $(a^{-2/3}b^{2/3})^{3/2}$

47. $\left(\frac{x^{1/2}}{y^2}\right)^4$

48. $\left(\frac{b^{-3/4}}{a^{-1/2}}\right)^8$

49. $\frac{x^{1/4} \cdot x^{-1/2}}{x^{2/3}}$

50. $\frac{b^{1/2} \cdot b^{-3/4}}{b^{1/4}}$

51. $\left(\frac{y^{2/3} \cdot y^{-5/6}}{y^{1/9}}\right)^9$

52. $\left(\frac{a^{1/3} \cdot a^{-2/3}}{a^{1/2}}\right)^4$

53. $\left(\frac{b^2 \cdot b^{-3/4}}{b^{-1/2}}\right)^{-1/2}$

54. $\frac{(x^{-5/6} \cdot x^3)^{-2/3}}{x^{4/3}}$

55. $(a^{2/3}b^2)^6(a^3b^3)^{1/3}$

56. $(x^3y^{-1/2})^{-2}(x^{-3}y^2)^{1/6}$

57. $(16x^{-2}y^4)^{-1/2}(xy^{1/2})$

58. $(27s^3t^{-6})^{1/3}(s^{-1/3}t^{5/6})^6$

59. $(x^{-2/3}y^{-3})^3(27x^{-3}y^6)^{-1/3}$

60. $(9x^{-2/3}y^{4/3})^{-1/2}(x^{2/3}y^{-8/3})^{1/2}$

61. $\frac{(4a^{4/3}b^{-2})^{-1/2}}{(a^{1/6}b^{-3/2})^2}$

62. $\frac{(4x^{-2}y^4)^{1/2}}{(8x^6y^{-3/2})^{-2/3}}$

63. $\left(\frac{x^{1/2}y^{-3/4}}{y^{2/3}}\right)^{-6}$

64. $\left(\frac{x^{1/2}y^{-5/4}}{y^{-3/4}}\right)^{-4}$

65. $\left(\frac{b^{-3}}{64a^{-1/2}}\right)^{-2/3}$

66. $\left(\frac{49c^{5/3}}{a^{-1/4}b^{5/6}}\right)^{-3/2}$

67. $y^{3/2}(y^{1/2} - y^{-1/2})$

68. $y^{3/5}(y^{2/5} + y^{-3/5})$

69. $a^{-1/4}(a^{5/4} - a^{9/4})$

70. $x^{4/3}(x^{2/3} + x^{-1/3})$

71. $x^n \cdot x^{n/2}$

72. $a^{n/2} \cdot a^{-n/3}$

73. $\frac{y^{n/2}}{y^{-n}}$

74. $\frac{b^{m/3}}{b^m}$

OBJECTIVE B

To write exponential expressions as radical expressions and to write radical expressions as exponential expressions



75. True or false? $8x^{1/3} = 2\sqrt[3]{x}$



76. True or false? $(\sqrt[3]{x})^5 = (x^5)^{1/3}$

For Exercises 77 to 92, rewrite the exponential expression as a radical expression.

77. $3^{1/4}$

78. $5^{1/2}$

79. $a^{3/2}$

80. $b^{4/3}$

81. $(2t)^{5/2}$

82. $(3x)^{2/3}$

83. $-2x^{2/3}$

84. $-3a^{2/5}$

85. $(a^2b)^{2/3}$

86. $(x^2y^3)^{3/4}$

87. $(a^2b^4)^{3/5}$

88. $(a^3b^7)^{3/2}$

89. $(4x - 3)^{3/4}$

90. $(3x - 2)^{1/3}$

91. $x^{-2/3}$

92. $b^{-3/4}$

For Exercises 93 to 108, rewrite the radical expression as an exponential expression.

93. $\sqrt{14}$

94. $\sqrt{7}$

95. $\sqrt[3]{x}$

96. $\sqrt[4]{y}$

97. $\sqrt[3]{x^4}$

98. $\sqrt[4]{a^3}$

99. $\sqrt[5]{b^3}$

100. $\sqrt[4]{b^5}$

101. $\sqrt{2x^2}$

102. $\sqrt[5]{4y^7}$

103. $-\sqrt{3x^5}$

104. $-\sqrt[4]{4x^5}$

105. $3x\sqrt[3]{y^2}$

106. $2y\sqrt{x^3}$

107. $\sqrt{a^2 - 2}$

108. $\sqrt{3 - y^2}$

OBJECTIVE C**To simplify radical expressions that are roots of perfect powers**

For Exercises 109 to 112, assume that x is a negative real number. State whether the expression simplifies to a positive number, a negative number, or a number that is not a real number.

109. $-\sqrt[3]{8x^{15}}$

110. $-\sqrt{9x^8}$

111. $\sqrt{-4x^{12}}$

112. $\sqrt[3]{-27x^9}$

For Exercises 113 to 136, simplify.

113. $\sqrt{x^{16}}$

114. $\sqrt{y^{14}}$

115. $-\sqrt{x^8}$

116. $-\sqrt{a^6}$

117. $\sqrt[3]{x^3y^9}$

118. $\sqrt[3]{a^6b^{12}}$

119. $-\sqrt[3]{x^{15}y^3}$

120. $-\sqrt[3]{a^9b^9}$

121. $\sqrt{16a^4b^{12}}$

122. $\sqrt{25x^8y^2}$

123. $\sqrt{-16x^4y^2}$

124. $\sqrt{-9a^4b^8}$

125. $\sqrt[3]{27x^9}$

126. $\sqrt[3]{8a^{21}b^6}$

127. $\sqrt[3]{-64x^9y^{12}}$

128. $\sqrt[3]{-27a^3b^{15}}$

129. $-\sqrt[4]{x^8y^{12}}$

130. $-\sqrt[4]{a^{16}b^4}$

131. $\sqrt[5]{x^{20}y^{10}}$

132. $\sqrt[5]{a^5b^{25}}$

133. $\sqrt[4]{81x^4y^{20}}$

134. $\sqrt[4]{16a^8b^{20}}$

135. $\sqrt[5]{32a^5b^{10}}$

136. $\sqrt[5]{-32x^{15}y^{20}}$

Applying the Concepts

137. Determine whether the following statements are true or false. If the statement is false, correct the right-hand side of the equation.

a. $\sqrt{(-2)^2} = -2$

b. $\sqrt[3]{(-3)^3} = -3$

c. $\sqrt[n]{a} = a^{1/n}$

d. $\sqrt[n]{a^n + b^n} = a + b$

e. $(a^{1/2} + b^{1/2})^2 = a + b$

f. $\sqrt[m]{a^n} = a^{m/n}$

138. Simplify.

a. $\sqrt[3]{\sqrt{x^6}}$

b. $\sqrt[4]{\sqrt{a^8}}$

c. $\sqrt{\sqrt{81y^8}}$

d. $\sqrt{\sqrt[n]{a^{4n}}}$

e. $\sqrt[n]{\sqrt[n]{b^{6n}}}$

f. $\sqrt{\sqrt[3]{x^{12}y^{24}}}$



139. If x is any real number, is $\sqrt{x^2} = x$ always true? Show why or why not.

SECTION

7.2

Operations on Radical Expressions

OBJECTIVE A

To simplify radical expressions 

Point of Interest

The Latin expression for irrational numbers was *numerus surdus*, which literally means “inaudible number.” A prominent 16th-century mathematician wrote of irrational numbers, “. . . just as an infinite number is not a number, so an irrational number is not a true number, but lies hidden in some sort of cloud of infinity.” In 1872, Richard Dedekind wrote a paper that established the first logical treatment of irrational numbers.

If a number is not a perfect power, its root can only be approximated; examples include $\sqrt{5}$ and $\sqrt[3]{3}$. These numbers are irrational numbers. Their decimal representations never terminate or repeat.

$$\sqrt{5} = 2.2360679\dots \quad \sqrt[3]{3} = 1.4422495\dots$$

A radical expression is in simplest form when the radicand contains no factor that is a perfect power. The Product Property of Radicals is used to simplify radical expressions whose radicands are not perfect powers.

The Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

HOW TO • 1

Simplify: $\sqrt{48}$

$$\sqrt{48} = \sqrt{16 \cdot 3}$$

$$= \sqrt{16} \sqrt{3}$$

$$= 4\sqrt{3}$$

• Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

• Use the Product Property of Radicals to write the expression as a product.

• Simplify $\sqrt{16}$.

Note that 48 must be written as the product of a perfect square and a factor that does not contain a perfect square. Therefore, it would not be correct to rewrite $\sqrt{48}$ as $\sqrt{4 \cdot 12}$ and simplify the expression as shown at the right. Although 4 is a perfect square factor of 48, 12 contains a perfect square ($12 = 4 \cdot 3$) and thus $\sqrt{12}$ can be simplified further. Remember to find the *largest* perfect power that is a factor of the radicand.

$$\begin{aligned}\sqrt{48} &= \sqrt{4 \cdot 12} \\ &= \sqrt{4} \sqrt{12} \\ &= 2\sqrt{12}\end{aligned}$$

Not in simplest form

HOW TO • 2

Simplify: $\sqrt{18x^2y^3}$

$$\sqrt{18x^2y^3} = \sqrt{9x^2y^2 \cdot 2y}$$

$$= \sqrt{9x^2y^2} \sqrt{2y}$$

$$= 3xy\sqrt{2y}$$

• Write the radicand as the product of a perfect square and factors that do not contain a perfect square.

• Use the Product Property of Radicals to write the expression as a product.

• Simplify.

HOW TO • 3Simplify: $\sqrt[3]{x^7}$

$$\sqrt[3]{x^7} = \sqrt[3]{x^6 \cdot x}$$

$$= \sqrt[3]{x^6} \sqrt[3]{x}$$

$$= x^2 \sqrt[3]{x}$$

- Write the radicand as the product of a perfect cube and a factor that does not contain a perfect cube.

- Use the Product Property of Radicals to write the expression as a product.

- Simplify.

HOW TO • 4Simplify: $\sqrt[4]{32x^7}$

$$\sqrt[4]{32x^7} = \sqrt[4]{16x^4(2x^3)}$$

$$= \sqrt[4]{16x^4} \sqrt[4]{2x^3}$$

$$= 2x \sqrt[4]{2x^3}$$

- Write the radicand as the product of a perfect fourth power and factors that do not contain a perfect fourth power.

- Use the Product Property of Radicals to write the expression as a product.

- Simplify.

EXAMPLE • 1Simplify: $\sqrt[4]{x^9}$ **Solution**

$$\begin{aligned}\sqrt[4]{x^9} &= \sqrt[4]{x^8 \cdot x} \\ &= \sqrt[4]{x^8} \sqrt[4]{x} \\ &= x^2 \sqrt[4]{x}\end{aligned}$$

- x^8 is a perfect fourth power.

YOU TRY IT • 1Simplify: $\sqrt[5]{x^7}$ **Your solution****EXAMPLE • 2**Simplify: $\sqrt[3]{-27a^5b^{12}}$ **Solution**

$$\begin{aligned}\sqrt[3]{-27a^5b^{12}} &= \sqrt[3]{-27a^3b^{12}(a^2)} \\ &= \sqrt[3]{-27a^3b^{12}} \sqrt[3]{a^2} \\ &= -3ab^4 \sqrt[3]{a^2}\end{aligned}$$

- $-27a^3b^{12}$ is a perfect cube.

YOU TRY IT • 2Simplify: $\sqrt[3]{-64x^8y^{18}}$ **Your solution**

Solutions on p. S22

OBJECTIVE B**To add or subtract radical expressions**

The Distributive Property is used to simplify the sum or difference of radical expressions that have the same radicand and the same index. For example,

$$3\sqrt{5} + 8\sqrt{5} = (3 + 8)\sqrt{5} = 11\sqrt{5}$$

$$2\sqrt[3]{3x} - 9\sqrt[3]{3x} = (2 - 9)\sqrt[3]{3x} = -7\sqrt[3]{3x}$$

Radical expressions that are in simplest form and have unlike radicands or different indices cannot be simplified by the Distributive Property. The expressions below cannot be simplified by the Distributive Property.

$$3\sqrt[4]{2} - 6\sqrt[4]{3}$$

$$2\sqrt[4]{4x} + 3\sqrt[3]{4x}$$

HOW TO • 5Simplify: $3\sqrt{32x^2} - 2x\sqrt{2} + \sqrt{128x^2}$

$$\begin{aligned}
 & 3\sqrt{32x^2} - 2x\sqrt{2} + \sqrt{128x^2} \\
 &= 3\sqrt{16x^2} \sqrt{2} - 2x\sqrt{2} + \sqrt{64x^2} \sqrt{2} \\
 &= 3 \cdot 4x\sqrt{2} - 2x\sqrt{2} + 8x\sqrt{2} \\
 &= 12x\sqrt{2} - 2x\sqrt{2} + 8x\sqrt{2} \\
 &= 18x\sqrt{2}
 \end{aligned}$$

- First simplify each term. Then combine like terms by using the Distributive Property.

EXAMPLE • 3Simplify: $5b\sqrt[4]{32a^7b^5} - 2a\sqrt[4]{162a^3b^9}$ **Solution**

$$\begin{aligned}
 & 5b\sqrt[4]{32a^7b^5} - 2a\sqrt[4]{162a^3b^9} \\
 &= 5b\sqrt[4]{16a^4b^4 \cdot 2a^3b} - 2a\sqrt[4]{81b^8 \cdot 2a^3b} \\
 &= 5b \cdot 2ab\sqrt[4]{2a^3b} - 2a \cdot 3b^2\sqrt[4]{2a^3b} \\
 &= 10ab^2\sqrt[4]{2a^3b} - 6ab^2\sqrt[4]{2a^3b} \\
 &= 4ab^2\sqrt[4]{2a^3b}
 \end{aligned}$$

YOU TRY IT • 3Simplify: $3xy\sqrt[3]{81x^5y} - \sqrt[3]{192x^8y^4}$ **Your solution**

Solution on p. S22

OBJECTIVE C**To multiply radical expressions**

The Product Property of Radicals is used to multiply radical expressions with the same index.

$$\sqrt{3x} \cdot \sqrt{5y} = \sqrt{3x \cdot 5y} = \sqrt{15xy}$$

HOW TO • 6Multiply: $\sqrt[3]{2a^5b} \sqrt[3]{16a^2b^2}$

$$\begin{aligned}
 \sqrt[3]{2a^5b} \sqrt[3]{16a^2b^2} &= \sqrt[3]{32a^7b^3} \\
 &= \sqrt[3]{8a^6b^3 \cdot 4a} \\
 &= 2a^2b \sqrt[3]{4a}
 \end{aligned}$$

- Use the Product Property of Radicals to multiply the radicands.
- Simplify.

HOW TO • 7Multiply: $\sqrt{2x}(\sqrt{8x} - \sqrt{3})$

$$\begin{aligned}
 \sqrt{2x}(\sqrt{8x} - \sqrt{3}) &= \sqrt{2x}(\sqrt{8x}) - \sqrt{2x}(\sqrt{3}) \\
 &= \sqrt{16x^2} - \sqrt{6x} \\
 &= 4x - \sqrt{6x}
 \end{aligned}$$

- Use the Distributive Property.
- Simplify.

HOW TO • 8Multiply: $(2\sqrt{5} - 3)(3\sqrt{5} + 4)$

$$\begin{aligned}
 (2\sqrt{5} - 3)(3\sqrt{5} + 4) &= 6(\sqrt{5})^2 + 8\sqrt{5} - 9\sqrt{5} - 12 \\
 &= 30 + 8\sqrt{5} - 9\sqrt{5} - 12 \\
 &= 18 - \sqrt{5}
 \end{aligned}$$

- Use the FOIL method to multiply the numbers.
- Combine like terms.

**Take Note**

The concept of conjugate is used in a number of different instances. Make sure you understand this idea.

The conjugate of $\sqrt{3} - 4$ is $\sqrt{3} + 4$.

The conjugate of $\sqrt{3} + 4$ is $\sqrt{3} - 4$.

The conjugate of $\sqrt{5a} + \sqrt{b}$ is $\sqrt{5a} - \sqrt{b}$.

The expressions $a + b$ and $a - b$ are **conjugates** of each other—that is, binomial expressions that differ only in the sign of a term. Recall that $(a + b)(a - b) = a^2 - b^2$. This identity is used to simplify conjugate radical expressions.

HOW TO • 9

Multiply: $(\sqrt{11} - 3)(\sqrt{11} + 3)$

$$(\sqrt{11} - 3)(\sqrt{11} + 3) = (\sqrt{11})^2 - 3^2 = 11 - 9 = 2$$

• The radical expressions are conjugates.

EXAMPLE • 4

Multiply: $\sqrt{3}(\sqrt{15} - \sqrt{21})$

Solution

$$\begin{aligned}\sqrt{3}(\sqrt{15} - \sqrt{21}) &= \sqrt{45} - \sqrt{63} \\ &= 3\sqrt{5} - 3\sqrt{7}\end{aligned}$$

YOU TRY IT • 4

Multiply: $5\sqrt{2}(\sqrt{6} + \sqrt{24})$

Your solution**EXAMPLE • 5**

Multiply: $(2 + 3\sqrt{5})(3 - \sqrt{5})$

Solution

$$\begin{aligned}(2 + 3\sqrt{5})(3 - \sqrt{5}) &= 6 - 2\sqrt{5} + 9\sqrt{5} - 3(\sqrt{5})^2 \\ &= 6 + 7\sqrt{5} - 3 \cdot 5 \\ &= 6 + 7\sqrt{5} - 15 \\ &= -9 + 7\sqrt{5}\end{aligned}$$

YOU TRY IT • 5

Multiply: $(4 - 2\sqrt{7})(1 + 3\sqrt{7})$

Your solution**EXAMPLE • 6**

Expand: $(3 - \sqrt{x+1})^2$

Solution

$$\begin{aligned}(3 - \sqrt{x+1})^2 &= (3 - \sqrt{x+1})(3 - \sqrt{x+1}) \\ &= 9 - 3\sqrt{x+1} - 3\sqrt{x+1} + (\sqrt{x+1})^2 \\ &= 9 - 6\sqrt{x+1} + (x+1) \\ &= x - 6\sqrt{x+1} + 10\end{aligned}$$

YOU TRY IT • 6

Expand: $(4 - \sqrt{2x})^2$

Your solution

OBJECTIVE D**To divide radical expressions**

The Quotient Property of Radicals is used to divide radical expressions with the same index.

The Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

HOW TO • 10

Divide: $\frac{\sqrt{5a^4b^7c^2}}{\sqrt{ab^3c}}$

$$\begin{aligned} \frac{\sqrt{5a^4b^7c^2}}{\sqrt{ab^3c}} &= \sqrt{\frac{5a^4b^7c^2}{ab^3c}} \\ &= \sqrt{5a^3b^4c} \\ &= \sqrt{a^2b^4 \cdot 5ac} = ab^2\sqrt{5ac} \end{aligned}$$

• Use the Quotient Property of Radicals.

• Simplify the radicand.

A radical expression is in simplest form when no radical exists in the denominator of an expression. The radical expressions at the right are *not* in simplest form.

$$\frac{3}{\sqrt{7}} \quad \frac{6}{\sqrt[3]{2x}} \quad \frac{6}{3 - \sqrt{5}}$$

Not in simplest form

The procedure used to remove a radical expression from the denominator is called **rationalizing the denominator**. The idea is to multiply the numerator and denominator by an expression that will result in a denominator that is a perfect root of the index.

HOW TO • 11

Simplify: $\frac{3}{\sqrt{7}}$

$$\begin{aligned} \frac{3}{\sqrt{7}} &= \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{3\sqrt{7}}{\sqrt{49}} = \frac{3\sqrt{7}}{7} \end{aligned}$$

• Multiply the numerator and denominator by $\sqrt{7}$.

• $\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = 7$. Because 49 is a perfect square, the denominator can now be written without a radical.

 Take Note

$\frac{\sqrt{7}}{\sqrt{7}} = 1$. Therefore, we are multiplying by 1 and not changing the value of the expression.

 Take Note

Because the index of the radical is 3, we must multiply by a factor that will produce a perfect third power. Ask, "What must $4x$ be multiplied by to produce a perfect third power?"

$$\begin{aligned} 4x \cdot ? &= 8x^3 \\ 4x \cdot 2x^2 &= 8x^3 \end{aligned}$$

We must multiply the numerator and denominator by $\sqrt[3]{2x^2}$.

HOW TO • 12

Simplify: $\frac{6}{\sqrt[3]{4x}}$

$$\begin{aligned} \frac{6}{\sqrt[3]{4x}} &= \frac{6}{\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} \\ &= \frac{6\sqrt[3]{2x^2}}{\sqrt[3]{8x^3}} = \frac{6\sqrt[3]{2x^2}}{2x} \\ &= \frac{3\sqrt[3]{2x^2}}{x} \end{aligned}$$

• Multiply the numerator and denominator by $\sqrt[3]{2x^2}$. See the Take Note at the left.

• $\sqrt[3]{4x} \cdot \sqrt[3]{2x^2} = \sqrt[3]{8x^3} = 2x$. Because $8x^3$ is a perfect cube, the denominator can now be written without a radical.

• Divide by the common factor.

Recall that the expressions $a + b$ and $a - b$ are conjugates and that $(a + b)(a - b) = a^2 - b^2$. This result can be used to simplify a square root radical expression that has two terms in the denominator.

HOW TO • 13 Simplify: $\frac{6}{3 - \sqrt{5}}$

$$\begin{aligned}\frac{6}{3 - \sqrt{5}} &= \frac{6}{3 - \sqrt{5}} \cdot \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{6(3 + \sqrt{5})}{3^2 - (\sqrt{5})^2} \\ &= \frac{6(3 + \sqrt{5})}{9 - 5} = \frac{6(3 + \sqrt{5})}{4} \\ &= \frac{3(3 + \sqrt{5})}{2} = \frac{9 + 3\sqrt{5}}{2}\end{aligned}$$

• Multiply the numerator and denominator by $3 + \sqrt{5}$, the conjugate of $3 - \sqrt{5}$.

EXAMPLE • 7

Simplify: $\sqrt{\frac{3}{2x}}$

Solution

$$\begin{aligned}\sqrt{\frac{3}{2x}} &= \frac{\sqrt{3}}{\sqrt{2x}} \\ &= \frac{\sqrt{3}}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} \\ &= \frac{\sqrt{6x}}{\sqrt{4x^2}} = \frac{\sqrt{6x}}{2x}\end{aligned}$$

• Quotient Property of Radicals

• Rationalize the denominator.

YOU TRY IT • 7

Simplify: $\sqrt{\frac{5}{6}}$

Your solution

EXAMPLE • 8

Simplify: $\frac{6}{\sqrt[4]{9x}}$

Solution

Ask “What must $9x$ be multiplied by to produce a perfect fourth power?”

$$9x \cdot ? = 81x^4$$

$$9x \cdot 9x^3 = 81x^4$$

Multiply the numerator and denominator by $\sqrt[4]{9x^3}$.

$$\frac{6}{\sqrt[4]{9x}} = \frac{6\sqrt[4]{9x^3}}{\sqrt[4]{9x} \cdot \sqrt[4]{9x^3}} = \frac{6\sqrt[4]{9x^3}}{\sqrt[4]{81x^4}} = \frac{6\sqrt[4]{9x^3}}{3x} = \frac{2\sqrt[4]{9x^3}}{x}$$

YOU TRY IT • 8

Simplify: $\frac{3x}{\sqrt[3]{3x^2}}$

Your solution

EXAMPLE • 9

Simplify: $\frac{1 - \sqrt{5}}{2 + 3\sqrt{5}}$

Solution

$$\begin{aligned}\frac{1 - \sqrt{5}}{2 + 3\sqrt{5}} &= \frac{1 - \sqrt{5}}{2 + 3\sqrt{5}} \cdot \frac{2 - 3\sqrt{5}}{2 - 3\sqrt{5}} \\ &= \frac{2 - 3\sqrt{5} - 2\sqrt{5} + 3\sqrt{25}}{2^2 - (3\sqrt{5})^2} \\ &= \frac{2 - 5\sqrt{5} + 3 \cdot 5}{4 - 9 \cdot 5} = \frac{17 - 5\sqrt{5}}{-41} = \frac{-17 + 5\sqrt{5}}{41}\end{aligned}$$

• The conjugate of $2 + 3\sqrt{5}$ is $2 - 3\sqrt{5}$.

YOU TRY IT • 9

Simplify: $\frac{3}{2 - \sqrt{x}}$

Your solution

7.2 EXERCISES**OBJECTIVE A** To simplify radical expressions

For Exercises 1 to 4, state whether the given expression is in simplest form.

1. $\sqrt{108}$

2. $\sqrt[3]{y^7}$

3. $\sqrt[4]{8ab^2c^3}$

4. $2b\sqrt[4]{8b^3}$

For Exercises 5 to 20, simplify.

5. $\sqrt{x^4y^3z^5}$

6. $\sqrt{x^3y^6z^9}$

7. $\sqrt{8a^3b^8}$

8. $\sqrt{24a^9b^6}$

9. $\sqrt{45x^2y^3z^5}$

10. $\sqrt{60xy^7z^{12}}$

11. $\sqrt[4]{48x^4y^5z^6}$

12. $\sqrt[4]{162x^9y^8z^2}$

13. $\sqrt[3]{a^{16}b^8}$

14. $\sqrt[3]{a^5b^8}$

15. $\sqrt[3]{-125x^2y^4}$

16. $\sqrt[3]{-216x^5y^9}$

17. $\sqrt[3]{a^4b^5c^6}$

18. $\sqrt[3]{a^8b^{11}c^{15}}$

19. $\sqrt[4]{16x^9y^5}$

20. $\sqrt[4]{64x^8y^{10}}$

OBJECTIVE B To add or subtract radical expressions

21. True or false? $\sqrt{9+a} + \sqrt{a+9} = 2\sqrt{a+9}$



22. True or false? $\sqrt{9+a} = 3 + \sqrt{a}$

For Exercises 23 to 48, simplify.

23. $2\sqrt{x} - 8\sqrt{x}$

24. $3\sqrt{y} + 12\sqrt{y}$

25. $\sqrt{8x} - \sqrt{32x}$

26. $\sqrt{27a} - \sqrt{8a}$

27. $\sqrt{18b} + \sqrt{75b}$

28. $2\sqrt{2x^3} + 4x\sqrt{8x}$

29. $3\sqrt{8x^2y^3} - 2x\sqrt{32y^3}$

30. $2\sqrt{32x^2y^3} - xy\sqrt{98y}$

31. $2a\sqrt{27ab^5} + 3b\sqrt{3a^3b}$

32. $\sqrt[3]{128} + \sqrt[3]{250}$

33. $\sqrt[3]{16} - \sqrt[3]{54}$

34. $2\sqrt[3]{3a^4} - 3a\sqrt[3]{81a}$

35. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5}$

36. $3\sqrt[3]{x^5y^7} - 8xy\sqrt[3]{x^2y^4}$

37. $3\sqrt[4]{32a^5} - a\sqrt[4]{162a}$

38. $2a\sqrt[4]{16ab^5} + 3b\sqrt[4]{256a^5b}$

39. $2\sqrt{50} - 3\sqrt{125} + \sqrt{98}$

40. $3\sqrt{108} - 2\sqrt{18} - 3\sqrt{48}$

41. $\sqrt{9b^3} - \sqrt{25b^3} + \sqrt{49b^3}$

42. $\sqrt{4x^7y^5} + 9x^2\sqrt{x^3y^5} - 5xy\sqrt{x^5y^3}$

43. $2x\sqrt{8xy^2} - 3y\sqrt{32x^3} + \sqrt{4x^3y^3}$

44. $5a\sqrt{3a^3b} + 2a^2\sqrt{27ab} - 4\sqrt{75a^5b}$

45. $\sqrt[3]{54xy^3} - 5\sqrt[3]{2xy^3} + y\sqrt[3]{128x}$

46. $2\sqrt[3]{24x^3y^4} + 4x\sqrt[3]{81y^4} - 3y\sqrt[3]{24x^3y}$

47. $2a\sqrt[4]{32b^5} - 3b\sqrt[4]{162a^4b} + \sqrt[4]{2a^4b^5}$

48. $6y\sqrt[4]{48x^5} - 2x\sqrt[4]{243xy^4} - 4\sqrt[4]{3x^5y^4}$

OBJECTIVE C To multiply radical expressions

For Exercises 49 to 83, simplify.

49. $\sqrt{8} \sqrt{32}$

50. $\sqrt{14} \sqrt{35}$

51. $\sqrt[3]{4} \sqrt[3]{8}$

52. $\sqrt[3]{6} \sqrt[3]{36}$

53. $\sqrt{x^2y^5} \sqrt{xy}$

54. $\sqrt{a^3b} \sqrt{ab^4}$

55. $\sqrt{2x^2y} \sqrt{32xy}$

56. $\sqrt{5x^3y} \sqrt{10x^3y^4}$

57. $\sqrt[3]{x^2y} \sqrt[3]{16x^4y^2}$

58. $\sqrt[3]{4a^2b^3} \sqrt[3]{8ab^5}$

59. $\sqrt[4]{12ab^3} \sqrt[4]{4a^5b^2}$

60. $\sqrt[4]{36a^2b^4} \sqrt[4]{12a^5b^3}$

61. $\sqrt{3}(\sqrt{27} - \sqrt{3})$

62. $\sqrt{10}(\sqrt{10} - \sqrt{5})$

63. $\sqrt{x}(\sqrt{x} - \sqrt{2})$

64. $\sqrt{y}(\sqrt{y} - \sqrt{5})$

65. $\sqrt{2x}(\sqrt{8x} - \sqrt{32})$

66. $\sqrt{3a}(\sqrt{27a^2} - \sqrt{a})$

67. $(3 - 2\sqrt{5})(2 + \sqrt{5})$

68. $(6 + 5\sqrt{2})(4 - 2\sqrt{2})$

69. $(-2 + \sqrt{7})(3 + 5\sqrt{7})$

70. $(5 - 2\sqrt{5})(7 - 3\sqrt{5})$

71. $(6 + 3\sqrt{2})(4 - 2\sqrt{2})$

72. $(10 - 3\sqrt{5})(3 + 2\sqrt{5})$

73. $(5 - 2\sqrt{7})(5 + 2\sqrt{7})$

74. $(4 + 2\sqrt{3})(4 - 2\sqrt{3})$

75. $(3 - \sqrt{2x})(1 + 5\sqrt{2x})$

76. $(7 + \sqrt{x})(4 - 2\sqrt{x})$

77. $(2 + 2\sqrt{x})(1 + 5\sqrt{x})$

78. $(3 - 2\sqrt{x})^2$

79. $(2 + \sqrt{x})^2$

80. $(5 - \sqrt{x + 2})^2$

81. $(4 - \sqrt{2x + 1})^2$

82. $(\sqrt{5} - 2\sqrt{7})(3\sqrt{5} + 2\sqrt{7})$

83. $(\sqrt{6} - 5\sqrt{3})(3\sqrt{6} + 4\sqrt{3})$

84. True or false? If $a > 0$, then $(\sqrt{a} - 1)(\sqrt{a} + 1) > a$.85. True or false? If $a > 0$, then $(\sqrt{a} + 1)^2 > a + 1$.**OBJECTIVE D****To divide radical expressions**

86. When is a radical expression in simplest form?



87. Explain what it means to rationalize the denominator of a radical expression and how to do so.



For Exercises 88 to 91, by what expression should the numerator and denominator be multiplied in order to rationalize the denominator?

88. $\frac{1}{\sqrt{6}}$

89. $\frac{7}{\sqrt[3]{2x^5}}$

90. $\frac{8x}{\sqrt[4]{27x}}$

91. $\frac{4}{\sqrt{3} - x}$

For Exercises 92 to 128, simplify.

92. $\frac{\sqrt{32x^2}}{\sqrt{2x}}$

93. $\frac{\sqrt{60y^4}}{\sqrt{12y}}$

94. $\frac{\sqrt{42a^3b^5}}{\sqrt{14a^2b}}$

95. $\frac{\sqrt{65ab^4}}{\sqrt{5ab}}$

96. $\frac{1}{\sqrt{5}}$

97. $\frac{1}{\sqrt{2}}$

98. $\frac{1}{\sqrt{2x}}$

99. $\frac{2}{\sqrt{3y}}$

100. $\frac{5}{\sqrt{5x}}$

101. $\frac{9}{\sqrt{3a}}$

102. $\sqrt{\frac{x}{5}}$

103. $\sqrt{\frac{y}{2}}$

104. $\frac{3}{\sqrt[3]{2}}$

105. $\frac{5}{\sqrt[3]{9}}$

106. $\frac{3}{\sqrt[3]{4x^2}}$

107. $\frac{5}{\sqrt[3]{3y}}$

108. $\frac{\sqrt{40x^3y^2}}{\sqrt{80x^2y^3}}$

109. $\frac{\sqrt{15a^2b^5}}{\sqrt{30a^5b^3}}$

110. $\frac{\sqrt{24a^2b}}{\sqrt{18ab^4}}$

111. $\frac{\sqrt{12x^3y}}{\sqrt{20x^4y}}$

112. $\frac{5}{\sqrt{3}-2}$

113. $\frac{-2}{1-\sqrt{2}}$

114. $\frac{-3}{2-\sqrt{3}}$

115. $\frac{-4}{3-\sqrt{2}}$

116. $\frac{2}{\sqrt{5}+2}$

117. $\frac{5}{2-\sqrt{7}}$

118. $\frac{3}{\sqrt{y}-2}$

119. $\frac{-7}{\sqrt{x}-3}$

120. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

121. $\frac{\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}}$

122. $\frac{2+3\sqrt{7}}{5-2\sqrt{7}}$

123. $\frac{2+3\sqrt{5}}{1-\sqrt{5}}$

124. $\frac{2\sqrt{3}-1}{3\sqrt{3}+2}$

125. $\frac{2\sqrt{a}-\sqrt{b}}{4\sqrt{a}+3\sqrt{b}}$

126. $\frac{2\sqrt{x}-4}{\sqrt{x}+2}$

127. $\frac{3\sqrt{y}-y}{\sqrt{y}+2y}$

128. $\frac{3\sqrt{x}-4\sqrt{y}}{3\sqrt{x}-2\sqrt{y}}$

Applying the Concepts

129. Determine whether the following statements are true or false. If the statement is false, correct the right-hand side of the equation.

a. $\sqrt[2]{3} \cdot \sqrt[3]{4} = \sqrt[5]{12}$

b. $\sqrt{3} \cdot \sqrt{3} = 3$

c. $\sqrt[3]{x} \cdot \sqrt[3]{x} = x$

d. $\sqrt{x} + \sqrt{y} = \sqrt{x+y}$

e. $\sqrt[2]{2} + \sqrt[3]{3} = \sqrt[5]{2+3}$

f. $8\sqrt[5]{a} - 2\sqrt[5]{a} = 6\sqrt[5]{a}$

130. Multiply: $(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2})$

131. Rewrite $\frac{\sqrt[4]{(a+b)^3}}{\sqrt{a+b}}$ as an expression with a single radical.

SECTION

7.3

Solving Equations Containing Radical Expressions

OBJECTIVE A

To solve a radical equation

An equation that contains a variable expression in a radicand is called a **radical equation**.

$$\left. \begin{array}{l} \sqrt[3]{2x-5} + x = 7 \\ \sqrt{x+1} - \sqrt{x} = 4 \end{array} \right\} \begin{array}{l} \text{Radical} \\ \text{equations} \end{array}$$

The following property is used to solve a radical equation.

The Property of Raising Each Side of an Equation to a Power

If two numbers are equal, then the same powers of the numbers are equal.

$$\text{If } a = b, \text{ then } a^n = b^n.$$

HOW TO • 1

Solve: $\sqrt{x-2} - 6 = 0$

$$\sqrt{x-2} - 6 = 0$$

$$\sqrt{x-2} = 6$$

• Isolate the radical by adding 6 to each side of the equation.

$$(\sqrt{x-2})^2 = 6^2$$

• Square each side of the equation.

$$x - 2 = 36$$

• Simplify and solve for x .

$$x = 38$$

Check: $\sqrt{x-2} - 6 = 0$

$$\sqrt{38-2} - 6 \quad | \quad 0$$

$$\sqrt{36} - 6 \quad | \quad 0$$

$$6 - 6 \quad | \quad 0$$

$$0 = 0$$

38 checks as a solution. The solution is 38.

HOW TO • 2

Solve: $\sqrt[3]{x+2} = -3$

$$\sqrt[3]{x+2} = -3$$

$$(\sqrt[3]{x+2})^3 = (-3)^3$$

• Cube each side of the equation.

$$x + 2 = -27$$

• Solve the resulting equation.

$$x = -29$$

Check: $\sqrt[3]{x+2} = -3$

$$\sqrt[3]{-29+2} \quad | \quad -3$$

$$\sqrt[3]{-27} \quad | \quad -3$$

$$-3 = -3$$

-29 checks as a solution. The solution is -29.

Raising each side of an equation to an *even* power may result in an equation that has extraneous solutions. (See Objective 6.5A.) Therefore, you *must* check proposed solutions of an equation if one of the steps in solving the equation was to raise each side to an even power.

Take Note

Note that

$$\begin{aligned}(2 - \sqrt{x})^2 &= (2 - \sqrt{x})(2 - \sqrt{x}) \\ &= 4 - 4\sqrt{x} + x\end{aligned}$$

Take Note

The proposed solutions of the equation were 1 and 25. However, 25 did not check as a solution. Therefore, it is an extraneous solution.

HOW TO • 3

Solve: $\sqrt{2x-1} + \sqrt{x} = 2$

$$\sqrt{2x-1} + \sqrt{x} = 2$$

$$\sqrt{2x-1} = 2 - \sqrt{x}$$

$$(\sqrt{2x-1})^2 = (2 - \sqrt{x})^2$$

$$2x - 1 = 4 - 4\sqrt{x} + x$$

$$x - 5 = -4\sqrt{x}$$

$$(x - 5)^2 = (-4\sqrt{x})^2$$

$$x^2 - 10x + 25 = 16x$$

$$x^2 - 26x + 25 = 0$$

$$(x - 25)(x - 1) = 0$$

$$x = 25 \text{ or } x = 1$$

Check:

$$\begin{array}{r|l}\sqrt{2x-1} + \sqrt{x} &= 2 \\ \hline \sqrt{2(25)-1} + \sqrt{25} & \left| \begin{array}{l} 2 \\ 7+5 \\ 12 \end{array} \right. \\ \hline & 12 \neq 2\end{array}$$

$$\begin{array}{r|l}\sqrt{2x-1} + \sqrt{x} &= 2 \\ \hline \sqrt{2(1)-1} + \sqrt{1} & \left| \begin{array}{l} 2 \\ 1+1 \\ 2 \end{array} \right. \\ \hline & 2 = 2\end{array}$$

• Solve for one of the radical expressions.

• Square each side. Recall that $(a - b)^2 = a^2 - 2ab + b^2$.

• Square each side again.

• Solve the quadratic equation by factoring.

25 does not check as a solution. 1 checks as a solution. **The solution is 1.** Here 25 is an extraneous solution.

EXAMPLE • 1

Solve: $\sqrt{x-1} + \sqrt{x+4} = 5$

Solution

$$\sqrt{x-1} + \sqrt{x+4} = 5$$

$$\sqrt{x+4} = 5 - \sqrt{x-1}$$

$$(\sqrt{x+4})^2 = (5 - \sqrt{x-1})^2$$

$$x + 4 = 25 - 10\sqrt{x-1} + x - 1$$

$$-20 = -10\sqrt{x-1}$$

$$2 = \sqrt{x-1}$$

$$2^2 = (\sqrt{x-1})^2$$

$$4 = x - 1$$

$$5 = x$$

The solution checks. **The solution is 5.**

YOU TRY IT • 1

Solve: $\sqrt{x} - \sqrt{x+5} = 1$

Your solution

EXAMPLE • 2

Solve: $\sqrt[3]{3x-1} = -4$

Solution

$$\sqrt[3]{3x-1} = -4$$

$$(\sqrt[3]{3x-1})^3 = (-4)^3$$

$$3x - 1 = -64$$

$$3x = -63$$

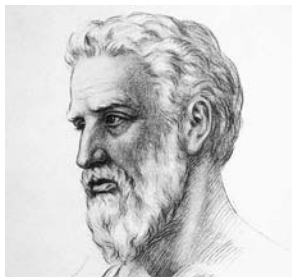
$$x = -21$$

The solution checks. **The solution is -21.**

YOU TRY IT • 2

Solve: $\sqrt[4]{x-8} = 3$

Your solution

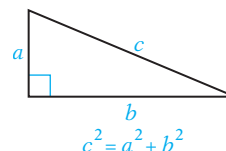
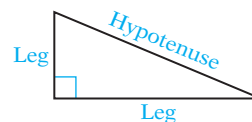
OBJECTIVE B**To solve application problems**

Pythagoras

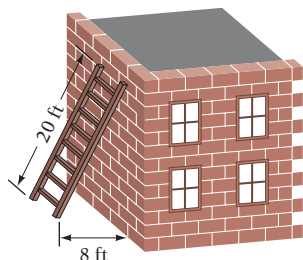
© Bettmann/Corbis

A right triangle contains one 90° angle. The side opposite the 90° angle is called the hypotenuse. The other two sides are called legs.

Pythagoras, a Greek mathematician, discovered that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. Recall that this is called the Pythagorean Theorem.

**EXAMPLE 3**

A ladder 20 ft long is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 8 ft from the building? Round to the nearest tenth.

**Strategy**

To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$$c^2 = a^2 + b^2$$

$$20^2 = 8^2 + b^2$$

$$400 = 64 + b^2$$

$$336 = b^2$$

$$\sqrt{336} = \sqrt{b^2}$$

$$\sqrt{336} = b$$

$$18.3 \approx b$$

The distance is approximately 18.3 ft.

- Pythagorean Theorem
- Replace c by 20 and a by 8.
- Solve for b .

- Take the square root of each side.

YOU TRY IT 3

Find the diagonal of a rectangle that is 6 cm long and 3 cm wide. Round to the nearest tenth.

Your strategy**Your solution**

EXAMPLE • 4

An object is dropped from a high building. Find the distance the object has fallen when its speed reaches 96 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object in feet per second and d is the distance in feet.

Strategy

To find the distance the object has fallen, replace v in the equation with the given value and solve for d .

Solution

$$\begin{aligned} v &= \sqrt{64d} \\ 96 &= \sqrt{64d} && \bullet \text{ Replace } v \text{ by } 96. \\ (96)^2 &= (\sqrt{64d})^2 && \bullet \text{ Square each side.} \\ 9216 &= 64d \\ 144 &= d \end{aligned}$$

The object has fallen 144 ft.

YOU TRY IT • 4

How far would a submarine periscope have to be above the water to locate a ship 5.5 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

Your strategy**Your solution****EXAMPLE • 5**

Find the length of a pendulum that makes one swing in 1.5 s. The equation for the time of one swing is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π . Round to the nearest hundredth.

Strategy

To find the length of the pendulum, replace T in the equation with the given value and solve for L .

Solution

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{32}} \\ 1.5 &= 2(3.14)\sqrt{\frac{L}{32}} && \bullet \text{ Replace } T \text{ by } 1.5 \text{ and } \pi \text{ by } 3.14. \\ \frac{1.5}{2(3.14)} &= \sqrt{\frac{L}{32}} && \bullet \text{ Divide each side by } 2(3.14). \\ \left[\frac{1.5}{2(3.14)}\right]^2 &= \left(\sqrt{\frac{L}{32}}\right)^2 && \bullet \text{ Square each side.} \\ \left(\frac{1.5}{6.28}\right)^2 &= \frac{L}{32} && \bullet \text{ Solve for } L. \\ 1.83 &\approx L && \text{Multiply each side by } 32. \end{aligned}$$

The length of the pendulum is 1.83 ft.

YOU TRY IT • 5

Find the distance required for a car to reach a velocity of 88 ft/s when the acceleration is 22 ft/s². Use the equation $v = \sqrt{2as}$, where v is the velocity in feet per second, a is the acceleration, and s is the distance in feet.

Your strategy**Your solution**

7.3 EXERCISES

OBJECTIVE A

To solve a radical equation

For Exercises 1 to 21, solve for x .

1. $\sqrt[3]{4x} = -2$

2. $\sqrt[3]{6x} = -3$

3. $\sqrt{3x - 2} = 5$

4. $\sqrt{3x + 9} - 12 = 0$

5. $\sqrt{4x - 3} + 9 = 4$

6. $\sqrt{2x - 5} + 4 = 1$

7. $\sqrt[3]{2x - 6} = 4$

8. $\sqrt[3]{x - 2} = 3$

9. $\sqrt[4]{3x} + 2 = 5$

10. $\sqrt[4]{4x + 1} = 2$

11. $\sqrt[3]{2x - 3} + 5 = 2$

12. $\sqrt[3]{x - 4} + 7 = 5$

13. $\sqrt{x} + 2 = x - 4$

14. $\sqrt{x + 1} + 5 = x + 4$

15. $\sqrt{x} + \sqrt{x - 5} = 5$

16. $\sqrt{x + 3} + \sqrt{x - 1} = 2$

17. $\sqrt{2x + 5} - \sqrt{2x} = 1$

18. $\sqrt{3x} - \sqrt{3x - 5} = 1$

19. $\sqrt{2x} - \sqrt{x - 1} = 1$

20. $\sqrt{2x - 5} + \sqrt{x + 1} = 3$

21. $\sqrt{2x + 2} + \sqrt{x} = 3$



22. The equation
- $\sqrt{x} = -4$
- has no solution. Why?

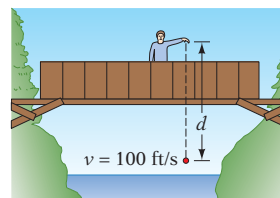


23. Without attempting to solve the equation, explain why
- $\sqrt{x} - \sqrt{x + 5} = 1$
- has no solution.
- Hint:*
- See Exercise 22.

OBJECTIVE B

To solve application problems

- 24.
- Physics**
- An object is dropped from a bridge. Find the distance the object has fallen when its speed reaches 100 ft/s. Use the equation
- $v = \sqrt{64d}$
- , where
- v
- is the speed in feet per second and
- d
- is the distance in feet.



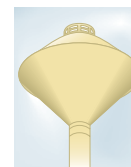
- 25.
- Physics**
- The time it takes for an object to fall a certain distance is given by the equation
- $t = \sqrt{\frac{2d}{g}}$
- , where
- t
- is the time in seconds,
- d
- is the distance in feet, and
- g
- is the acceleration due to gravity. If an astronaut above the moon's surface drops an object, how far will it have fallen in 3 s? The acceleration on the moon's surface is 5.5 feet per second per second.



© iStockphoto.com/technotr

- 26.
- Sailing**
- The total recommended area
- A
- , in square feet, of the sails for a certain sailboat is given by
- $A = 16\sqrt[3]{d^2}$
- , where
- d
- is the displacement of the hull in cubic feet. If a sailboat has a total of 400 ft
- ²
- of sail, what is the displacement of the hull of the sailboat?

27. **Water Tanks** A 6-foot-high conical water tank is filled to the top. When a valve at the bottom of the tank is opened, the height h , in feet, of the water in the tank is given by $h = (88.18 - 3.18t)^{2/5}$, where t is the time in seconds after the valve is opened.
- Find the height of the water 10 s after the valve is opened. Round to the nearest tenth.
 - How long will it take to empty the tank? Round to the nearest tenth.



28. **Water Tanks** The velocity v , in feet per second, of the water pouring out of a small hole in the bottom of a cylindrical tank is given by $v = \sqrt{64h + 10}$, where h is the height, in feet, of the water in the tank. What is the height of the water in the tank when the velocity of the water leaving the tank is 14 ft/s? Round to the nearest tenth.

29. **Pendulums** Find the length of a pendulum that makes one swing in 3 s. The equation for the time of one swing of a pendulum is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.



30. **Satellites** Read the article at the right. At what height above Earth's surface is the A-Train in orbit? Use the equation $v = \sqrt{\frac{4 \times 10^{14}}{h + 6.4 \times 10^6}}$, where v is the speed of the satellite in meters per second and h is the height, in meters, above Earth's surface. Round to the nearest ten thousand.

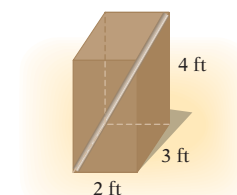
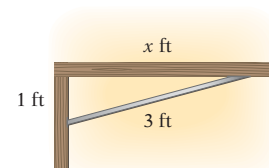
In the News

NASA to Add OCO to A-Train

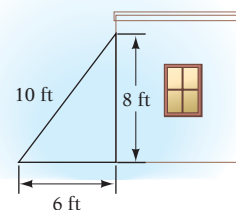
The 2009 launch of the Orbiting Carbon Observatory adds a sixth satellite to the "A-Train" of satellites that orbit Earth at about 7500 m/s, providing scientists with a wealth of data that can be used to study climate change.

Source: Jet Propulsion Laboratory

31. **Television** High definition television (HDTV) gives consumers a wider viewing area, more like a film screen in a theater. A regular television with a 27-inch diagonal measurement has a screen 16.2 in. tall. An HDTV screen with the same 16.2-inch height would have a diagonal measuring 33 in. How many inches wider is the HDTV screen? Round to the nearest hundredth.
32. **Construction** A carpenter is inserting a 3-foot brace between two beams as shown in the figure. How far from the vertical beam will the brace reach? Round to the nearest tenth.
33. **Moving Boxes** A moving box has a base that measures 2 ft by 3 ft, and the box is 4 ft tall. Find the length of the longest pole that could be placed in the box. Round to the nearest tenth.



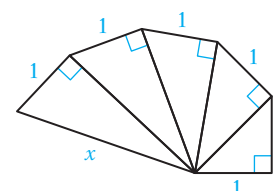
34. A 10-foot ladder is resting against a wall, with the bottom of the ladder 6 ft from the wall. The top of the ladder begins sliding down the wall at a constant rate of 2 ft/s. Is the bottom of the ladder sliding away from the wall at the same rate?



Applying the Concepts

35. Solve: $\sqrt{3x - 2} = \sqrt{2x - 3} + \sqrt{x - 1}$

36. **Geometry** Find the length of the side labeled x .



SECTION

7.4

Complex Numbers

OBJECTIVE A

To simplify a complex number

The radical expression $\sqrt{-4}$ is not a real number because there is no real number whose square is -4 . However, the solution of an algebraic equation is sometimes the square root of a negative number.

For example, the equation $x^2 + 1 = 0$ does not have a real number solution because there is no real number whose square is -1 .

$$\begin{array}{rcl} x^2 + 1 & = & 0 \\ x^2 & = & -1 \end{array}$$

Around the 17th century, a new number, called an **imaginary number**, was defined so that a negative number would have a square root. The letter i was chosen to represent the number whose square is -1 .

$$i^2 = -1$$

An imaginary number is defined in terms of i .



Point of Interest

The first written occurrence of an imaginary number was in a book published in 1545 by Hieronimo Cardan, where he wrote (in our modern notation) $5 + \sqrt{-15}$. He went on to say that the number “is as refined as it is useless.” It was not until the 20th century that applications of complex numbers were found.

Definition of $\sqrt{-a}$

If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$.

$$\sqrt{-a} = i\sqrt{a}$$

Here are some examples.

$$\begin{aligned} \sqrt{-16} &= i\sqrt{16} = 4i \\ \sqrt{-12} &= i\sqrt{12} = 2i\sqrt{3} \\ \sqrt{-21} &= i\sqrt{21} \\ \sqrt{-1} &= i\sqrt{1} = i \end{aligned}$$

It is customary to write i in front of a radical to avoid confusing $\sqrt{a}i$ with $\sqrt{ai}$.

The real numbers and imaginary numbers make up the complex numbers.

Complex Number

A **complex number** is a number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The number a is the **real part** of $a + bi$, and the number b is the **imaginary part**.

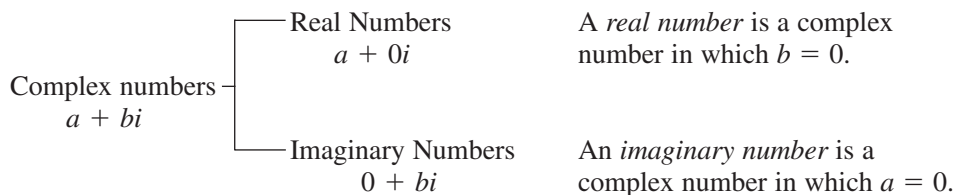


Take Note

The *imaginary part* of a complex number is a real number. As another example, the imaginary part of $6 - 8i$ is -8 .

Examples of complex numbers are shown at the right.

Real Part	Imaginary Part
$a + bi$	
$3 + 2i$	
$8 - 10i$	

**Tips for Success**

Be sure you understand how to simplify expressions such as those in Example 1 and You Try It 1, as it is a prerequisite for solving quadratic equations in Chapter 8.

HOW TO • 1Simplify: $7 - \sqrt{-50}$

$$\begin{aligned}
 7 - \sqrt{-50} &= 7 - i\sqrt{50} && \bullet \text{ Use the definition of } \sqrt{-a} \text{ to write } \sqrt{-50} = i\sqrt{50}. \\
 &= 7 - i\sqrt{25 \cdot 2} && \bullet \text{ Simplify the radical expression.} \\
 &= 7 - 5i\sqrt{2}
 \end{aligned}$$

EXAMPLE • 1Simplify: $\sqrt{-80}$ **Solution**

$$\sqrt{-80} = i\sqrt{80} = i\sqrt{16 \cdot 5} = 4i\sqrt{5}$$

YOU TRY IT • 1Simplify: $\sqrt{-45}$ **Your solution****EXAMPLE • 2**

Evaluate $-b + \sqrt{b^2 - 4ac}$ when $a = 2$, $b = -2$, and $c = 3$. Write the result as a complex number.

Solution

$$\begin{aligned}
 &-b + \sqrt{b^2 - 4ac} \\
 &-(-2) + \sqrt{(-2)^2 - 4(2)(3)} = 2 + \sqrt{4 - 24} \\
 &= 2 + \sqrt{-20} = 2 + i\sqrt{20} \\
 &= 2 + i\sqrt{4 \cdot 5} = 2 + 2i\sqrt{5}
 \end{aligned}$$

YOU TRY IT • 2

Evaluate $-b + \sqrt{b^2 - 4ac}$ when $a = 1$, $b = 6$, and $c = 25$. Write the result as a complex number.

Your solution

Solutions on p. S23

OBJECTIVE B**To add or subtract complex numbers****Integrating Technology**

See the Keystroke Guide: *Complex Numbers* for instructions on using a graphing calculator to perform operations on complex numbers.

Addition and Subtraction of Complex Numbers

To add two complex numbers, add the real parts and add the imaginary parts. To subtract two complex numbers, subtract the real parts and subtract the imaginary parts.

$$\begin{aligned}
 (a + bi) + (c + di) &= (a + c) + (b + d)i \\
 (a + bi) - (c + di) &= (a - c) + (b - d)i
 \end{aligned}$$

HOW TO • 2Add: $(3 - 5i) + (2 + 3i)$

$$(3 - 5i) + (2 + 3i) = (3 + 2) + (-5 + 3)i = 5 - 2i$$

- Add the real parts and add the imaginary parts of the complex number.

HOW TO • 3Subtract: $(5 + 6i) - (7 - 3i)$

$$(5 + 6i) - (7 - 3i) = (5 - 7) + [6 - (-3)]i = -2 + 9i$$

- Subtract the real parts and subtract the imaginary parts of the complex number.

The additive inverse of the complex number $a + bi$ is $-a - bi$. The sum of these two numbers is 0.

$$(a + bi) + (-a - bi) = (a - a) + (b - b)i = 0 + 0i = 0$$

EXAMPLE • 3Simplify: $(3 + 2i) + (6 - 5i)$ **Solution**

$$(3 + 2i) + (6 - 5i) = 9 - 3i$$

YOU TRY IT • 3Simplify: $(-4 + 2i) - (6 - 8i)$ **Your solution****EXAMPLE • 4**Simplify: $(9 - \sqrt{-8}) - (5 + \sqrt{-32})$ **Solution**

$$\begin{aligned} (9 - \sqrt{-8}) - (5 + \sqrt{-32}) &= (9 - i\sqrt{8}) - (5 + i\sqrt{32}) \\ &= (9 - i\sqrt{4 \cdot 2}) - (5 + i\sqrt{16 \cdot 2}) \\ &= (9 - 2i\sqrt{2}) - (5 + 4i\sqrt{2}) \\ &= 4 - 6i\sqrt{2} \end{aligned}$$

YOU TRY IT • 4Simplify: $(16 - \sqrt{-45}) - (3 + \sqrt{-20})$ **Your solution**

Solutions on p. S23

OBJECTIVE C**To multiply complex numbers**

When multiplying complex numbers, we often find that the term i^2 is a part of the product. Recall that $i^2 = -1$.

HOW TO • 4Simplify: $2i \cdot 3i$

$$\begin{aligned} 2i \cdot 3i &= 6i^2 \\ &= 6(-1) \\ &= -6 \end{aligned}$$

- Multiply the imaginary numbers.
- Replace i^2 by -1 .
- Simplify.

**Take Note**

This example illustrates an important point. When working with an expression that contains a square root of a negative number, always rewrite the number as the product of a real number and i before continuing.

HOW TO • 5Simplify: $\sqrt{-6} \cdot \sqrt{-24}$

$$\begin{aligned}\sqrt{-6} \cdot \sqrt{-24} &= i\sqrt{6} \cdot i\sqrt{24} \\ &= i^2 \sqrt{144} \\ &= -\sqrt{144} \\ &= -12\end{aligned}$$

- Write each radical as the product of a real number and i .
- Multiply the imaginary numbers.
- Replace i^2 by -1 .
- Simplify the radical expression.

Note from the last example that it would have been incorrect to multiply the radicands of the two radical expressions. To illustrate,

$$\sqrt{-6} \cdot \sqrt{-24} = \sqrt{(-6)(-24)} = \sqrt{144} = 12, \text{ not } -12$$

The Product Property of Radicals does *not* hold true when both radicands are negative and the index is an even number.

HOW TO • 6Simplify: $4i(3 - 2i)$

$$\begin{aligned}4i(3 - 2i) &= 12i - 8i^2 \\ &= 12i - 8(-1) \\ &= 8 + 12i\end{aligned}$$

- Use the Distributive Property to remove parentheses.
- Replace i^2 by -1 .
- Write the answer in the form $a + bi$.

The product of two complex numbers is defined as follows.

Multiplication of Complex Numbers

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

One way to remember this rule is to think of the FOIL method.

HOW TO • 7Simplify: $(2 + 4i)(3 - 5i)$

$$\begin{aligned}(2 + 4i)(3 - 5i) &= 6 - 10i + 12i - 20i^2 \\ &= 6 + 2i - 20i^2 \\ &= 6 + 2i - 20(-1) \\ &= 26 + 2i\end{aligned}$$

- Use the FOIL method to find the product.
- Replace i^2 by -1 .
- Write the answer in the form $a + bi$.

The conjugate of $a + bi$ is $a - bi$.

The product of complex conjugates, $(a + bi)(a - bi)$, is the real number $a^2 + b^2$.

$$\begin{aligned}(a + bi)(a - bi) &= a^2 - b^2i^2 \\ &= a^2 - b^2(-1) \\ &= a^2 + b^2\end{aligned}$$

HOW TO • 8Simplify: $(2 + 3i)(2 - 3i)$

$$\begin{aligned}(2 + 3i)(2 - 3i) &= 2^2 + 3^2 \\ &= 4 + 9 \\ &= 13\end{aligned}$$

- $a = 2, b = 3$
The product of the conjugates is $2^2 + 3^2$.

Note that the product of a complex number and its conjugate is a real number.

EXAMPLE • 5Simplify: $(2i)(-5i)$ **Solution**

$$(2i)(-5i) = -10i^2 = (-10)(-1) = 10$$

YOU TRY IT • 5Simplify: $(-3i)(-10i)$ **Your solution****EXAMPLE • 6**Simplify: $\sqrt{-10} \cdot \sqrt{-5}$ **Solution**

$$\begin{aligned}\sqrt{-10} \cdot \sqrt{-5} &= i\sqrt{10} \cdot i\sqrt{5} \\ &= i^2\sqrt{50} = -\sqrt{25 \cdot 2} = -5\sqrt{2}\end{aligned}$$

YOU TRY IT • 6Simplify: $-\sqrt{-8} \cdot \sqrt{-5}$ **Your solution****EXAMPLE • 7**Simplify: $3i(2 - 4i)$ **Solution**

$$\begin{aligned}3i(2 - 4i) &= 6i - 12i^2 && \bullet \text{ The Distributive Property} \\ &= 6i - 12(-1) \\ &= 12 + 6i\end{aligned}$$

YOU TRY IT • 7Simplify: $-6i(3 + 4i)$ **Your solution****EXAMPLE • 8**Simplify: $(3 - 4i)(2 + 5i)$ **Solution**

$$\begin{aligned}(3 - 4i)(2 + 5i) &= 6 + 15i - 8i - 20i^2 && \bullet \text{ FOIL} \\ &= 6 + 7i - 20i^2 \\ &= 6 + 7i - 20(-1) = 26 + 7i\end{aligned}$$

YOU TRY IT • 8Simplify: $(4 - 3i)(2 - i)$ **Your solution****EXAMPLE • 9**Expand: $(3 + 4i)^2$ **Solution**

$$\begin{aligned}(3 + 4i)^2 &= (3 + 4i)(3 + 4i) \\ &= 9 + 12i + 12i + 16i^2 \\ &= 9 + 24i + 16(-1) \\ &= 9 + 24i - 16 \\ &= -7 + 24i\end{aligned}$$

YOU TRY IT • 9Expand: $(5 - 3i)^2$ **Your solution****EXAMPLE • 10**Simplify: $(4 + 5i)(4 - 5i)$ **Solution**

$$\begin{aligned}(4 + 5i)(4 - 5i) &= 4^2 + 5^2 && \bullet \text{ Conjugates} \\ &= 16 + 25 = 41\end{aligned}$$

YOU TRY IT • 10Simplify: $(3 + 6i)(3 - 6i)$ **Your solution**

OBJECTIVE D**To divide complex numbers**

A rational expression containing one or more complex numbers is in simplest form when no imaginary number remains in the denominator.

HOW TO • 9Simplify: $\frac{2 - 3i}{2i}$

$$\begin{aligned}\frac{2 - 3i}{2i} &= \frac{2 - 3i}{2i} \cdot \frac{i}{i} \\ &= \frac{2i - 3i^2}{2i^2} \\ &= \frac{2i - 3(-1)}{2(-1)} \\ &= \frac{3 + 2i}{-2} = -\frac{3}{2} - i\end{aligned}$$

- Multiply the numerator and denominator by $\frac{i}{i}$.

- Replace i^2 by -1 .

- Simplify. Write the answer in the form $a + bi$.

HOW TO • 10Simplify: $\frac{3 + 2i}{1 + i}$

$$\begin{aligned}\frac{3 + 2i}{1 + i} &= \frac{3 + 2i}{1 + i} \cdot \frac{1 - i}{1 - i} \\ &= \frac{3 - 3i + 2i - 2i^2}{1^2 + 1^2} \\ &= \frac{3 - i - 2(-1)}{1 + 1} \\ &= \frac{5 - i}{2} = \frac{5}{2} - \frac{1}{2}i\end{aligned}$$

- Multiply the numerator and denominator by the conjugate of $1 + i$.

- Replace i^2 by -1 and simplify.

- Write the answer in the form $a + bi$.

EXAMPLE • 11Simplify: $\frac{5 + 4i}{3i}$ **Solution**

$$\begin{aligned}\frac{5 + 4i}{3i} &= \frac{5 + 4i}{3i} \cdot \frac{i}{i} = \frac{5i + 4i^2}{3i^2} \\ &= \frac{5i + 4(-1)}{3(-1)} = \frac{-4 + 5i}{-3} = \frac{4}{3} - \frac{5}{3}i\end{aligned}$$

YOU TRY IT • 11Simplify: $\frac{2 - 3i}{4i}$ **Your solution****EXAMPLE • 12**Simplify: $\frac{5 - 3i}{4 + 2i}$ **Solution**

$$\begin{aligned}\frac{5 - 3i}{4 + 2i} &= \frac{5 - 3i}{4 + 2i} \cdot \frac{4 - 2i}{4 - 2i} \\ &= \frac{20 - 10i - 12i + 6i^2}{4^2 + 2^2} \\ &= \frac{20 - 22i + 6(-1)}{16 + 4} \\ &= \frac{14 - 22i}{20} = \frac{7 - 11i}{10} = \frac{7}{10} - \frac{11}{10}i\end{aligned}$$

YOU TRY IT • 12Simplify: $\frac{2 + 5i}{3 - 2i}$ **Your solution**

7.4 EXERCISES

OBJECTIVE A

To simplify a complex number



1. What is an imaginary number? What is a complex number?



2. Are all real numbers also complex numbers? Are all complex numbers also real numbers?

3. Fill in the blank with $\sqrt{a}$ or $i\sqrt{a}$.
If $a > 0$, then $\sqrt{-a} = \underline{\hspace{1cm}}$.4. Does $\sqrt{a^2} = (\sqrt{a})^2$ for all values of a ?

For Exercises 5 to 12, simplify.

5. $\sqrt{-25}$

6. $\sqrt{-64}$

7. $\sqrt{-98}$

8. $\sqrt{-72}$

9. $3 + \sqrt{-45}$

10. $7 - \sqrt{-63}$

11. $-6 - \sqrt{-100}$

12. $4 + \sqrt{-49}$

For Exercises 13 to 24, evaluate $-b + \sqrt{b^2 - 4ac}$ for the given values of a , b , and c .
Write the result as a complex number.

13. $a = 1, b = 4, c = 5$

14. $a = 1, b = -6, c = 13$

15. $a = 2, b = -4, c = 10$

16. $a = 4, b = -12, c = 45$

17. $a = 3, b = -8, c = 6$

18. $a = 3, b = -2, c = 9$

19. $a = 4, b = 2, c = 7$

20. $a = 4, b = 5, c = 10$

21. $a = -2, b = 5, c = -6$

22. $a = -1, b = 4, c = -29$

23. $a = -3, b = 4, c = -6$

24. $a = -5, b = 1, c = -5$

OBJECTIVE B

To add or subtract complex numbers

For Exercises 25 to 34, simplify.

25. $(2 + 4i) + (6 - 5i)$

26. $(6 - 9i) + (4 + 2i)$

27. $(-2 - 4i) - (6 - 8i)$

28. $(3 - 5i) + (8 - 2i)$

29. $(8 - 2i) - (2 + 4i)$

30. $(5 - 5i) - (11 - 6i)$

31. $5 + (6 - 4i)$

32. $-7 + (3 + 5i)$

33. $3i - (6 + 5i)$

34. $(7 + 3i) - 8i$



35. If the sum of two complex numbers is an imaginary number, what must be true of the complex numbers?



36. If the sum of two complex numbers is a real number, what must be true of the complex numbers?

OBJECTIVE C**To multiply complex numbers**

For Exercises 37 to 54, simplify.

37. $(7i)(-9i)$

38. $(-6i)(-4i)$

39. $\sqrt{-2}\sqrt{-8}$

40. $\sqrt{-5}\sqrt{-45}$

41. $(5 + 2i)(5 - 2i)$

42. $(3 + 8i)(3 - 8i)$

43. $2i(6 + 2i)$

44. $-3i(4 - 5i)$

45. $-i(4 - 3i)$

46. $i(6 + 2i)$

47. $(5 - 2i)(3 + i)$

48. $(2 - 4i)(2 - i)$

49. $(6 + 5i)(3 + 2i)$

50. $(4 - 7i)(2 + 3i)$

51. $(2 + 5i)^2$

52. $(3 - 4i)^2$

53. $\left(\frac{6}{5} + \frac{3}{5}i\right)\left(\frac{2}{3} - \frac{1}{3}i\right)$

54. $(2 - i)\left(\frac{2}{5} + \frac{1}{5}i\right)$



55. True or false? For all real numbers
- a
- and
- b
- , the product
- $(a + bi)(a - bi)$
- is a positive real number.



56. Given that
- $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = i$
- (try to verify this statement), what is a square root of
- i
- ?

OBJECTIVE D**To divide complex numbers**

For Exercises 57 to 71, simplify.

57. $\frac{3}{i}$

58. $\frac{4}{5i}$

59. $\frac{2 - 3i}{-4i}$

60. $\frac{16 + 5i}{-3i}$

61. $\frac{4}{5 + i}$

62. $\frac{6}{5 + 2i}$

63. $\frac{2}{2 - i}$

64. $\frac{5}{4 - i}$

65. $\frac{1 - 3i}{3 + i}$

66. $\frac{2 + 12i}{5 + i}$

67. $\frac{3i}{1 + 4i}$

68. $\frac{-2i}{2 - 3i}$

69. $\frac{2 - 3i}{3 + i}$

70. $\frac{3 + 5i}{1 - i}$

71. $\frac{5 + 3i}{3 - i}$



72. True or false? The quotient of two imaginary numbers is an imaginary number.



73. True or false? The reciprocal of an imaginary number is an imaginary number.

Applying the Concepts

74. Evaluate i^n for $n = 0, 1, 2, 3, 4, 5, 6$, and 7 . Make a conjecture about the value of i^n for any natural number. Using your conjecture, evaluate i^{76} .
75. a. Is $3i$ a solution of $2x^2 + 18 = 0$?
b. Is $3 + i$ a solution of $x^2 - 6x + 10 = 0$?

FOCUS ON PROBLEM SOLVING

Another Look at Polya's Four-Step Process

Polya's four general steps to follow when attempting to solve a problem are to understand the problem, devise a plan, carry out the plan, and review the solution. (See the Focus on Problem Solving in the chapter entitled "Review of Real Numbers.") In the process of devising a plan (Step 2), it may be appropriate to write a mathematical expression or an equation. We will illustrate this with the following problem.

Number the letters of the alphabet in sequence from 1 to 26. (See the list at the left.) Find a word for which the product of the numerical values of the letters of the word equals 1,000,000. We will agree that a "word" is any sequence of letters that contains at least one vowel but is not necessarily in the dictionary.

A =	1
B =	2
C =	3
D =	4
E =	5
F =	6
G =	7
H =	8
I =	9
J =	10
K =	11
L =	12
M =	13
N =	14
O =	15
P =	16
Q =	17
R =	18
S =	19
T =	20
U =	21
V =	22
W =	23
X =	24
Y =	25
Z =	26

Understand the Problem

Consider REZB. The product of the values of the letters is $18 \cdot 5 \cdot 26 \cdot 2 = 4680$. This "word" is a sequence of letters with at least one vowel. However, the product of the numerical values of the letters is not 1,000,000. Thus this word does not solve our problem.

Devise a Plan

Actually, we should have known that the product of the values of the letters in REZB could not equal 1,000,000. The letter R has a factor of 9, and the letter Z has a factor of 13. Neither of these two numbers is a factor of 1,000,000. Consequently, R and Z cannot be letters in the word we are trying to find. This observation leads to an important consideration: The value of each of the letters that make up our word must be a factor of 1,000,000. To find these letters, consider the prime factorization of 1,000,000.

$$1,000,000 = 2^6 \cdot 5^6$$

Looking at the prime factorization, we note that only letters whose numerical values contain 2 or 5 as factors are possible candidates. These letters are B, D, E, H, J, P, T, and Y. One additional point: Because 1 times any number is the number, the letter A can be part of any word we construct.

Our task is now to construct a word from these letters such that the product of the numerical values is 1,000,000. From the prime factorization above, we must have 2 as a factor six times and 5 as a factor six times.

Carry Out the Plan

We must construct a word with the characteristics described in our plan. Here is a possibility:

THEBEYE

Review the Solution

You should multiply the values of all the letters and verify that the product is 1,000,000. To ensure that you have an understanding of the problem, find other "words" that satisfy the conditions of the problem.

PROJECTS AND GROUP ACTIVITIES

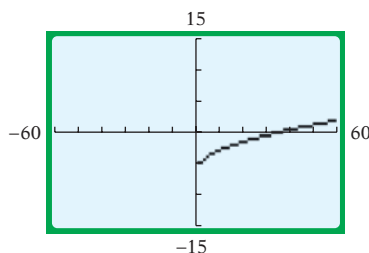
Solving Radical Equations with a Graphing Calculator



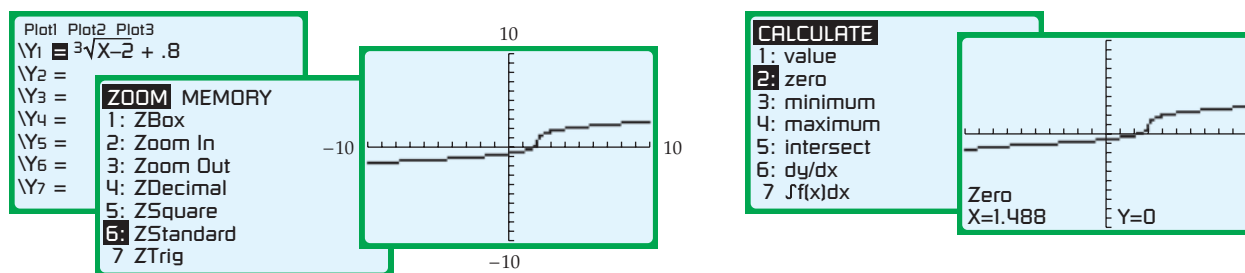
The radical equation $\sqrt{x-2} - 6 = 0$ was solved algebraically at the beginning of Section 7.3. To solve $\sqrt{x-2} - 6 = 0$ with a graphing calculator, use the left side of the equation to write an equation in two variables.

$$y = \sqrt{x-2} - 6$$

The graph of $y = \sqrt{x-2} - 6$ is shown below. The solution set of the equation $y = \sqrt{x-2} - 6$ is the set of ordered pairs (x, y) whose coordinates make the equation a true statement. The x -coordinate at which $y = 0$ is the solution of the equation $\sqrt{x-2} - 6 = 0$. The solution is the x -intercept of the curve given by the equation $y = \sqrt{x-2} - 6$. The solution is 38.



The solution of the radical equation $\sqrt[3]{x-2} = -0.8$ is 1.488. A graphing calculator can be used to find any rational or irrational solution to a predetermined degree of accuracy. The graph of $y = \sqrt[3]{x-2} + 0.8$ is shown below. The graph intersects the x -axis at $x = 1.488$.



Integrating Technology

See the Keystroke Guide: *Graph* for instructions on using a graphing calculator to graph a function, and *Zero* for instructions on finding the x -intercepts of a function.

To graph the equation $y = \sqrt[3]{x-2} + 0.8$ on a TI-84, enter

Y= MATH 4 $\text{X,T,}\theta,n$ $-$ 2 $)$ $+$.8 ZOOM 6

Enter 2ND CALC 2 to find the x -intercept of the equation.

You were to solve Exercises 1 to 21 in Section 7.3 by algebraic means. Try some of those exercises again, this time using a graphing calculator.

Use a graphing calculator to find the solutions of the equations in Exercises 1 to 3. Round to the nearest thousandth.

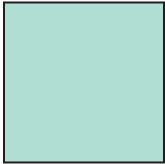
- $\sqrt{x+0.3} = 1.3$
- $\sqrt[3]{x+1.2} = -1.1$
- $\sqrt[4]{3x-1.5} = 1.4$

The Golden Rectangle

Rectangle A

Take the time to look closely at each of the four rectangles shown at the left. Which rectangle seems to be the most pleasant to look at? Record the responses of all the students in your class. Which rectangle was named most frequently as the most appealing?

The length of rectangle A is three times the width. The ratio of the length to the width is 3 to 1.



Rectangle B

Rectangle B is a square. Its length is equal to its width. The ratio of the length to the width is 1 to 1.

The ratio of the length of rectangle C to its width is 1.2 to 1.



Rectangle C

The shape of rectangle D is referred to as the golden rectangle. In a golden rectangle, the ratio of the length to the width is approximately 1.6 to 1.

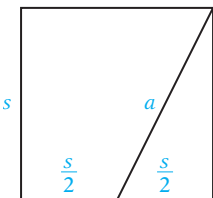
Did most students in the class select the golden rectangle as the one most pleasant to look at? The early Greeks considered a golden rectangle to be the most pleasing to the eye. Therefore, they used it extensively in their art and architecture. The Parthenon in Athens, Greece, is an example of the use of the golden rectangle in Greek architecture. It was built about 435 B.C. as a temple to the goddess Athena.



Rectangle D



© Reed Kaestner/Corbis



1. Construct a golden rectangle. Beginning with a square with each side of length s , draw a line segment from the midpoint of the base of the square to a vertex, as shown. Call this distance a . Make a rectangle whose width is that of the square and whose length is $\frac{s}{2} + a$.
2. Find the area of the rectangle in terms of s .
3. Find the exact ratio of the length to the width.
4. Find some examples of the golden rectangle in art and architecture. (*Note:* The Internet is a wonderful source of information on the golden rectangle. Simply enter “golden rectangle” in a search engine. You will be directed to hundreds of websites where you can learn about this rectangle’s appearance in art, architecture, music, nature, and mathematics.)

CHAPTER 7

SUMMARY

KEY WORDS

$a^{1/n}$ is the n th root of a . [7.1A, p. 398]

The expression $\sqrt[n]{a}$ is another symbol for the n th root of a . In the expression $\sqrt[n]{a}$, the symbol $\sqrt[n]{}$ is called a *radical*, n is the *index* of the radical, and a is the *radicand*. [7.1B, p. 400]

The symbol $\sqrt{}$ is used to indicate the positive square root, or *principal square root*, of a number. [7.1C, p. 402]

The expressions $a + b$ and $a - b$ are called *conjugates* of each other. The product of conjugates of the form $(a + b)(a - b)$ is $a^2 - b^2$. [7.2C, p. 411]

The procedure used to remove a radical from the denominator of a radical expression is called *rationalizing the denominator*. [7.2D, p. 412]

A *radical equation* is an equation that contains a variable expression in a radicand. [7.3A, p. 418]

A *complex number* is a number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. For the complex number $a + bi$, a is the *real part* of the complex number and b is the *imaginary part* of the complex number. [7.4A, p. 424]

EXAMPLES

$$16^{1/2} = 4 \text{ because } 4^2 = 16.$$

$$125^{1/3} = \sqrt[3]{125} = 5$$

The index is 3, and the radicand is 125.

$$\sqrt{16} = 4$$

$$-\sqrt{16} = -4$$

$$(x + 3)(x - 3) = x^2 - 3^2 = x^2 - 9$$

$$\begin{aligned} \frac{2}{1 - \sqrt{3}} &= \frac{2}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \\ &= \frac{2(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})} \\ &= \frac{2 + 2\sqrt{3}}{1 - 3} = -1 - \sqrt{3} \end{aligned}$$

$$\sqrt{x - 2} - 3 = 6 \text{ is a radical equation.}$$

$$3 + 2i \text{ is a complex number.}$$

3 is the real part and 2 is the imaginary part of the complex number.

ESSENTIAL RULES AND PROCEDURES

Rule for Rational Exponents [7.1A, p. 398]

If m and n are positive integers and $a^{1/n}$ is a real number, then $a^{m/n} = (a^{1/n})^m$.

EXAMPLES

$$8^{2/3} = (8^{1/3})^2 = 2^2 = 4$$

Definition of n th Root of a [7.1B, p. 400]

If a is a real number, then $a^{1/n} = \sqrt[n]{a}$.

$$x^{1/3} = \sqrt[3]{x}$$

Writing Exponential Expressions as Radical Expressions

[7.1B, p. 400]

If $a^{1/n}$ is a real number, then $a^{m/n} = a^{m \cdot 1/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$.

The expression $a^{m/n}$ can also be written $(\sqrt[n]{a})^m$.

$$b^{3/4} = \sqrt[4]{b^3}$$

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$$

Product Property of Radicals [7.2A, p. 408]

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
and $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

$$\sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7} = 3\sqrt{7}$$

Quotient Property of Radicals [7.2D, p. 412]

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are positive real numbers and $b \neq 0$, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \text{ and } \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[3]{\frac{5}{27}} = \frac{\sqrt[3]{5}}{\sqrt[3]{27}} = \frac{\sqrt[3]{5}}{3}$$

Property of Raising Each Side of an Equation to a Power

[7.3A, p. 418]

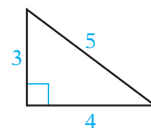
If $a = b$, then $a^n = b^n$.

$$\text{If } x = 4, \text{ then } x^2 = 16.$$

Pythagorean Theorem [7.3B, p. 420]

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

$$c^2 = a^2 + b^2$$



$$5^2 = 3^2 + 4^2$$

Definition of $\sqrt{-a}$ [7.4A, p. 424]

If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$: $\sqrt{-a} = i\sqrt{a}$.

$$\sqrt{-8} = i\sqrt{8} = 2i\sqrt{2}$$

Addition and Subtraction of Complex Numbers [7.4B, p. 425]

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

$$\begin{aligned} (2 + 4i) + (3 + 6i) \\ &= (2 + 3) + (4 + 6)i \\ &= 5 + 10i \end{aligned}$$

$$\begin{aligned} (4 + 3i) - (7 + 4i) \\ &= (4 - 7) + (3 - 4)i \\ &= -3 - i \end{aligned}$$

Multiplication of Complex Numbers [7.4C, p. 427]

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

One way to remember this rule is to think of the FOIL method.

$$\begin{aligned} (2 - 3i)(5 + 4i) \\ &= 10 + 8i - 15i - 12i^2 \\ &= 10 - 7i - 12(-1) \\ &= 22 - 7i \end{aligned}$$

CHAPTER 7**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How do you write an expression that contains a rational exponent as a radical expression?
2. In Chapter 7, why is it assumed that all variables represent positive numbers?
3. How can you tell when a radical expression is in simplest form?
4. How do you rationalize the denominator of a radical expression that has two terms in the denominator?
5. When can you add two radical expressions?
6. Why do you have to isolate the radical when solving an equation containing a radical expression?
7. How can you tell if a solution is an extraneous solution?
8. How do you add two complex numbers?
9. How can $\sqrt{36} + \sqrt{-36}$ be written as a complex number?
10. How do you find the product of two variable expressions with the same base and fractional exponents?

CHAPTER 7

REVIEW EXERCISES

1. Simplify: $(16x^{-4}y^{12})^{1/4}(100x^6y^{-2})^{1/2}$
2. Solve: $\sqrt[4]{3x - 5} = 2$
3. Multiply: $(6 - 5i)(4 + 3i)$
4. Rewrite $7y\sqrt[3]{x^2}$ as an exponential expression.
5. Multiply: $(\sqrt{3} + 8)(\sqrt{3} - 2)$
6. Solve: $\sqrt{4x + 9} + 10 = 11$
7. Divide: $\frac{x^{-3/2}}{x^{7/2}}$
8. Simplify: $\frac{8}{\sqrt{3y}}$
9. Simplify: $\sqrt[3]{-8a^6b^{12}}$
10. Simplify: $\sqrt{50a^4b^3} - ab\sqrt{18a^2b}$
11. Simplify: $\frac{14}{4 - \sqrt{2}}$
12. Simplify: $\frac{5 + 2i}{3i}$
13. Simplify: $\sqrt{18a^3b^6}$
14. Subtract: $(17 + 8i) - (15 - 4i)$
15. Simplify: $3x\sqrt[3]{54x^8y^{10}} - 2x^2y\sqrt[3]{16x^5y^7}$
16. Multiply: $\sqrt[3]{16x^4y}\sqrt[3]{4xy^5}$
17. Multiply: $i(3 - 7i)$
18. Simplify: $\frac{(4a^{-2/3}b^4)^{-1/2}}{(a^{-1/6}b^{3/2})^2}$

19. Simplify: $\sqrt[5]{-64a^8b^{12}}$

20. Divide: $\frac{5 + 9i}{1 - i}$

21. Multiply: $\sqrt{-12}\sqrt{-6}$

22. Solve: $\sqrt{x - 5} + \sqrt{x + 6} = 11$

23. Simplify: $\sqrt[4]{81a^8b^{12}}$

24. Simplify: $\frac{9}{\sqrt[3]{3x}}$

25. Subtract: $(-8 + 3i) - (4 - 7i)$

26. Expand: $(2 + \sqrt{2x - 1})^2$

27. Simplify: $4x\sqrt{12x^2y} + \sqrt{3x^4y} - x^2\sqrt{27y}$

28. Simplify: $81^{-1/4}$

29. Simplify: $(a^{16})^{-5/8}$

30. Simplify: $-\sqrt{49x^6y^{16}}$

31. Rewrite $4a^{2/3}$ as a radical expression.

32. Simplify: $(9x^2y^4)^{-1/2}(x^6y^6)^{1/3}$

33. Simplify: $\sqrt[4]{x^6y^8z^{10}}$

34. Simplify: $\sqrt{54} + \sqrt{24}$

35. Simplify: $\sqrt{48x^5y} - x\sqrt{80x^2y}$

36. Simplify: $\sqrt{32}\sqrt{50}$

37. Simplify: $\sqrt{3x}(3 + \sqrt{3x})$

38. Simplify: $\frac{\sqrt{125x^6}}{\sqrt{5x^3}}$

39. Simplify: $\frac{2 - 3\sqrt{7}}{6 - \sqrt{7}}$

40. Simplify: $\sqrt{-36}$

41. Evaluate $-b + \sqrt{b^2 - 4ac}$ when $a = 1$, $b = -8$, and $c = 25$.

42. Evaluate $-b + \sqrt{b^2 - 4ac}$ when $a = 1$, $b = 2$, and $c = 9$.

43. Add: $(5 + 2i) + (4 - 3i)$

44. Simplify: $(3 + 2\sqrt{5})(3 - 2\sqrt{5})$

45. Simplify: $(3 - 9i) - 7$

46. Expand: $(4 - i)^2$

47. Simplify: $\frac{-6}{i}$

48. Divide: $\frac{7}{2 - i}$

49. Solve: $\sqrt{2x - 7} + 2 = 5$

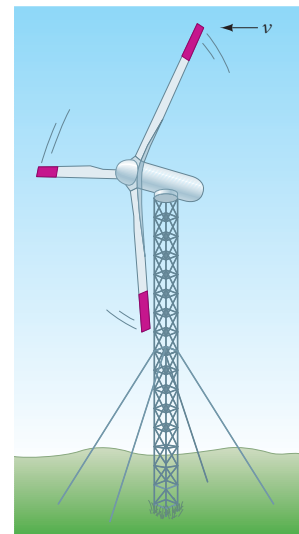
50. Solve: $\sqrt[3]{9x} = -6$

51. **Geometry** Find the width of a rectangle that has a diagonal of 13 in. and a length of 12 in.

52. **Energy** The velocity of the wind determines the amount of power generated by a windmill. A typical equation for this relationship is $v = 4.05 \sqrt[3]{P}$, where v is the velocity in miles per hour and P is the power in watts. Find the amount of power generated by a 20-mph wind. Round to the nearest whole number.

53. **Automotive Technology** Find the distance required for a car to reach a velocity of 88 ft/s when the acceleration is 16 ft/s². Use the equation $v = \sqrt{2as}$, where v is the velocity in feet per second, a is the acceleration, and s is the distance in feet.

54. **Home Maintenance** A 12-foot ladder is leaning against a house in preparation for washing the windows. How far from the house is the bottom of the ladder when the top of the ladder touches the house 10 ft above the ground? Round to the nearest hundredth.



CHAPTER 7

TEST

1. Write $\frac{1}{2} \sqrt[4]{x^3}$ as an exponential expression.
2. Simplify:
 $\sqrt[3]{54x^7y^3} - x\sqrt[3]{128x^4y^3} - x^2\sqrt[3]{2xy^3}$
3. Write $3y^{2/5}$ as a radical expression.
4. Multiply: $(2 + 5i)(4 - 2i)$
5. Expand: $(3 - 2\sqrt{x})^2$
6. Simplify: $\frac{r^{2/3} r^{-1}}{r^{-1/2}}$
7. Solve: $\sqrt{x + 12} - \sqrt{x} = 2$
8. Multiply: $\sqrt[4]{4a^5b^3} \sqrt[4]{8a^3b^7}$
9. Multiply: $\sqrt{3x}(\sqrt{x} - \sqrt{25x})$
10. Subtract: $(5 - 2i) - (8 - 4i)$
11. Simplify: $\sqrt{32x^4y^7}$
12. Multiply: $(2\sqrt{3} + 4)(3\sqrt{3} - 1)$
13. Simplify: $\sqrt{-5} \sqrt{-20}$
14. Simplify: $\frac{4 - 2\sqrt{5}}{2 - \sqrt{5}}$

15. Add: $\sqrt{18a^3} + a\sqrt{50a}$

16. Multiply: $(\sqrt{a} - 3\sqrt{b})(2\sqrt{a} + 5\sqrt{b})$

17. Simplify: $\frac{(2x^{1/3}y^{-2/3})^6}{(x^{-4}y^8)^{1/4}}$

18. Simplify: $\frac{10x}{\sqrt[3]{5x^2}}$

19. Simplify: $\frac{2 + 3i}{1 - 2i}$

20. Solve: $\sqrt[3]{2x - 2} + 4 = 2$

21. Simplify: $\left(\frac{4a^4}{b^2}\right)^{-3/2}$

22. Simplify: $\sqrt[3]{27a^4b^3c^7}$

23. Divide: $\frac{\sqrt{32x^5y}}{\sqrt{2xy^3}}$

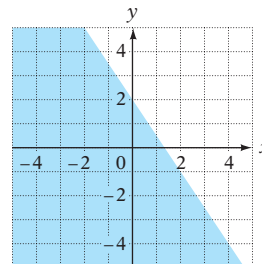
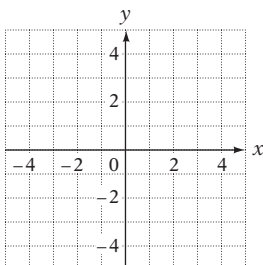
24. Simplify: $\frac{5x}{\sqrt{5x}}$

25. **Physics** An object is dropped from a high building. Find the distance the object has fallen when its speed reaches 192 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object in feet per second and d is the distance in feet.

CUMULATIVE REVIEW EXERCISES

1. Identify the property that justifies the statement $(a + 2)b = ab + 2b$.
2. Given $f(x) = 3x^2 - 2x + 1$, evaluate $f(-3)$.
3. Solve: $5 - \frac{2}{3}x = 4$
4. Solve: $2[4 - 2(3 - 2x)] = 4(1 - x)$
5. Solve: $2 + |4 - 3x| = 5$
6. Solve: $6x - 3(2x + 2) > 3 - 3(x + 2)$
Write the solution set in set-builder notation.
7. Solve: $|2x + 3| \leq 9$
8. Factor: $81x^2 - y^2$
9. Factor: $x^5 + 2x^3 - 3x$
10. Find the equation of the line that passes through the points $(2, 3)$ and $(-1, 2)$.
11. Find the value of the determinant:

$$\begin{vmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 3 & 1 & -2 \end{vmatrix}$$
12. Solve $P = \frac{R - C}{n}$ for C .
13. Simplify: $(2^{-1}x^2y^{-6})(2^{-1}y^{-4})^{-2}$
14. Multiply: $\frac{x^2y^3}{x^2 + 2x - 8} \cdot \frac{2x^2 - 7x + 6}{xy^4}$
15. Subtract: $\sqrt{40x^3} - x\sqrt{90x}$
16. Solve: $\frac{x}{x - 2} - 2x = \frac{-3}{x - 2}$
17. Graph $3x - 2y = -6$, and find the slope and y-intercept.
18. Graph the solution set of $3x + 2y \leq 4$.



19. Divide: $\frac{2i}{3-i}$

20. Solve: $\sqrt[3]{3x-4} + 5 = 1$

21. Add: $\frac{x}{2x-3} + \frac{4}{x+4}$

22. Solve by using Cramer's Rule:
 $2x - y = 4$
 $-2x + 3y = 5$

23. **Stamps** A collection of 30 stamps consists of 13¢ stamps and 18¢ stamps. The total value of the stamps is \$4.85. Find the number of 18¢ stamps.

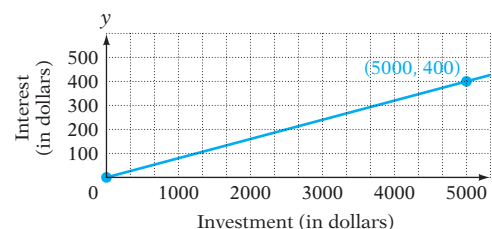


24. **Uniform Motion** A sales executive traveled 25 mi by car and then an additional 625 mi by plane. The rate of the plane was five times greater than the rate of the car. The total time of the trip was 3 h. Find the rate of the plane.

25. **Astronomy** How long does it take light to travel to Earth from the moon when the moon is 232,500 mi from Earth? Light travels at a rate of 1.86×10^5 mi/s.

26. **Oceanography** How far would a submarine periscope have to be above the water to locate a ship 7 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest tenth of a foot.

27. **Investments** The graph shows the amount invested and the annual income from an investment. Find the slope of the line between the two points shown on the graph. Then write a sentence that states the meaning of the slope.



Quadratic Equations

CHAPTER

8

OBJECTIVES

SECTION 8.1

- A** To solve a quadratic equation by factoring
- B** To write a quadratic equation given its solutions
- C** To solve a quadratic equation by taking square roots

SECTION 8.2

- A** To solve a quadratic equation by completing the square

SECTION 8.3

- A** To solve a quadratic equation by using the quadratic formula

SECTION 8.4

- A** To solve an equation that is quadratic in form
- B** To solve a radical equation that is reducible to a quadratic equation
- C** To solve a rational equation that is reducible to a quadratic equation

SECTION 8.5

- A** To solve a nonlinear inequality

SECTION 8.6

- A** To solve application problems

ARE YOU READY?

Take the Chapter 8 Prep Test to find out if you are ready to learn to:

- Solve a quadratic equation by factoring, by taking square roots, by completing the square, and by using the quadratic formula
- Solve an equation that is reducible to a quadratic equation
- Solve a nonlinear inequality

PREP TEST

Do these exercises to prepare for Chapter 8.

1. Simplify: $\sqrt{18}$

2. Simplify: $\sqrt{-9}$

3. Simplify: $\frac{3x-2}{x-1} - 1$

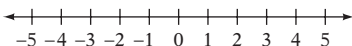
4. Evaluate $b^2 - 4ac$ when $a = 2$, $b = -4$, and $c = 1$.

5. Is $4x^2 + 28x + 49$ a perfect square trinomial?

6. Factor: $4x^2 - 4x + 1$

7. Factor: $9x^2 - 4$

8. Graph $\{x|x < -1\} \cap \{x|x < 4\}$.



9. Solve: $x(x-1) = x+15$

10. Solve: $\frac{4}{x-3} = \frac{16}{x}$

SECTION

8.1

Solving Quadratic Equations by Factoring or by Taking Square Roots

OBJECTIVE A

To solve a quadratic equation by factoring 

Recall that a *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$.

$$\text{Quadratic Equations} \begin{cases} 3x^2 - x + 2 = 0, & a = 3, & b = -1, & c = 2 \\ -x^2 + 4 = 0, & a = -1, & b = 0, & c = 4 \\ 6x^2 - 5x = 0, & a = 6, & b = -5, & c = 0 \end{cases}$$

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. Because the degree of the polynomial $ax^2 + bx + c$ is 2, a quadratic equation is also called a **second-degree equation**.

As we discussed earlier, quadratic equations sometimes can be solved by using the Principle of Zero Products. This method is reviewed here.

The Principle of Zero Products

If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products states that if the product of two factors is zero, then at least one of the factors must be zero.

 Take Note

Recall that the steps involved in solving a quadratic equation by factoring are

1. Write the equation in standard form.
2. Factor.
3. Use the Principle of Zero Products to set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

HOW TO • 1Solve by factoring: $3x^2 = 2 - 5x$

$$\begin{aligned} 3x^2 &= 2 - 5x \\ 3x^2 + 5x - 2 &= 0 \\ (3x - 1)(x + 2) &= 0 \\ 3x - 1 &= 0 & x + 2 &= 0 \\ 3x &= 1 & x &= -2 \\ x &= \frac{1}{3} \end{aligned}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products to write two equations.
- Solve each equation.

$\frac{1}{3}$ and -2 check as solutions. The solutions are $\frac{1}{3}$ and -2 .

HOW TO • 2Solve by factoring: $x^2 - 6x = -9$

$$\begin{aligned} x^2 - 6x &= -9 \\ x^2 - 6x + 9 &= 0 \\ (x - 3)(x - 3) &= 0 \\ x - 3 &= 0 & x - 3 &= 0 \\ x &= 3 & x &= 3 \end{aligned}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products.
- Solve each equation.

3 checks as a solution. The solution is 3.

 Take Note

When a quadratic equation has two solutions that are the same number, the solution is called a **double root** of the equation. The number 3 is a double root of $x^2 - 6x = -9$.

EXAMPLE • 1Solve by factoring: $2x(x - 3) = x + 4$ **Solution**

$$\begin{aligned}
 2x(x - 3) &= x + 4 \\
 2x^2 - 6x &= x + 4 \\
 2x^2 - 7x - 4 &= 0 & \bullet \text{ Write in standard form.} \\
 (2x + 1)(x - 4) &= 0 & \bullet \text{ Solve by factoring.} \\
 2x + 1 = 0 & \quad x - 4 = 0 \\
 2x = -1 & \quad x = 4 \\
 x = -\frac{1}{2} &
 \end{aligned}$$

The solutions are $-\frac{1}{2}$ and 4.**YOU TRY IT • 1**Solve by factoring: $2x^2 = 7x - 3$ **Your solution****EXAMPLE • 2**Solve for x by factoring: $x^2 - 4ax - 5a^2 = 0$ **Solution**This is a literal equation. Solve for x in terms of a .

$$\begin{aligned}
 x^2 - 4ax - 5a^2 &= 0 \\
 (x + a)(x - 5a) &= 0 \\
 x + a = 0 & \quad x - 5a = 0 \\
 x = -a & \quad x = 5a
 \end{aligned}$$

The solutions are $-a$ and $5a$.**YOU TRY IT • 2**Solve for x by factoring: $x^2 - 3ax - 4a^2 = 0$ **Your solution**

Solutions on p. S23

OBJECTIVE B**To write a quadratic equation given its solutions**As shown below, the solutions of the equation $(x - r_1)(x - r_2) = 0$ are r_1 and r_2 .

$$\begin{array}{lcl}
 (x - r_1)(x - r_2) = 0 & & \\
 x - r_1 = 0 & \quad & x - r_2 = 0 \\
 x = r_1 & & x = r_2 \\
 \text{Check: } \frac{(x - r_1)(x - r_2) = 0}{\begin{array}{l|l} (r_1 - r_1)(r_1 - r_2) & 0 \\ 0 \cdot (r_1 - r_2) & 0 \\ \hline 0 & 0 \end{array}} & & \frac{(x - r_1)(x - r_2) = 0}{\begin{array}{l|l} (r_2 - r_1)(r_2 - r_2) & 0 \\ (r_2 - r_1) \cdot 0 & 0 \\ \hline 0 & 0 \end{array}}
 \end{array}$$

Using the equation $(x - r_1)(x - r_2) = 0$ and the fact that r_1 and r_2 are solutions of this equation, it is possible to write a quadratic equation given its solutions.**HOW TO • 3**Write a quadratic equation that has solutions 4 and -5 .

$$\begin{aligned}
 (x - r_1)(x - r_2) &= 0 \\
 (x - 4)[x - (-5)] &= 0 & \bullet \text{ Replace } r_1 \text{ by } 4 \text{ and } r_2 \text{ by } -5. \\
 (x - 4)(x + 5) &= 0 & \bullet \text{ Simplify.} \\
 x^2 + x - 20 &= 0 & \bullet \text{ Multiply.}
 \end{aligned}$$

HOW TO 4

Write a quadratic equation with integer coefficients and solutions

$$\frac{2}{3} \text{ and } \frac{1}{2}.$$

$$(x - r_1)(x - r_2) = 0$$

$$\left(x - \frac{2}{3}\right)\left(x - \frac{1}{2}\right) = 0$$

- Replace r_1 by $\frac{2}{3}$ and r_2 by $\frac{1}{2}$.

$$x^2 - \frac{7}{6}x + \frac{1}{3} = 0$$

- Multiply.

$$6\left(x^2 - \frac{7}{6}x + \frac{1}{3}\right) = 6 \cdot 0$$

- Multiply each side of the equation by the LCM of the denominators.

$$6x^2 - 7x + 2 = 0$$

EXAMPLE 3Write a quadratic equation with integer coefficients and solutions $\frac{1}{2}$ and -4 .**Solution**

$$(x - r_1)(x - r_2) = 0$$

$$\left(x - \frac{1}{2}\right)[x - (-4)] = 0 \quad \bullet r_1 = \frac{1}{2}, r_2 = -4$$

$$\left(x - \frac{1}{2}\right)(x + 4) = 0$$

$$x^2 + \frac{7}{2}x - 2 = 0$$

$$2\left(x^2 + \frac{7}{2}x - 2\right) = 2 \cdot 0$$

$$2x^2 + 7x - 4 = 0$$

YOU TRY IT 3Write a quadratic equation with integer coefficients and solutions 3 and $-\frac{1}{2}$.**Your solution**

Solution on p. S24

OBJECTIVE C**To solve a quadratic equation by taking square roots**The solution of the quadratic equation $x^2 = 16$ is shown at the right.

$$\begin{aligned} x^2 &= 16 \\ x^2 - 16 &= 0 \\ (x - 4)(x + 4) &= 0 \\ x - 4 &= 0 & x + 4 &= 0 \\ x &= 4 & x &= -4 \end{aligned}$$

The solution can also be found by taking the square root of each side of the equation and writing the positive and negative square roots of the radicand. The notation ± 4 means $x = 4$ or $x = -4$.

$$\begin{aligned} x^2 &= 16 \\ \sqrt{x^2} &= \sqrt{16} \\ x &= \pm 4 \end{aligned}$$

The solutions are 4 and -4 .

HOW TO 5Solve by taking square roots: $3x^2 = 54$

$$3x^2 = 54$$

$$x^2 = 18$$

$$\sqrt{x^2} = \sqrt{18}$$

$$x = \pm\sqrt{18}$$

$$x = \pm 3\sqrt{2}$$

- Solve for x^2 .
- Take the square root of each side of the equation.
- Simplify.

The solutions are $3\sqrt{2}$ and $-3\sqrt{2}$. • $3\sqrt{2}$ and $-3\sqrt{2}$ check as solutions.

Solving a quadratic equation by taking the square root of each side of the equation can lead to solutions that are complex numbers.

HOW TO • 6 Solve by taking square roots: $2x^2 + 18 = 0$

$$2x^2 + 18 = 0$$

$$2x^2 = -18$$

$$x^2 = -9$$

$$\sqrt{x^2} = \sqrt{-9}$$

$$x = \pm\sqrt{-9}$$

$$x = \pm 3i$$

• Solve for x^2 .

• Take the square root of each side of the equation.

• Simplify.

Check:	$2x^2 + 18 = 0$	$2x^2 + 18 = 0$
	$\frac{2(3i)^2 + 18}{2(-9) + 18} \bigg 0$	$\frac{2(-3i)^2 + 18}{2(-9) + 18} \bigg 0$
	$\frac{-18 + 18}{0} \bigg 0$	$\frac{-18 + 18}{0} \bigg 0$
	$0 = 0$	$0 = 0$

The solutions are $3i$ and $-3i$.



Tips for Success

Always check the solution of an equation.

An equation containing the square of a binomial can be solved by taking square roots.

HOW TO • 7 Solve by taking square roots: $(x + 2)^2 - 24 = 0$

$$(x + 2)^2 - 24 = 0$$

$$(x + 2)^2 = 24$$

$$\sqrt{(x + 2)^2} = \sqrt{24}$$

$$x + 2 = \pm\sqrt{24}$$

$$x + 2 = \pm 2\sqrt{6}$$

$$x + 2 = 2\sqrt{6}$$

$$x = -2 + 2\sqrt{6}$$

$$x + 2 = -2\sqrt{6}$$

$$x = -2 - 2\sqrt{6}$$

• Solve for $(x + 2)^2$.

• Take the square root of each side of the equation. Then simplify.

• Solve for x .

The solutions are $-2 + 2\sqrt{6}$ and $-2 - 2\sqrt{6}$.

EXAMPLE • 4

Solve by taking square roots: $3(x - 2)^2 + 12 = 0$

Solution

$$3(x - 2)^2 + 12 = 0$$

$$3(x - 2)^2 = -12$$

$$(x - 2)^2 = -4$$

$$\sqrt{(x - 2)^2} = \sqrt{-4}$$

$$x - 2 = \pm\sqrt{-4}$$

$$x - 2 = \pm 2i$$

$$x - 2 = 2i$$

$$x = 2 + 2i$$

$$x - 2 = -2i$$

$$x = 2 - 2i$$

• Solve for $(x - 2)^2$.

• Take the square root of each side of the equation.

• Solve for x .

The solutions are $2 + 2i$ and $2 - 2i$.

YOU TRY IT • 4

Solve by taking square roots: $2(x + 1)^2 - 24 = 0$

Your solution

8.1 EXERCISES

OBJECTIVE A

To solve a quadratic equation by factoring



1. Explain why the restriction $a \neq 0$ is necessary in the definition of a quadratic equation.



2. What does the Principle of Zero Products state? How is it used to solve a quadratic equation?

For Exercises 3 to 6, write the quadratic equation in standard form with the coefficient of x^2 positive. Name the values of a , b , and c .

3. $2x^2 - 4x = 5$

4. $x^2 = 3x + 1$

5. $5x = 4x^2 + 6$

6. $3x^2 = 7$



For Exercises 7 to 10, solve for x .

7. $(x - 3)(x - 5) = 0$

8. $(x + 2)(x - 4) = 0$

9. $(x + 7)(x + 1) = 0$

10. $x(x + 3) = 0$

For Exercises 11 to 41, solve by factoring.

11. $x^2 - 4x = 0$

12. $y^2 + 6y = 0$

13. $t^2 - 25 = 0$

14. $p^2 - 81 = 0$

15. $s^2 - s - 6 = 0$

16. $v^2 + 4v - 5 = 0$

17. $y^2 - 6y + 9 = 0$

18. $x^2 + 10x + 25 = 0$

19. $9z^2 - 18z = 0$

20. $4y^2 + 20y = 0$

21. $r^2 - 3r = 10$

22. $p^2 + 5p = 6$

23. $v^2 + 10 = 7v$

24. $t^2 - 16 = 15t$

25. $2x^2 - 9x - 18 = 0$

26. $3y^2 - 4y - 4 = 0$

27. $4z^2 - 9z + 2 = 0$

28. $2s^2 - 9s + 9 = 0$

29. $3w^2 + 11w = 4$

30. $2r^2 + r = 6$

31. $6x^2 = 23x + 18$

32. $6x^2 = 7x - 2$

33. $4 - 15u - 4u^2 = 0$

34. $3 - 2y - 8y^2 = 0$

35. $x + 18 = x(x - 6)$

36. $t + 24 = t(t + 6)$

37. $4s(s + 3) = s - 6$

38. $3v(v - 2) = 11v + 6$

39. $u^2 - 2u + 4 = (2u - 3)(u + 2)$

40. $(3v - 2)(2v + 1) = 3v^2 - 11v - 10$

41. $(3x - 4)(x + 4) = x^2 - 3x - 28$

For Exercises 42 to 50, solve for x by factoring.

42. $x^2 + 14ax + 48a^2 = 0$

43. $x^2 - 9bx + 14b^2 = 0$

44. $x^2 + 9xy - 36y^2 = 0$

45. $x^2 - 6cx - 7c^2 = 0$

46. $x^2 - ax - 20a^2 = 0$

47. $2x^2 + 3bx + b^2 = 0$

48. $3x^2 - 4cx + c^2 = 0$

49. $3x^2 - 14ax + 8a^2 = 0$

50. $3x^2 - 11xy + 6y^2 = 0$

OBJECTIVE B**To write a quadratic equation given its solutions**

For Exercises 51 to 80, write a quadratic equation that has integer coefficients and has as solutions the given pair of numbers.

51. 2 and 5

52. 3 and 1

53. -2 and -4

54. -1 and -3

55. 6 and -1

56. -2 and 5

57. 3 and -3

58. 5 and -5

59. 4 and 4

60. 2 and 2

61. 0 and 5

62. 0 and -2

63. 0 and 3

64. 0 and -1

65. 3 and $\frac{1}{2}$

66. 2 and $\frac{2}{3}$

67. $-\frac{3}{4}$ and 2

68. $-\frac{1}{2}$ and 5

69. $-\frac{5}{3}$ and -2

70. $-\frac{3}{2}$ and -1

71. $-\frac{2}{3}$ and $\frac{2}{3}$

72. $-\frac{1}{2}$ and $\frac{1}{2}$

73. $\frac{1}{2}$ and $\frac{1}{3}$

74. $\frac{3}{4}$ and $\frac{2}{3}$

75. $\frac{6}{5}$ and $-\frac{1}{2}$

76. $\frac{3}{4}$ and $-\frac{3}{2}$

77. $-\frac{1}{4}$ and $-\frac{1}{2}$

78. $-\frac{5}{6}$ and $-\frac{2}{3}$

79. $\frac{3}{5}$ and $-\frac{1}{10}$

80. $\frac{7}{2}$ and $-\frac{1}{4}$

81. If 0 is a solution of $ax^2 + bx + c = 0$, what is the value of c ?82. If u and v are solutions of $x^2 + bx + c = 0$, what are the values of b and c ?**OBJECTIVE C****To solve a quadratic equation by taking square roots**

For Exercises 83 to 112, solve by taking square roots.

83. $y^2 = 49$

84. $x^2 = 64$

85. $z^2 = -4$

86. $v^2 = -16$

87. $s^2 - 4 = 0$

88. $r^2 - 36 = 0$

89. $4x^2 - 81 = 0$

90. $9x^2 - 16 = 0$

91. $y^2 + 49 = 0$

92. $z^2 + 16 = 0$

93. $v^2 - 48 = 0$

94. $s^2 - 32 = 0$

95. $r^2 - 75 = 0$

96. $u^2 - 54 = 0$

97. $z^2 + 18 = 0$

98. $t^2 + 27 = 0$

99. $(x - 1)^2 = 36$

100. $(x + 2)^2 = 25$

101. $3(y + 3)^2 = 27$

102. $4(s - 2)^2 = 36$

103. $5(z + 2)^2 = 125$

104. $2(y - 3)^2 = 18$

105. $\left(v - \frac{1}{2}\right)^2 = \frac{1}{4}$

106. $\left(r + \frac{2}{3}\right)^2 = \frac{1}{9}$

107. $(x + 5)^2 - 6 = 0$

108. $(t - 1)^2 - 15 = 0$

109. $(v - 3)^2 + 45 = 0$

110. $(x + 5)^2 + 32 = 0$

111. $\left(u + \frac{2}{3}\right)^2 - 18 = 0$

112. $\left(z - \frac{1}{2}\right)^2 - 20 = 0$



For Exercises 113 to 116, assume that a and b are positive real numbers. In each case, state how many real or complex number solutions the equation has.

113. $(x - a)^2 + b = 0$

114. $x^2 + a = 0$

115. $(x - a)^2 = 0$

116. $(x - a)^2 - b = 0$

Applying the Concepts

For Exercises 117 to 124, write a quadratic equation that has as solutions the given pair of numbers.

117. $\sqrt{2}$ and $-\sqrt{2}$

118. $\sqrt{5}$ and $-\sqrt{5}$

119. i and $-i$

120. $2i$ and $-2i$

121. $2\sqrt{2}$ and $-2\sqrt{2}$

122. $3\sqrt{2}$ and $-3\sqrt{2}$

123. $i\sqrt{2}$ and $-i\sqrt{2}$

124. $2i\sqrt{3}$ and $-2i\sqrt{3}$

For Exercises 125 to 128, solve for x .

125. $(2x - 1)^2 = (2x + 3)^2$

126. $(x - 4)^2 = (x + 2)^2$

127. $(2x + 1)^2 = (x - 7)^2$

128. $(x + 3)^2 = (2x - 3)^2$



129. Use the Principle of Zero Products to explain why an equation with solutions -2 , 3 , and 5 could not be a quadratic equation.

SECTION

8.2

Solving Quadratic Equations
by Completing the Square

OBJECTIVE A

To solve a quadratic equation by completing the square



Recall that a perfect-square trinomial is the square of a binomial.

Perfect-Square Trinomial		Square of a Binomial
$x^2 + 8x + 16$	$=$	$(x + 4)^2$
$x^2 - 10x + 25$	$=$	$(x - 5)^2$
$x^2 + 2ax + a^2$	$=$	$(x + a)^2$

For each perfect-square trinomial, the square of $\frac{1}{2}$ of the coefficient of x equals the constant term.

$$\left(\frac{1}{2} \text{ coefficient of } x\right)^2 = \text{constant term}$$

$$x^2 + 8x + 16, \quad \left(\frac{1}{2} \cdot 8\right)^2 = 16$$

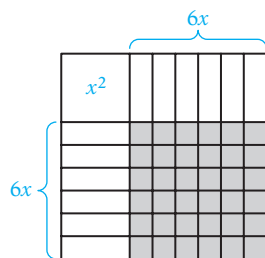
$$x^2 - 10x + 25, \quad \left[\frac{1}{2}(-10)\right]^2 = 25$$

$$x^2 + 2ax + a^2, \quad \left(\frac{1}{2} \cdot 2a\right)^2 = a^2$$



Point of Interest

Early attempts to solve quadratic equations were primarily geometric. The Persian mathematician al-Khwarizmi (c. A.D. 800) essentially completed a square of $x^2 + 12x$ as follows.



This relationship can be used to write the constant term for a perfect-square trinomial. Adding to a binomial the constant term that makes it a perfect-square trinomial is called **completing the square**.

HOW TO • 1

Complete the square on $x^2 + 12x$. Write the resulting perfect-square trinomial as the square of a binomial.

$$\left[\frac{1}{2}(12)\right]^2 = (6)^2 = 36$$

$$x^2 + 12x + 36$$

$$x^2 + 12x + 36 = (x + 6)^2$$

• Find the constant term.

• Complete the square on $x^2 + 12x$ by adding the constant term.

• Write the resulting perfect-square trinomial as the square of a binomial.

HOW TO • 2

Complete the square on $z^2 - 3z$. Write the resulting perfect-square trinomial as the square of a binomial.

$$\left[\frac{1}{2} \cdot (-3)\right]^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$z^2 - 3z + \frac{9}{4}$$

$$z^2 - 3z + \frac{9}{4} = \left(z - \frac{3}{2}\right)^2$$

• Find the constant term.

• Complete the square on $z^2 - 3z$ by adding the constant term.

• Write the resulting perfect-square trinomial as the square of a binomial.

Any quadratic equation can be solved by completing the square. Add to each side of the equation the term that completes the square. Rewrite the equation in the form $(x + a)^2 = b$. Then take the square root of each side of the equation.

**Point of Interest**

Mathematicians have studied quadratic equations for centuries. Many of the initial equations were a result of trying to solve a geometry problem. One of the most famous, which dates from around 500 B.C., is “squaring the circle.” The question was “Is it possible to construct a square whose area is that of a given circle?” For these early mathematicians, to *construct* meant to draw with only a straightedge and a compass. It was approximately 2300 years later that mathematicians were able to prove that such a construction is impossible.

HOW TO 3Solve by completing the square: $x^2 - 6x - 15 = 0$

$$x^2 - 6x - 15 = 0$$

$$x^2 - 6x = 15$$

$$x^2 - 6x + 9 = 15 + 9$$

$$(x - 3)^2 = 24$$

$$\sqrt{(x - 3)^2} = \sqrt{24}$$

$$x - 3 = \pm\sqrt{24}$$

$$x - 3 = \pm 2\sqrt{6}$$

$$x - 3 = 2\sqrt{6}$$

$$x = 3 + 2\sqrt{6}$$

$$x - 3 = -2\sqrt{6}$$

$$x = 3 - 2\sqrt{6}$$

Check:

$$\begin{array}{r} x^2 - 6x - 15 = 0 \\ (3 + 2\sqrt{6})^2 - 6(3 + 2\sqrt{6}) - 15 \quad | \quad 0 \\ 9 + 12\sqrt{6} + 24 - 18 - 12\sqrt{6} - 15 \quad | \quad 0 \\ 0 = 0 \end{array}$$

$$\begin{array}{r} x^2 - 6x - 15 = 0 \\ (3 - 2\sqrt{6})^2 - 6(3 - 2\sqrt{6}) - 15 \quad | \quad 0 \\ 9 - 12\sqrt{6} + 24 - 18 + 12\sqrt{6} - 15 \quad | \quad 0 \\ 0 = 0 \end{array}$$

The solutions are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$.

• Add 15 to each side of the equation.

• Complete the square. Add

$$\left[\frac{1}{2}(-6)\right]^2 = (-3)^2 = 9 \text{ to each side of the equation.}$$

• Factor the perfect-square trinomial.

• Take the square root of each side of the equation.

• Solve for x .**Take Note**

The exact solutions of the equation $x^2 - 6x - 15 = 0$ are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$. Approximate solutions of the equation are 7.899 and -1.899.

In the example above, the solutions of the equation $x^2 - 6x - 15 = 0$ are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$. These are the exact solutions. However, in some situations it may be preferable to have decimal approximations of the solutions of a quadratic equation. Approximate solutions can be found by using a calculator and then rounding to the desired degree of accuracy.

$$3 + 2\sqrt{6} \approx 7.899 \text{ and } 3 - 2\sqrt{6} \approx -1.899$$

To the nearest thousandth, the approximate solutions of the equation $x^2 - 6x - 15 = 0$ are 7.899 and -1.899.

HOW TO 4Solve $2x^2 - x - 2 = 0$ by completing the square. Find the exact solutions, and approximate the solutions to the nearest thousandth.

In order for us to complete the square on an expression, the coefficient of the squared term must be 1. After adding the constant term to each side of the equation, multiply each side of the equation by $\frac{1}{2}$.

$$2x^2 - x - 2 = 0$$

$$2x^2 - x = 2$$

• Add 2 to each side of the equation.

$$\frac{1}{2}(2x^2 - x) = \frac{1}{2} \cdot 2$$

• Multiply each side of the equation by $\frac{1}{2}$.



Integrating Technology

A graphing calculator can be used to check a proposed solution of a quadratic equation. For the equation at the right, enter the expression $2x^2 - x - 2$ in Y_1 . Then evaluate the function for $\frac{1 + \sqrt{17}}{4}$ and $\frac{1 - \sqrt{17}}{4}$. The value of each should be zero. See the Keystroke Guide: *Evaluating Functions*.

$$x^2 - \frac{1}{2}x = 1$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = 1 + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{17}{16}$$

$$\sqrt{\left(x - \frac{1}{4}\right)^2} = \sqrt{\frac{17}{16}}$$

$$x - \frac{1}{4} = \pm \frac{\sqrt{17}}{4}$$

$$x - \frac{1}{4} = \frac{\sqrt{17}}{4}$$

$$x = \frac{1}{4} + \frac{\sqrt{17}}{4}$$

$$x - \frac{1}{4} = -\frac{\sqrt{17}}{4}$$

$$x = \frac{1}{4} - \frac{\sqrt{17}}{4}$$

The exact solutions are $\frac{1 + \sqrt{17}}{4}$ and $\frac{1 - \sqrt{17}}{4}$.

$$\frac{1 + \sqrt{17}}{4} \approx 1.281 \quad \frac{1 - \sqrt{17}}{4} \approx -0.781$$

To the nearest thousandth, the solutions are 1.281 and -0.781.

Check of exact solutions:

$2x^2 - x - 2 = 0$	$2x^2 - x - 2 = 0$
$2\left(\frac{1 + \sqrt{17}}{4}\right)^2 - \frac{1 + \sqrt{17}}{4} - 2$	$2\left(\frac{1 - \sqrt{17}}{4}\right)^2 - \frac{1 - \sqrt{17}}{4} - 2$
$2\left(\frac{18 + 2\sqrt{17}}{16}\right) - \frac{1 + \sqrt{17}}{4} - 2$	$2\left(\frac{18 - 2\sqrt{17}}{16}\right) - \frac{1 - \sqrt{17}}{4} - 2$
$\frac{18 + 2\sqrt{17}}{8} - \frac{2 + 2\sqrt{17}}{8} - 2$	$\frac{18 - 2\sqrt{17}}{8} - \frac{2 - 2\sqrt{17}}{8} - 2$
$\frac{16}{8} - 2$	$\frac{16}{8} - 2$
$0 = 0$	$0 = 0$



Tips for Success

This is a new skill and one that is difficult for many students. Be sure to do all you need to do in order to be successful at solving equations by completing the square: Read through the introductory material, work through the HOW TO examples, study the paired Examples, do the You Try Its, and check your solutions against the ones in the back of the book. See *AIM for Success* in the Preface.

The above example illustrates all the steps required in solving a quadratic equation by completing the square.

Procedure for Solving a Quadratic Equation by Completing the Square

1. Write the equation in the form $ax^2 + bx = -c$.
2. Multiply both sides of the equation by $\frac{1}{a}$.
3. Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
4. Factor the perfect-square trinomial.
5. Take the square root of each side of the equation.
6. Solve the resulting equation for x .
7. Check the solutions.

EXAMPLE • 1Solve by completing the square: $4x^2 - 8x + 1 = 0$ **Solution**

$$4x^2 - 8x + 1 = 0$$

$$4x^2 - 8x = -1$$

- Write in the form $ax^2 + bx = -c$.

$$\frac{1}{4}(4x^2 - 8x) = \frac{1}{4}(-1)$$

- Multiply both sides by $\frac{1}{a}$.

$$x^2 - 2x = -\frac{1}{4}$$

$$x^2 - 2x + 1 = -\frac{1}{4} + 1$$

- Complete the square.

$$(x - 1)^2 = \frac{3}{4}$$

- Factor.

$$\sqrt{(x - 1)^2} = \sqrt{\frac{3}{4}}$$

- Take square roots.

$$x - 1 = \pm \frac{\sqrt{3}}{2}$$

$$x - 1 = \frac{\sqrt{3}}{2}$$

$$x - 1 = -\frac{\sqrt{3}}{2}$$

- Solve for x .

$$x = 1 + \frac{\sqrt{3}}{2}$$

$$x = 1 - \frac{\sqrt{3}}{2}$$

$$= \frac{2 + \sqrt{3}}{2}$$

$$= \frac{2 - \sqrt{3}}{2}$$

The solutions are $\frac{2 + \sqrt{3}}{2}$ and $\frac{2 - \sqrt{3}}{2}$.

YOU TRY IT • 1Solve by completing the square: $4x^2 - 4x - 1 = 0$ **Your solution****EXAMPLE • 2**Solve by completing the square: $x^2 + 4x + 5 = 0$ **Solution**

$$x^2 + 4x + 5 = 0$$

$$x^2 + 4x = -5$$

$$x^2 + 4x + 4 = -5 + 4$$

- Complete the square.

$$(x + 2)^2 = -1$$

- Factor.

$$\sqrt{(x + 2)^2} = \sqrt{-1}$$

- Take square roots.

$$x + 2 = \pm i$$

$$x + 2 = i$$

$$x + 2 = -i$$

- Solve for x .

$$x = -2 + i$$

$$x = -2 - i$$

The solutions are $-2 + i$ and $-2 - i$.

YOU TRY IT • 2Solve by completing the square: $x^2 + 4x + 8 = 0$ **Your solution**

8.2 EXERCISES**OBJECTIVE A****To solve a quadratic equation by completing the square**

For Exercises 1 to 48, solve by completing the square.

1. $x^2 - 4x - 5 = 0$

2. $y^2 + 6y + 5 = 0$

3. $v^2 + 8v - 9 = 0$

4. $w^2 - 2w - 24 = 0$

5. $z^2 - 6z + 9 = 0$

6. $u^2 + 10u + 25 = 0$

7. $r^2 + 4r - 7 = 0$

8. $s^2 + 6s - 1 = 0$

9. $x^2 - 6x + 7 = 0$

10. $y^2 + 8y + 13 = 0$

11. $z^2 - 2z + 2 = 0$

12. $t^2 - 4t + 8 = 0$

13. $s^2 - 5s - 24 = 0$

14. $v^2 + 7v - 44 = 0$

15. $x^2 + 5x - 36 = 0$

16. $y^2 - 9y + 20 = 0$

17. $p^2 - 3p + 1 = 0$

18. $r^2 - 5r - 2 = 0$

19. $t^2 - t - 1 = 0$

20. $u^2 - u - 7 = 0$

21. $y^2 - 6y = 4$

22. $w^2 + 4w = 2$

23. $x^2 = 8x - 15$

24. $z^2 = 4z - 3$

25. $v^2 = 4v - 13$

26. $x^2 = 2x - 17$

27. $p^2 + 6p = -13$

28. $x^2 + 4x = -20$

29. $y^2 - 2y = 17$

30. $x^2 + 10x = 7$

31. $z^2 = z + 4$

32. $r^2 = 3r - 1$

33. $x^2 + 13 = 2x$

34. $6v^2 - 7v = 3$

35. $4x^2 - 4x + 5 = 0$

36. $4t^2 - 4t + 17 = 0$

37. $9x^2 - 6x + 2 = 0$

38. $9y^2 - 12y + 13 = 0$

39. $2s^2 = 4s + 5$

40. $3u^2 = 6u + 1$

41. $2r^2 = 3 - r$

42. $2x^2 = 12 - 5x$

43. $y - 2 = (y - 3)(y + 2)$

44. $8s - 11 = (s - 4)(s - 2)$

45. $6t - 2 = (2t - 3)(t - 1)$

46. $2z + 9 = (2z + 3)(z + 2)$

47. $(x - 4)(x + 1) = x - 3$

48. $(y - 3)^2 = 2y + 10$

For Exercises 49 to 54, solve by completing the square. Approximate the solutions to the nearest thousandth.

49. $z^2 + 2z = 4$

50. $t^2 - 4t = 7$

51. $2x^2 = 4x - 1$

52. $3y^2 = 5y - 1$

53. $4z^2 + 2z = 1$

54. $4w^2 - 8w = 3$



55. For what values of c does the equation $x^2 + 4x + c = 0$ have real number solutions?



56. For what values of c does the equation $x^2 - 6x + c = 0$ have complex number solutions?

Applying the Concepts

For Exercises 57 to 59, solve for x by completing the square.

57. $x^2 - ax - 2a^2 = 0$

58. $x^2 + 3ax - 4a^2 = 0$

59. $x^2 + 3ax - 10a^2 = 0$

60. **Sports** The height h (in feet) of a baseball above the ground t seconds after it is hit can be approximated by the equation $h = -16t^2 + 70t + 4$. Using this equation, determine when the ball will hit the ground. Round to the nearest hundredth. (Hint: The ball hits the ground when $h = 0$.)

61. **Sports** After a baseball is hit, there are two equations that can be considered. One gives the height h (in feet) the ball is above the ground t seconds after it is hit. The second is the distance s (in feet) the ball is from home plate t seconds after it is hit. A model of this situation is given by $h = -16t^2 + 70t + 4$ and $s = 44.5t$. Using this model, determine whether the ball will clear a fence 325 ft from home plate.



AP Images



62. Explain how to complete the square on $x^2 + bx$.

SECTION

8.3

Solving Quadratic Equations by Using the Quadratic Formula

OBJECTIVE A

To solve a quadratic equation by using the quadratic formula 

A general formula known as the **quadratic formula** can be derived by applying the method of completing the square to the standard form of a quadratic equation. This formula can be used to solve any quadratic equation. The equation $ax^2 + bx + c = 0$ is solved by completing the square as follows.

Add the opposite of the constant term to each side of the equation.

$$\begin{aligned} ax^2 + bx + c &= 0 \\ ax^2 + bx + c + (-c) &= 0 + (-c) \end{aligned}$$

Multiply each side of the equation by the reciprocal of a , the coefficient of x^2 .

$$\begin{aligned} ax^2 + bx &= -c \\ \frac{1}{a}(ax^2 + bx) &= \frac{1}{a}(-c) \end{aligned}$$

Complete the square by adding $\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$ to each side of the equation.

$$\begin{aligned} x^2 + \frac{b}{a}x &= -\frac{c}{a} \\ x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= \frac{b^2}{4a^2} - \frac{c}{a} \end{aligned}$$

Simplify the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \left(\frac{c}{a} \cdot \frac{4a}{4a}\right)$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Factor the perfect-square trinomial on the left side of the equation.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Take the square root of each side of the equation.

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} \text{Solve for } x. \quad x + \frac{b}{2a} &= \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

$$\begin{aligned} x + \frac{b}{2a} &= -\frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$



Point of Interest

Although mathematicians have studied quadratic equations since around 500 B.C., it was not until the 18th century that the formula was written as it is today. Of further note, the word *quadratic* has the same Latin root as the word *square*.

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula is frequently written as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Take Note

The solutions of this quadratic equation are rational numbers. When this happens, the equation could have been solved by factoring and using the Principle of Zero Products. This may be easier than applying the quadratic formula.

HOW TO • 1

Solve by using the quadratic formula: $2x^2 + 5x + 3 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)} \\ &= \frac{-5 \pm \sqrt{25 - 24}}{4} \\ &= \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4} \\ x &= \frac{-5 + 1}{4} = \frac{-4}{4} = -1 \quad x = \frac{-5 - 1}{4} = \frac{-6}{4} = -\frac{3}{2} \end{aligned}$$

The solutions are -1 and $-\frac{3}{2}$.

- The equation $2x^2 + 5x + 3 = 0$ is in standard form. $a = 2$, $b = 5$, $c = 3$
- Replace a , b , and c in the quadratic formula with these values.

HOW TO • 2

Solve $3x^2 = 4x + 6$ by using the quadratic formula. Find the exact solutions, and approximate the solutions to the nearest thousandth.

$$\begin{aligned} 3x^2 &= 4x + 6 \\ 3x^2 - 4x - 6 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)} \\ &= \frac{4 \pm \sqrt{16 - (-72)}}{6} \\ &= \frac{4 \pm \sqrt{88}}{6} = \frac{4 \pm 2\sqrt{22}}{6} \\ &= \frac{2(2 \pm \sqrt{22})}{2 \cdot 3} = \frac{2 \pm \sqrt{22}}{3} \end{aligned}$$

- Write the equation in standard form. Subtract $4x$ and 6 from each side of the equation. $a = 3$, $b = -4$, $c = -6$
- Replace a , b , and c in the quadratic formula with these values.

Check:

$$\begin{array}{l|l} 3x^2 = 4x + 6 & \\ \hline 3\left(\frac{2 + \sqrt{22}}{3}\right)^2 & 4\left(\frac{2 + \sqrt{22}}{3}\right) + 6 \\ 3\left(\frac{4 + 4\sqrt{22} + 22}{9}\right) & \frac{8}{3} + \frac{4\sqrt{22}}{3} + \frac{18}{3} \\ 3\left(\frac{26 + 4\sqrt{22}}{9}\right) & \frac{26}{3} + \frac{4\sqrt{22}}{3} \\ \frac{26 + 4\sqrt{22}}{3} & = \frac{26 + 4\sqrt{22}}{3} \end{array}$$

$$\begin{array}{l|l} 3x^2 = 4x + 6 & \\ \hline 3\left(\frac{2 - \sqrt{22}}{3}\right)^2 & 4\left(\frac{2 - \sqrt{22}}{3}\right) + 6 \\ 3\left(\frac{4 - 4\sqrt{22} + 22}{9}\right) & \frac{8}{3} - \frac{4\sqrt{22}}{3} + \frac{18}{3} \\ 3\left(\frac{26 - 4\sqrt{22}}{9}\right) & \frac{26}{3} - \frac{4\sqrt{22}}{3} \\ \frac{26 - 4\sqrt{22}}{3} & = \frac{26 - 4\sqrt{22}}{3} \end{array}$$

The exact solutions are $\frac{2 + \sqrt{22}}{3}$ and $\frac{2 - \sqrt{22}}{3}$.

$$\frac{2 + \sqrt{22}}{3} \approx 2.230 \quad \frac{2 - \sqrt{22}}{3} \approx -0.897$$

To the nearest thousandth, the solutions are 2.230 and -0.897 .



Integrating Technology

See the Projects and Group Activities at the end of this chapter for instructions on using a graphing calculator to solve a quadratic equation. Instructions are also provided in the Keystroke Guide: Zero.

HOW TO • 3

Solve by using the quadratic formula: $4x^2 = 8x - 13$

$$\begin{aligned}
 4x^2 - 8x + 13 &= 0 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 13}}{2 \cdot 4} \\
 &= \frac{8 \pm \sqrt{64 - 208}}{8} = \frac{8 \pm \sqrt{-144}}{8} \\
 &= \frac{8 \pm 12i}{8} = \frac{2 \pm 3i}{2}
 \end{aligned}$$

The solutions are $1 + \frac{3}{2}i$ and $1 - \frac{3}{2}i$.

- Write the equation in standard form.
- Use the quadratic formula.
- $a = 4, b = -8, c = 13$

Of the three preceding examples, the first two had real number solutions; the last one had complex number solutions.

In the quadratic formula, the quantity $b^2 - 4ac$ is called the **discriminant**. When a, b , and c are real numbers, the discriminant determines whether a quadratic equation will have a double root, two real number solutions that are not equal, or two complex number solutions.

The Effect of the Discriminant on the Solutions of a Quadratic Equation

1. If $b^2 - 4ac = 0$, the equation has two equal real number solutions, a double root.
2. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions.
3. If $b^2 - 4ac < 0$, the equation has two complex number solutions.

HOW TO • 4

Use the discriminant to determine whether $x^2 - 4x - 5 = 0$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

$$\begin{aligned}
 b^2 - 4ac \\
 (-4)^2 - 4(1)(-5) &= 16 + 20 = 36 \\
 36 &> 0
 \end{aligned}$$

- Evaluate the discriminant.
- $a = 1, b = -4, c = -5$

Because $b^2 - 4ac > 0$, the equation has two unequal real number solutions.

EXAMPLE • 1

Solve by using the quadratic formula:
 $2x^2 - x + 5 = 0$

Solution

$$\begin{aligned}
 2x^2 - x + 5 &= 0 \\
 a = 2, b = -1, c = 5 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2 \cdot 2} \\
 &= \frac{1 \pm \sqrt{1 - 40}}{4} = \frac{1 \pm \sqrt{-39}}{4} \\
 &= \frac{1 \pm i\sqrt{39}}{4}
 \end{aligned}$$

The solutions are $\frac{1}{4} + \frac{\sqrt{39}}{4}i$ and $\frac{1}{4} - \frac{\sqrt{39}}{4}i$.

YOU TRY IT • 1

Solve by using the quadratic formula:
 $x^2 - 2x + 10 = 0$

Your solution

EXAMPLE • 2

Solve by using the quadratic formula:

$$2x^2 = (x - 2)(x - 3)$$

Solution

$$2x^2 = (x - 2)(x - 3)$$

$$2x^2 = x^2 - 5x + 6$$

• Write in standard form.

$$x^2 + 5x - 6 = 0$$

$$a = 1, b = 5, c = -6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(1)(-6)}}{2 \cdot 1}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2}$$

$$= \frac{-5 \pm 7}{2}$$

$$x = \frac{-5 + 7}{2} \quad x = \frac{-5 - 7}{2}$$

$$= \frac{2}{2} = 1 \quad = \frac{-12}{2} = -6$$

The solutions are 1 and -6.

YOU TRY IT • 2

Solve by using the quadratic formula:

$$4x^2 = 4x - 1$$

Your solution**EXAMPLE • 3**

Use the discriminant to determine whether $4x^2 - 2x + 5 = 0$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

Solution

$$a = 4, b = -2, c = 5$$

$$\begin{aligned} b^2 - 4ac &= (-2)^2 - 4(4)(5) \\ &= 4 - 80 \\ &= -76 \end{aligned}$$

$$-76 < 0$$

Because the discriminant is less than zero, the equation has two complex number solutions.

YOU TRY IT • 3

Use the discriminant to determine whether $3x^2 - x - 1 = 0$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

Your solution

8.3 EXERCISES

OBJECTIVE A

To solve a quadratic equation by using the quadratic formula



1. Write the quadratic formula. What does each variable in the formula represent?



2. Write the expression that appears under the radical symbol in the quadratic formula. What is this quantity called? What can it be used to determine?

For Exercises 3 to 29, solve by using the quadratic formula.

3. $x^2 - 3x - 10 = 0$

4. $z^2 - 4z - 8 = 0$

5. $y^2 + 5y - 36 = 0$

6. $z^2 - 3z - 40 = 0$

7. $w^2 = 8w + 72$

8. $t^2 = 2t + 35$

9. $v^2 = 24 - 5v$

10. $x^2 = 18 - 7x$

11. $2y^2 + 5y - 1 = 0$

12. $4p^2 - 7p + 1 = 0$

13. $8s^2 = 10s + 3$

14. $12t^2 = 5t + 2$

15. $x^2 = 14x - 4$

16. $v^2 = 12v - 24$

17. $2z^2 - 2z - 1 = 0$

18. $6w^2 = 9w - 1$

19. $z^2 + 2z + 2 = 0$

20. $p^2 - 4p + 5 = 0$

21. $y^2 - 2y + 5 = 0$

22. $x^2 + 6x + 13 = 0$

23. $s^2 - 4s + 13 = 0$

24. $t^2 - 6t + 10 = 0$

25. $2w^2 - 2w - 5 = 0$

26. $2v^2 + 8v + 3 = 0$

27. $2x^2 + 6x + 5 = 0$

28. $2y^2 + 2y + 13 = 0$

29. $4t^2 - 6t + 9 = 0$

For Exercises 30 to 35, solve by using the quadratic formula. Approximate the solutions to the nearest thousandth.

30. $x^2 - 6x - 6 = 0$

31. $p^2 - 8p + 3 = 0$

32. $r^2 - 2r = 4$

33. $w^2 + 4w = 1$

34. $3t^2 = 7t + 1$

35. $2y^2 = y + 5$

For Exercises 36 to 41, use the discriminant to determine whether the quadratic equation has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

36. $2z^2 - z + 5 = 0$

37. $3y^2 + y + 1 = 0$

38. $9x^2 - 12x + 4 = 0$

39. $4x^2 + 20x + 25 = 0$

40. $2v^2 - 3v - 1 = 0$

41. $3w^2 + 3w - 2 = 0$



42. Suppose $a > 0$ and $c < 0$. Does the equation $ax^2 + bx + c = 0$ (i) always have real number solutions, (ii) never have real number solutions, or (iii) sometimes have real number solutions, depending on the value of b ?



43. If $a > 0$ and $c > 0$, what is the smallest value of b in $ax^2 + bx + c = 0$ that will guarantee that the equation will have real number solutions?

Applying the Concepts

44. **Sports** The height h , in feet, of a baseball above the ground t seconds after it is hit by a Little Leaguer can be approximated by the equation $h = -0.01t^2 + 2t + 3.5$. Does the ball reach a height of 100 ft?

45. **Sports** The height h , in feet, of an arrow shot upward can be approximated by the equation $h = 128t - 16t^2$, where t is the time in seconds. Does the arrow reach a height of 275 ft?



46. Name the three methods of solving a quadratic equation that have been discussed. What are the advantages and disadvantages of each?

For what values of p do the quadratic equations in Exercises 47 and 48 have two unequal real number solutions? Write the answer in set-builder notation.

47. $x^2 - 6x + p = 0$

48. $x^2 + 10x + p = 0$

For what values of p do the quadratic equations in Exercises 49 and 50 have two complex number solutions? Write the answer in interval notation.

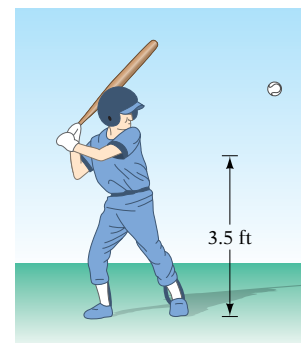
49. $x^2 - 2x + p = 0$

50. $x^2 + 4x + p = 0$

51. Find all values of x that satisfy the equation $x^2 + ix + 2 = 0$.

52. Show that the equation $x^2 + bx - 1 = 0$, $b \in \text{real numbers}$, always has real number solutions regardless of the value of b .

53. Show that the equation $2x^2 + bx - 2 = 0$, $b \in \text{real numbers}$, always has real number solutions regardless of the value of b .



SECTION

8.4

Solving Equations That Are Reducible to Quadratic Equations

OBJECTIVE A

To solve an equation that is quadratic in form 

Certain equations that are not quadratic can be expressed in quadratic form by making suitable substitutions. An equation is **quadratic in form** if it can be written as $au^2 + bu + c = 0$.

The equation $x^4 - 4x^2 - 5 = 0$ is quadratic in form.

$$\begin{aligned} x^4 - 4x^2 - 5 &= 0 \\ (x^2)^2 - 4(x^2) - 5 &= 0 \\ u^2 - 4u - 5 &= 0 & \bullet \text{ Let } x^2 = u. \end{aligned}$$

The equation $y - y^{1/2} - 6 = 0$ is quadratic in form.

$$\begin{aligned} y - y^{1/2} - 6 &= 0 \\ (y^{1/2})^2 - (y^{1/2}) - 6 &= 0 \\ u^2 - u - 6 &= 0 & \bullet \text{ Let } y^{1/2} = u. \end{aligned}$$

The key to recognizing equations that are quadratic in form is as follows. When the equation is written in standard form, the exponent on one variable term is $\frac{1}{2}$ the exponent on the other variable term.

HOW TO • 1

Solve: $z + 7z^{1/2} - 18 = 0$

$$\begin{aligned} z + 7z^{1/2} - 18 &= 0 & \bullet \text{ The equation is quadratic in form.} \\ (z^{1/2})^2 + 7(z^{1/2}) - 18 &= 0 \\ u^2 + 7u - 18 &= 0 & \bullet \text{ Let } z^{1/2} = u. \\ (u - 2)(u + 9) &= 0 & \bullet \text{ Solve by factoring.} \\ u - 2 = 0 & \quad u + 9 = 0 \\ u = 2 & \quad u = -9 \\ z^{1/2} = 2 & \quad z^{1/2} = -9 & \bullet \text{ Replace } u \text{ by } z^{1/2}. \\ \sqrt{z} = 2 & \quad \sqrt{z} = -9 \\ (\sqrt{z})^2 = 2^2 & \quad (\sqrt{z})^2 = (-9)^2 & \bullet \text{ Solve for } z. \\ z = 4 & \quad z = 81 \end{aligned}$$

Check each solution.

Check:	$z + 7z^{1/2} - 18 = 0$	$z + 7z^{1/2} - 18 = 0$
	$4 + 7(4)^{1/2} - 18 \quad \quad 0$	$81 + 7(81)^{1/2} - 18 \quad \quad 0$
	$4 + 7 \cdot 2 - 18 \quad \quad 0$	$81 + 7 \cdot 9 - 18 \quad \quad 0$
	$4 + 14 - 18 \quad \quad 0$	$81 + 63 - 18 \quad \quad 0$
	$0 = 0$	$126 \neq 0$

4 checks as a solution, but 81 does not check as a solution.

The solution is 4.

 Take Note

When each side of an equation is squared, the resulting equation may have a solution that is not a solution of the original equation.

EXAMPLE • 1

Solve: $x^4 + x^2 - 12 = 0$

Solution

$$\begin{aligned}
 x^4 + x^2 - 12 &= 0 \\
 (x^2)^2 + (x^2) - 12 &= 0 \\
 u^2 + u - 12 &= 0 \\
 (u - 3)(u + 4) &= 0
 \end{aligned}$$

$$\begin{array}{ll}
 u - 3 = 0 & u + 4 = 0 \\
 u = 3 & u = -4
 \end{array}$$

Replace u by x^2 .

$$\begin{array}{ll}
 x^2 = 3 & x^2 = -4 \\
 \sqrt{x^2} = \sqrt{3} & \sqrt{x^2} = \sqrt{-4} \\
 x = \pm\sqrt{3} & x = \pm 2i
 \end{array}$$

The solutions are $\sqrt{3}$, $-\sqrt{3}$, $2i$, and $-2i$.**YOU TRY IT • 1**

Solve: $x - 5x^{1/2} + 6 = 0$

Your solution

Solution on p. S25

OBJECTIVE B**To solve a radical equation that is reducible to a quadratic equation**

Certain equations containing radicals can be expressed as quadratic equations.

HOW TO • 2

Solve: $\sqrt{x+2} + 4 = x$

$$\sqrt{x+2} + 4 = x$$

$$\sqrt{x+2} = x - 4$$

$$(\sqrt{x+2})^2 = (x-4)^2$$

$$x+2 = x^2 - 8x + 16$$

$$0 = x^2 - 9x + 14$$

$$0 = (x-7)(x-2)$$

$$\begin{array}{ll}
 x - 7 = 0 & x - 2 = 0 \\
 x = 7 & x = 2
 \end{array}$$

Check each solution.

$$\begin{array}{rcl}
 \text{Check: } \sqrt{x+2} + 4 = x & & \\
 \sqrt{7+2} + 4 & | & 7 \\
 \sqrt{9} + 4 & | & 7 \\
 3 + 4 & | & 7 \\
 7 & = & 7
 \end{array}$$

$$\begin{array}{rcl}
 \sqrt{x+2} + 4 = x & & \\
 \sqrt{2+2} + 4 & | & 2 \\
 \sqrt{4} + 4 & | & 2 \\
 2 + 4 & | & 2 \\
 6 & \neq & 2
 \end{array}$$

7 checks as a solution, but 2 does not check as a solution.

The solution is 7.

- Solve for the radical expression.
- Square each side of the equation.
- Simplify.
- Write the equation in standard form.
- Solve for x .

EXAMPLE • 2

Solve: $\sqrt{7y - 3} + 3 = 2y$

Solution

$$\sqrt{7y - 3} + 3 = 2y$$

$$\sqrt{7y - 3} = 2y - 3$$

$$(\sqrt{7y - 3})^2 = (2y - 3)^2$$

$$7y - 3 = 4y^2 - 12y + 9$$

$$0 = 4y^2 - 19y + 12$$

$$0 = (4y - 3)(y - 4)$$

$$4y - 3 = 0$$

$$4y = 3$$

$$y = \frac{3}{4}$$

$$y - 4 = 0$$

$$y = 4$$

4 checks as a solution.

$\frac{3}{4}$ does not check as a solution.

The solution is 4.

• Solve for the radical.

• Square each side.

• Write in standard form.

• Solve by factoring.

YOU TRY IT • 2

Solve: $\sqrt{2x + 1} + x = 7$

Your solution**EXAMPLE • 3**

Solve: $\sqrt{2y + 1} - \sqrt{y} = 1$

Solution

$$\sqrt{2y + 1} - \sqrt{y} = 1$$

Solve for one of the radical expressions.

$$\sqrt{2y + 1} = \sqrt{y} + 1$$

$$(\sqrt{2y + 1})^2 = (\sqrt{y} + 1)^2$$

$$2y + 1 = y + 2\sqrt{y} + 1$$

$$y = 2\sqrt{y}$$

$$y^2 = (2\sqrt{y})^2$$

$$y^2 = 4y$$

$$y^2 - 4y = 0$$

$$y(y - 4) = 0$$

$$y = 0$$

$$y - 4 = 0$$

$$y = 4$$

0 and 4 check as solutions.

The solutions are 0 and 4.

• Square each side.

• Square each side.

YOU TRY IT • 3

Solve: $\sqrt{2x - 1} + \sqrt{x} = 2$

Your solution

OBJECTIVE C**To solve a rational equation that is reducible to a quadratic equation**

After each side of a rational equation has been multiplied by the LCM of the denominators, the resulting equation may be a quadratic equation.

HOW TO • 3 Solve: $\frac{1}{r} + \frac{1}{r+1} = \frac{3}{2}$

$$\frac{1}{r} + \frac{1}{r+1} = \frac{3}{2}$$

$$2r(r+1)\left(\frac{1}{r} + \frac{1}{r+1}\right) = 2r(r+1) \cdot \frac{3}{2}$$

- Multiply each side of the equation by the LCM of the denominators.

$$2(r+1) + 2r = r(r+1) \cdot 3$$

$$2r + 2 + 2r = 3r(r+1)$$

$$4r + 2 = 3r^2 + 3r$$

$$0 = 3r^2 - r - 2$$

$$0 = (3r+2)(r-1)$$

- Write the equation in standard form.
- Solve for r by factoring.

$$3r + 2 = 0 \quad r - 1 = 0$$

$$3r = -2 \quad r = 1$$

$$r = -\frac{2}{3}$$

$-\frac{2}{3}$ and 1 check as solutions.

The solutions are $-\frac{2}{3}$ and 1.

EXAMPLE • 4

Solve: $\frac{9}{x-3} = 2x + 1$

Solution

$$\frac{9}{x-3} = 2x + 1$$

$$(x-3)\frac{9}{x-3} = (x-3)(2x+1)$$

$$9 = 2x^2 - 5x - 3$$

$$0 = 2x^2 - 5x - 12$$

$$0 = (2x+3)(x-4)$$

- Clear denominators.
- Write in standard form.
- Solve by factoring.

$$2x + 3 = 0 \quad x - 4 = 0$$

$$2x = -3 \quad x = 4$$

$$x = -\frac{3}{2}$$

The solutions are $-\frac{3}{2}$ and 4.

YOU TRY IT • 4

Solve: $3y + \frac{25}{3y-2} = -8$

Your solution

8.4 EXERCISES

OBJECTIVE A

To solve an equation that is quadratic in form

 For Exercises 1 to 4, state whether the equation could be solved by writing it as a quadratic equation of the form $u^2 - 8u - 20 = 0$.

1. $x^{10} - 8x^5 - 20 = 0$ 2. $x^{16} - 8x^4 - 20 = 0$ 3. $x^{1/10} - 8x^{1/5} - 20 = 0$ 4. $x^{2/5} - 8x^{1/5} - 20 = 0$

For Exercises 5 to 22, solve.

5. $x^4 - 13x^2 + 36 = 0$

6. $y^4 - 5y^2 + 4 = 0$

7. $z^4 - 6z^2 + 8 = 0$

8. $t^4 - 12t^2 + 27 = 0$

9. $p - 3p^{1/2} + 2 = 0$

10. $v - 7v^{1/2} + 12 = 0$

11. $x - x^{1/2} - 12 = 0$

12. $w - 2w^{1/2} - 15 = 0$

13. $z^4 + 3z^2 - 4 = 0$

14. $y^4 + 5y^2 - 36 = 0$

15. $x^4 + 12x^2 - 64 = 0$

16. $x^4 - 81 = 0$

17. $p + 2p^{1/2} - 24 = 0$

18. $v + 3v^{1/2} - 4 = 0$

19. $y^{2/3} - 9y^{1/3} + 8 = 0$

20. $z^{2/3} - z^{1/3} - 6 = 0$

21. $9w^4 - 13w^2 + 4 = 0$

22. $4y^4 - 7y^2 - 36 = 0$

OBJECTIVE B

To solve a radical equation that is reducible to a quadratic equation

 23. For which of Exercises 25 to 42 will the first step in solving the equation be to square each side of the equation?

 24. For which of Exercises 25 to 42 will it be necessary to square each side of the equation twice?

For Exercises 25 to 42, solve.

25. $\sqrt{x+1} + x = 5$

26. $\sqrt{x-4} + x = 6$

27. $x = \sqrt{x} + 6$

28. $\sqrt{2y-1} = y - 2$

29. $\sqrt{3w+3} = w + 1$

30. $\sqrt{2s+1} = s - 1$

31. $\sqrt{4y+1} - y = 1$

32. $\sqrt{3s+4} + 2s = 12$

33. $\sqrt{10x+5} - 2x = 1$

34. $\sqrt{t+8} = 2t + 1$

35. $\sqrt{p+11} = 1 - p$

36. $x - 7 = \sqrt{x - 5}$

37. $\sqrt{x-1} - \sqrt{x} = -1$

38. $\sqrt{y+1} = \sqrt{y+5}$

39. $\sqrt{2x-1} = 1 - \sqrt{x-1}$

40. $\sqrt{x+6} + \sqrt{x+2} = 2$

41. $\sqrt{t+3} + \sqrt{2t+7} = 1$

42. $\sqrt{5-2x} = \sqrt{2-x} + 1$

OBJECTIVE C**To solve a rational equation that is reducible to a quadratic equation**

43. To solve Exercise 47, the first step will be to multiply each side of the equation by what expression?



44. To solve Exercise 52, the first step will be to multiply each side of the equation by what expression?

For Exercises 45 to 56, solve.

45. $x = \frac{10}{x-9}$

46. $z = \frac{5}{z-4}$

47. $\frac{y-1}{y+2} + y = 1$

48. $\frac{2p-1}{p-2} + p = 8$

49. $\frac{3r+2}{r+2} - 2r = 1$

50. $\frac{2v+3}{v+4} + 3v = 4$

51. $\frac{2}{2x+1} + \frac{1}{x} = 3$

52. $\frac{3}{s} - \frac{2}{2s-1} = 1$

53. $\frac{16}{z-2} + \frac{16}{z+2} = 6$

54. $\frac{2}{y+1} + \frac{1}{y-1} = 1$

55. $\frac{t}{t-2} + \frac{2}{t-1} = 4$

56. $\frac{4t+1}{t+4} + \frac{3t-1}{t+1} = 2$

Applying the Concepts

57. Solve: $(x^2 - 7)^{1/2} = (x - 1)^{1/2}$

58. Solve: $(\sqrt{x} - 2)^2 - 5\sqrt{x} + 14 = 0$ (Hint: Let $u = \sqrt{x} - 2$.)

59. Solve: $(\sqrt{x} + 3)^2 - 4\sqrt{x} - 17 = 0$ (Hint: Let $u = \sqrt{x} + 3$.)

- 60.
- Mathematics**
- The fourth power of a number is twenty-five less than ten times the square of the number. Find the number.

SECTION

8.5

Quadratic Inequalities and Rational Inequalities

OBJECTIVE A

To solve a nonlinear inequality

A **quadratic inequality** is one that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where $a \neq 0$. The symbols $\leq$ and $\geq$ can also be used. The solution set of a quadratic inequality can be found by solving a compound inequality.

To solve $x^2 - 3x - 10 > 0$, first $x^2 - 3x - 10 > 0$
factor the trinomial. $(x + 2)(x - 5) > 0$

There are two cases for which the product of the factors will be positive: (1) **both factors are positive**, or (2) **both factors are negative**.

$$\begin{aligned} (1) \quad & x + 2 > 0 \quad \text{and} \quad x - 5 > 0 \\ (2) \quad & x + 2 < 0 \quad \text{and} \quad x - 5 < 0 \end{aligned}$$

Solve each pair of compound inequalities.

$$\begin{aligned} (1) \quad & x + 2 > 0 \quad \text{and} \quad x - 5 > 0 \\ & x > -2 \quad \quad \quad x > 5 \\ & \{x | x > -2\} \cap \{x | x > 5\} = \{x | x > 5\} \end{aligned}$$

$$\begin{aligned} (2) \quad & x + 2 < 0 \quad \text{and} \quad x - 5 < 0 \\ & x < -2 \quad \quad \quad x < 5 \\ & \{x | x < -2\} \cap \{x | x < 5\} = \{x | x < -2\} \end{aligned}$$

Because the two cases for which the product will be positive are connected by *or*, the solution set is the union of the solution sets of the individual inequalities.

$$\{x | x > 5\} \cup \{x | x < -2\} = \{x | x > 5 \text{ or } x < -2\}$$

Although the solution set of any quadratic inequality can be found by using the method outlined above, a graphical method is often easier to use.

HOW TO 1

Solve and graph the solution set of $x^2 - x - 6 < 0$.

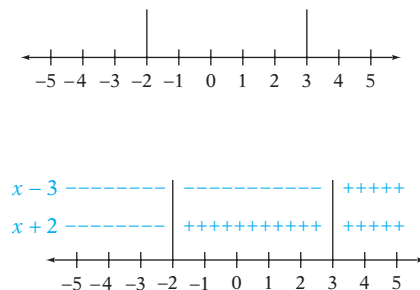
Factor the trinomial.

$$\begin{aligned} x^2 - x - 6 &< 0 \\ (x - 3)(x + 2) &< 0 \end{aligned}$$

On a number line, draw vertical lines indicating the numbers that make each factor equal to zero.

$$\begin{aligned} x - 3 &= 0 & x + 2 &= 0 \\ x &= 3 & x &= -2 \end{aligned}$$

For each factor, place plus signs above the number line for those regions where the factor is positive and minus signs where the factor is negative.

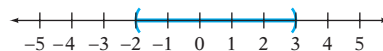


Because $x^2 - x - 6 < 0$, the solution set will be the regions where **one factor is positive and the other factor is negative**.

Write the solution set.

$$\{x | -2 < x < 3\}$$

The graph of the solution set of $x^2 - x - 6 < 0$ is shown at the right.

 **Take Note**

For each factor, choose a number in each region. For example: When $x = -4$, $x - 3$ is negative; when $x = 1$, $x - 3$ is negative; and when $x = 4$, $x - 3$ is positive. When $x = -4$, $x + 2$ is negative; when $x = 1$, $x + 2$ is positive; and when $x = 4$, $x + 2$ is positive.

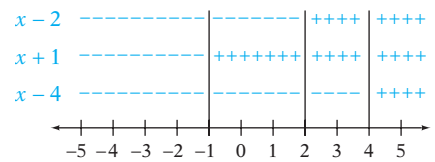
HOW TO 2Solve and graph the solution set of $(x - 2)(x + 1)(x - 4) > 0$.

On a number line, identify for each factor the regions where the factor is positive and those where the factor is negative.

There are two regions where the product of the three factors is positive.

Write the solution set.

The graph of the solution set of $(x - 2)(x + 1)(x - 4) > 0$ is shown at the right.



$$\{x \mid -1 < x < 2 \text{ or } x > 4\}$$

**HOW TO 3**Solve: $\frac{2x - 5}{x - 4} \leq 1$

Rewrite the inequality so that zero appears on the right side of the inequality.

Simplify.

On a number line, identify for each factor of the numerator and each factor of the denominator the regions where the factor is positive and those where the factor is negative.

The region where the quotient of the two factors is negative is between 1 and 4.

Write the solution set.

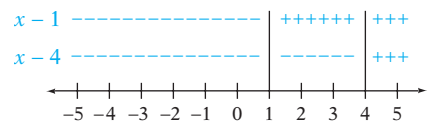
Note that 1 is part of the solution set but 4 is not because the denominator of the rational expression is zero when $x = 4$.

$$\frac{2x - 5}{x - 4} \leq 1$$

$$\frac{2x - 5}{x - 4} - 1 \leq 0$$

$$\frac{2x - 5}{x - 4} - \frac{x - 4}{x - 4} \leq 0$$

$$\frac{x - 1}{x - 4} \leq 0$$



$$\{x \mid 1 \leq x < 4\}$$

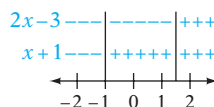
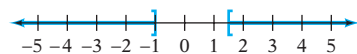
EXAMPLE 1

Solve and graph the solution set of $2x^2 - x - 3 \geq 0$.

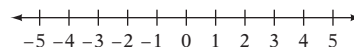
Solution

$$\begin{aligned} 2x^2 - x - 3 &\geq 0 \\ (2x - 3)(x + 1) &\geq 0 \end{aligned}$$

$$\{x \mid x \leq -1 \text{ or } x \geq \frac{3}{2}\}$$

**YOU TRY IT 1**

Solve and graph the solution set of $2x^2 - x - 10 \leq 0$.

Your solution

8.5 EXERCISES

OBJECTIVE A

To solve a nonlinear inequality



1. If
- $(x - 3)(x - 5) > 0$
- , what must be true of the values of
- $x - 3$
- and
- $x - 5$
- ?



2. For the inequality
- $\frac{x-2}{x-3} \leq 1$
- , which of the values 1, 2, and 3 is not a possible element of the solution set? Why?

For Exercises 3 to 6, for the given values of x , state whether the inequality is true or false.

- 3.
- $(x - 3)(x + 2)(x - 4) > 0$
- a.
- $x = 2$
- b.
- $x = -2$
- c.
- $x = -3$

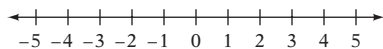
- 4.
- $\frac{(x + 1)(x - 5)}{x - 3} \geq 0$
- a.
- $x = 2$
- b.
- $x = 3$
- c.
- $x = -1$

- 5.
- $(x + 5)(x - 6) \leq 0$
- a.
- $x < 6$
- b.
- $-5 \leq x \leq 6$

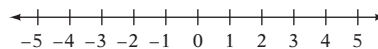
- 6.
- $\frac{x - 4}{x + 3} > 0$
- a.
- $x < -3$
- b.
- $x > 4$

For Exercises 7 to 22, solve and graph the solution set.

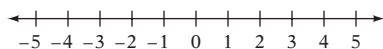
- 7.
- $(x - 4)(x + 2) > 0$



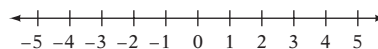
- 8.
- $(x + 1)(x - 3) > 0$



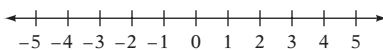
- 9.
- $x^2 - 3x + 2 \geq 0$



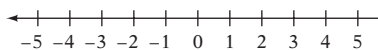
- 10.
- $x^2 + 5x + 6 > 0$



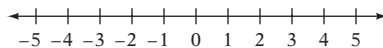
- 11.
- $x^2 - x - 12 < 0$



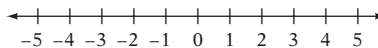
- 12.
- $x^2 + x - 20 < 0$



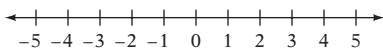
- 13.
- $(x - 1)(x + 2)(x - 3) < 0$



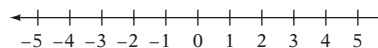
- 14.
- $(x + 4)(x - 2)(x + 1) > 0$



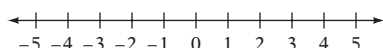
- 15.
- $(x + 4)(x - 2)(x - 1) \geq 0$



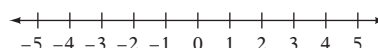
- 16.
- $(x - 1)(x + 5)(x - 2) \leq 0$



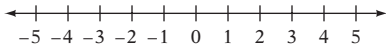
- 17.
- $\frac{x - 4}{x + 2} > 0$



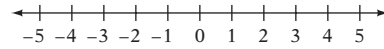
- 18.
- $\frac{x + 2}{x - 3} > 0$



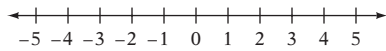
19. $\frac{x-3}{x+1} \leq 0$



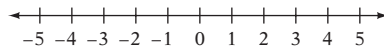
20. $\frac{x-1}{x} > 0$



21. $\frac{(x-1)(x+2)}{x-3} \leq 0$



22. $\frac{(x+3)(x-1)}{x-2} \geq 0$



For Exercises 23 to 37, solve.

23. $x^2 - 16 > 0$

24. $x^2 - 4 \geq 0$

25. $x^2 - 9x \leq 36$

26. $x^2 + 4x > 21$

27. $4x^2 - 8x + 3 < 0$

28. $2x^2 + 11x + 12 \geq 0$

29. $\frac{3}{x-1} < 2$

30. $\frac{x}{(x-1)(x+2)} \geq 0$

31. $\frac{x-2}{(x+1)(x-1)} \leq 0$

32. $\frac{1}{x} < 2$

33. $\frac{x}{2x-1} \geq 1$

34. $\frac{x}{2x-3} \leq 1$

35. $\frac{x}{2-x} \leq -3$

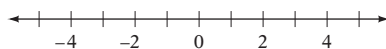
36. $\frac{3}{x-2} > \frac{2}{x+2}$

37. $\frac{3}{x-5} > \frac{1}{x+1}$

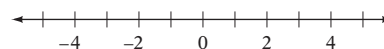
Applying the Concepts

For Exercises 38 to 43, graph the solution set.

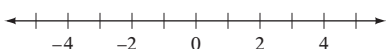
38. $(x+2)(x-3)(x+1)(x+4) > 0$



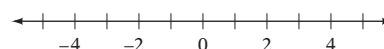
39. $(x-1)(x+3)(x-2)(x-4) \geq 0$



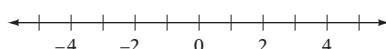
40. $(x^2 + 2x - 8)(x^2 - 2x - 3) < 0$



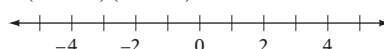
41. $(x^2 + 2x - 3)(x^2 + 3x + 2) \geq 0$



42. $(x^2 + 1)(x^2 - 3x + 2) > 0$



43. $\frac{x^2(3-x)(2x+1)}{(x+4)(x+2)} \geq 0$



SECTION

8.6

Applications of Quadratic Equations

OBJECTIVE A

To solve application problems



There are various applications of quadratic equations.

HOW TO 1

Assuming no air resistance, the height h , in feet, of a soccer ball x feet from where it is kicked at an angle of 45° to the ground can be given by

$$h = -\frac{32}{v^2}x^2 + x, \text{ where } v \text{ is the initial speed of the soccer ball in feet per second.}$$

If a soccer ball is kicked with an initial speed of 50 ft/s, how far from where the ball was kicked is the height of the ball 15 ft? Round to the nearest tenth.

Strategy

To find where the soccer ball will be 15 ft above the ground, use the equation

$$h = -\frac{32}{v^2}x^2 + x. \text{ Substitute 15 for } h \text{ and 50 for } v, \text{ and then solve for } x.$$

Solution

$$h = -\frac{32}{v^2}x^2 + x$$

$$15 = -\frac{32}{50^2}x^2 + x$$

- Replace h by 15 and v by 50.

$$0 = -0.0128x^2 + x - 15$$

- Write the equation in standard form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Solve by using the quadratic formula.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-0.0128)(-15)}}{2(-0.0128)}$$

- $a = -0.0128, b = 1, c = -15$

$$= \frac{-1 \pm \sqrt{0.232}}{-0.0256} \approx \frac{-1 \pm 0.48166}{-0.0256}$$

$$x = \frac{-1 + 0.48166}{-0.0256} \quad \text{or} \quad x = \frac{-1 - 0.48166}{-0.0256}$$

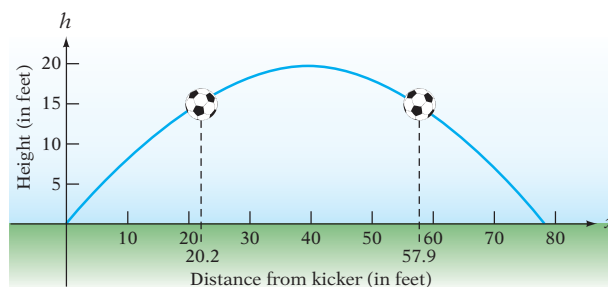
$$= \frac{-0.51834}{-0.0256} \quad = \frac{-1.48166}{-0.0256}$$

$$\approx 20.2$$

$$\approx 57.9$$

The ball is 15 ft above the ground when it is 20.2 ft and 57.9 ft from where it was kicked.

A drawing of the flight of the ball is shown below. Note that the ball is 15 ft high at two locations, 20.2 ft from the kicker and 57.9 ft from the kicker.



EXAMPLE • 1

A small pipe takes 2 h longer to empty a tank than does a larger pipe. After working together for 1 h, the larger pipe quits. It takes the smaller pipe 1 more hour to empty the tank. How long would it take each pipe, working alone, to empty the tank?

Strategy

- This is a work problem.
- The unknown time for the larger pipe to empty the tank working alone: t
- The unknown time for the smaller pipe to empty the tank working alone: $t + 2$
- The larger pipe operates for 1 h. The smaller pipe operates for 2 h.

	Rate	•	Time	=	Part
Larger pipe	$\frac{1}{t}$	•	1	=	$\frac{1}{t}$
Smaller pipe	$\frac{1}{t+2}$	•	2	=	$\frac{2}{t+2}$

- The sum of the part of the task completed by the larger pipe and the part completed by the smaller pipe equals 1.

Solution

$$\begin{aligned}\frac{1}{t} + \frac{2}{t+2} &= 1 \\ t(t+2)\left(\frac{1}{t} + \frac{2}{t+2}\right) &= t(t+2) \cdot 1 \\ (t+2) + 2t &= t^2 + 2t \\ 0 &= t^2 - t - 2 \\ 0 &= (t+1)(t-2) \\ t+1 &= 0 & t-2 &= 0 \\ t &= -1 & t &= 2\end{aligned}$$

Because time cannot be negative, $t = -1$ is not possible. **It takes the larger pipe, working alone, 2 h to empty the tank.**

The time for the smaller pipe to empty the tank working alone is $t + 2$.

$$\begin{aligned}t + 2 \\ 2 + 2 = 4\end{aligned} \quad \bullet \text{ Replace } t \text{ by } 2.$$

It takes the smaller pipe, working alone, 4 h to empty the tank.

YOU TRY IT • 1

It takes William 3 h longer than it does Olivia to detail a car. Working together, they can detail the car in 2 h. How long would it take William, working alone, to detail the car?

Your strategy**Your solution**

EXAMPLE • 2

In 8 h, two campers rowed 15 mi down a river and then rowed back to their campsite. The rate of the river's current was 1 mph. Find the rate at which the campers row in still water.

Strategy

- Unknown rowing rate of the campers: r

	<i>Distance</i>	$\div$	<i>Rate</i>	$=$	<i>Time</i>
Down river	15	$\div$	$r + 1$	$=$	$\frac{15}{r + 1}$
Up river	15	$\div$	$r - 1$	$=$	$\frac{15}{r - 1}$

- The total time of the trip was 8 h.

Solution

$$\begin{aligned}\frac{15}{r + 1} + \frac{15}{r - 1} &= 8 \\ (r + 1)(r - 1)\left(\frac{15}{r + 1} + \frac{15}{r - 1}\right) &= (r + 1)(r - 1) \cdot 8 \\ 15(r - 1) + 15(r + 1) &= (r^2 - 1)8 \\ 30r &= 8r^2 - 8 \\ 0 &= 8r^2 - 30r - 8 \\ 0 &= 2(4r + 1)(r - 4) \\ 4r + 1 = 0 &\quad r - 4 = 0 \\ r = -\frac{1}{4} &\quad r = 4\end{aligned}$$

The rate cannot be negative, so the solution $-\frac{1}{4}$ is not possible. **The rowing rate was 4 mph.**

EXAMPLE • 3

The length of a rectangle is 5 in. more than the width. The area is 36 in^2 . Find the width of the rectangle.

Strategy

- Width of the rectangle: w
- Length of the rectangle: $w + 5$
- Use the formula $A = lw$.

Solution

$$\begin{aligned}A &= lw \\ 36 &= (w + 5)w \\ 36 &= w^2 + 5w \\ 0 &= w^2 + 5w - 36 \\ 0 &= (w + 9)(w - 4) \\ w + 9 = 0 &\quad w - 4 = 0 \\ w = -9 &\quad w = 4\end{aligned}$$

The width cannot be negative. **The width of the rectangle is 4 in.**

YOU TRY IT • 2

The rate of a jet in calm air is 250 mph. Flying with the wind, the jet can fly 1200 mi in 2 h less time than is required to make the return trip against the wind. Find the rate of the wind.

Your strategy**Your solution****YOU TRY IT • 3**

The base of a triangle is 4 in. more than twice the height. The area is 35 in^2 . Find the height of the triangle.

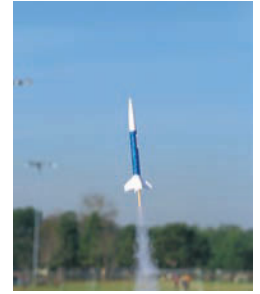
Your strategy**Your solution**

8.6 EXERCISES

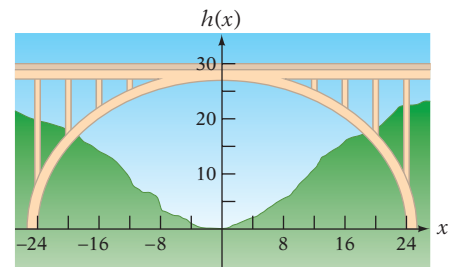
OBJECTIVE A

To solve application problems

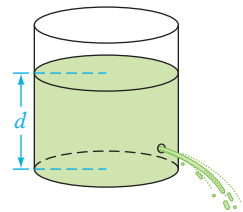
- Safety** A car with good tire tread can stop in less distance than a car with poor tread. The formula for the stopping distance d , in feet, of a car with good tread on dry cement is approximated by $d = 0.04v^2 + 0.5v$, where v is the speed of the car. If the driver must be able to stop within 60 ft, what is the maximum safe speed, to the nearest mile per hour, of the car?
- Rockets** A model rocket is launched with an initial velocity of 200 ft/s. The height h , in feet, of the rocket t seconds after the launch is given by $h = -16t^2 + 200t$. How many seconds after the launch will the rocket be 300 ft above the ground? Round to the nearest hundredth of a second.
- Physics** The height of a projectile fired upward is given by the formula $s = v_0t - 16t^2$, where s is the height in feet, v_0 is the initial velocity, and t is the time in seconds. Find the time for a projectile to return to Earth if it has an initial velocity of 200 ft/s.
- Physics** The height of a projectile fired upward is given by the formula $s = v_0t - 16t^2$, where s is the height in feet, v_0 is the initial velocity, and t is the time in seconds. Find the time for a projectile to reach a height of 64 ft if it has an initial velocity of 128 ft/s. Round to the nearest hundredth of a second.
- Construction** The height h , in feet, of an arch is given by the equation $h(x) = -\frac{3}{64}x^2 + 27$, where $|x|$ is the distance in feet from the center of the arch.
 - What is the maximum height of the arch?
 - What is the height of the arch 8 ft to the right of the center?
 - How far from the center is the arch 8 ft tall? Round to the nearest hundredth.



© Bill Aron/PhotoEdit, Inc.



- Physics** Using Torricelli's Principle, it can be shown that the depth d of a liquid in a bottle with a hole of area 0.5 cm^2 in its side can be approximated by $d = 0.0034t^2 - 0.52518t + 20$, where t is the time since a stopper was removed from the hole. When will the depth be 10 cm? Round to the nearest tenth of a second.



- Sports** The Water Cube was built in Beijing, China, to house the National Swimming Center for the 2008 Olympics. Although not actually a cube (its height is not equal to its length and width), the Water Cube is designed to look like a "cube" of water molecules. The volume of the 31-meter-high Water Cube is $971,199 \text{ m}^3$. Find the length of a side of its square base. Recall that $V = LWH$.



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- Suppose a large pipe and a smaller pipe, working together, can empty a swimming pool in 4 h. Is it possible for the larger pipe, working alone, to empty the tank in less than 4 h?

9. **Tanks** A small pipe can fill a tank in 6 min more time than it takes a larger pipe to fill the same tank. Working together, the pipes can fill the tank in 4 min. How long would it take each pipe, working alone, to fill the tank?
10. **Landscaping** It takes a small sprinkler 16 min longer to soak a lawn than it takes a larger sprinkler. Working together, the sprinklers can soak the lawn in 6 min. How long would it take each sprinkler, working alone, to soak the lawn?
11. **Parallel Processing** Parallel processing is the simultaneous use of more than one computer to run a program. Suppose one computer, working alone, takes 4 h longer than a second computer to run a program that can determine whether a large number is prime. After both computers work together for 1 h, the faster computer crashes. The slower computer continues working for another 2 h before completing the program. How long would it take the faster computer, working alone, to run the program?
12. **Payroll** It takes one printer, working alone, 6 h longer to print a weekly payroll than it takes a second printer. Working together, the two printers can print the weekly payroll in 4 h. How long would it take each printer, working alone, to print the payroll?
13. **Wood Flooring** It takes an apprentice carpenter 2 h longer to install a small section of wood floor than it does a more experienced carpenter. After the carpenters work together for 2 h, the experienced carpenter leaves for another job. It then takes the apprentice carpenter 2 h longer to complete the installation. How long would it take the apprentice carpenter, working alone, to install the floor? Round to the nearest tenth.
14. **Uniform Motion** A cruise ship made a trip of 100 mi in 8 h. The ship traveled the first 40 mi at a constant rate before increasing its speed by 5 mph. Then it traveled another 60 mi at the increased speed. Find the rate of the cruise ship for the first 40 mi.
15. **Uniform Motion** The Concorde's speed in calm air was 1320 mph. Flying with the wind, the Concorde could fly from New York to London, a distance of approximately 4000 mi, in 0.5 h less time than is required to make the return trip. Find the rate of the wind to the nearest mile per hour.
16. **Uniform Motion** A car travels 120 mi. A second car, traveling 10 mph faster than the first car, makes the same trip in 1 h less time. Find the speed of each car.
17. **Air Force One** The Air Force uses the designation VC-25 for the plane on which the president of the United States flies. When the president is on the plane, its call sign is Air Force One. The plane's speed in calm air is 630 mph. Flying with the jet stream, the plane can fly from Washington, D.C., to London, a distance of approximately 3660 mi, in 1.75 h less time than is required to make the return trip. Find the rate of the jet stream to the nearest mile per hour.
18. **Uniform Motion** For a portion of the Green River in Utah, the rate of the river's current is 4 mph. A tour guide can row 5 mi down this river and back in 3 h. Find the rowing rate of the guide in calm water.

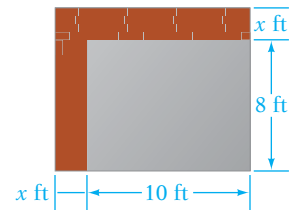
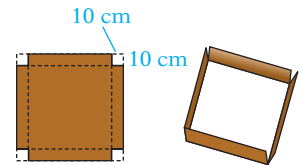
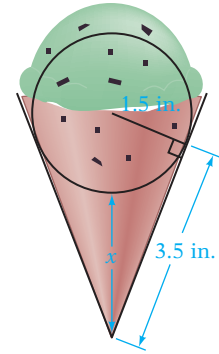
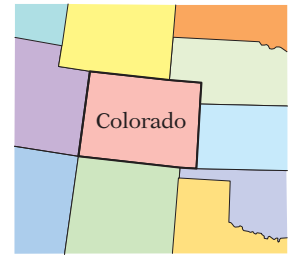


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19. **Geography** The state of Colorado is almost perfectly rectangular, with its north border 111 mi longer than its west border. If the state encompasses 104,000 mi², estimate the dimensions of Colorado. Round to the nearest mile.
20. **Geometry** The length of a rectangle is 2 ft less than three times the width of the rectangle. The area of the rectangle is 65 ft². Find the length and width of the rectangle.
21. **Geometry** The length of the base of a triangle is 1 cm less than five times the height of the triangle. The area of the triangle is 21 cm². Find the height of the triangle and the length of the base of the triangle.
22. **Ice Cream** A perfectly spherical scoop of mint chocolate chip ice cream is placed in a cone, as shown in the figure. How far is the bottom of the scoop of ice cream from the bottom of the cone? Round to the nearest tenth. (*Hint*: A line segment from the center of the ice cream to the point at which the ice cream touches the cone is perpendicular to the edge of the cone.)
23. **Geometry** A square piece of cardboard is formed into a box by cutting 10-centimeter squares from each of the four corners and then folding up the sides, as shown in the figure. If the volume V of the box is to be 49,000 cm³, what size square piece of cardboard is needed? Recall that $V = LWH$.
24. **Construction** A homeowner hires a mason to lay a brick border around a cement patio that measures 8 ft by 10 ft. If the total area of the patio and border is 168 ft², what is the width of the border?
25. **Fencing** A dog trainer has 80 ft of fencing with which to create a rectangular work area for dogs. If the trainer wants to enclose an area of 300 ft², what will be the dimensions of the work area?



26. **Sports** Read the article at the right.
- The screen behind the east end zone is a rectangle with length 13 ft less than three times its width. Find the length and width of the east end zone screen.
 - The screen behind the west end zone is a rectangle with length 1 ft less than twice its width. Find the length and width of the west end zone screen.

In the News

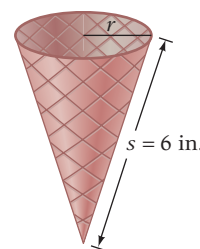
Dolphins on the Big Screen

The Miami Dolphins' stadium, Landshark Stadium, is home to one of the world's largest digital displays—a 6850-square-foot screen installed behind the east end zone. A smaller screen, of area 4950 ft², sits behind the west end zone.

Source: Business Wire

Applying the Concepts

27. **Geometry** The surface area of the ice cream cone shown at the right is given by $A = \pi r^2 + \pi rs$, where r is the radius of the circular top of the cone and s is the slant height of the cone. If the surface area of the cone is 11.25π in² and the slant height is 6 in., find the radius of the cone.



FOCUS ON PROBLEM SOLVING

Inductive and
Deductive
Reasoning

Consider the following sums of odd positive integers.

- $1 + 3 = 4 = 2^2$
- Sum of the first two odd numbers is 2^2 .
- $1 + 3 + 5 = 9 = 3^2$
- Sum of the first three odd numbers is 3^2 .
- $1 + 3 + 5 + 7 = 16 = 4^2$
- Sum of the first four odd numbers is 4^2 .
- $1 + 3 + 5 + 7 + 9 = 25 = 5^2$
- Sum of the first five odd numbers is 5^2 .

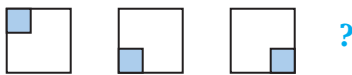
1. Make a conjecture about the value of the sum

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$$

without adding the numbers.

If the pattern continues, a possible conjecture is that the sum of the first eight odd positive integers is $8^2 = 64$. By adding the numbers, you can verify that this is correct. Inferring that the pattern established by a few cases is valid for all cases is an example of **inductive reasoning**.

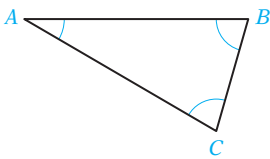
2. Use inductive reasoning to find the next figure in the sequence below.



The fact that a pattern *appears* to be true does not *prove* it is true for all cases. For example, consider the polynomial $n^2 - n + 41$. If we begin substituting positive integer values for n and evaluating the expression, we produce the following table.

n	$n^2 - n + 41$
1	$1^2 - 1 + 41 = 41$ 41 is a prime number.
2	$2^2 - 2 + 41 = 43$ 43 is a prime number.
3	$3^2 - 3 + 41 = 47$ 47 is a prime number.
4	$4^2 - 4 + 41 = 53$ 53 is a prime number.

Even if we try a number like 30, we have $30^2 - 30 + 41 = 911$, and 911 is a prime number. Thus it appears that the value of this polynomial is a prime number for any value of n . However, the conjecture is not true. For instance, if $n = 41$, we have $41^2 - 41 + 41 = 41^2 = 1681$, which is not a prime number because it is divisible by 41. This illustrates that inductive reasoning may lead to incorrect conclusions and that an inductive proof must be available to prove conjectures. There is such a proof, called *mathematical induction*, that you may study in a future math course.

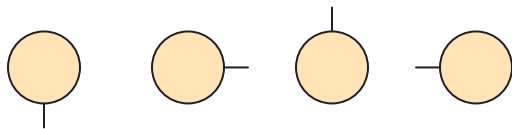


$$\angle A + \angle B + \angle C = 180^\circ$$

Now consider the true statement that the sum of the measures of the interior angles of a triangle is 180° . The figure at the left is a triangle; therefore, the sum of the measures of the interior angles must be 180° . This is an example of *deductive reasoning*. **Deductive reasoning** uses a rule or statement of fact to reach a conclusion.

3. Use deductive reasoning to complete the following sentence. All even numbers are divisible by 2. Because 14,386 is an even number, . . .

4. a. Use the pattern given below to find the three missing terms in 64.
A 2 3 4 B 6 7 8 C 10 11 12 D 14 15 16...
- b. Did you use inductive or deductive reasoning?
5. a. Draw the next figure in the sequence:



- b. Did you use inductive or deductive reasoning?
6. a. If $\diamond\diamond\diamond\diamond = \ddagger\ddagger\ddagger$ and $\ddagger\ddagger\ddagger = \nabla\nabla\nabla$, how many $\diamond$'s equal $\nabla\nabla\nabla\nabla\nabla\nabla$?
- b. Did you use inductive or deductive reasoning?
7. a. If $\spadesuit\spadesuit = \blacklozenge\blacklozenge\blacklozenge\blacklozenge$, and $\blacklozenge\blacklozenge = \clubsuit\clubsuit$, and $\clubsuit\clubsuit\clubsuit\clubsuit = \heartsuit$, how many $\heartsuit$'s equal $\spadesuit\spadesuit\spadesuit\spadesuit\spadesuit\spadesuit$?
- b. Did you use inductive or deductive reasoning?



Dale C. Spartas/Corbis

- For Exercises 8 to 10, determine whether inductive or deductive reasoning is being used.
8. The Atlanta Braves have won eight games in a row. Therefore, the Atlanta Braves will win their next game.
9. All quadrilaterals have four sides. A square is a quadrilateral. Therefore, a square has four sides.
10. Every English setter likes to hunt. Duke is an English setter, so Duke likes to hunt.

PROJECTS AND GROUP ACTIVITIES

Using a Graphing Calculator to Solve a Quadratic Equation

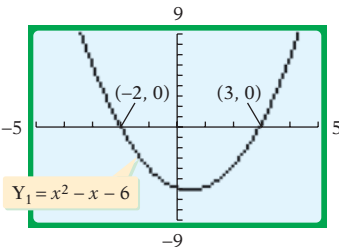


FIGURE 1

Recall that an x -intercept of the graph of an equation is a point at which the graph crosses the x -axis. For the graph in Figure 1, the x -intercepts are $(-2, 0)$ and $(3, 0)$.

Recall also that to find the x -intercept of a graph, set $y = 0$ and then solve for x . For the equation in Figure 1, if we set $y = 0$, the resulting equation is $0 = x^2 - x - 6$, which is a quadratic equation. Solving this equation by factoring, we have

$$\begin{aligned} 0 &= x^2 - x - 6 \\ 0 &= (x + 2)(x - 3) \\ x + 2 &= 0 & x - 3 &= 0 \\ x &= -2 & x &= 3 \end{aligned}$$

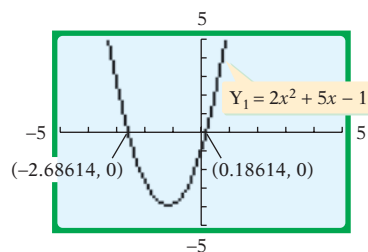


FIGURE 2

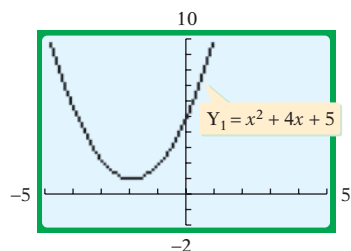


FIGURE 3

Thus the solutions of the equation are the x -coordinates of the x -intercepts of the graph.

This connection between the solutions of an equation and the x -intercepts of its graph enables us to find approximations of the real number solutions of an equation graphically. For example, to use a TI-84 to approximate the solutions of $2x^2 + 5x - 1 = 0$ graphically, use the following keystrokes to graph $y = 2x^2 + 5x - 1$ and find the decimal approximations of the x -coordinates of the x -intercepts. Use the window shown in the graph in Figure 2.

$Y=$ CLEAR 2 X,T,θ,n x^2 + 5 X,T,θ,n - 1 GRAPH 2ND CALC 2

Use the arrow keys to move to the left of the leftmost x -intercept. Press ENTER. Use the arrow keys to move to a point just to the right of the leftmost x -intercept. Press ENTER twice. The x -coordinate at the bottom of the screen is the approximation of one solution of the equation. To find the other solution, use the same procedure but move the cursor first to the left and then to the right of the rightmost x -intercept.

Attempting to find the solutions of an equation graphically will not necessarily yield all the solutions. Because the x -coordinates of the x -intercepts of a graph are *real* numbers, only real number solutions can be found. For instance, consider the equation $x^2 + 4x + 5 = 0$. The graph of $y = x^2 + 4x + 5$ is shown in Figure 3. Note that the graph has no x -intercepts and that consequently it has no real number solutions. However, $x^2 + 4x + 5 = 0$ does have complex number solutions that can be obtained by using the quadratic formula. They are $-2 + i$ and $-2 - i$.

Use a graphing calculator to approximate the solutions of the equations in Exercises 1 to 4.

1. $x^2 + 6x + 1 = 0$
2. $-x^2 + x + 3 = 0$
3. $2x^2 + 4x - 1 = 0$
4. $x^2 + 4x + 5 = 0$

CHAPTER 8

SUMMARY

KEY WORDS

A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation is also called a *second-degree equation*. A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. [8.1A, p. 446]

When a quadratic equation has two solutions that are the same number, the solution is called a *double root* of the equation. [8.1A, p. 446]

EXAMPLES

$3x^2 + 4x - 7 = 0$, $x^2 - 1 = 0$, and $4x^2 + 8x = 0$ are quadratic equations. They are all written in standard form.

$x^2 - 4x + 4 = 0$
 $(x - 2)(x - 2) = 0$
 $x - 2 = 0 \quad x - 2 = 0$
 $x = 2 \quad x = 2$
 2 is a double root.

For an equation of the form $ax^2 + bx + c = 0$, the quantity $b^2 - 4ac$ is called the *discriminant*. [8.3A, p. 462]

$$\begin{aligned} 2x^2 - 3x + 5 &= 0 \\ a &= 2, b = -3, c = 5 \\ b^2 - 4ac &= (-3)^2 - 4(2)(5) \\ &= 9 - 40 = -31 \end{aligned}$$

An equation is *quadratic in form* if it can be written as $au^2 + bu + c = 0$. [8.4A, p. 466]

$$\begin{aligned} 6x^4 - 5x^2 - 4 &= 0 \\ 6(x^2)^2 - 5(x^2) - 4 &= 0 \\ 6u^2 - 5u - 4 &= 0 \end{aligned}$$

The equation is quadratic in form.

A *quadratic inequality* is one that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where $a \neq 0$. The symbols $\leq$ and $\geq$ can also be used. [8.5A, p. 472]

$3x^2 + 5x - 8 \leq 0$ is a quadratic inequality.

ESSENTIAL RULES AND PROCEDURES

Principle of Zero Products [8.1A, p. 446]

If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLES

$$\begin{aligned} (x - 3)(x + 4) &= 0 \\ x - 3 &= 0 & x + 4 &= 0 \\ x &= 3 & x &= -4 \end{aligned}$$

To Solve a Quadratic Equation by Factoring [8.1A, p. 446]

1. Write the equation in standard form.
2. Factor the polynomial.
3. Use the Principle of Zero Products to set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

$$\begin{aligned} x^2 + 2x &= 35 \\ x^2 + 2x - 35 &= 0 \\ (x - 5)(x + 7) &= 0 \\ x - 5 &= 0 & x + 7 &= 0 \\ x &= 5 & x &= -7 \end{aligned}$$

To Write a Quadratic Equation Given Its Solutions

[8.1B, p. 447]

Use the equation $(x - r_1)(x - r_2) = 0$. Replace r_1 with one solution and r_2 with the other solution. Then multiply the two factors.

Write a quadratic equation that has solutions -3 and 6 .

$$\begin{aligned} (x - r_1)(x - r_2) &= 0 \\ [x - (-3)](x - 6) &= 0 \\ (x + 3)(x - 6) &= 0 \\ x^2 - 3x - 18 &= 0 \end{aligned}$$

To Solve a Quadratic Equation by Taking Square Roots

[8.1C, pp. 448, 449]

1. Solve for x^2 or for $(x + a)^2$.
2. Take the square root of each side of the equation.
3. Simplify.
4. Check the solutions.

$$\begin{aligned}
 (x + 2)^2 - 9 &= 0 \\
 (x + 2)^2 &= 9 \\
 \sqrt{(x + 2)^2} &= \sqrt{9} \\
 x + 2 &= \pm\sqrt{9} \\
 x + 2 &= \pm 3 \\
 x + 2 = 3 \quad x + 2 &= -3 \\
 x = 1 \quad x &= -5
 \end{aligned}$$

To Complete the Square [8.2A, p. 454]

Add to a binomial of the form $x^2 + bx$ the square of $\frac{1}{2}$ of the coefficient of x , making it a perfect-square trinomial.

To complete the square on $x^2 - 8x$,
 add $\left[\frac{1}{2}(-8)\right]^2 = 16$: $x^2 - 8x + 16$.

To Solve a Quadratic Equation by Completing the Square

[8.2A, p. 456]

1. Write the equation in the form $ax^2 + bx = -c$.
2. Multiply both sides of the equation by $\frac{1}{a}$.
3. Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
4. Factor the perfect-square trinomial.
5. Take the square root of each side of the equation.
6. Solve the resulting equation for x .
7. Check the solutions.

$$\begin{aligned}
 x^2 + 4x - 1 &= 0 \\
 x^2 + 4x &= 1 \\
 x^2 + 4x + 4 &= 1 + 4 \\
 (x + 2)^2 &= 5 \\
 \sqrt{(x + 2)^2} &= \sqrt{5} \\
 x + 2 &= \pm\sqrt{5} \\
 x + 2 = \sqrt{5} \quad x + 2 &= -\sqrt{5} \\
 x = -2 + \sqrt{5} \quad x &= -2 - \sqrt{5}
 \end{aligned}$$

Quadratic Formula [8.3A, p. 460]

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$\begin{aligned}
 2x^2 - 3x + 4 &= 0 \\
 a = 2, b = -3, c &= 4 \\
 x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)} \\
 &= \frac{3 \pm \sqrt{-23}}{4} = \frac{3 \pm i\sqrt{23}}{4} \\
 &= \frac{3}{4} \pm \frac{\sqrt{23}}{4}i
 \end{aligned}$$

The Effect of the Discriminant on the Solutions of a Quadratic Equation [8.3A, p. 462]

1. If $b^2 - 4ac = 0$, the equation has two equal real number solutions, a double root.
2. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions.
3. If $b^2 - 4ac < 0$, the equation has two complex number solutions.

$$\begin{aligned}
 x^2 + 8x + 16 &= 0 \text{ has a double root} \\
 \text{because } b^2 - 4ac &= 8^2 - 4(1)(16) = 0. \\
 2x^2 + 3x - 5 &= 0 \text{ has two unequal real} \\
 \text{number solutions because} \\
 b^2 - 4ac &= 3^2 - 4(2)(-5) = 49. \\
 3x^2 + 2x + 4 &= 0 \text{ has two complex} \\
 \text{number solutions because} \\
 b^2 - 4ac &= 2^2 - 4(3)(4) = -44.
 \end{aligned}$$

CHAPTER 8**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How do you write a quadratic equation if you are given the solutions?
2. What does the $\pm$ symbol mean?
3. Complete the square on the expression $x^2 - 18x$.
4. Given exact solutions $4 + 3\sqrt{2}$ and $4 - 3\sqrt{2}$, what are the approximate solutions? Round to the nearest ten-thousandth.
5. What is the procedure for solving a quadratic equation by completing the square?
6. What is the quadratic formula?
7. What does the discriminant tell you about the solutions to a quadratic equation?
8. Why is it especially important to check the solutions to radical equations?
9. What is a quadratic inequality?
10. In a quadratic inequality, how does the inequality symbol indicate whether or not to include the endpoints of the solution set?
11. In a rational inequality, why is it important to check the elements of the solution set?

CHAPTER 8

REVIEW EXERCISES

1. Solve by factoring: $2x^2 - 3x = 0$
2. Solve for x by factoring: $6x^2 + 9cx = 6c^2$
3. Solve by taking square roots:
 $x^2 = 48$
4. Solve by taking square roots:
 $\left(x + \frac{1}{2}\right)^2 + 4 = 0$
5. Solve by completing the square:
 $x^2 + 4x + 3 = 0$
6. Solve by completing the square:
 $7x^2 - 14x + 3 = 0$
7. Solve by using the quadratic formula:
 $12x^2 - 25x + 12 = 0$
8. Solve by using the quadratic formula:
 $x^2 - x + 8 = 0$
9. Write a quadratic equation that has integer coefficients and has solutions 0 and -3 .
10. Write a quadratic equation that has integer coefficients and has solutions $\frac{3}{4}$ and $-\frac{2}{3}$.
11. Solve by completing the square:
 $x^2 - 2x + 8 = 0$
12. Solve by completing the square:
 $(x - 2)(x + 3) = x - 10$
13. Solve by using the quadratic formula:
 $3x(x - 3) = 2x - 4$
14. Use the discriminant to determine whether $3x^2 - 5x + 3 = 0$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.
15. Solve: $(x + 3)(2x - 5) < 0$
16. Solve: $(x - 2)(x + 4)(2x + 3) \leq 0$

17. Solve: $x^{2/3} + x^{1/3} - 12 = 0$

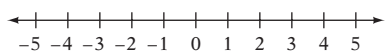
18. Solve: $2(x - 1) + 3\sqrt{x - 1} - 2 = 0$

19. Solve: $3x = \frac{9}{x - 2}$

20. Solve: $\frac{3x + 7}{x + 2} + x = 3$

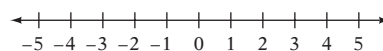
21. Solve and graph the solution set:

$$\frac{x - 2}{2x - 3} \geq 0$$



22. Solve and graph the solution set:

$$\frac{(2x - 1)(x + 3)}{x - 4} \leq 0$$



23. Solve: $x = \sqrt{x} + 2$

24. Solve: $2x = \sqrt{5x + 24} + 3$

25. Solve: $\frac{x - 2}{2x + 3} - \frac{x - 4}{x} = 2$

26. Solve: $1 - \frac{x + 4}{2 - x} = \frac{x - 3}{x + 2}$

27. Write a quadratic equation that has integer coefficients and has solutions $\frac{1}{3}$ and -3 .

28. Solve by factoring: $2x^2 + 9x = 5$

29. Solve: $2(x + 1)^2 - 36 = 0$

30. Solve by using the quadratic formula:
 $x^2 + 6x + 10 = 0$

31. Solve: $\frac{2}{x - 4} + 3 = \frac{x}{2x - 3}$

32. Solve: $x^4 - 28x^2 + 75 = 0$

33. Solve: $\sqrt{2x - 1} + \sqrt{2x} = 3$

34. Solve: $2x^{2/3} + 3x^{1/3} - 2 = 0$

35. Solve: $\sqrt{3x - 2} + 4 = 3x$

36. Solve by completing the square:
 $x^2 - 10x + 7 = 0$

37. Solve: $\frac{2x}{x - 4} + \frac{6}{x + 1} = 11$

38. Solve by using the quadratic formula:
 $9x^2 - 3x = 1$

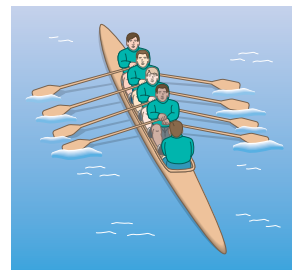
39. Solve: $2x = 4 - 3\sqrt{x - 1}$

40. Solve: $1 - \frac{x + 3}{3 - x} = \frac{x - 4}{x + 3}$

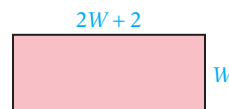
41. Use the discriminant to determine whether $2x^2 - 5x = 6$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

42. Solve: $x^2 - 3x \leq 10$

43. **Sports** To prepare for an upcoming race, a sculling crew rowed 16 mi down a river and back in 6 h. If the rate of the river's current is 2 mph, find the sculling crew's rate of rowing in calm water.



44. **Geometry** The length of a rectangle is 2 cm more than twice the width. The area of the rectangle is 60 cm^2 . Find the length and width of the rectangle.



45. **Integers** The sum of the squares of three consecutive even integers is fifty-six. Find the three integers.

46. **Computers** An older computer requires 12 min longer to print the payroll than does a newer computer. Together the computers can print the payroll in 8 min. Find the time required for the new computer, working alone, to print the payroll.

47. **Uniform Motion** A car travels 200 mi. A second car, making the same trip, travels 10 mph faster than the first car and makes the trip in 1 h less time. Find the speed of each car.

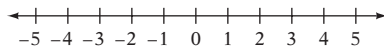
CHAPTER 8**TEST**

1. Solve by factoring: $3x^2 + 10x = 8$
2. Solve by factoring: $6x^2 - 5x - 6 = 0$
3. Write a quadratic equation that has integer coefficients and has solutions 3 and -3 .
4. Write a quadratic equation that has integer coefficients and has solutions $\frac{1}{2}$ and -4 .
5. Solve by taking square roots:
 $3(x - 2)^2 - 24 = 0$
6. Solve by completing the square:
 $x^2 - 6x - 2 = 0$
7. Solve by completing the square:
 $3x^2 - 6x = 2$
8. Solve by using the quadratic formula:
 $2x^2 - 2x = 1$
9. Solve by using the quadratic formula:
 $x^2 + 4x + 12 = 0$
10. Solve: $2x + 7x^{1/2} - 4 = 0$
11. Solve: $x^4 - 4x^2 + 3 = 0$
12. Solve: $\sqrt{2x + 1} + 5 = 2x$

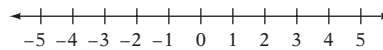
13. Solve: $\sqrt{x-2} = \sqrt{x} - 2$

14. Solve: $\frac{2x}{x-3} + \frac{5}{x-1} = 1$

15. Solve and graph the solution set of
 $(x-2)(x+4)(x-4) < 0$.

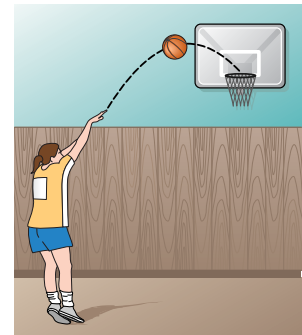


16. Solve and graph the solution set of
 $\frac{2x-3}{x+4} \leq 0$.



17. Use the discriminant to determine whether $9x^2 + 24x = -16$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

18. **Sports** A basketball player shoots at a basket that is 25 ft away. The height h , in feet, of the ball above the ground after t seconds is given by $h = -16t^2 + 32t + 6.5$. How many seconds after the ball is released does it hit the basket, which is 10 ft off the ground? Round to the nearest hundredth.



19. **Woodworking** It takes Clive 6 h longer to stain a bookcase than it does Cora. Working together, they can stain the bookcase in 4 h. Working alone, how long would it take Cora to stain the bookcase?

20. **Uniform Motion** The rate of a river's current is 2 mph. A canoe was rowed 6 mi down the river and back in 4 h. Find the rowing rate in calm water.

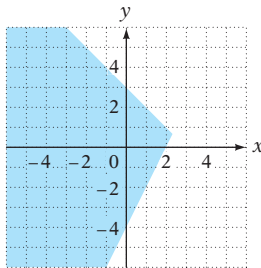
CUMULATIVE REVIEW EXERCISES

1. Evaluate $2a^2 - b^2 \div c^2$ when $a = 3$, $b = -4$, and $c = -2$.
2. Solve: $\frac{2x - 3}{4} - \frac{x + 4}{6} = \frac{3x - 2}{8}$
3. Find the slope of the line containing the points $(3, -4)$ and $(-1, 2)$.
4. Find the equation of the line that contains the point $(1, 2)$ and is parallel to the line $x - y = 1$.
5. Factor: $-3x^3y + 6x^2y^2 - 9xy^3$
6. Factor: $6x^2 - 7x - 20$
7. Factor: $a^n x + a^n y - 2x - 2y$
8. Divide: $(3x^3 - 13x^2 + 10) \div (3x - 4)$
9. Simplify: $\frac{x^2 + 2x + 1}{8x^2 + 8x} \cdot \frac{4x^3 - 4x^2}{x^2 - 1}$
10. Find the distance between the points $(-2, 3)$ and $(2, 5)$.
11. Solve $S = \frac{n}{2}(a + b)$ for b .
12. Simplify: $-2i(7 - 4i)$
13. Simplify: $a^{-1/2}(a^{1/2} - a^{3/2})$
14. Simplify: $\frac{\sqrt[3]{8x^4y^5}}{\sqrt[3]{16xy^6}}$
15. Solve: $\frac{x}{x + 2} - \frac{4x}{x + 3} = 1$
16. Solve: $\frac{x}{2x + 3} - \frac{3}{4x^2 - 9} = \frac{x}{2x - 3}$
17. Solve: $x^4 - 6x^2 + 8 = 0$
18. Solve: $\sqrt{3x + 1} - 1 = x$

19. Solve: $|3x - 2| < 8$

20. Find the x - and y -intercepts of the graph of $6x - 5y = 15$.

21. Graph the solution set:
 $x + y \leq 3$
 $2x - y < 4$

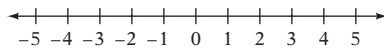


22. Solve by using Cramer's Rule.
 $x + y + z = 2$
 $-x + 2y - 3z = -9$
 $x - 2y - 2z = -1$

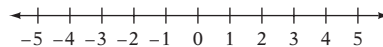
23. Given $f(x) = \frac{2x - 3}{x^2 - 1}$, find $f(-2)$.

24. Find the domain of the function
 $f(x) = \frac{x - 2}{x^2 - 2x - 15}$.

25. Solve and graph the solution set of
 $x^3 + x^2 - 6x < 0$.



26. Solve and graph the solution set of
 $\frac{(x - 1)(x - 5)}{x + 3} \geq 0$.

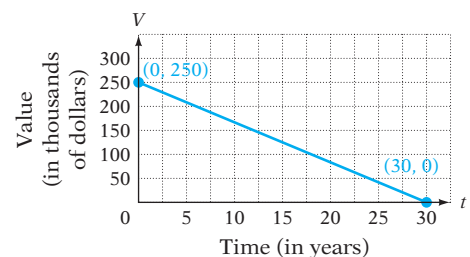


27. **Mechanics** A piston rod for an automobile is $9\frac{3}{8}$ in. long, with a tolerance of $\frac{1}{64}$ in. Find the lower and upper limits of the length of the piston rod.

28. **Geometry** The length of the base of a triangle is $(x + 8)$ ft. The height is $(2x - 4)$ ft. Find the area of the triangle in terms of the variable x .

29. Use the discriminant to determine whether $2x^2 + 4x + 3 = 0$ has two equal real number solutions, two unequal real number solutions, or two complex number solutions.

30. **Depreciation** The graph shows the relationship between the cost of a building and the depreciation allowed for income tax purposes. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



Functions and Relations

OBJECTIVES

SECTION 9.1

- A** To graph a quadratic function
- B** To find the x -intercepts of a parabola
- C** To find the minimum or maximum of a quadratic function
- D** To solve application problems

SECTION 9.2

- A** To graph functions

SECTION 9.3

- A** To perform operations on functions
- B** To find the composition of two functions

SECTION 9.4

- A** To determine whether a function is one-to-one
- B** To find the inverse of a function

ARE YOU READY?

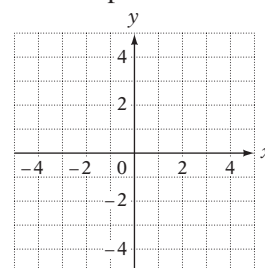
Take the Chapter 9 Prep Test to find out if you are ready to learn to:

- Graph functions
- Find the minimum and maximum values of a quadratic function
- Perform operations on functions
- Find the composition of two functions
- Determine whether a function is one-to-one
- Find the inverse of a function

PREP TEST

Do these exercises to prepare for Chapter 9.

1. Evaluate $-\frac{b}{2a}$ for $b = -4$ and $a = 2$.
2. Given $y = -x^2 + 2x + 1$, find the value of y when $x = -2$.
3. Given $f(x) = x^2 - 3x + 2$, find $f(-4)$.
4. Evaluate $p(r) = r^2 - 5$ when $r = 2 + h$.
5. Solve: $0 = 3x^2 - 7x - 6$
6. Solve by using the quadratic formula: $0 = x^2 - 4x + 1$
7. Solve $x = 2y + 4$ for y .
8. Find the domain and range of the relation $\{(-2, 4), (3, 5), (4, 6), (6, 5)\}$. Is the relation a function?
9. What value is excluded from the domain of $f(x) = \frac{3}{x-8}$?
10. Graph: $x = -2$



SECTION

9.1

Properties of Quadratic Functions

OBJECTIVE A

To graph a quadratic function



Recall that a linear function is one that can be expressed by the equation $f(x) = mx + b$. The graph of a linear function has certain characteristics. It is a straight line with slope m and y -intercept $(0, b)$. A **quadratic function** is one that can be expressed by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. The graph of this function, called a **parabola**, also has certain characteristics. The graph of a quadratic function can be drawn by finding ordered pairs that belong to the function.

HOW TO 1

Graph $f(x) = x^2 - 2x - 3$.

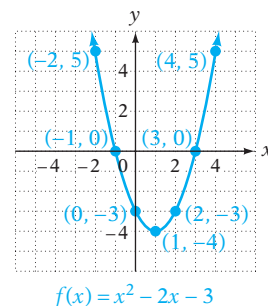
By evaluating the function for various values of x , find enough ordered pairs to determine the shape of the graph.

Take Note

Sometimes the value of the independent variable is called the **input** because it is *put in* place of the independent variable. The result of evaluating the function is called the **output**.

An **input/output table** shows the results of evaluating a function for various values of the independent variable. An input/output table for $f(x) = x^2 - 2x - 3$ is shown at the right.

x	$f(x) = x^2 - 2x - 3$	$f(x)$	(x, y)
-2	$f(-2) = (-2)^2 - 2(-2) - 3$	5	$(-2, 5)$
-1	$f(-1) = (-1)^2 - 2(-1) - 3$	0	$(-1, 0)$
0	$f(0) = (0)^2 - 2(0) - 3$	-3	$(0, -3)$
1	$f(1) = (1)^2 - 2(1) - 3$	-4	$(1, -4)$
2	$f(2) = (2)^2 - 2(2) - 3$	-3	$(2, -3)$
3	$f(3) = (3)^2 - 2(3) - 3$	0	$(3, 0)$
4	$f(4) = (4)^2 - 2(4) - 3$	5	$(4, 5)$



Because the value of $f(x) = x^2 - 2x - 3$ is a real number for all values of x , the domain of f is all real numbers. From the graph, it appears that no value of y is less than -4 . Thus the range is $\{y | y \geq -4\}$. The range can also be determined algebraically, as shown below, by completing the square.

Take Note

In completing the square, 1 is both added and subtracted. Because $1 - 1 = 0$, the expression $x^2 - 2x - 3$ is not changed. Note that

$$\begin{aligned}(x - 1)^2 - 4 &= (x^2 - 2x + 1) - 4 \\ &= x^2 - 2x - 3\end{aligned}$$

which is the original expression.

$$f(x) = x^2 - 2x - 3$$

$$= (x^2 - 2x) - 3$$

$$= (x^2 - 2x + 1) - 1 - 3$$

$$= (x - 1)^2 - 4$$

• **Group the variable terms.**

• **Complete the square on $x^2 - 2x$. Add and subtract**

$$\left[\frac{1}{2}(-2)\right]^2 = 1 \text{ to and from } x^2 - 2x.$$

• **Factor and combine like terms.**

Because the square of a real number is always nonnegative, we have

$$(x - 1)^2 \geq 0$$

$$(x - 1)^2 - 4 \geq -4$$

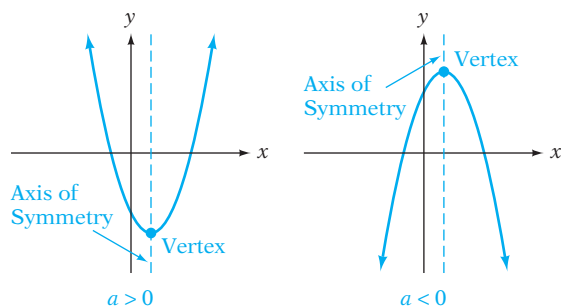
• **Subtract 4 from each side of the inequality.**

$$f(x) \geq -4$$

• **$f(x) = (x - 1)^2 - 4$**

$$y \geq -4$$

From the last inequality, the range is $\{y | y \geq -4\}$.



In general, the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, resembles a “cup” shape as shown at the left. When $a > 0$, the parabola opens up and the **vertex of the parabola** is the point with the smallest y -coordinate. When $a < 0$, the parabola opens down and the vertex is the point with the largest y -coordinate. The **axis of symmetry of a parabola** is the vertical line that passes through the vertex of the parabola and is parallel to the y -axis. To understand the axis of symmetry, think of folding the graph along that vertical line. The two portions of the graph will match up.

The vertex and axis of symmetry of a parabola can be found by completing the square.

HOW TO 2

Find the coordinates of the vertex and the equation of the axis of symmetry for the graph of $F(x) = x^2 + 4x + 3$.

To find the coordinates of the vertex, complete the square.

$$F(x) = x^2 + 4x + 3$$

$$= (x^2 + 4x) + 3$$

$$= (x^2 + 4x + 4) - 4 + 3$$

$$= (x + 2)^2 - 1$$

• **Group the variable terms.**

• **Complete the square on $x^2 + 4x$. Add and subtract $\left[\frac{1}{2}(4)\right]^2 = 4$ to and from $x^2 + 4x$.**

• **Factor and combine like terms.**

Because a , the coefficient of x^2 , is positive ($a = 1$), the parabola opens up and the vertex is the point with the smallest y -coordinate. Because $(x + 2)^2 \geq 0$ for all values of x , the smallest y -coordinate occurs when $(x + 2)^2 = 0$. The quantity $(x + 2)^2$ is equal to zero when $x = -2$. Therefore, **the x -coordinate of the vertex is -2 .**

To find the y -coordinate of the vertex, evaluate the function at $x = -2$.

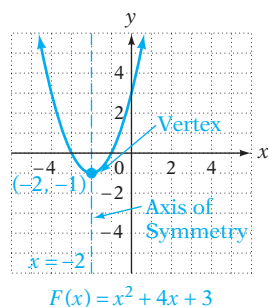
$$F(x) = (x + 2)^2 - 1$$

$$F(-2) = (-2 + 2)^2 - 1 = -1$$

The y -coordinate of the vertex is -1 .

From the results above, **the coordinates of the vertex are $(-2, -1)$.**

The axis of symmetry is the vertical line that passes through the vertex. The equation of the vertical line that passes through the point $(-2, -1)$ is $x = -2$. **The equation of the axis of symmetry is $x = -2$.**



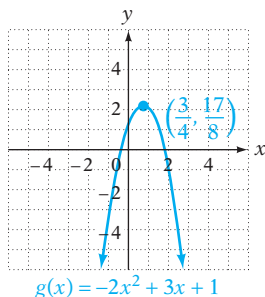
$$F(x) = x^2 + 4x + 3$$

By following the process illustrated in the preceding example and completing the square on $f(x) = ax^2 + bx + c$, we can find a formula for the coordinates of the vertex and the equation of the axis of symmetry of a parabola.

Vertex and Axis of Symmetry of a Parabola

Let $f(x) = ax^2 + bx + c$ be the equation of a parabola. The coordinates of the vertex are

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right). \text{ The equation of the axis of symmetry is } x = -\frac{b}{2a}.$$

**HOW TO • 3**

Find the coordinates of the vertex of the parabola whose equation is $g(x) = -2x^2 + 3x + 1$. Then graph the equation.

$$x\text{-coordinate of the vertex: } -\frac{b}{2a} = -\frac{3}{2(-2)} = \frac{3}{4}$$

• **From the equation**
 $g(x) = -2x^2 + 3x + 1$,
 $a = -2, b = 3$.

$$y\text{-coordinate of the vertex: } g(x) = -2x^2 + 3x + 1$$

$$g\left(\frac{3}{4}\right) = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 1$$

$$g\left(\frac{3}{4}\right) = \frac{17}{8}$$

• **Evaluate the function at the value of the x-coordinate of the vertex.**

The vertex is $\left(\frac{3}{4}, \frac{17}{8}\right)$.

Because a is negative, the graph opens down. Find a few ordered pairs that belong to the function and then sketch the graph, as shown at the left.

Take Note

Once the coordinates of the vertex are found, the range of the quadratic function can be determined.

Once the y-coordinate of the vertex is known, the range of the function can be determined. Here, the graph of g opens down, so the y-coordinate of the vertex is the largest value of y . Therefore, the range of g is $\left\{y \mid y \leq \frac{17}{8}\right\}$. The domain is $\{x \mid x \in \text{real numbers}\}$.

EXAMPLE • 1

Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola whose equation is $y = -x^2 + 4x + 1$. Then graph the equation.

Solution

x -coordinate of vertex:

$$-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

y -coordinate of vertex:

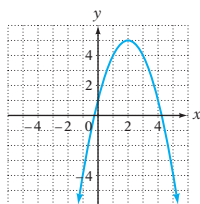
$$y = -x^2 + 4x + 1$$

$$= -(2)^2 + 4(2) + 1$$

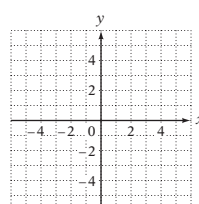
$$= 5$$

Vertex: $(2, 5)$

Axis of symmetry: $x = 2$

**YOU TRY IT • 1**

Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola whose equation is $y = 4x^2 + 4x + 1$. Then graph the equation.

Your solution**EXAMPLE • 2**

Find the domain and range of $f(x) = 0.5x^2 - 3$. Then graph the equation.

Solution

x -coordinate of vertex:

$$-\frac{b}{2a} = -\frac{0}{2(0.5)} = 0$$

y -coordinate of vertex:

$$f(x) = 0.5x^2 - 3$$

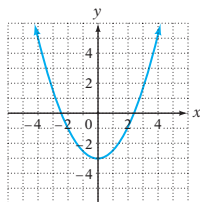
$$f(0) = 0.5(0)^2 - 3$$

$$= -3$$

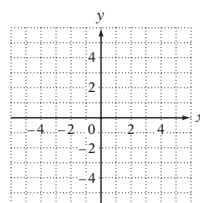
Vertex: $(0, -3)$

The domain is $\{x \mid x \in \text{real numbers}\}$.

The range is $\{y \mid y \geq -3\}$.

**YOU TRY IT • 2**

Find the domain and range of $f(x) = -x^2 - 2x - 1$. Then graph the equation.

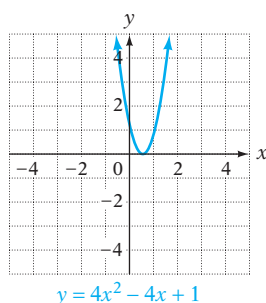
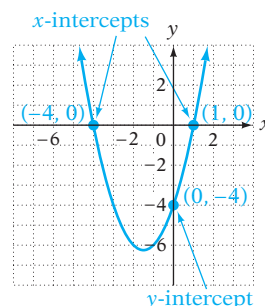
Your solution

OBJECTIVE B

To find the x -intercepts of a parabola

Recall that a point at which a graph crosses the x - or y -axis is called an *intercept* of the graph. The x -intercepts of the graph of an equation occur when $y = 0$; the y -intercepts occur when $x = 0$.

The graph of $y = x^2 + 3x - 4$ is shown at the right. The points whose coordinates are $(-4, 0)$ and $(1, 0)$ are the x -intercepts of the graph. The point whose coordinates are $(0, -4)$ is the y -intercept of the graph.



HOW TO 4

Find the x -intercepts of the graph of $y = 4x^2 - 4x + 1$.

To find the x -intercepts, let $y = 0$ and then solve for x .

$$y = 4x^2 - 4x + 1$$

$$0 = 4x^2 - 4x + 1$$

$$0 = (2x - 1)(2x - 1)$$

• Let $y = 0$.

• Solve for x by factoring and using the Principle of Zero Products.

$$2x - 1 = 0$$

$$2x - 1 = 0$$

$$2x = 1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x = \frac{1}{2}$$

The x -intercept is $\left(\frac{1}{2}, 0\right)$.



Integrating Technology

See the Keystroke Guide: Zero for instructions on using a graphing calculator to find the x -intercepts of the graph of a parabola.

In the preceding example, the parabola has only one x -intercept. In this case, the parabola is said to be *tangent* to the x -axis at $x = \frac{1}{2}$.

HOW TO 5

Find the x -intercepts of the graph of $y = 2x^2 - x - 6$.

To find the x -intercepts, let $y = 0$ and solve for x .

$$y = 2x^2 - x - 6$$

$$0 = 2x^2 - x - 6$$

$$0 = (2x + 3)(x - 2)$$

• Let $y = 0$.

• Solve for x by factoring and using the Principle of Zero Products.

$$2x + 3 = 0$$

$$x - 2 = 0$$

$$x = -\frac{3}{2}$$

$$x = 2$$

The x -intercepts are $\left(-\frac{3}{2}, 0\right)$ and $(2, 0)$.



Take Note

A zero of a function is the x -coordinate of an x -intercept of the graph of the function. Because the x -intercepts of the graph of $f(x) = 2x^2 - x - 6$ are

$\left(-\frac{3}{2}, 0\right)$ and $(2, 0)$, the

zeros are $-\frac{3}{2}$ and 2.

If the equation above, $y = 2x^2 - x - 6$, is written in function notation as $f(x) = 2x^2 - x - 6$, then to find the x -intercepts you would let $f(x) = 0$ and solve for x . A value of x for which $f(x) = 0$ is called a **zero of a function**. Thus $-\frac{3}{2}$ and 2 are zeros of $f(x) = 2x^2 - x - 6$.



Integrating Technology

See the Keystroke Guide: *Zero* for instructions on using a graphing calculator to find the zeros of a function.



Take Note

The graph of $f(x) = x^2 - 2x - 1$ is shown at the right. Note that the zeros are the x -intercepts of the graph of f .

HOW TO 6

Find the zeros of $f(x) = x^2 - 2x - 1$.

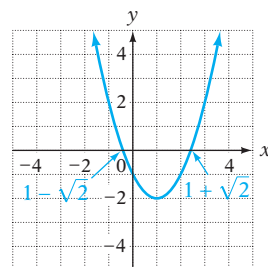
To find the zeros, let $f(x) = 0$ and solve for x .

$$\begin{aligned} f(x) &= x^2 - 2x - 1 \\ 0 &= x^2 - 2x - 1 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm \sqrt{8}}{2} \\ &= \frac{2 \pm 2\sqrt{2}}{2} \\ &= 1 \pm \sqrt{2} \end{aligned}$$

• Because $x^2 - 2x - 1$ does not easily factor, use the quadratic formula to solve for x .

• $a = 1, b = -2, c = -1$

The zeros of the function are $1 - \sqrt{2}$ and $1 + \sqrt{2}$.

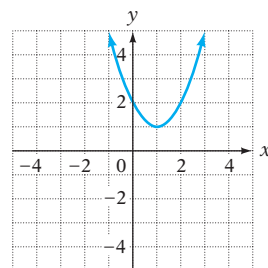


The preceding examples suggest that there is a relationship among the x -intercepts of the graph of a function, the zeros of a function, and the solutions of an equation. In fact, those three concepts are different ways of discussing the same number. The choice depends on the focus of the discussion. If we are discussing graphing, then the intercept is our focus; if we are discussing functions, then the zero of the function is our focus; and if we are discussing equations, the solution of the equation is our focus.

The x -axis is a real number line. Therefore, if a graph crosses the x -axis, it must have a real number zero. It is also true that if a graph does not cross the x -axis, it does not have a real number zero. The zeros in this case are complex numbers.

The graph of $f(x) = x^2 - 2x + 2$ is shown at the right. The graph has no x -intercepts. Thus f has no real zeros.

Using the quadratic formula, the complex number solutions of $x^2 - 2x + 2 = 0$ are $1 - i$ and $1 + i$. Thus the zeros of $f(x) = x^2 - 2x + 2$ are the complex numbers $1 - i$ and $1 + i$.



Tips for Success

The paragraph at the right begins with the word “Recall.” This signals that the content refers to material presented earlier in the text. The ideas presented here will be more meaningful if you return to the discussion of the word *discriminant* on page 462 and review the concepts presented there.

The Effect of the Discriminant on the Number of x -Intercepts of the Graph of a Parabola

1. If $b^2 - 4ac = 0$, the parabola has one x -intercept.
2. If $b^2 - 4ac > 0$, the parabola has two x -intercepts.
3. If $b^2 - 4ac < 0$, the parabola has no x -intercepts.

HOW TO • 7

Use the discriminant to determine the number of x -intercepts of the graph of the parabola whose equation is $y = 2x^2 - x + 2$.

$$b^2 - 4ac$$

$$(-1)^2 - 4(2)(2) = 1 - 16 = -15$$

$$-15 < 0$$

• Evaluate the discriminant.

• $a = 2, b = -1, c = 2$

The discriminant is less than zero. Therefore, the parabola has no x -intercepts.

EXAMPLE • 3

Find the x -intercepts of the graph of $y = 2x^2 - 5x + 2$.

Solution

$$y = 2x^2 - 5x + 2$$

$$0 = 2x^2 - 5x + 2$$

$$0 = (2x - 1)(x - 2)$$

• Let $y = 0$.

• Solve for x by factoring.

$$2x - 1 = 0 \quad x - 2 = 0$$

$$2x = 1 \quad x = 2$$

$$x = \frac{1}{2}$$

The x -intercepts are $\left(\frac{1}{2}, 0\right)$ and $(2, 0)$.

EXAMPLE • 4

Find the zeros of $f(x) = x^2 + 4x + 5$.

Solution

$$f(x) = x^2 + 4x + 5$$

$$0 = x^2 + 4x + 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

• Let $f(x) = 0$.

• Use the quadratic formula.

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} \quad \bullet a = 1, b = 4, c = 5$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2} = -2 \pm i$$

The zeros of the function are $-2 + i$ and $-2 - i$.

EXAMPLE • 5

Use the discriminant to determine the number of x -intercepts of the graph of $y = x^2 - 6x + 9$.

Solution

$$b^2 - 4ac$$

$$(-6)^2 - 4(1)(9)$$

$$= 36 - 36 = 0$$

• Evaluate the discriminant.

• $a = 1, b = -6, c = 9$

The discriminant is equal to zero.

The parabola has one x -intercept.

YOU TRY IT • 3

Find the x -intercepts of the graph of $y = x^2 + 3x + 4$.

Your solution**YOU TRY IT • 4**

Find the zeros of $g(x) = x^2 - x + 6$.

Your solution**YOU TRY IT • 5**

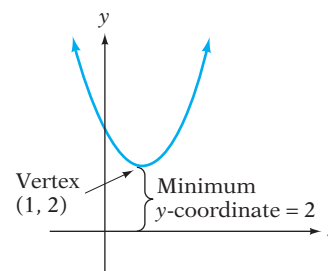
Use the discriminant to determine the number of x -intercepts of the graph of $y = x^2 - x - 6$.

Your solution

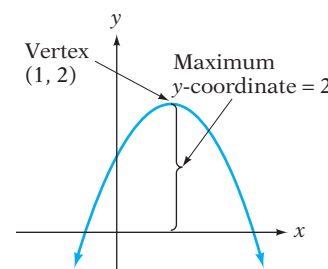
OBJECTIVE C**To find the minimum or maximum of a quadratic function****Tips for Success**

After studying this objective, you should be able to describe in words what the minimum or maximum value of a quadratic function is, draw a graph of a quadratic function with either a maximum or a minimum value, determine from a quadratic equation whether it has a minimum or a maximum value, and calculate algebraically the minimum or maximum value of a quadratic function.

The graph of $f(x) = x^2 - 2x + 3$ is shown at the right. Because a is positive, the parabola opens up. The vertex of the parabola is the lowest point on the parabola. It is the point that has the minimum y -coordinate. This point represents the **minimum value of the function**.



The graph of $f(x) = -x^2 + 2x + 1$ is shown at the right. Because a is negative, the parabola opens down. The vertex of the parabola is the highest point on the parabola. It is the point that has the maximum y -coordinate. This point represents the **maximum value of the function**.



To find the minimum or maximum value of a quadratic function, first find the x -coordinate of the vertex. Then evaluate the function at that value.

HOW TO 8

Find the maximum or minimum value of $f(x) = -2x^2 + 4x + 3$.

$$x = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1$$

$$f(x) = -2x^2 + 4x + 3$$

$$\begin{aligned} f(1) &= -2(1)^2 + 4(1) + 3 \\ &= -2 + 4 + 3 \\ &= 5 \end{aligned}$$

The maximum value of the function is 5.

- Find the x -coordinate of the vertex.
 $a = -2, b = 4$

- Evaluate the function at $x = 1$.

- Because $a < 0$, the graph of f opens down. Therefore, the function has a maximum value.

**Integrating Technology**

See the Projects and Group Activities at the end of this chapter for instructions on using a graphing calculator to find the minimum or maximum value of a function.

EXAMPLE 6

Find the maximum or minimum value of $f(x) = 2x^2 - 3x + 1$.

Solution

$$x = -\frac{b}{2a} = -\frac{-3}{2(2)} = \frac{3}{4}$$

$$f(x) = 2x^2 - 3x + 1$$

$$\begin{aligned} f\left(\frac{3}{4}\right) &= 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1 \\ &= \frac{9}{8} - \frac{9}{4} + 1 = -\frac{1}{8} \end{aligned}$$

Because a is positive, the graph opens up. The function has a minimum value.

The minimum value of the function is $-\frac{1}{8}$.

- The x -coordinate of the vertex

$$\bullet x = \frac{3}{4}$$

YOU TRY IT 6

Find the maximum or minimum value of $f(x) = -3x^2 + 4x - 1$.

Your solution

OBJECTIVE D

To solve application problems



© Jim Craig/Corbis

HOW TO 9

A carpenter is forming a rectangular floor for a storage shed. The perimeter of the rectangle is 44 ft. What dimensions of the rectangle would give the floor a maximum area? What is the maximum area?

We are given the perimeter of the rectangle, and we want to find the dimensions of the rectangle that will yield the maximum area for the floor. Use the equation for the perimeter of a rectangle.

$$\begin{aligned} P &= 2L + 2W \\ 44 &= 2L + 2W && \bullet P = 44 \\ 22 &= L + W && \bullet \text{Divide both sides of the equation by 2.} \\ 22 - L &= W && \bullet \text{Solve the equation for } W. \end{aligned}$$

Now use the equation for the area of a rectangle. Use substitution to express the area in terms of L .

$$\begin{aligned} A &= LW \\ A &= L(22 - L) && \bullet \text{From the equation above, } W = 22 - L. \text{ Substitute } 22 - L \text{ for } W. \\ A &= 22L - L^2 && \bullet \text{The area of the rectangle is } 22L - L^2. \end{aligned}$$

To find the length of the rectangle, find the L -coordinate of the vertex of the function $f(L) = -L^2 + 22L$.

$$L = -\frac{b}{2a} = -\frac{22}{2(-1)} = 11 \quad \bullet \text{For the equation } f(L) = -L^2 + 22L, \text{ } a = -1 \text{ and } b = 22.$$

The length of the rectangle is 11 ft.

To find the width, replace L in $22 - L$ by the L -coordinate of the vertex and evaluate.

$$\begin{aligned} W &= 22 - L \\ W &= 22 - 11 = 11 && \bullet \text{Replace } L \text{ by } 11 \text{ and evaluate.} \end{aligned}$$

The width of the rectangle is 11 ft.

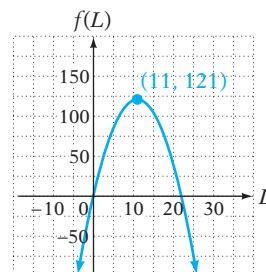
The dimensions of the rectangle that would give the floor a maximum area are 11 ft by 11 ft.

To find the maximum area of the floor, evaluate $f(L) = -L^2 + 22L$ at the L -coordinate of the vertex.

$$\begin{aligned} f(L) &= -L^2 + 22L \\ f(11) &= -(11)^2 + 22(11) && \bullet \text{Evaluate the function at } 11. \\ &= -121 + 242 = 121 \end{aligned}$$

The maximum area of the floor is 121 ft².

The graph of $f(L) = -L^2 + 22L$ is shown at the right. Note that the vertex of the parabola is (11, 121). For any value of L less than 11, the area of the floor will be less than 121 ft². For any value of L greater than 11, the area of the floor will be less than 121 ft². The maximum value of the function is 121, and the maximum value occurs when $L = 11$.


Point of Interest

Calculus is a branch of mathematics that demonstrates, among other things, how to find the maximum or minimum values of functions other than quadratic functions. These are very important problems in applied mathematics. For instance, an automotive engineer wants to design a car whose shape will *minimize* the effects of air resistance. The same engineer tries to *maximize* the efficiency of the car's engine. Similarly, an economist may try to determine what business practices will *minimize* cost and *maximize* profit.

EXAMPLE • 7

A mining company has determined that the cost c , in dollars per ton, of mining a mineral is given by $c(x) = 0.2x^2 - 2x + 12$, where x is the number of tons of the mineral mined. Find the number of tons of the mineral that should be mined to minimize the cost. What is the minimum cost per ton?

Strategy

- To find the number of tons of the mineral that should be mined to minimize the cost, find the x -coordinate of the vertex.
- To find the minimum cost, evaluate $c(x)$ at the x -coordinate of the vertex.

Solution

$$c(x) = 0.2x^2 - 2x + 12$$

$$\bullet a = 0.2, b = -2, \\ c = 12$$

$$x = -\frac{b}{2a} = -\frac{(-2)}{2(0.2)} = 5$$

$$\bullet \text{The } x\text{-coordinate} \\ \text{of the vertex}$$

To minimize the cost, 5 tons of the mineral should be mined.

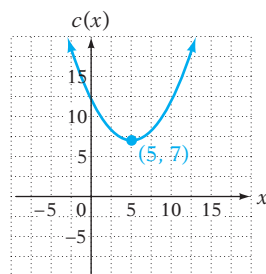
$$c(x) = 0.2x^2 - 2x + 12$$

$$\begin{aligned} c(5) &= 0.2(5)^2 - 2(5) + 12 \\ &= 5 - 10 + 12 \\ &= 7 \end{aligned}$$

$$\bullet x = 5$$

The minimum cost is \$7 per ton.

Note: The graph of $c(x) = 0.2x^2 - 2x + 12$ is shown below. The vertex of the parabola is $(5, 7)$. For any value of x less than 5, the cost per ton is greater than \$7. For any value of x greater than 5, the cost per ton is greater than \$7. Seven is the minimum value of the function, and the minimum value occurs when $x = 5$.

**YOU TRY IT • 7**

The height s , in feet, of a ball thrown straight up is given by $s(t) = -16t^2 + 64t$, where t is the time in seconds. Find the time it takes the ball to reach its maximum height. What is the maximum height?

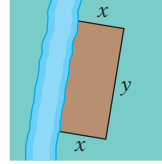
Your strategy**Your solution**

EXAMPLE • 8

Find two numbers whose difference is 10 and whose product is a minimum. What is the minimum product of the two numbers?

YOU TRY IT • 8

A rectangular fence is being constructed along a stream to enclose a picnic area. If there are 100 ft of fence available, what dimensions of the rectangle will produce the maximum area for picnicking?

**Strategy**

- Let x represent one number. Because the difference between the two numbers is 10,

$$x + 10$$

represents the other number.

[Note: $(x + 10) - (x) = 10$]

Then their product is represented by

$$x(x + 10) = x^2 + 10x$$

- To find one of the two numbers, find the x -coordinate of the vertex of $f(x) = x^2 + 10x$.
- To find the other number, replace x in $x + 10$ by the x -coordinate of the vertex and evaluate.
- To find the minimum product, evaluate the function at the x -coordinate of the vertex.

Your strategy**Solution**

$$f(x) = x^2 + 10x$$

$$\bullet a = 1, b = 10, c = 0$$

$$x = -\frac{b}{2a} = -\frac{10}{2(1)} = -5$$

$$\bullet \text{ One number is } -5.$$

$$\begin{aligned} x + 10 \\ -5 + 10 = 5 \end{aligned}$$

$$\bullet \text{ The other number is } 5.$$

The numbers are -5 and 5 .

$$\begin{aligned} f(x) &= x^2 + 10x \\ f(-5) &= (-5)^2 + 10(-5) \\ &= 25 - 50 \\ &= -25 \end{aligned}$$

The minimum product of the two numbers is -25 .

Your solution

9.1 EXERCISES

OBJECTIVE A

To graph a quadratic function



1. What is a quadratic function?



2. Describe the graph of a parabola.



3. What is the vertex of a parabola?



4. What is the axis of symmetry of the graph of a parabola?

5. The equation of the axis of symmetry of a parabola is
- $x = -5$
- . What is the
- x
- coordinate of the vertex of the parabola?

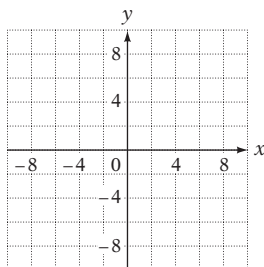
6. The equation of the axis of symmetry of a parabola is
- $x = 8$
- . What is the
- x
- coordinate of the vertex of the parabola?

7. The coordinates of the vertex of a parabola are
- $(7, -9)$
- . What is the equation of the axis of symmetry of the parabola?

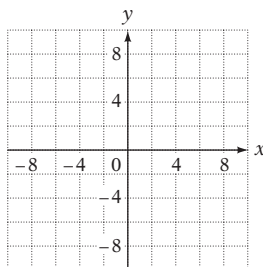
8. The coordinates of the vertex of a parabola are
- $(-4, 10)$
- . What is the equation of the axis of symmetry of the parabola?

For Exercises 9 to 23, find the coordinates of the vertex and the equation of the axis of symmetry for the parabola given by the equation. Then graph the equation.

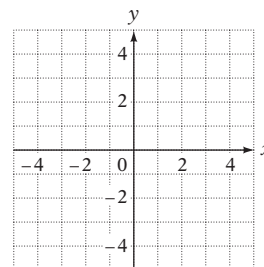
9. $y = x^2 - 2x - 4$



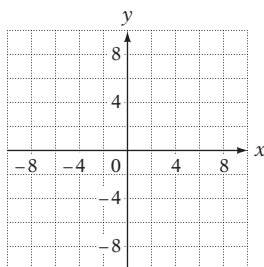
10. $y = x^2 + 4x - 4$



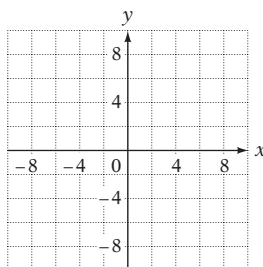
11. $y = -x^2 + 2x - 3$



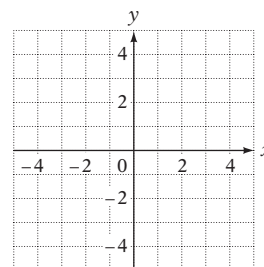
12. $y = -x^2 + 4x - 5$



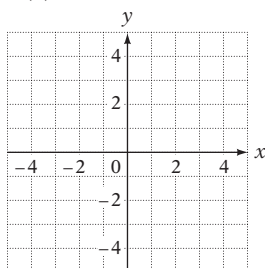
13. $f(x) = x^2 - x - 6$



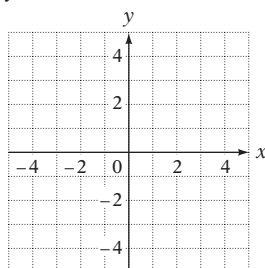
14. $G(x) = x^2 - x - 2$



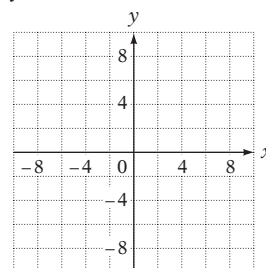
15. $F(x) = x^2 - 3x + 2$



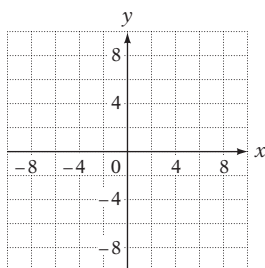
16. $y = 2x^2 - 4x + 1$



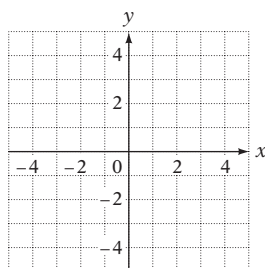
17. $y = -2x^2 + 6x$



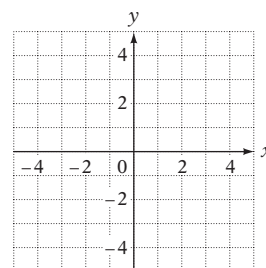
18. $y = \frac{1}{2}x^2 + 4$



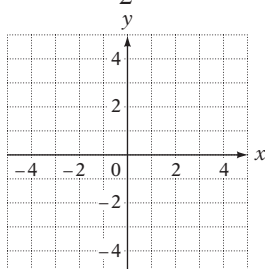
19. $y = -\frac{1}{4}x^2 - 1$



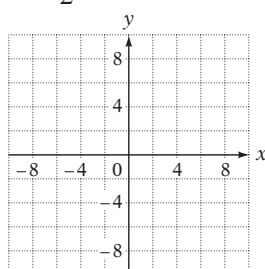
20. $h(x) = \frac{1}{2}x^2 - x + 1$



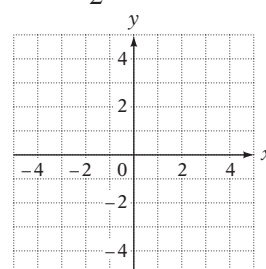
21. $P(x) = -\frac{1}{2}x^2 + 2x - 3$



22. $y = \frac{1}{2}x^2 + 2x - 6$



23. $y = -\frac{1}{2}x^2 + x - 3$

24. Is 3 in the range of the function given by $f(x) = (x + 2)^2 + 4$?25. What are the domain and range of the function given by $f(x) = (x - 1)^2 + 2$?

For Exercises 26 to 31, state the domain and range of the function.

26. $f(x) = 2x^2 - 4x - 5$

27. $f(x) = 2x^2 + 8x + 3$

28. $f(x) = -2x^2 - 3x + 2$

29. $f(x) = -x^2 + 6x - 9$

30. $f(x) = -x^2 - 4x - 5$

31. $f(x) = x^2 + 4x - 3$

OBJECTIVE B**To find the x -intercepts of a parabola**

32. **a.** What is an x -intercept of the graph of a parabola?
b. How many x -intercepts can the graph of a parabola have?



33. **a.** What is the y -intercept of the graph of a parabola?
b. How many y -intercepts can the graph of a parabola have?



34. The zeros of $f(x) = x^2 - 2x - 3$ are -1 and 3 . **a.** What are the x -intercepts of the graph of f ? **b.** What are the solutions of $x^2 - 2x - 3 = 0$?



35. The x -intercepts of the graph of $f(x) = x^2 + 3x - 4$ are $(-4, 0)$ and $(1, 0)$.
a. What are the zeros of f ? **b.** What are the solutions of $x^2 + 3x - 4 = 0$?

For Exercises 36 to 47, find the x -intercepts of the graph of the parabola given by each equation.

36. $y = x^2 - 4$

37. $y = x^2 - 9$

38. $y = 2x^2 - 4x$

39. $y = 3x^2 + 6x$

40. $y = x^2 - x - 2$

41. $y = x^2 - 2x - 8$

42. $y = 2x^2 - x - 1$

43. $y = 2x^2 - 5x - 3$

44. $y = x^2 + 2x - 1$

45. $y = x^2 + 4x - 3$

46. $y = x^2 + 6x + 10$

47. $y = -x^2 - 4x - 5$

For Exercises 48 to 65, find the zeros of f .

48. $f(x) = x^2 + 3x + 2$

49. $f(x) = x^2 - 4x - 5$

50. $f(x) = 2x^2 - x - 3$

51. $f(x) = 3x^2 - 2x - 8$

52. $f(x) = x^2 - 6x + 9$

53. $h(x) = 4x^2 - 4x + 1$

54. $f(x) = 2x^2 - 3x$

55. $f(x) = -3x^2 + 4x$

56. $f(x) = 2x^2 - 32$

57. $f(x) = -3x^2 + 12$

58. $f(x) = 2x^2 - 4$

59. $f(x) = 2x^2 - 54$

60. $f(x) = x^2 - 4x + 1$

61. $f(x) = x^2 - 2x - 17$

62. $f(x) = x^2 - 6x + 4$

63. $f(x) = x^2 + 4x + 5$

64. $f(x) = x^2 - 8x + 20$

65. $f(x) = x^2 + 4x + 13$

For Exercises 66 to 77, use the discriminant to determine the number of x -intercepts of the graph.

66. $y = 2x^2 + 2x - 1$

67. $y = -x^2 - x + 3$

68. $y = x^2 - 8x + 16$

69. $y = x^2 - 10x + 25$

70. $y = -3x^2 - x - 2$

71. $y = -2x^2 + x - 1$

72. $y = -2x^2 + x + 1$

73. $y = 4x^2 - x - 2$

74. $y = 2x^2 + x + 1$

75. $y = 2x^2 + x + 4$

76. $y = -3x^2 + 2x - 8$

77. $y = 4x^2 + 2x - 5$



78. Let $f(x) = (x - 3)^2 + a$. For what values of a will the graph of f have **a.** two x -intercepts, **b.** one x -intercept, and **c.** no x -intercepts?



79. Let $f(x) = -(x + 1)^2 + a$. For what values of a will the graph of f have **a.** two x -intercepts, **b.** one x -intercept, and **c.** no x -intercepts?

OBJECTIVE C

To find the minimum or maximum of a quadratic function



80. What is the minimum or maximum value of a quadratic function?



81. Describe how to find the minimum or maximum value of a quadratic function.



82. Does the function have a minimum or a maximum value?

a. $f(x) = -x^2 + 6x - 1$ **b.** $f(x) = 2x^2 - 4$ **c.** $f(x) = -5x^2 + x$



83. Does the function have a minimum or a maximum value?

a. $f(x) = 3x^2 - 2x + 4$ **b.** $f(x) = -x^2 + 9$ **c.** $f(x) = 6x^2 - 3x$

For Exercises 84 to 95, find the minimum or maximum value of the quadratic function.

84. $f(x) = x^2 - 2x + 3$

85. $f(x) = 2x^2 + 4x$

86. $f(x) = -2x^2 + 4x - 3$

87. $f(x) = -2x^2 + 4x - 5$

88. $f(x) = -2x^2 - 3x + 4$

89. $f(x) = -2x^2 - 3x$

90. $f(x) = 2x^2 + 3x - 8$

91. $f(x) = 3x^2 + 3x - 2$

92. $f(x) = -3x^2 + x - 6$

93. $f(x) = -x^2 - x + 2$

94. $f(x) = x^2 - 5x + 3$

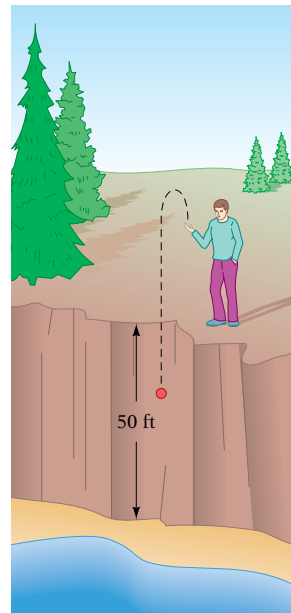
95. $f(x) = 3x^2 + 5x + 2$

OBJECTIVE D**To solve application problems**

96. **Physics** The height s , in feet, of a rock thrown upward at an initial speed of 64 ft/s from a cliff 50 ft above an ocean beach is given by the function $s(t) = -16t^2 + 64t + 50$, where t is the time in seconds. Find the maximum height above the beach that the rock will attain.

97. **Business** A manufacturer of microwave ovens believes that the revenue R , in dollars, the company receives is related to the price P , in dollars, of an oven by the function $R(P) = 125P - 0.25P^2$. What price will give the maximum revenue?

98. **Business** A tour operator believes that the profit P , in dollars, from selling x tickets is given by $P(x) = 40x - 0.25x^2$. Using this model, what is the maximum profit the tour operator can expect?



99. **Weightlessness** Read the article at the right. Suppose the height h , in meters, of the airplane is modeled by the equation $h(t) = -1.42t^2 + 119t + 6000$, where t is the number of seconds elapsed after the plane enters its parabolic path. Show that the maximum height of the airplane is approximately the value given in the article. Round values to the nearest unit.



100. **Weightlessness** The airplane described in Exercise 99 is used to prepare Russian cosmonauts for their work in the weightless environment of space travel. NASA uses a similar technique to train American astronauts. Suppose the height h , in feet, of NASA's airplane is modeled by the equation $h(t) = -6.63t^2 + 431t + 25,000$, where t is the number of seconds elapsed after the plane enters its parabolic path. Find the maximum height of the airplane. Round to the nearest thousand feet.

In the News**Zero-G Painting Sells Sky High**

A painting completed while the artist floated weightless in zero gravity sold at auction for over \$300,000. The British artist flew from the Star City cosmonaut training center in an airplane that produces a weightless environment by flying in a series of parabolic paths. During its flight, the plane may reach a maximum height of about 8500 m above Earth.

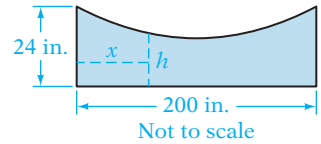
Sources: BBC News,
www.msnbc.msn.com,
www.space-travellers.com

101. **Construction** The suspension cable that supports a small footbridge hangs in the shape of a parabola. The height h , in feet, of the cable above the bridge is given by the function $h(x) = 0.25x^2 - 0.8x + 25$, where x is the distance in feet from one end of the bridge. What is the minimum height of the cable above the bridge?



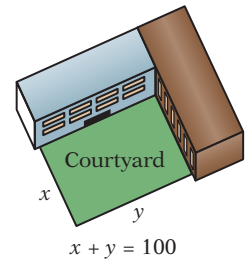


- 102. Telescopes** An equation that models the thickness h , in inches, of the mirror in the telescope at the Palomar Mountain Observatory is given by $h(x) = 0.000379x^2 - 0.0758x + 24$, where x is measured in inches from the edge of the mirror. Find the minimum thickness of the mirror.

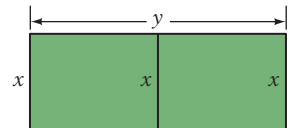


- 103. Mathematics** Find two numbers whose sum is 20 and whose product is a maximum.
- 104. Mathematics** Find two numbers whose product is a maximum and whose sum is 1.
- 105. Mathematics** Find two numbers whose difference is 14 and whose product is a minimum.
- 106. Mathematics** Find two numbers whose product is a maximum and whose difference is 1.
- 107. Ranching** A rancher has 200 ft of fencing with which to build a rectangular corral alongside an existing fence. Determine the dimensions of the corral that will maximize the enclosed area.

- 108. Construction** A courtyard at the corner of two buildings is to be enclosed using 100 ft of redwood fencing. Find the dimensions of the courtyard that will maximize the area.



- 109. Recreation** A large lot in a park is going to be split into two softball fields, and each field will be enclosed with a fence. The parks and recreation department has 2100 ft of fencing to enclose the fields. What dimensions will enclose the greatest area?



- 110.** If the height h , in feet, of a ball t seconds after it has been tossed directly upward is given by $h(t) = -16(t - 2)^2 + 40$, what is the maximum height the ball will attain?

Applying the Concepts

- 111.** One root of the quadratic equation $2x^2 - 5x + k = 0$ is 4. What is the other root?
- 112.** What is the value of k if the vertex of the graph of $y = x^2 - 8x + k$ is a point on the x -axis?
- 113.** The zeros of $f(x) = mx^2 + nx + 1$ are -2 and 3 . What are the zeros of $g(x) = nx^2 + mx - 1$?

SECTION

9.2

Graphs of Functions

OBJECTIVE A

To graph functions



The graphs of the polynomial functions $L(x) = mx + b$ (a straight line) and $Q(x) = ax^2 + bx + c$, $a \neq 0$ (a parabola) were discussed in previous objectives. The graphs of other functions can also be drawn by finding ordered pairs that belong to the function, plotting the points that correspond to the ordered pairs, and then drawing a curve through the points.

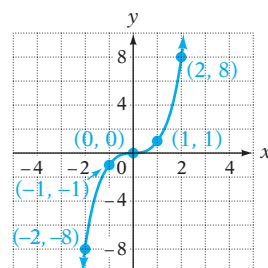
 Take Note

The units along the x -axis are different from those along the y -axis. If the units along the x -axis were the same as those along the y -axis, the graph would appear narrower than the one shown.

HOW TO • 1 Graph: $F(x) = x^3$

Select several values of x and evaluate the function.

x	$F(x) = x^3$	$F(x)$	(x, y)
-2	$(-2)^3$	-8	$(-2, -8)$
-1	$(-1)^3$	-1	$(-1, -1)$
0	0^3	0	$(0, 0)$
1	1^3	1	$(1, 1)$
2	2^3	8	$(2, 8)$

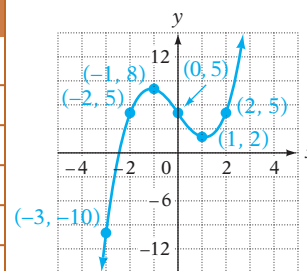


Plot the ordered pairs and draw a graph through the points.

HOW TO • 2 Graph: $g(x) = x^3 - 4x + 5$

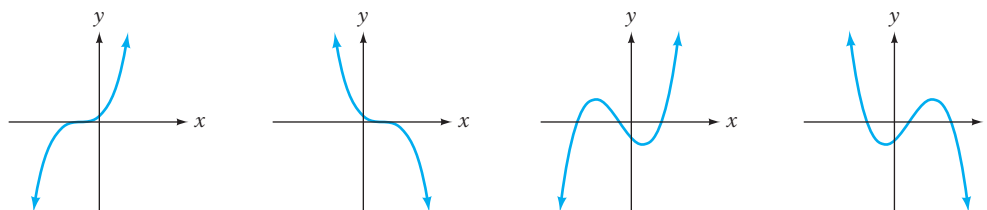
Select several values of x and evaluate the function.

x	$g(x) = x^3 - 4x + 5$	$g(x)$	(x, y)
-3	$(-3)^3 - 4(-3) + 5$	-10	$(-3, -10)$
-2	$(-2)^3 - 4(-2) + 5$	5	$(-2, 5)$
-1	$(-1)^3 - 4(-1) + 5$	8	$(-1, 8)$
0	$(0)^3 - 4(0) + 5$	5	$(0, 5)$
1	$(1)^3 - 4(1) + 5$	2	$(1, 2)$
2	$(2)^3 - 4(2) + 5$	5	$(2, 5)$



Plot the ordered pairs and draw a graph through the points.

Note from the graphs of the two cubic functions above that the shapes of the graphs can be quite different. The following graphs of typical cubic polynomial functions show the general shape of a cubic polynomial.



As the degree of a polynomial increases, the graph of the polynomial function can change significantly. In such cases, it may be necessary to plot many points before an accurate graph can be drawn. Only polynomials of degree 3 are considered here.



Integrating Technology

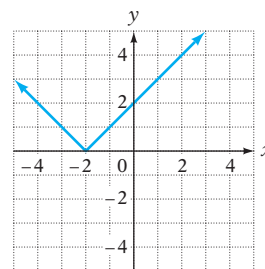
See the Keystroke Guide: *Graph* for instructions on using a graphing calculator to graph a function.

HOW TO • 3

Graph: $f(x) = |x + 2|$

This is an **absolute-value function**.

x	$f(x) = x + 2 $	$f(x)$	(x, y)
-3	$ -3 + 2 $	1	$(-3, 1)$
-2	$ -2 + 2 $	0	$(-2, 0)$
-1	$ -1 + 2 $	1	$(-1, 1)$
0	$ 0 + 2 $	2	$(0, 2)$
1	$ 1 + 2 $	3	$(1, 3)$
2	$ 2 + 2 $	4	$(2, 4)$



In general, the graph of the absolute value of a linear polynomial is V-shaped.

HOW TO • 4

Graph: $R(x) = \sqrt{2x - 4}$

This is a **radical function**. Because the square root of a negative number is not a real number, the domain of this function requires that $2x - 4 \geq 0$. Solve this inequality for x .

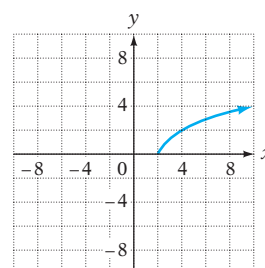
$$2x - 4 \geq 0$$

$$2x \geq 4$$

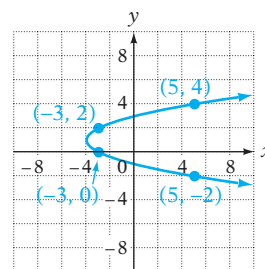
$$x \geq 2$$

The domain is $\{x | x \geq 2\}$. This means that only values of x that are greater than or equal to 2 can be chosen as values at which to evaluate the function. In this case, some of the y -coordinates must be approximated.

x	$R(x) = \sqrt{2x - 4}$	$R(x)$	(x, y)
2	$\sqrt{2(2) - 4}$	0	$(2, 0)$
3	$\sqrt{2(3) - 4}$	1.41	$(3, 1.41)$
4	$\sqrt{2(4) - 4}$	2	$(4, 2)$
5	$\sqrt{2(5) - 4}$	2.45	$(5, 2.45)$
6	$\sqrt{2(6) - 4}$	2.83	$(6, 2.83)$



Recall that a function is a special type of relation in which no two ordered pairs have the same first coordinate. Graphically, this means that the graph of a function cannot pass through two points that have the same x -coordinate and different y -coordinates. For instance, the graph at the right is not the graph of a function because there are ordered pairs with the same x -coordinate and different y -coordinates.

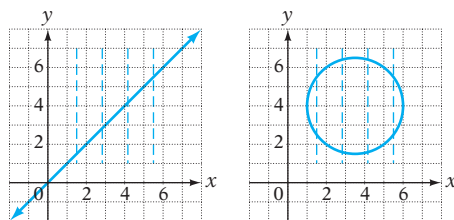


The last graph illustrates a general statement that can be made about whether a graph defines a function. It is called the **vertical-line test**.

Vertical-Line Test

A graph defines a function if any vertical line intersects the graph at no more than one point.

For example, the graph of a nonvertical straight line is the graph of a function. Any vertical line intersects the graph no more than once. The graph of a circle, however, is not the graph of a function. There are vertical lines that intersect the graph at more than one point.

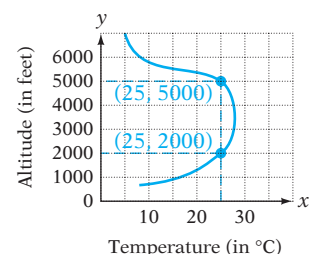


Some practical applications can be modeled by a graph that is not the graph of a function. Below is an example of such an application.



Nik Wheeler/Terra/Corbis

One of the causes of smog is an inversion layer of air in which temperatures at higher altitudes are warmer than those at lower altitudes. The graph at the right shows the altitudes at which various temperatures were recorded. As shown by the dashed lines in the graph, there are two altitudes at which the temperature was 25°C . This means that there are two ordered pairs (shown in the graph) with the same first coordinate but different second coordinates. The graph does not define a function.



When a graph does define a function, the domain and range can be estimated from the graph.

Take Note

To determine the domain, think of collapsing the graph onto the x -axis and determining the interval on the x -axis that the graph covers. To determine the range, think of collapsing the graph onto the y -axis and determining the interval on the y -axis that the graph covers.

HOW TO 5

Determine the domain and range of the function given by the graph at the right. Write the answer in set-builder notation.

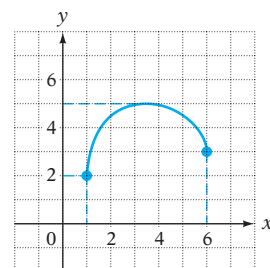
The solid dots on the graph indicate its beginning and ending points.

The domain is the set of x -coordinates.

Domain: $\{x \mid 1 \leq x \leq 6\}$

The range is the set of y -coordinates.

Range: $\{y \mid 2 \leq y \leq 5\}$



Take Note

In the first example, the domain and range are given in set-builder notation. In the second example, the domain and range are given in interval notation. Either method may be used.

HOW TO 6

Determine the domain and range of the function given by the graph at the right. Write the answer in interval notation.

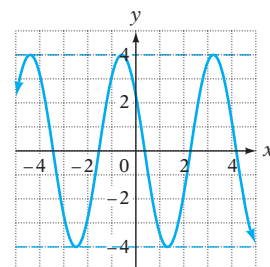
The arrows on the graph indicate that the graph continues in the same manner.

The domain is the set of x -coordinates.

Domain: $(-\infty, \infty)$

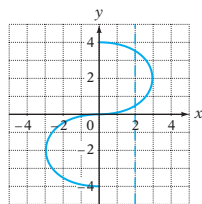
The range is the set of y -coordinates.

Range: $[-4, 4]$



EXAMPLE • 1

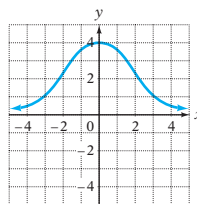
Use the vertical-line test to determine whether the graph shown is the graph of a function.

Solution

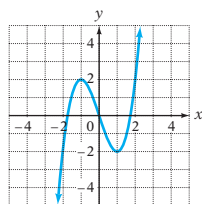
A vertical line intersects the graph more than once.
The graph is not the graph of a function.

YOU TRY IT • 1

Use the vertical-line test to determine whether the graph shown is the graph of a function.

Your solution**EXAMPLE • 2**

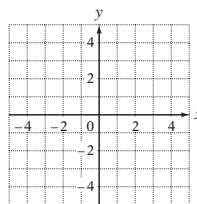
Graph and write in set-builder notation the domain and range of the function defined by $f(x) = x^3 - 3x$.

Solution

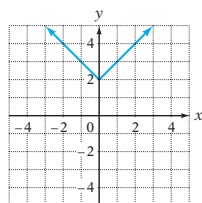
Domain: $\{x|x \in \text{real numbers}\}$
Range: $\{y|y \in \text{real numbers}\}$

YOU TRY IT • 2

Graph and write in set-builder notation the domain and range of the function defined by $f(x) = -\frac{1}{2}x^3 + 2x$.

Your solution**EXAMPLE • 3**

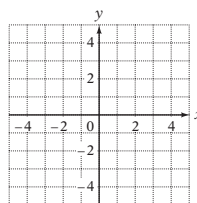
Graph and write in interval notation the domain and range of the function defined by $f(x) = |x| + 2$.

Solution

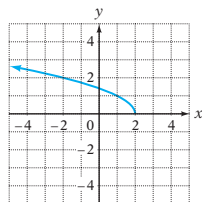
Domain: $(-\infty, \infty)$
Range: $[2, \infty)$

YOU TRY IT • 3

Graph and write in interval notation the domain and range of the function defined by $f(x) = |x - 3|$.

Your solution**EXAMPLE • 4**

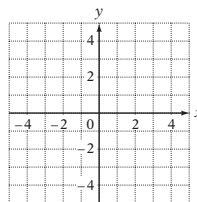
Graph and write in set-builder notation the domain and range of the function defined by $f(x) = \sqrt{2 - x}$.

Solution

Domain: $\{x|x \leq 2\}$
Range: $\{y|y \geq 0\}$

YOU TRY IT • 4

Graph and write in set-builder notation the domain and range of the function defined by $f(x) = -\sqrt{x - 1}$.

Your solution

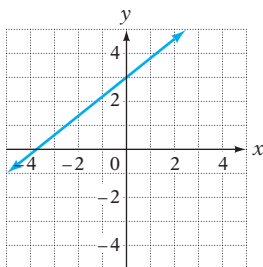
9.2 EXERCISES

OBJECTIVE A

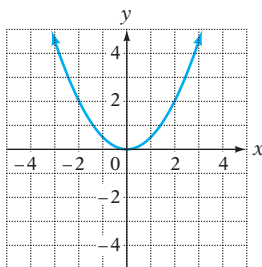
To graph functions

For Exercises 1 to 6, use the vertical-line test to determine whether the graph is the graph of a function.

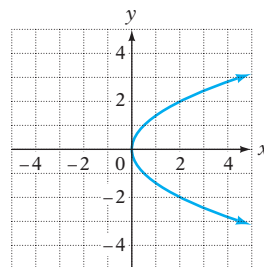
1.



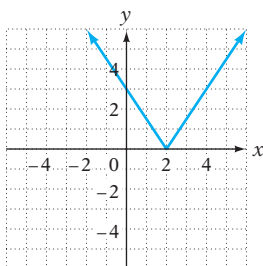
2.



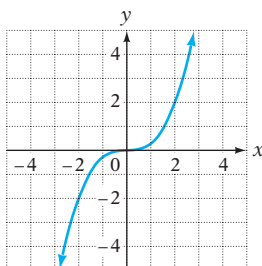
3.



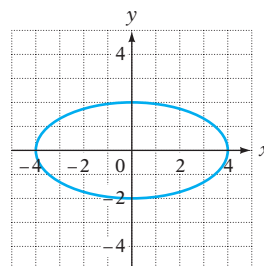
4.



5.



6.



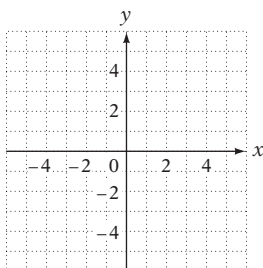
7. Is 4 in the domain of $f(x) = \sqrt{3 - x}$?



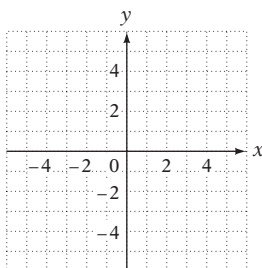
8. Is 4 in the range of $f(x) = |2x + 5| + 1$?

For Exercises 9 to 20, graph the function and state its domain and range in set-builder notation.

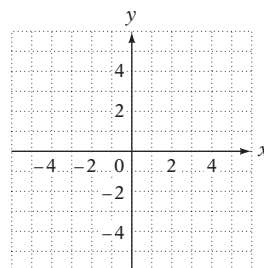
9. $f(x) = 1 - x^3$



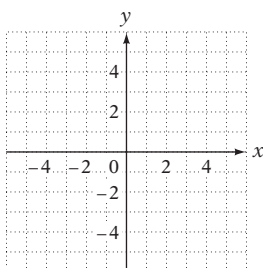
10. $f(x) = x^3 - 1$



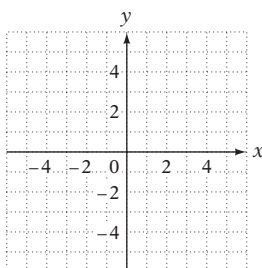
11. $f(x) = 3|2 - x|$



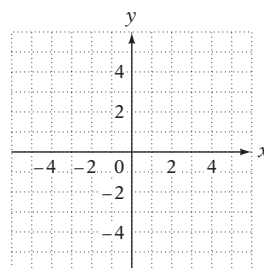
12. $f(x) = |2x - 1|$



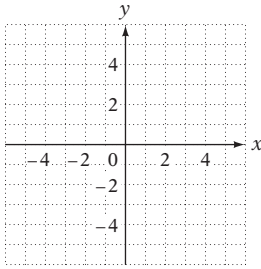
13. $f(x) = \sqrt{4 - x}$



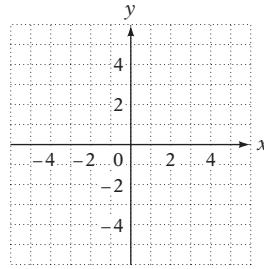
14. $f(x) = \sqrt{1 + x}$



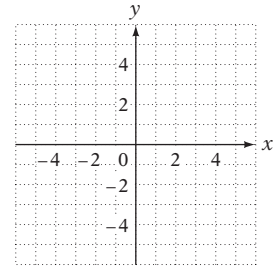
15. $f(x) = x^3 + 4x^2 + 4x$



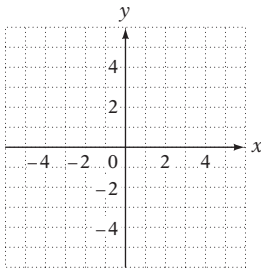
16. $f(x) = x^3 - x^2 - x + 1$



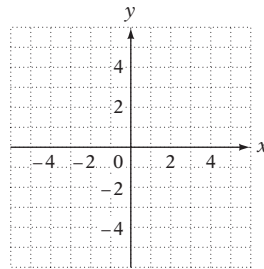
17. $f(x) = |2x + 2|$



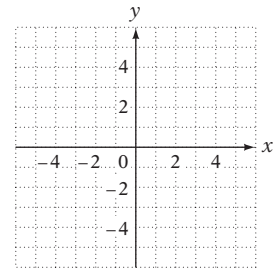
18. $f(x) = 2|x + 1|$



19. $f(x) = -\sqrt{x + 2}$



20. $f(x) = -\sqrt{x - 3}$



Applying the Concepts

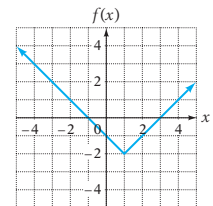
21. If $f(x) = \sqrt{x - 2}$ and $f(a) = 4$, find a .

22. If $f(x) = \sqrt{x + 5}$ and $f(a) = 3$, find a .

23. $f(a, b)$ = the sum of a and b
 $g(a, b)$ = the product of a and b
 Find $f(2, 5) + g(2, 5)$.

24. The graph of the function f is shown at the right. For this function, which of the following are true?

(i) $f(4) = 1$ (ii) $f(0) = 3$ (iii) $f(-3) = 2$



25. Let $f(x)$ be the digit in the x th decimal place of the repeating decimal $0.\overline{387}$. For example, $f(3) = 7$ because 7 is the digit in the third decimal place. Find $f(14)$.

26. Given $f(x) = (x + 1)(x - 1)$, for what values of x is $f(x)$ negative? Write your answer in set-builder notation.

27. Given $f(x) = (x + 2)(x - 2)$, for what values of x is $f(x)$ negative? Write your answer in set-builder notation.

28. Given $f(x) = -|x + 3|$, for what value of x is $f(x)$ greatest?

29. Given $f(x) = |2x - 2|$, for what value of x is $f(x)$ smallest?

SECTION

9.3

Algebra of Functions

OBJECTIVE A

To perform operations on functions



The operations of addition, subtraction, multiplication, and division of functions are defined as follows.

Operations on Functions

If f and g are functions and x is an element of the domain of each function, then

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) & (f \cdot g)(x) &= f(x) \cdot g(x) \\ (f - g)(x) &= f(x) - g(x) & \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0\end{aligned}$$

HOW TO • 1

Given $f(x) = x^2 + 1$ and $g(x) = 3x - 2$, find $(f + g)(3)$ and

$$(f \cdot g)(-1).$$

$$\begin{aligned}(f + g)(3) &= f(3) + g(3) \\ &= [(3)^2 + 1] + [3(3) - 2] \\ &= 10 + 7 = 17\end{aligned}$$

$$\begin{aligned}(f \cdot g)(-1) &= f(-1) \cdot g(-1) \\ &= [(-1)^2 + 1] \cdot [3(-1) - 2] \\ &= 2 \cdot (-5) = -10\end{aligned}$$

Using $f(x) = x^2 + 1$ and $g(x) = 3x - 2$ from the last example, let $S(x)$ be the sum of the two functions. Then

$$\begin{aligned}S(x) &= (f + g)(x) = f(x) + g(x) & \bullet \text{ The definition of addition of functions} \\ &= [x^2 + 1] + [3x - 2] & \bullet f(x) = x^2 + 1, g(x) = 3x - 2 \\ S(x) &= x^2 + 3x - 1\end{aligned}$$

Now evaluate $S(3)$.

$$\begin{aligned}S(x) &= x^2 + 3x - 1 \\ S(3) &= (3)^2 + 3(3) - 1 \\ &= 9 + 9 - 1 \\ &= 17 = (f + g)(3)\end{aligned}$$

Note that $S(3) = 17$ and $(f + g)(3) = 17$. This shows that adding $f(x) + g(x)$ and then evaluating is the same as evaluating $f(x)$ and $g(x)$ and then adding. The same is true for the other operations on functions. For instance, let $P(x)$ be the product of the functions $f(x) = x^2 + 1$ and $g(x) = 3x - 2$. Then

$$\begin{aligned}P(x) &= (f \cdot g)(x) = f(x) \cdot g(x) \\ &= (x^2 + 1)(3x - 2) \\ &= 3x^3 - 2x^2 + 3x - 2 \\ P(-1) &= 3(-1)^3 - 2(-1)^2 + 3(-1) - 2 \\ &= -3 - 2 - 3 - 2 \\ &= -10\end{aligned}$$

• Note that $P(-1) = -10$ and $(f \cdot g)(-1) = -10$.

HOW TO • 2

Given $f(x) = 2x^2 - 5x + 3$ and $g(x) = x^2 - 1$, find $\left(\frac{f}{g}\right)(1)$.

$$\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)}$$

$$= \frac{2(1)^2 - 5(1) + 3}{(1)^2 - 1} = \frac{0}{0}$$

• Not a real number

Because $\frac{0}{0}$ is not defined, the expression $\left(\frac{f}{g}\right)(1)$ cannot be evaluated.

EXAMPLE • 1

Given $f(x) = x^2 - x + 1$ and $g(x) = x^3 - 4$, find $(f - g)(3)$.

Solution

$$\begin{aligned}(f - g)(3) &= f(3) - g(3) \\ &= (3^2 - 3 + 1) - (3^3 - 4) \\ &= 7 - 23 \\ &= -16 \\ (f - g)(3) &= -16\end{aligned}$$

YOU TRY IT • 1

Given $f(x) = x^2 + 2x$ and $g(x) = 5x - 2$, find $(f + g)(-2)$.

Your solution**EXAMPLE • 2**

Given $f(x) = x^2 + 2$ and $g(x) = 2x + 3$, find $(f \cdot g)(-2)$.

Solution

$$\begin{aligned}(f \cdot g)(-2) &= f(-2) \cdot g(-2) \\ &= [(-2)^2 + 2] \cdot [2(-2) + 3] \\ &= 6(-1) \\ &= -6 \\ (f \cdot g)(-2) &= -6\end{aligned}$$

YOU TRY IT • 2

Given $f(x) = 4 - x^2$ and $g(x) = 3x - 4$, find $(f \cdot g)(3)$.

Your solution**EXAMPLE • 3**

Given $f(x) = x^2 + 4x + 4$ and $g(x) = x^3 - 2$, find $\left(\frac{f}{g}\right)(3)$.

Solution

$$\begin{aligned}\left(\frac{f}{g}\right)(3) &= \frac{f(3)}{g(3)} \\ &= \frac{3^2 + 4(3) + 4}{3^3 - 2} \\ &= \frac{25}{25} \\ &= 1 \\ \left(\frac{f}{g}\right)(3) &= 1\end{aligned}$$

YOU TRY IT • 3

Given $f(x) = x^2 - 4$ and $g(x) = x^2 + 2x + 1$, find $\left(\frac{f}{g}\right)(4)$.

Your solution

OBJECTIVE B

To find the composition of two functions



Jose Fuste Raga/Latitude/Corbis

Composition of functions is another way in which functions can be combined. This method of combining functions uses the output of one function as the input for a second function.

Suppose a forest fire is started by lightning striking a tree, and the spread of the fire can be approximated by a circle whose radius r , in feet, is given by $r(t) = 24\sqrt{t}$, where t is the number of hours after the tree is struck by the lightning. The area of the fire is the area of a circle and is given by the formula $A(r) = \pi r^2$. Because the area of the fire depends on the radius of the circle and the radius depends on the time since the tree was struck, there is a relationship between the area of the fire and time. This relationship can be found by evaluating the formula for the area of a circle using $r(t) = 24\sqrt{t}$.

$$\begin{aligned}
 A(r) &= \pi r^2 \\
 A[r(t)] &= \pi [r(t)]^2 && \bullet \text{ Replace } r \text{ by } r(t). \\
 &= \pi [24\sqrt{t}]^2 && \bullet r(t) = 24\sqrt{t} \\
 &= 576\pi t && \bullet \text{ Simplify.}
 \end{aligned}$$

The result is the function $A(t) = 576\pi t$, which gives the area of the fire in terms of the time since the lightning struck. For instance, when $t = 3$, we have

$$\begin{aligned}
 A(t) &= 576\pi t \\
 A(3) &= 576\pi(3) \\
 &\approx 5429
 \end{aligned}$$

Three hours after the lightning strikes, the area of the fire is approximately 5429 ft².

The function produced above, in which one function was evaluated at another function, is referred to as the *composition* of A with r . The notation $A \circ r$ is used to denote this composition of functions. That is,

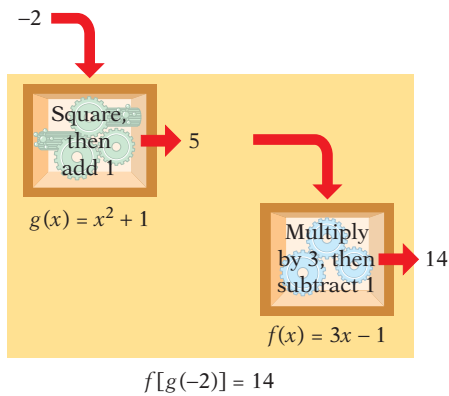
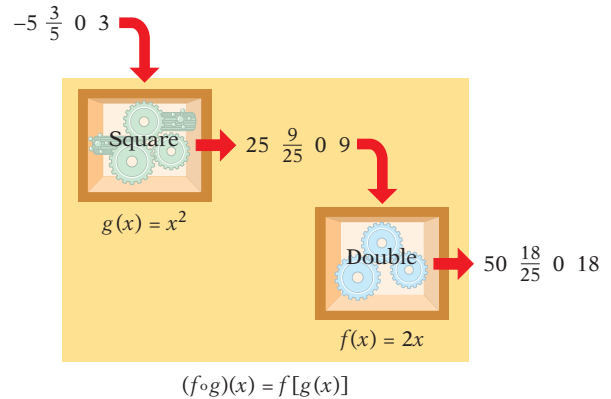
$$(A \circ r)(t) = 576\pi t$$

Definition of the Composition of Two Functions

Let f and g be two functions such that $g(x)$ is in the domain of f for all x in the domain of g . Then the **composition of the two functions**, denoted by $f \circ g$, is the function whose value at x is given by $(f \circ g)(x) = f[g(x)]$.

The function defined by $(f \circ g)(x)$ is also called the **composite** of f and g and represents a **composite function**. We read $(f \circ g)(x)$ or $f[g(x)]$ as “ f of g of x .”

The function machine at the right illustrates the composition of the two functions $g(x) = x^2$ and $f(x) = 2x$. Note that a composite function combines two functions. First, one function pairs an input with an output. Then that output is used as the input for a second function, which in turn produces a final output.



Consider $f(x) = 3x - 1$ and $g(x) = x^2 + 1$. The expression $(f \circ g)(-2)$ or, equivalently, $f[g(-2)]$ means to evaluate the function f at $g(-2)$.

$$\begin{aligned} g(x) &= x^2 + 1 \\ g(-2) &= (-2)^2 + 1 && \bullet \text{ Evaluate } g \text{ at } -2. \\ g(-2) &= 5 \\ f(x) &= 3x - 1 \\ f(5) &= 3(5) - 1 = 14 && \bullet \text{ Evaluate } f \text{ at } g(-2) = 5. \end{aligned}$$

If we apply our function machine analogy, the composition of functions looks something like the figure at the left.

We can find a general expression for $f[g(x)]$ by evaluating f at $g(x)$. For instance, using $f(x) = 3x - 1$ and $g(x) = x^2 + 1$, we have

$$\begin{aligned} f(x) &= 3x - 1 \\ f[g(x)] &= 3[g(x)] - 1 && \bullet \text{ Replace } x \text{ by } g(x). \\ &= 3[x^2 + 1] - 1 && \bullet \text{ Replace } g(x) \text{ by } x^2 + 1. \\ &= 3x^2 + 2 && \bullet \text{ Simplify.} \end{aligned}$$

The requirement in the definition of the composition of two functions that $g(x)$ be in the domain of f for all x in the domain of g is important. For instance, let

$$f(x) = \frac{1}{x-1} \quad \text{and} \quad g(x) = 3x - 5$$

When $x = 2$,

$$\begin{aligned} g(2) &= 3(2) - 5 = 1 \\ f[g(2)] &= f(1) = \frac{1}{1-1} = \frac{1}{0} && \bullet \text{ This is not a real number.} \end{aligned}$$

In this case, $g(2)$ is not in the domain of f . Thus the composition is not defined at 2.

HOW TO 3

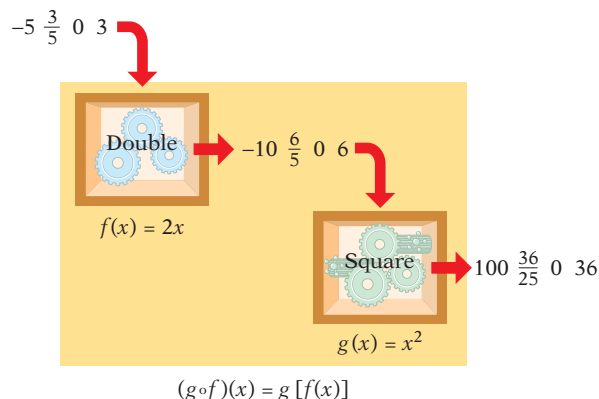
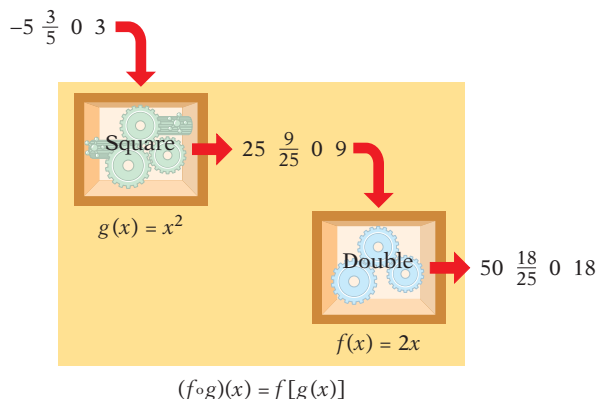
Given $f(x) = x^3 - x + 1$ and $g(x) = 2x^2 - 10$, evaluate $(g \circ f)(2)$.

$$\begin{aligned} f(2) &= (2)^3 - (2) + 1 = 7 \\ (g \circ f)(2) &= g[f(2)] \\ &= g(7) \\ &= 2(7)^2 - 10 = 88 \end{aligned}$$

HOW TO • 4Given $f(x) = 3x - 2$ and $g(x) = x^2 - 2x$, find $(f \circ g)(x)$.

$$\begin{aligned}
 (f \circ g)(x) &= f[g(x)] \\
 &= 3(x^2 - 2x) - 2 \\
 &= 3x^2 - 6x - 2
 \end{aligned}$$

When we evaluate compositions of functions, the order in which the functions are applied is important. In the two diagrams below, the order of the *square* function, $g(x) = x^2$, and the *double* function, $f(x) = 2x$, is interchanged. Note that the final outputs are different. Therefore, $(f \circ g)(x) \neq (g \circ f)(x)$.

**EXAMPLE • 4**Given $f(x) = x^2 - x$ and $g(x) = 3x - 2$, find $f[g(3)]$.**Solution**

$$\begin{aligned}
 g(x) &= 3x - 2 \\
 g(3) &= 3(3) - 2 = 9 - 2 = 7 && \bullet \text{ Evaluate } g \text{ at } 3. \\
 f(x) &= x^2 - x && \bullet \text{ Evaluate } f \text{ at } \\
 f[g(3)] &= f(7) = 7^2 - 7 = 42 && g(3) = 7.
 \end{aligned}$$

YOU TRY IT • 4Given $f(x) = 1 - 2x$ and $g(x) = x^2$, find $f[g(-1)]$.**Your solution****EXAMPLE • 5**Given $s(t) = t^2 + 3t - 1$ and $v(t) = 2t + 1$, determine $s[v(t)]$.**Solution**

$$\begin{aligned}
 s(t) &= t^2 + 3t - 1 && \bullet \text{ Evaluate } s \text{ at } v(t). \\
 s[v(t)] &= (2t + 1)^2 + 3(2t + 1) - 1 \\
 &= (4t^2 + 4t + 1) + 6t + 3 - 1 \\
 &= 4t^2 + 10t + 3
 \end{aligned}$$

YOU TRY IT • 5Given $L(s) = s + 1$ and $M(s) = s^3 + 1$, determine $M[L(s)]$.**Your solution**

9.3 EXERCISES

OBJECTIVE A

To perform operations on functions



1. Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x + 4}$. For the given value of x , is it possible to find $(f + g)(x)$?

a. 0 b. -4 c. 1 d. -5



2. Let $f(1) = -5$ and $g(1) = 3$. Find each of the following.

a. $(f + g)(1)$ b. $(f - g)(1)$ c. $(f \cdot g)(1)$ d. $\left(\frac{f}{g}\right)(1)$

For Exercises 3 to 10, let $f(x) = 2x^2 - 3$ and $g(x) = -2x + 4$. Find:

3. $f(2) - g(2)$ 4. $f(3) - g(3)$ 5. $f(0) + g(0)$ 6. $f(1) + g(1)$

7. $(f \cdot g)(2)$ 8. $(f \cdot g)(-1)$ 9. $\left(\frac{f}{g}\right)(4)$ 10. $\left(\frac{g}{f}\right)(-3)$

For Exercises 11 to 18, let $f(x) = 2x^2 + 3x - 1$ and $g(x) = 2x - 4$. Find:

11. $f(1) + g(1)$ 12. $f(-3) + g(-3)$ 13. $f(4) - g(4)$

14. $f(-2) - g(-2)$ 15. $(f \cdot g)(1)$ 16. $(f \cdot g)(-2)$

17. $\left(\frac{f}{g}\right)(2)$ 18. $(f \cdot g)\left(\frac{1}{2}\right)$

For Exercises 19 to 21, let $f(x) = x^2 + 3x - 5$ and $g(x) = x^3 - 2x + 3$. Find:

19. $f(2) - g(2)$ 20. $(f \cdot g)(-3)$ 21. $\left(\frac{f}{g}\right)(-2)$

OBJECTIVE B

To find the composition of two functions



22. Explain the meaning of the notation $f[g(3)]$.



23. Explain the meaning of the expression $(f \circ g)(-2)$.



24. Let $f(x) = x^2 + 1$ and $g(x) = \frac{x}{x^2 - 4}$.

a. Is 2 in the domain of $f \circ g$? b. Is 2 in the domain of $g \circ f$?



25. Suppose $g(4) = -3$ and $(f \circ g)(4) = 5$. What is the value of $f(-3)$?

Given $f(x) = 2x - 3$ and $g(x) = 4x - 1$, evaluate the composite functions in Exercises 26 to 31.

26. $f[g(0)]$ 27. $g[f(0)]$ 28. $f[g(2)]$ 29. $g[f(-2)]$ 30. $f[g(x)]$ 31. $g[f(x)]$

Given $h(x) = 2x + 4$ and $f(x) = \frac{1}{2}x + 2$, evaluate the composite functions in Exercises 32 to 37.

32. $h[f(0)]$ 33. $f[h(0)]$ 34. $h[f(2)]$ 35. $f[h(-1)]$ 36. $h[f(x)]$ 37. $f[h(x)]$

Given $g(x) = x^2 + 3$ and $h(x) = x - 2$, evaluate the composite functions in Exercises 38 to 43.

38. $g[h(0)]$ 39. $h[g(0)]$ 40. $g[h(4)]$ 41. $h[g(-2)]$ 42. $g[h(x)]$ 43. $h[g(x)]$

Given $f(x) = x^2 + x + 1$ and $h(x) = 3x + 2$, evaluate the composite functions in Exercises 44 to 49.

44. $f[h(0)]$ 45. $h[f(0)]$ 46. $f[h(-1)]$ 47. $h[f(-2)]$ 48. $f[h(x)]$ 49. $h[f(x)]$

Given $f(x) = x - 2$ and $g(x) = x^3$, evaluate the composite functions in Exercises 50 to 55.

50. $f[g(2)]$ 51. $f[g(-1)]$ 52. $g[f(2)]$ 53. $g[f(-1)]$ 54. $f[g(x)]$ 55. $g[f(x)]$

56. **Oil Spills** Suppose the spread of an oil leak from a tanker can be approximated by a circle with the tanker at its center and radius r , in feet. The radius of the spill t hours after the beginning of the leak is given by $r(t) = 45t$.

- Find the area of the spill as a function of time.
- What is the area of the spill after 3 h? Round to the nearest whole number.

57. **Manufacturing** Suppose the manufacturing cost, in dollars, per digital camera is given by the function $M(x) = \frac{50x + 10,000}{x}$. A camera store will sell the cameras by marking up the manufacturing cost per camera, $M(x)$, by 60%.

- Express the selling price of a camera as a function of the number of cameras to be manufactured. That is, find $S \circ M$.
- Find $(S \circ M)(5000)$.
- Write a sentence that explains the meaning of the answer to part b.



58. **Manufacturing** The number of electric scooters e that a factory can produce per day is a function of the number of hours h it operates and is given by $e(h) = 250h$ for $0 \leq h \leq 10$. The daily cost c to manufacture e electric scooters is given by the function $c(e) = 0.05e^2 + 60e + 1000$.

- Find $(c \circ e)(h)$.
- Evaluate $(c \circ e)(10)$.
- Write a sentence that explains the meaning of the answer to part b.



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Robert Brenner/PhotoEdit, Inc.



- 59. Electric Cars** Read the article at the right. The garage's income for the conversion of vehicles is given by $I(n) = 12,500n$, where n is the number of vehicles converted. The number of vehicles the garage has converted is given by $n(m) = 4m$, where m is the number of months.

- Find $(I \circ n)(m)$.
- Evaluate $(I \circ n)(3)$.
- Write a sentence that explains the meaning of the answer to part b.



- 60. Electric Cars** A company in Santa Cruz, California, sells kits for converting cars to electric power. The company's income, in dollars, from the sale of conversion kits is given by $I(n) = 10,000n$, where n is the number of conversion kits sold. The company sells an average of 9 kits per week.

- Find the income the company receives from conversion kits as a function of the number of weeks w .
- What is the company's income from the sale of conversion kits after 1 year?

- 61. Automobile Rebates** A car dealership offers a \$1500 rebate and a 10% discount off the price of a new car. Let p be the sticker price of a new car on the dealer's lot, r the price after the rebate, and d the discounted price. Then $r(p) = p - 1500$ and $d(p) = 0.90p$.

- Write a composite function for the dealer taking the rebate first and then the discount.
- Write a composite function for the dealer taking the discount first and then the rebate.
- Which composite function would you prefer the dealer use when you buy a new car?

In the News

Going Electric

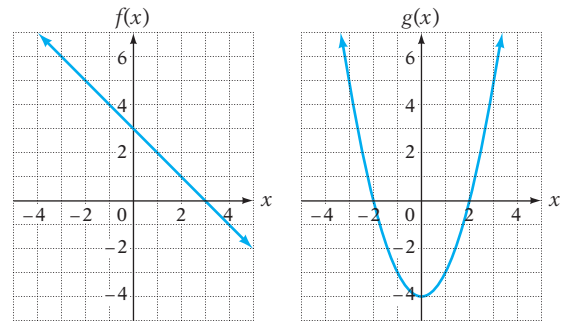
If you don't want to wait for car manufacturers to come out with a reasonably priced electric car, you can take your gasoline-powered vehicle to a garage right here in Walton, Kansas, that charges \$12,500 per vehicle to convert cars and trucks to electric power. The garage is able to convert four vehicles per month.

Source: Associated Press

Applying the Concepts

The graphs of f and g are shown at the right. Use the graphs to determine the values of the composite functions in Exercises 62 to 67.

62. $f[g(-1)]$ 63. $g[f(1)]$ 64. $(g \circ f)(2)$
65. $(f \circ g)(3)$ 66. $g[f(3)]$ 67. $f[g(0)]$



For Exercises 68 to 73, let $g(x) = x^2 - 1$. Find:

68. $g(2 + h)$ 69. $g(3 + h) - g(3)$ 70. $g(-1 + h) - g(-1)$
71. $\frac{g(1 + h) - g(1)}{h}$ 72. $\frac{g(-2 + h) - g(-2)}{h}$ 73. $\frac{g(a + h) - g(a)}{h}$

For Exercises 74 to 79, let $f(x) = 2x$, $g(x) = 3x - 1$, and $h(x) = x - 2$. Find:

74. $f(g[h(2)])$ 75. $g(h[f(1)])$ 76. $h(g[f(-1)])$
77. $f(h[g(0)])$ 78. $f(g[h(x)])$ 79. $g(f[h(x)])$

SECTION

9.4

One-to-One and Inverse Functions

OBJECTIVE A

To determine whether a function is one-to-one



Recall that a function is a set of ordered pairs in which no two ordered pairs that have the same first coordinate have different second coordinates. This means that given any x , there is only one y that can be paired with that x . A **one-to-one function** satisfies the additional condition that given any y , there is only one x that can be paired with that y . One-to-one functions are commonly written as 1–1.

One-to-One Function

A function f is a 1–1 function if for any a and b in the domain of f , $f(a) = f(b)$ implies $a = b$.

This definition states that if the y -coordinates of an ordered pair are equal, $f(a) = f(b)$, then the x -coordinates must be equal, $a = b$.

The function defined by $f(x) = 2x + 1$ is a 1–1 function. To show this, determine $f(a)$ and $f(b)$. Then form the equation $f(a) = f(b)$.

$$f(a) = 2a + 1 \qquad f(b) = 2b + 1$$

$$f(a) = f(b)$$

$$2a + 1 = 2b + 1$$

$$2a = 2b$$

$$a = b$$

• Subtract 1 from each side of the equation.

• Divide each side of the equation by 2.

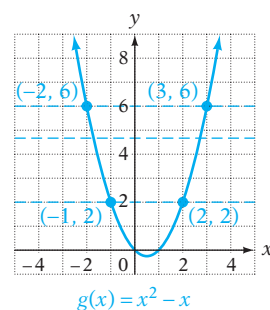
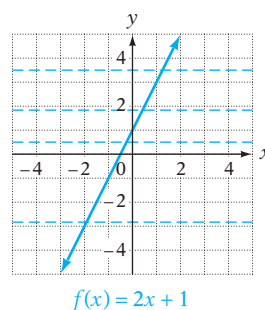
Because $f(a) = f(b)$ implies $a = b$, the function is a 1–1 function.

Consider the function defined by $g(x) = x^2 - x$. Evaluate g at -2 and 3 .

$$g(-2) = (-2)^2 - (-2) = 6 \qquad g(3) = 3^2 - 3 = 6$$

Note that $g(-2) = 6$ and $g(3) = 6$, but $-2 \neq 3$. Thus g is not a 1–1 function.

The graphs of $f(x) = 2x + 1$ and $g(x) = x^2 - x$ are shown at the right. Note that a horizontal line intersects the graph of f at no more than one point. However, a horizontal line intersects the graph of g at more than one point.



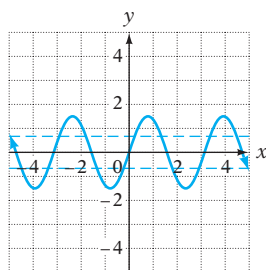
Looking at the graph of f , note that for each y -coordinate there is only one x -coordinate. Thus f is a 1–1 function. From the graph of g , however, there are *two* x -coordinates for a given y -coordinate. For instance, $(-2, 6)$ and $(3, 6)$ are the coordinates of two points on the graph for which the y -coordinates are the same and the x -coordinates are different. Therefore, g is not a 1–1 function.

Horizontal-Line Test

The graph of a function represents the graph of a 1–1 function if any horizontal line intersects the graph at no more than one point.

EXAMPLE • 1

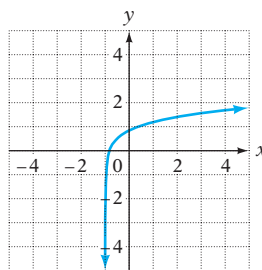
Determine whether the graph is the graph of a 1–1 function.

**Solution**

Because a horizontal line intersects the graph more than once, the graph is not the graph of a 1–1 function.

YOU TRY IT • 1

Determine whether the graph is the graph of a 1–1 function.

**Your solution**

Solution on p. S28

OBJECTIVE B**To find the inverse of a function**

The **inverse** of a function f is the set of ordered pairs formed by reversing the coordinates of each ordered pair of f .

For instance, let $f(x) = 3x$ with domain $\{-2, -1, 0, 1, 2\}$. The ordered pairs of f are $\{(-2, -6), (-1, -3), (0, 0), (1, 3), (2, 6)\}$. Reversing the coordinates of the ordered pairs of f gives $\{(-6, -2), (-3, -1), (0, 0), (3, 1), (6, 2)\}$. This set of ordered pairs is a function and is called the *inverse function* of f . The inverse function of f is denoted by f^{-1} .

From the ordered pairs of f and f^{-1} , we have

Domain of f : $\{-2, -1, 0, 1, 2\}$

Range of f : $\{-6, -3, 0, 3, 6\}$

Domain of f^{-1} : $\{-6, -3, 0, 3, 6\}$

Range of f^{-1} : $\{-2, -1, 0, 1, 2\}$

The domain of f^{-1} is the range of f , and the range of f^{-1} is the domain of f .

**Take Note**

It is important to note that $f^{-1}(x)$ is *not* the reciprocal

of $f(x)$. $f^{-1}(x) = \frac{x}{3}$; the

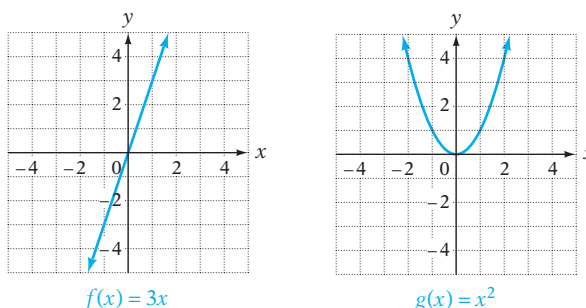
reciprocal of f is $\frac{1}{f(x)} = \frac{1}{3x}$.

Note that $f^{-1}(6) = 2$, whereas evaluating the reciprocal of f at $x = 6$ gives $\frac{1}{18}$.

Because the function f multiplies an element of the domain by 3, a reasonable guess would be that the *inverse* function would *divide* an element by 3. To test this, we can write $f^{-1}(x) = \frac{x}{3}$, where the symbol $f^{-1}(x)$ is read “ f inverse of x .” Using $\{-6, -3, 0, 3, 6\}$ as the domain of f^{-1} , the ordered pairs of f^{-1} are $\{(-6, -2), (-3, -1), (0, 0), (3, 1), (6, 2)\}$. This is the same result we obtained by interchanging the coordinates of the ordered pairs of f .

Now let $g(x) = x^2$ with domain $\{-2, -1, 0, 1, 2\}$. The ordered pairs of g are $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$. Reversing the coordinates of the ordered pairs of g gives $\{(4, -2), (1, -1), (0, 0), (1, 1), (4, 2)\}$. This set of ordered pairs *is not* a function. There are ordered pairs with the same first coordinate and different second coordinates. This illustrates that not every function has an inverse function.

The graphs of $f(x) = 3x$ and $g(x) = x^2$, with the set of real numbers as the domain, are shown below.



By the horizontal-line test, f is a 1–1 function but g is not a 1–1 function.

Condition for an Inverse Function

A function f has an inverse function f^{-1} if and only if f is a 1–1 function.

HOW TO • 1

Find the inverse of the function

$\{(-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8)\}$.

Reverse the coordinates of the ordered pairs of the function.

$\{(-8, -2), (-1, -1), (0, 0), (1, 1), (8, 2)\}$

Because no two ordered pairs have the same first coordinate, this is a function. **The inverse function is $\{(-8, -2), (-1, -1), (0, 0), (1, 1), (8, 2)\}$.**

We can use the fact that the inverse of a function is found by interchanging x and y to determine the inverse of a function that is given by an equation.

HOW TO • 2

Find the inverse of $f(x) = 3x + 6$.

$$f(x) = 3x + 6$$

$$y = 3x + 6$$

$$x = 3y + 6$$

$$x - 6 = 3y$$

$$\frac{1}{3}x - 2 = y$$

$$f^{-1}(x) = \frac{1}{3}x - 2$$

- Replace $f(x)$ by y .
- Interchange x and y .
- Solve for y .

- Replace y by $f^{-1}(x)$.

The inverse of the function $f(x) = 3x + 6$ is $f^{-1}(x) = \frac{1}{3}x - 2$.

**Take Note**

Inverse functions can be likened to, for instance, multiplying a number by 5 and then dividing the result by 5; you will be back to the original number. Evaluating f at a number and then evaluating f^{-1} at the output of f returns you to the original number.

The composition of a function and its inverse has a special property.

Composition of Inverse Functions Property

If f is a function and f^{-1} is its inverse function, then $f^{-1}[f(x)] = x$ and $f[f^{-1}(x)] = x$.

In words, $f^{-1}[f(x)] = x$ states that if $f(a) = b$, then $f^{-1}(b) = a$, the original number used to evaluate f . A similar statement can be made for $f[f^{-1}(x)] = x$.

For instance, consider $f(x) = 3x + 6$ and $f^{-1}(x) = \frac{1}{3}x - 2$ from the previous example.

Choose *any* value of x in the domain of f , say, $x = 4$. Then $f(4) = 3 \cdot 4 + 6 = 18$. Now evaluate f^{-1} at 18. $f^{-1}(18) = \frac{1}{3}(18) - 2 = 4$, the original value of x that was chosen.

The Composition of Inverse Functions Property can be used to determine whether two functions are inverses of each other.

HOW TO 3

Are $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x + 2$ inverses of each other?

To determine whether the functions are inverses, use the Composition of Inverse Functions Property.

$$\begin{aligned} f[g(x)] &= 2\left(\frac{1}{2}x + 2\right) - 4 & g[f(x)] &= \frac{1}{2}(2x - 4) + 2 \\ &= x + 4 - 4 & &= x - 2 + 2 \\ &= x & &= x \end{aligned}$$

Because $f[g(x)] = x$ and $g[f(x)] = x$, the functions are inverses of each other.

Inverse functions occur naturally in many real-world situations.

HOW TO 4

The equation $F(x) = \frac{9}{5}x + 32$ converts a Celsius temperature x to a Fahrenheit temperature $F(x)$. Find the inverse of this function, and write a sentence that explains its meaning.

$$\begin{aligned} F(x) &= \frac{9}{5}x + 32 \\ y &= \frac{9}{5}x + 32 & \bullet \text{ Replace } F(x) \text{ by } y. \\ x &= \frac{9}{5}y + 32 & \bullet \text{ Interchange } x \text{ and } y. \\ x - 32 &= \frac{9}{5}y & \bullet \text{ Solve for } y. \\ \frac{5}{9}(x - 32) &= y \\ F^{-1}(x) &= \frac{5}{9}(x - 32) & \bullet \text{ Replace } y \text{ by } F^{-1}(x). \end{aligned}$$

The inverse is $F^{-1}(x) = \frac{5}{9}(x - 32)$. This equation gives a formula for converting a Fahrenheit temperature to a Celsius temperature.

EXAMPLE • 2Find the inverse of $f(x) = 2x - 3$.**Solution**

$$\begin{aligned}
 f(x) &= 2x - 3 \\
 y &= 2x - 3 && \bullet \text{ Replace } f(x) \text{ by } y. \\
 x &= 2y - 3 && \bullet \text{ Interchange } x \text{ and } y. \\
 x + 3 &= 2y && \bullet \text{ Solve for } y. \\
 \frac{x}{2} + \frac{3}{2} &= y \\
 f^{-1}(x) &= \frac{x}{2} + \frac{3}{2} && \bullet \text{ Replace } y \text{ by } f^{-1}(x).
 \end{aligned}$$

The inverse of the function is given by

$$f^{-1}(x) = \frac{x}{2} + \frac{3}{2}.$$

YOU TRY IT • 2Find the inverse of $f(x) = \frac{1}{2}x + 4$.**Your solution****EXAMPLE • 3**Are $f(x) = 3x - 6$ and $g(x) = \frac{1}{3}x + 2$ inverses of each other?**Solution**

$$\begin{aligned}
 f[g(x)] &= 3\left(\frac{1}{3}x + 2\right) - 6 = x + 6 - 6 = x \\
 g[f(x)] &= \frac{1}{3}(3x - 6) + 2 = x - 2 + 2 = x
 \end{aligned}$$

Yes, the functions are inverses of each other.

YOU TRY IT • 3Are $f(x) = 2x - 6$ and $g(x) = \frac{1}{2}x - 3$ inverses of each other?**Your solution****EXAMPLE • 4**To convert feet to inches, we can use the formula $f(x) = 12x$, where x is the number of feet and $f(x)$ is the number of inches. Find $f^{-1}(x)$, and write a sentence that explains its meaning.**Solution**

$$\begin{aligned}
 f(x) &= 12x \\
 y &= 12x && \bullet \text{ Replace } f(x) \text{ by } y. \\
 x &= 12y && \bullet \text{ Interchange } x \text{ and } y. \\
 \frac{x}{12} &= y && \bullet \text{ Solve for } y. \\
 \frac{x}{12} &= f^{-1}(x) && \bullet \text{ Replace } y \text{ by } f^{-1}(x).
 \end{aligned}$$

The inverse is $f^{-1}(x) = \frac{x}{12}$. This equation gives a formula for converting inches to feet.**YOU TRY IT • 4**The speed of sound s , in feet per second, depends on the temperature of the air and can be approximated by $s(x) = \frac{3}{5}x + 1125$, where x is the air temperature in degrees Celsius. Find $s^{-1}(x)$, and write a sentence that explains its meaning.**Your solution**

9.4 EXERCISES

OBJECTIVE A

To determine whether a function is one-to-one



1. What is a 1-1 function?

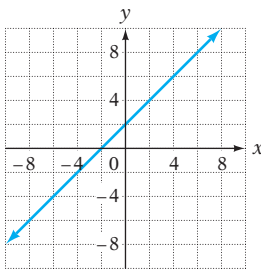


2. What is the horizontal-line test?

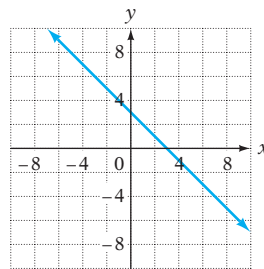
3. a. Suppose f is a function and $f(3) = 5$. Is it possible that there exists another value c for which $f(c) = 5$?b. Suppose f is a 1-1 function and $f(3) = 5$. Is it possible that there exists another value c for which $f(c) = 5$?4. The domain of a 1-1 function f has n elements. How many elements are in the range of f ?

For Exercises 5 to 16, determine whether the graph represents the graph of a 1-1 function.

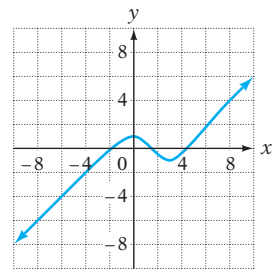
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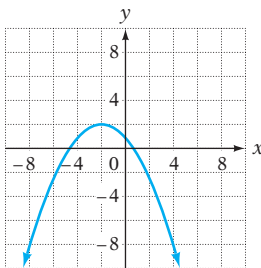
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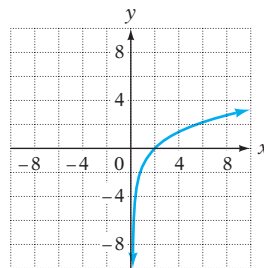
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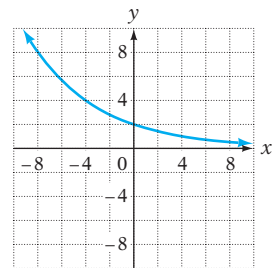
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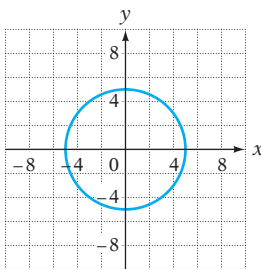
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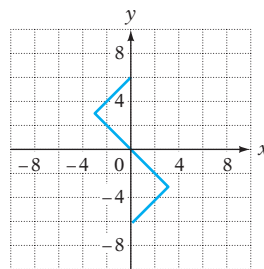
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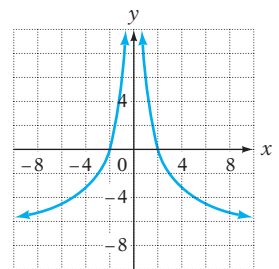
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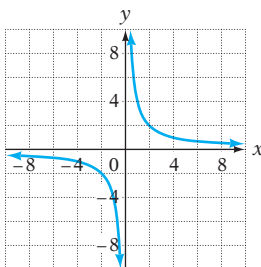
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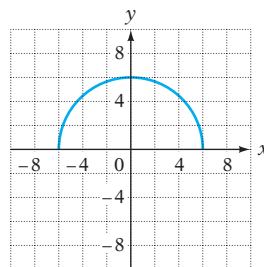
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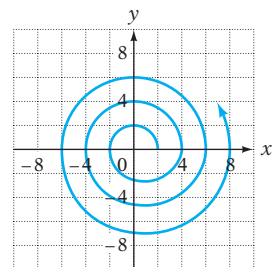
14.



15.



16.



OBJECTIVE B**To find the inverse of a function**

-  17. How are the ordered pairs of the inverse of a function related to the function?
-  18. Why is it that not all functions have an inverse function?
-  19. Suppose the domain of f is $\{1, 2, 3, 4, 5\}$, and the range of f is $\{3, 7, 11\}$. Does f have an inverse function?
-  20. Suppose that f is a 1–1 function and that $f(3) = -2$. Which of the following is (are) possible? (i) $f^{-1}(-2) = -2$ (ii) $f^{-1}(-2) = 3$ (iii) $f^{-1}(-2) = 6$
-  21. Let f be a 1–1 function with $f(3) = 4$, $f(4) = 5$, and $f(6) = 7$. What is the value of **a.** $f^{-1}(5)$, **b.** $f^{-1}(4)$, and **c.** $f^{-1}(7)$?
-  22. Let f be a 1–1 function with $f(-3) = 3$, $f(-4) = 7$, and $f(0) = 8$. What is the value of **a.** $f^{-1}(7)$, **b.** $f^{-1}(3)$, and **c.** $f^{-1}(8)$?

For Exercises 23 to 30, find the inverse of the function. If the function does not have an inverse function, write “no inverse.”

23. $\{(1, 0), (2, 3), (3, 8), (4, 15)\}$ 24. $\{(1, 0), (2, 1), (-1, 0), (-2, 0)\}$
25. $\{(3, 5), (-3, -5), (2, 5), (-2, -5)\}$ 26. $\{(-5, -5), (-3, -1), (-1, 3), (1, 7)\}$
27. $\{(0, -2), (-1, 5), (3, 3), (-4, 6)\}$ 28. $\{(-2, -2), (0, 0), (2, 2), (4, 4)\}$
29. $\{(-2, -3), (-1, 3), (0, 3), (1, 3)\}$ 30. $\{(2, 0), (1, 0), (3, 0), (4, 0)\}$

For Exercises 31 to 48, find $f^{-1}(x)$.

31. $f(x) = 4x - 8$ 32. $f(x) = 3x + 6$ 33. $f(x) = 2x + 4$
34. $f(x) = x - 5$ 35. $f(x) = \frac{1}{2}x - 1$ 36. $f(x) = \frac{1}{3}x + 2$
37. $f(x) = -2x + 2$ 38. $f(x) = -3x - 9$ 39. $f(x) = \frac{2}{3}x + 4$
40. $f(x) = \frac{3}{4}x - 4$ 41. $f(x) = -\frac{1}{3}x + 1$ 42. $f(x) = -\frac{1}{2}x + 2$

43. $f(x) = 2x - 5$

44. $f(x) = 3x + 4$

45. $f(x) = 5x - 2$

46. $f(x) = 4x - 2$

47. $f(x) = 6x - 3$

48. $f(x) = -8x + 4$

For Exercises 49 to 51, let $f(x) = 3x - 5$. Find:

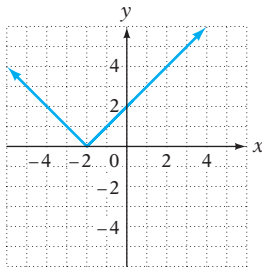
49. $f^{-1}(0)$

50. $f^{-1}(2)$

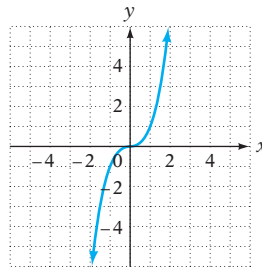
51. $f^{-1}(4)$

For Exercises 52 to 54, state whether the graph is the graph of a function. If it is the graph of a function, does the function have an inverse?

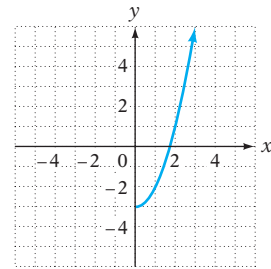
52.



53.



54.



For Exercises 55 to 62, use the Composition of Inverse Functions Property to determine whether the functions are inverses.

55. $f(x) = 4x; g(x) = \frac{x}{4}$

56. $g(x) = x + 5; h(x) = x - 5$

57. $f(x) = 3x; h(x) = \frac{1}{3x}$

58. $h(x) = x + 2; g(x) = 2 - x$

59. $g(x) = 3x + 2; f(x) = \frac{1}{3}x - \frac{2}{3}$

60. $h(x) = 4x - 1; f(x) = \frac{1}{4}x + \frac{1}{4}$

61. $f(x) = \frac{1}{2}x - \frac{3}{2}; g(x) = 2x + 3$

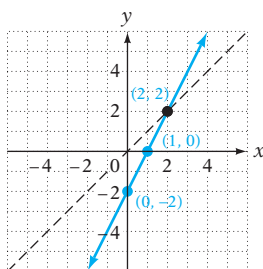
62. $g(x) = -\frac{1}{2}x - \frac{1}{2}; h(x) = -2x + 1$

63. To convert ounces to pounds, we can use the formula $f(x) = \frac{x}{16}$, where x is the number of ounces and $f(x)$ is the number of pounds. Find $f^{-1}(x)$, and write a sentence that explains its meaning.
64. To convert pounds to kilograms, we can use the formula $f(x) = \frac{x}{2.2}$, where x is the number of pounds and $f(x)$ is the number of kilograms. Find $f^{-1}(x)$, and write a sentence that explains its meaning.
65. The function given by $f(x) = x + 30$ converts a dress size x in the United States to a dress size $f(x)$ in France. Find the inverse of this function, and write a sentence that explains its meaning.
66. The function given by $f(x) = x - 31$ converts a shoe size x in France to a shoe size $f(x)$ in the United States. Find the inverse of this function, and write a sentence that explains its meaning.
67. The target heart rate $f(x)$ for a certain athlete is given by $f(x) = 90x + 65$, where x is the training intensity percent. Find the inverse of this function, and write a sentence that explains its meaning.
68. A data messaging service charges \$5 per month plus \$.10 per message. The function given by $f(x) = 0.1x + 5$ gives the monthly cost $f(x)$ for sending x messages. Find the inverse of this function, and write a sentence that explains its meaning.

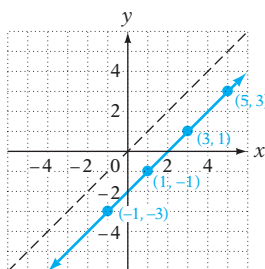
Applying the Concepts

Recall that interchanging the x - and y -coordinates of each ordered pair of a 1–1 function forms the inverse function. Given the graphs in Exercises 69 to 74, interchange the x - and y -coordinates of the given ordered pairs and then plot the new ordered pairs. Draw a smooth graph through the ordered pairs. The graph is the graph of the inverse function.

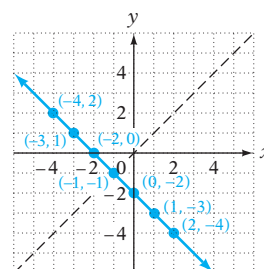
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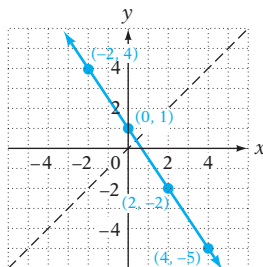
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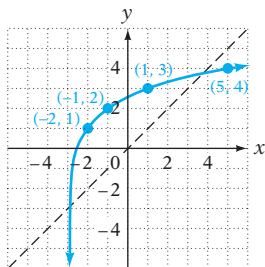
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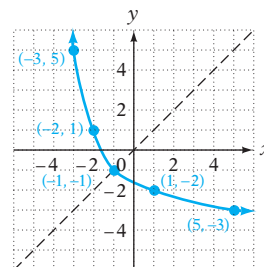
72.



73.



74.



Each of the tables in Exercises 75 and 76 defines a function. Is the inverse of the function a function? Explain your answer.

75. **Grading Scale**

Score	Grade
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

76. **First-Class Postage Rates**

Weight	Cost
$0 < w < 1$	\$.42
$1 < w \leq 2$	\$.59
$2 < w \leq 3$	\$.76
$3 < w \leq 3.5$	\$.93



77. Is the inverse of a constant function a function? Explain your answer.



78. The graphs of all functions given by $f(x) = mx + b$, $m \neq 0$, are straight lines. Are all of these functions 1–1 functions? If so, explain why. If not, give an example of a linear function that is not 1–1.

Mathematics is based on the concept of proof, which in turn is based on principles of logic. It was Aristotle (384 B.C. to 322 B.C.) who took the logical principles that had been developed over time and set them in a structured form. From this structured form, several methods of proof were developed.

A deductive proof is one that proceeds in a carefully ordered sequence of steps. It is the type of proof you may have seen in a geometry class. Deductive proofs can be modeled as shown at the left.

Here is an example of a deductive proof. It proves that the product of a negative number and a positive number is a negative number. The proof is based on the fact that every real number has exactly one additive inverse. For instance, the additive inverse of 4 is -4 and the additive inverse of $-\frac{2}{3}$ is $\frac{2}{3}$.

Theorem: If a and b are positive real numbers, then $(-a)b = -(ab)$.

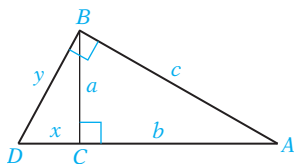
Proof: $(-a)b + ab = (-a + a)b$ • **Distributive Property**
 $= 0 \cdot b$ • **Inverse Property of Addition**
 $= 0$ • **Multiplication Property of Zero**

Since $(-a)b + ab = 0$, $(-a)b$ is the additive inverse of ab . But $-(ab)$ is also the additive inverse of ab and, because every number has exactly one additive inverse, $(-a)b = -(ab)$.

Here is another example of a deductive proof. It is a proof of part of the Pythagorean Theorem.

Theorem: If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Proof: Let $\triangle DBA$ be a right triangle and let BC be the altitude to the hypotenuse of $\triangle DBA$, as shown at the left. Then $\triangle BCD$ and $\triangle BCA$ are right triangles.



1. $\triangle DBA$ is a right triangle with height BC .
2. $\triangle DBC$ is similar to $\triangle BAC$ and $\triangle DBA$ is similar to $\triangle BCA$.
3. $\frac{a}{b} = \frac{x}{a}$
4. $a^2 = bx$
5. $\frac{x+b}{c} = \frac{c}{b}$
6. $bx + b^2 = c^2$
7. $a^2 + b^2 = c^2$

Given.

Theorem from geometry: the altitude to the hypotenuse of a right triangle forms triangles that are similar to each other.

The ratios of corresponding sides of similar triangles are equal.

Multiply each side by ab .

The ratios of corresponding sides of similar triangles are equal.

Multiply each side by cb .

Substitution Property from (4).

Because the first statement of the proof is true and each subsequent statement of the proof is true, the first statement implies that the last statement is true. Therefore, we have proved: If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

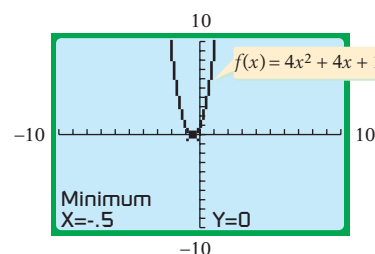
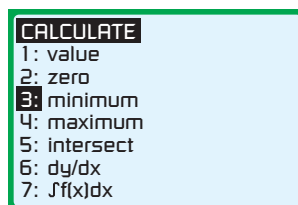
PROJECTS AND GROUP ACTIVITIES

Finding the Maximum or Minimum of a Function Using a Graphing Calculator



Recall that for a quadratic function, the maximum or minimum value of the function is the y -coordinate of the vertex of the graph of the function. We determined algebraically that the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. It is also possible to use a graphing calculator to approximate a maximum or minimum value. The method used to find the value depends on the calculator model. However, it is generally true for most calculators that the point at which the function has a maximum or minimum must be shown on the screen. If it is not, you must select a different viewing window.

For a TI-84 graphing calculator, press **2ND** **CALC** 3 to determine a minimum value or **2ND** **CALC** 4 to determine a maximum value. Move the cursor to a point on the curve that is to the left of the minimum (or maximum). Press **ENTER**. Move the cursor to a point on the curve that is to the right of the minimum (or maximum). Press **ENTER** twice. The minimum (or maximum) value is shown as the y -coordinate at the bottom of the screen.



- Find the maximum or minimum value of $f(x) = -x^2 - 3x + 2$ by using a graphing calculator.
- Find the maximum or minimum value of $f(x) = x^2 - 4x - 2$ by using a graphing calculator.
- Find the maximum or minimum value of $f(x) = -\sqrt{2}x^2 - \pi x + 2$ by using a graphing calculator.
- Find the maximum or minimum value of $f(x) = \frac{x^2}{\sqrt{5}} - \sqrt{7}x - \sqrt{11}$ by using a graphing calculator.
- Business** A manufacturer of camera lenses estimates that the average monthly cost C of production is given by the function $C(x) = 0.1x^2 - 20x + 2000$, where x is the number of lenses produced each month. Find the number of lenses the company should produce in order to minimize the average cost.
- Compensation** The net annual income I , in dollars, of a family physician can be modeled by the equation $I(x) = -290(x - 48)^2 + 148,000$, where x is the age of the physician and $27 \leq x \leq 70$.
 - Find the age at which the physician's income will be a maximum.
 - Find the maximum income.

Business Applications of Maximum and Minimum Values of Quadratic Functions

A company's revenue R is the total amount of money the company earns by selling its products. The cost C is the total amount of money the company spends to manufacture and sell its products. A company's profit P is the difference between the revenue and the cost.

$$P = R - C$$

A company's revenue and cost may be represented by equations.



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HOW TO 1

The owners of a travel agency are selling tickets for a tour. The travel agency must pay a cost per person of \$10. A maximum of 180 people can be accommodated on the tour. For every \$.25 increase in the price of a ticket over the \$10 cost, the owners estimate that they will sell one ticket less. Therefore, the price per ticket can be represented by

$$\begin{aligned} p &= 10 + 0.25(180 - x) \\ &= 55 - 0.25x \end{aligned}$$

where x represents the number of tickets sold. Determine the maximum profit and the price per ticket that yields the maximum profit.

Solution

The cost is represented by $10x$ and the revenue is represented by $x(55 - 0.25x)$, where x represents the number of tickets sold.

Use the equation $P = R - C$ to represent the profit.

$$\begin{aligned} P &= R - C \\ P &= x(55 - 0.25x) - 10x \\ P &= -0.25x^2 + 45x \end{aligned}$$

The graph of $P = -0.25x^2 + 45x$ is a parabola that opens down. Find the x -coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{45}{2(-0.25)} = 90$$

To find the maximum profit, evaluate $P = -0.25x^2 + 45x$ at 90.

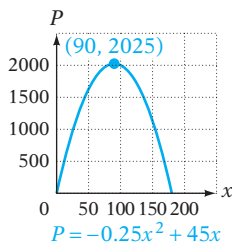
$$\begin{aligned} P &= -0.25x^2 + 45x \\ P &= -0.25(90^2) + 45(90) \\ P &= 2025 \end{aligned}$$

To find the price per ticket, evaluate $p = 55 - 0.25x$ at 90.

$$\begin{aligned} p &= 55 - 0.25x \\ p &= 55 - 0.25(90) \\ p &= 32.50 \end{aligned}$$

The travel agency can expect a maximum profit of \$2025 when 90 people take the tour at a ticket price of \$32.50 per person.

The graph of the profit function is shown at the left.



- Business** A company's total monthly cost, in dollars, for producing and selling n DVDs per month is $C = 25n + 4000$. The company's revenue, in dollars, from selling all n DVDs is $R = 275n - 0.2n^2$.
 - How many DVDs must the company produce and sell each month in order to make the maximum monthly profit?
 - What is the maximum monthly profit?
- Business** A shipping company has determined that its cost to deliver x packages per week is $1000 + 6x$. The price per package that the company charges when it sends x packages per week is $24 - 0.02x$. Determine the company's maximum weekly profit and the price per package that yields the maximum profit.

CHAPTER 9

SUMMARY

KEY WORDS

A *quadratic function* is one that can be expressed by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. [9.1A, p. 496]

EXAMPLES

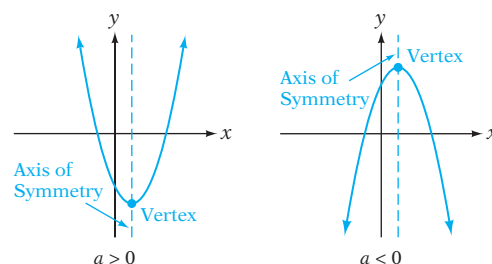
$f(x) = 4x^2 + 3x - 5$ is a quadratic function.

The value of the independent variable is sometimes called the *input*. The result of evaluating the function is called the *output*. An *input/output table* shows the results of evaluating a function for various values of the independent variable. [9.1A, p. 496]

$$f(x) = x^2 + 4x - 6$$

x	y
-2	-10
-1	-9
0	-6
1	-1
2	6

The graph of a quadratic function is a *parabola*. When $a > 0$, the parabola opens up and the *vertex* of the parabola is the point with the smallest y -coordinate. When $a < 0$, the parabola opens down and the *vertex* of the parabola is the point with the largest y -coordinate. The *axis of symmetry* is the vertical line that passes through the vertex of the parabola and is parallel to the y -axis. [9.1A, pp. 496, 497]



A point at which a graph crosses the x -axis is called an *x -intercept* of the graph. The x -intercepts occur when $y = 0$. [9.1B, p. 499]

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x - 2)(x - 3) &= 0 \\ x - 2 = 0 &\quad x - 3 = 0 \\ x = 2 &\quad x = 3 \end{aligned}$$

The x -intercepts are $(2, 0)$ and $(3, 0)$.

A value of x for which $f(x) = 0$ is called a *zero* of the function. [9.1B, p. 499]

$$\begin{aligned} f(x) &= x^2 - 7x + 10 \\ 0 &= x^2 - 7x + 10 \\ 0 &= (x - 2)(x - 5) \\ x - 2 = 0 &\quad x - 5 = 0 \\ x = 2 &\quad x = 5 \end{aligned}$$

2 and 5 are zeros of the function.

The graph of a quadratic function has a *minimum value* if $a > 0$ and a *maximum value* if $a < 0$. [9.1C, p. 502]

The graph of $f(x) = 3x^2 - x + 4$ has a minimum value.

The graph of $f(x) = -x^2 - 6x + 5$ has a maximum value.

A function is a *one-to-one function* if, for any a and b in the domain of f , $f(a) = f(b)$ implies $a = b$. This means that given any y , there is only one x that can be paired with that y . One-to-one functions are commonly written as 1–1. [9.4A, p. 526]

A non-constant linear function is a 1–1 function.

A quadratic function is not a 1–1 function.

The *inverse of a function* is the set of ordered pairs formed by reversing the coordinates of each ordered pair of the function. [9.4B, p. 527]

The inverse of the function $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$ is $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)\}$.

ESSENTIAL RULES AND PROCEDURES

Vertex and Axis of Symmetry of a Parabola [9.1A, p. 497]

Let $f(x) = ax^2 + bx + c$ be the equation of a parabola. The coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. The equation of the axis of symmetry is $x = -\frac{b}{2a}$.

EXAMPLES

$$f(x) = x^2 - 2x - 4$$

$$a = 1, b = -2$$

$$-\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

$$f(1) = 1^2 - 2(1) - 4 = -5$$

The coordinates of the vertex are $(1, -5)$.

The axis of symmetry is $x = 1$.

Effect of the Discriminant on the Number of x -Intercepts of a Parabola [9.1B, p. 500]

1. If $b^2 - 4ac = 0$, the parabola has one x -intercept.
2. If $b^2 - 4ac > 0$, the parabola has two x -intercepts.
3. If $b^2 - 4ac < 0$, the parabola has no x -intercepts.

$x^2 + 8x + 16 = 0$ has one x -intercept because $b^2 - 4ac = 8^2 - 4(1)(16) = 0$.

$2x^2 + 3x - 5 = 0$ has two x -intercepts because $b^2 - 4ac = 3^2 - 4(2)(-5) = 49$.

$3x^2 + 2x + 4 = 0$ has no x -intercepts because $b^2 - 4ac = 2^2 - 4(3)(4) = -44$.

To Find the Minimum or Maximum Value of a Quadratic Function [9.1C, p. 502]

First find the x -coordinate of the vertex. Then evaluate the function at that value.

$$f(x) = x^2 - 2x - 4$$

$$a = 1, b = -2$$

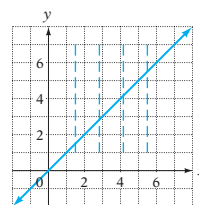
$$-\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

$$f(1) = 1^2 - 2(1) - 4 = -5$$

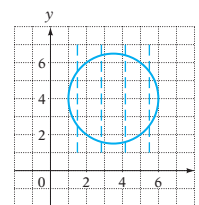
$a > 0$. The minimum value of the function is -5 .

Vertical-Line Test [9.2A, p. 513]

A graph defines a function if any vertical line intersects the graph at no more than one point.



A function



Not a function

Operations on Functions [9.3A, p. 518]

If f and g are functions and x is an element of the domain of each function, then

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$

Given $f(x) = x + 2$ and $g(x) = 2x$, then

$$\begin{aligned}(f + g)(4) &= f(4) + g(4) \\ &= (4 + 2) + 2(4) = 6 + 8 \\ &= 14\end{aligned}$$

$$\begin{aligned}(f - g)(4) &= f(4) - g(4) \\ &= (4 + 2) - 2(4) = 6 - 8 \\ &= -2\end{aligned}$$

$$\begin{aligned}(f \cdot g)(4) &= f(4) \cdot g(4) \\ &= (4 + 2) \cdot 2(4) = 6 \cdot 8 \\ &= 48\end{aligned}$$

$$\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{4 + 2}{2(4)} = \frac{6}{8} = \frac{3}{4}$$

Definition of the Composition of Two Functions [9.3B, p. 520]

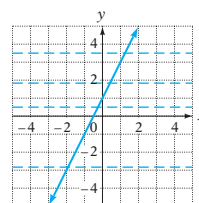
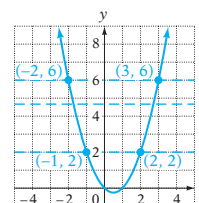
The composition of two functions f and g , symbolized by $f \circ g$, is the function whose value at x is given by $(f \circ g)(x) = f[g(x)]$.

Given $f(x) = x - 4$ and $g(x) = 4x$, then

$$\begin{aligned}(f \circ g)(2) &= f[g(2)] \\ &= f(8) \quad \text{because } g(2) = 8 \\ &= 8 - 4 = 4\end{aligned}$$

Horizontal-Line Test [9.4A, p. 527]

The graph of a function represents the graph of a 1–1 function if any horizontal line intersects the graph at no more than one point.

A 1-1
functionNot a 1-1
function**Condition for an Inverse Function** [9.4B, p. 528]

A function f has an inverse function if and only if f is a 1–1 function.

The function $f(x) = x^2$ does not have an inverse function. When $y = 4$, $x = 2$ or -2 ; therefore, the function $f(x)$ is not a 1–1 function.

Composition of Inverse Functions Property [9.4B, p. 529]

$$f^{-1}[f(x)] = x \text{ and } f[f^{-1}(x)] = x.$$

$$f(x) = 2x - 3 \quad f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$

$$f^{-1}[f(x)] = \frac{1}{2}(2x - 3) + \frac{3}{2} = x$$

$$f[f^{-1}(x)] = 2\left(\frac{1}{2}x + \frac{3}{2}\right) - 3 = x$$

CHAPTER 9**CONCEPT REVIEW**

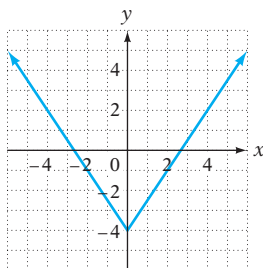
Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. Given a quadratic function, how do you find the vertex and axis of symmetry of its graph?
2. Given its vertex and axis of symmetry, can you accurately graph a parabola?
3. Suppose you know the vertex of a parabola and whether it opens up or down. How can you determine the range of the function?
4. What does the discriminant tell you about the x -intercepts of a parabola?
5. For the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, how does the coefficient of the x^2 term change the shape of the parabola?
6. What is the vertical-line test?
7. What is the shape of the graph of the absolute value of a linear polynomial?
8. Describe the four basic operations on functions.
9. What does the notation $(f \circ g)(x)$ represent?
10. How does the horizontal-line test determine a 1–1 function?
11. What condition must be met for a function to have an inverse?

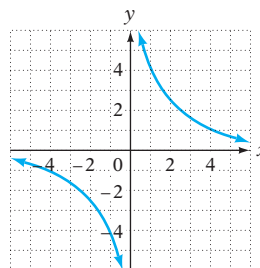
CHAPTER 9

REVIEW EXERCISES

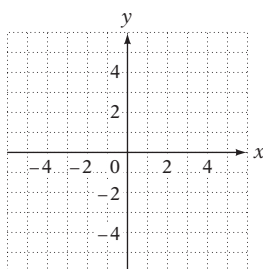
1. Is the graph below the graph of a function?



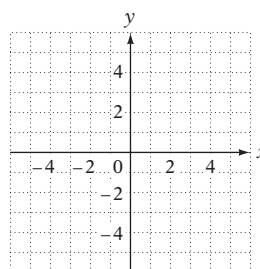
2. Is the graph below the graph of a 1-1 function?



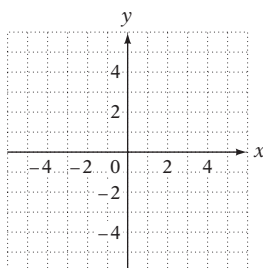
3. Graph
- $f(x) = 3x^3 - 2$
- . State the domain and range.



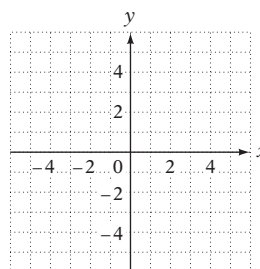
4. Graph
- $f(x) = \sqrt{x+4}$
- . State the domain and range.



5. Graph
- $f(x) = |x| - 3$
- . State the domain and range.



6. Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola whose equation is
- $y = x^2 - 2x + 3$
- . Then graph the equation.



7. Use the discriminant to determine the number of
- x
- intercepts of the graph of
- $y = -3x^2 + 4x + 6$
- .

8. Use the discriminant to determine the number of
- x
- intercepts of the graph of
- $f(x) = 2x^2 + x + 5$
- .

9. Find the
- x
- intercepts of the graph of
- $y = 3x^2 + 9x$
- .

10. Find the
- x
- intercepts of the graph of
- $f(x) = x^2 - 6x + 7$
- .

11. Find the zeros of
- $f(x) = 2x^2 - 7x - 15$
- .

12. Find the zeros of
- $f(x) = x^2 - 2x + 10$
- .

13. Find the maximum value of $f(x) = -2x^2 + 4x + 1$.
14. Find the minimum value of $f(x) = x^2 - 7x + 8$.
15. Given $f(x) = x^2 + 4$ and $g(x) = 4x - 1$, find $f[g(0)]$.
16. Given $f(x) = 6x + 8$ and $g(x) = 4x + 2$, find $g[f(-1)]$.
17. Given $f(x) = 3x^2 - 4$ and $g(x) = 2x + 1$, find $f[g(x)]$.
18. Given $f(x) = 2x^2 + x - 5$ and $g(x) = 3x - 1$, find $g[f(x)]$.

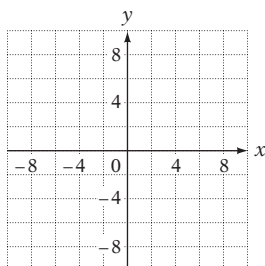
For Exercises 19 to 22, use $f(x) = x^2 + 2x - 3$ and $g(x) = x^2 - 2$.

19. Evaluate: $(f + g)(2)$
20. Evaluate: $(f - g)(-4)$
21. Evaluate: $(f \cdot g)(-4)$
22. Evaluate: $\left(\frac{f}{g}\right)(3)$
23. Find the inverse of $f(x) = -6x + 4$.
24. Find the inverse of $f(x) = \frac{2}{3}x - 12$.
25. Use the Composition of Inverse Functions Property to determine whether $f(x) = -\frac{1}{4}x + \frac{5}{4}$ and $g(x) = -4x + 5$ are inverses of each other.
26. Use the Composition of Inverse Functions Property to determine whether $f(x) = \frac{1}{2}x$ and $g(x) = 2x + 1$ are inverses of each other.
27. **Deep-sea Diving** The pressure p , in pounds per square inch, on a scuba diver x feet below the surface of the water can be approximated by $p(x) = 0.4x + 15$. Find $p^{-1}(x)$, and write a sentence that explains its meaning.
28. **Business** The monthly profit P , in dollars, for a company from the sale of x youth baseball gloves is given by $P(x) = -x^2 + 100x + 2500$. How many gloves should be made each month to maximize profit? What is the maximum monthly profit?
29. **Geometry** The perimeter of a rectangle is 28 ft. What dimensions would give the rectangle a maximum area?

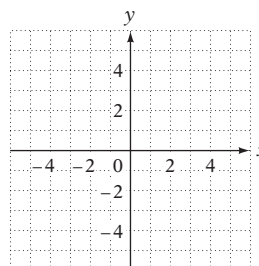
CHAPTER 9

TEST

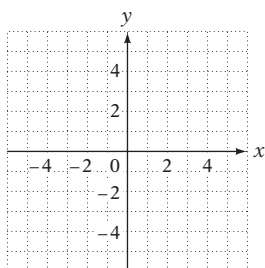
1. Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola whose equation is $y = x^2 - 6x + 4$. Then graph the equation.



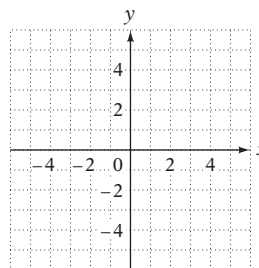
2. Graph $f(x) = \left| \frac{1}{2}x \right| - 2$. State the domain and range.



3. Graph $f(x) = -\sqrt{3 - x}$. State the domain and range.



4. Graph $f(x) = x^3 - 3x + 2$. State the domain and range.



5. Use the discriminant to determine the number of x -intercepts of the graph of $f(x) = 3x^2 - 4x + 6$.

6. Find the coordinates of the x -intercepts of the graph of $y = 3x^2 - 7x - 6$.

7. Find the maximum value of $f(x) = -x^2 + 8x - 7$.

8. Find the domain and range of $f(x) = 2x^2 + 4x - 5$.

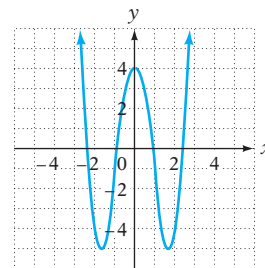
9. Given $f(x) = x^2 + 2x - 3$ and $g(x) = x^3 - 1$, find $(f - g)(2)$.

10. Given $f(x) = x^3 + 1$ and $g(x) = 2x - 3$, find $(f \cdot g)(-3)$.

11. Given $f(x) = 4x - 5$ and $g(x) = x^2 + 3x + 4$, find $\left(\frac{f}{g}\right)(-2)$.

12. Given $f(x) = x^2 + 4$ and $g(x) = 2x^2 + 2x + 1$, find $(f - g)(-4)$.

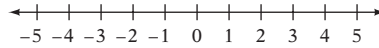
13. Given $f(x) = 2x - 7$ and $g(x) = x^2 - 2x - 5$, find $f[g(2)]$.
14. Given $f(x) = x^2 + 1$ and $g(x) = x^2 + x + 1$, find $g[f(-2)]$.
15. Given $f(x) = x^2 - 1$ and $g(x) = 3x + 2$, find $g[f(x)]$.
16. Given $f(x) = 2x^2 - 7$ and $g(x) = x - 1$, find $f[g(x)]$.
17. Does the function $\{(1, 4), (2, 5), (3, 6), (4, 5), (5, 4)\}$ have an inverse function?
18. Find the inverse of the function $\{(2, 6), (3, 5), (4, 4), (5, 3)\}$.
19. Find the inverse of $f(x) = 4x - 2$.
20. Find the inverse of $f(x) = \frac{1}{4}x - 4$.
21. Use the Composition of Inverse Functions Property to determine whether $f(x) = \frac{1}{2}x + 2$ and $g(x) = 2x - 4$ are inverses of each other.
22. Use the Composition of Inverse Functions Property to determine whether $f(x) = \frac{2}{3}x - 2$ and $g(x) = \frac{3}{2}x + 3$ are inverses of each other.
23. Determine whether the graph at the right is the graph of a 1–1 function.
24. **Delivery Service** A company that delivers documents in a city via bicycle messenger charges \$5 per message plus \$1.25 per mile for the service. The total cost $C(x)$ to deliver a document to a location x miles away is given by $C(x) = 1.25x + 5$. Find $C^{-1}(x)$, and write a sentence that explains its meaning.
25. **Business** The daily production cost $C(x)$, in dollars, to manufacture small speakers is given by $C(x) = x^2 - 50x + 675$, where x is the number of speakers produced per day. How many speakers should be made per day to minimize the daily production cost? What is the minimum daily production cost?
26. **Mathematics** Find two numbers whose sum is 28 and whose product is a maximum.
27. **Geometry** The perimeter of a rectangle is 200 cm. What dimensions would give the rectangle a maximum area? What is the maximum area?



CUMULATIVE REVIEW EXERCISES

1. Evaluate $-3a + \left| \frac{3b - ab}{3b - c} \right|$ when $a = 2$, $b = 2$, and $c = -2$.

2. Graph $\{x | x < -3\} \cap \{x | x > -4\}$.



3. Solve: $\frac{3x - 1}{6} - \frac{5 - x}{4} = \frac{5}{6}$

4. Solve: $4x - 2 < -10$ or $3x - 1 > 8$
Write the solution set in set-builder notation.

5. Solve: $|8 - 2x| \geq 0$

6. Simplify: $\left(\frac{3a^3b}{2a} \right)^2 \left(\frac{a^2}{-3b^2} \right)^3$

7. Multiply: $(x - 4)(2x^2 + 4x - 1)$

8. Solve by using the addition method:
 $6x - 2y = -3$
 $4x + y = 5$

9. Factor: $x^3y + x^2y^2 - 6xy^3$

10. Solve: $(b + 2)(b - 5) = 2b + 14$

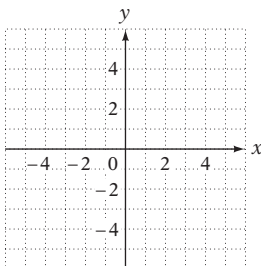
11. Solve: $x^2 - 2x > 15$

12. Subtract: $\frac{x^2 + 4x - 5}{2x^2 - 3x + 1} - \frac{x}{2x - 1}$

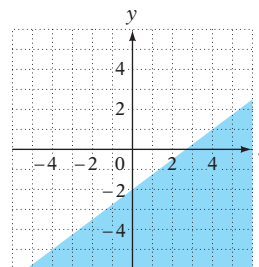
13. Solve: $\frac{5}{x^2 + 7x + 12} = \frac{9}{x + 4} - \frac{2}{x + 3}$

14. Divide: $\frac{4 - 6i}{2i}$

15. Graph $f(x) = \frac{1}{4}x^2$. Find the vertex and axis of symmetry.



16. Graph the solution set of $3x - 4y \geq 8$.



17. Find the equation of the line containing the points whose coordinates are $(-3, 4)$ and $(2, -6)$.
18. Find the equation of the line that contains the point whose coordinates are $(-3, 1)$ and is perpendicular to the line $2x - 3y = 6$.
19. Solve: $3x^2 = 3x - 1$
20. Solve: $\sqrt{8x + 1} = 2x - 1$
21. Find the minimum value of $f(x) = 2x^2 - 3$.
22. Find the range of $f(x) = |3x - 4|$ if the domain is $\{0, 1, 2, 3\}$.
23. Is this set of ordered pairs a function?
 $\{(-3, 0), (-2, 0), (-1, 1), (0, 1)\}$
24. Solve: $\sqrt[3]{5x - 2} = 2$
25. Given $g(x) = 3x - 5$ and $h(x) = \frac{1}{2}x + 4$, find $g[h(2)]$.
26. Find the inverse of $f(x) = -3x + 9$.
27. **Mixtures** Find the cost per pound of a tea mixture made from 30 lb of tea costing \$4.50 per pound and 45 lb of tea costing \$3.60 per pound.
28. **Mixtures** How many pounds of an 80% copper alloy must be mixed with 50 lb of a 20% copper alloy to make an alloy that is 40% copper?
29. **Mixtures** Six ounces of an insecticide are mixed with 16 gal of water to make a spray for spraying an orange grove. How much additional insecticide is required if it is to be mixed with 28 gal of water?
30. **Tanks** A large pipe can fill a tank in 8 min less time than it takes a smaller pipe to fill the same tank. Working together, both pipes can fill the tank in 3 min. How long would it take the larger pipe, working alone, to fill the tank?
31. **Physics** The distance d that a spring stretches varies directly as the force f used to stretch the spring. If a force of 50 lb can stretch a spring 30 in., how far can a force of 40 lb stretch the spring?
32. **Music** The frequency of vibration f in a pipe in an open pipe organ varies inversely as the length L of the pipe. If the air in a pipe 2 m long vibrates 60 times per minute, find the frequency in a pipe that is 1.5 m long.



Exponential and Logarithmic Functions

OBJECTIVES

SECTION 10.1

- A** To evaluate an exponential function
- B** To graph an exponential function

SECTION 10.2

- A** To find the logarithm of a number
- B** To use the Properties of Logarithms to simplify expressions containing logarithms
- C** To use the Change-of-Base Formula

SECTION 10.3

- A** To graph a logarithmic function

SECTION 10.4

- A** To solve an exponential equation
- B** To solve a logarithmic equation

SECTION 10.5

- A** To solve application problems

ARE YOU READY?

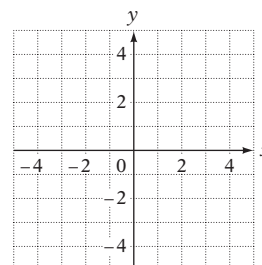
Take the Chapter 10 Prep Test to find out if you are ready to learn to:

- Evaluate an exponential function
- Graph an exponential function and a logarithmic function
- Find the logarithm of a number
- Use the Properties of Logarithms
- Use the Change-of-Base Formula
- Solve an exponential equation and a logarithmic equation

PREP TEST

Do these exercises to prepare for Chapter 10.

1. Simplify: 3^{-2}
2. Simplify: $\left(\frac{1}{2}\right)^{-4}$
3. Complete: $\frac{1}{8} = 2^?$
4. Evaluate $f(x) = x^4 + x^3$ for $x = -1$ and $x = 3$.
5. Solve: $3x + 7 = x - 5$
6. Solve: $16 = x^2 - 6x$
7. Evaluate $A(1 + i)^n$ for $A = 5000$, $i = 0.04$, and $n = 6$. Round to the nearest hundredth.
8. Graph: $f(x) = x^2 - 1$



SECTION

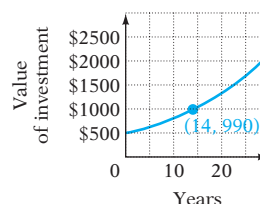
10.1 Exponential Functions

OBJECTIVE A

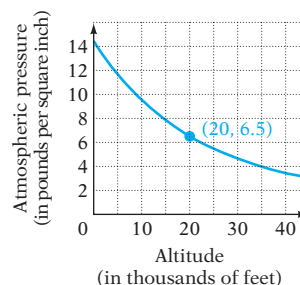
To evaluate an exponential function



The growth of a \$500 savings account that earns 5% annual interest compounded daily is shown in the graph at the right. In approximately 14 years, the savings account contains approximately \$1000, twice the initial amount. The growth of this savings account is an example of an exponential function.



The pressure of the atmosphere at a certain altitude is shown in the graph at the right. This is another example of an exponential function. From the graph, we read that the air pressure is approximately 6.5 lb/in² at an altitude of 20,000 ft.



Definition of an Exponential Function

The **exponential function** with base b is defined by

$$f(x) = b^x$$

where $b > 0$, $b \neq 1$, and x is any real number.

In the definition of an exponential function, b , the base, is required to be positive. If the base were a negative number, the value of the function would be a complex number for some values of x . For instance, the value of $f(x) = (-4)^x$ when $x = \frac{1}{2}$ is $f\left(\frac{1}{2}\right) = (-4)^{1/2} = \sqrt{-4} = 2i$. To avoid complex number values of a function, the base of the exponential function is always a positive number.



Integrating Technology

A graphing calculator can be used to evaluate an exponential expression at an irrational number.

$4^{\sqrt{2}}$ 7.102993301

HOW TO 1

Evaluate $f(x) = 2^x$ at $x = 3$ and $x = -2$.

$$f(3) = 2^3 = 8$$

• Substitute 3 for x and simplify.

$$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

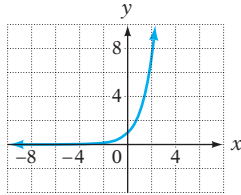
• Substitute -2 for x and simplify.

To evaluate an exponential expression at an irrational number such as $\sqrt{2}$, we obtain an approximation to the value of the function by approximating the irrational number. For instance, the value of $f(x) = 4^x$ at $x = \sqrt{2}$ can be approximated by using an approximation of $\sqrt{2}$.

$$f(\sqrt{2}) = 4^{\sqrt{2}} \approx 4^{1.4142} \approx 7.1029$$

**Take Note**

The natural exponential function is an extremely important function. It is used extensively in applied problems in virtually all disciplines, from archaeology to zoology. Leonhard Euler (1707–1783) was the first to use the letter e as the base of the natural exponential function.



Because $f(x) = b^x$ ($b > 0, b \neq 1$) can be evaluated at both rational and irrational numbers, the domain of f is all real numbers. And because $b^x > 0$ for all values of x , the range of f is the positive real numbers.

A frequently used base in applications of exponential functions is an irrational number designated by e . **The number e is approximately 2.71828183.** It is an irrational number, so it has a nonterminating, nonrepeating decimal representation.

Natural Exponential Function

The function defined by $f(x) = e^x$ is called the **natural exponential function**.

The e^x key on a calculator can be used to evaluate the natural exponential function. The graph of $y = e^x$ is shown at the left.

EXAMPLE • 1

Evaluate $f(x) = \left(\frac{1}{2}\right)^x$ at $x = 2$ and $x = -3$.

Solution

$$f(x) = \left(\frac{1}{2}\right)^x$$

$$f(2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad \bullet x = 2$$

$$f(-3) = \left(\frac{1}{2}\right)^{-3} = 2^3 = 8 \quad \bullet x = -3$$

YOU TRY IT • 1

Evaluate $f(x) = \left(\frac{2}{3}\right)^x$ at $x = 3$ and $x = -2$.

Your solution**EXAMPLE • 2**

Evaluate $f(x) = 2^{3x-1}$ at $x = 1$ and $x = -1$.

Solution

$$f(x) = 2^{3x-1}$$

$$f(1) = 2^{3(1)-1} = 2^2 = 4 \quad \bullet x = 1$$

$$f(-1) = 2^{3(-1)-1} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16} \quad \bullet x = -1$$

YOU TRY IT • 2

Evaluate $f(x) = 2^{2x+1}$ at $x = 0$ and $x = -2$.

Your solution**EXAMPLE • 3**

Evaluate $f(x) = e^{2x}$ at $x = 1$ and $x = -1$. Round to the nearest ten-thousandth.

Solution

$$f(x) = e^{2x}$$

$$f(1) = e^{2 \cdot 1} = e^2 \approx 7.3891 \quad \bullet x = 1$$

$$f(-1) = e^{2(-1)} = e^{-2} \approx 0.1353 \quad \bullet x = -1$$

YOU TRY IT • 3

Evaluate $f(x) = e^{2x-1}$ at $x = 2$ and $x = -2$. Round to the nearest ten-thousandth.

Your solution

OBJECTIVE B**To graph an exponential function****Integrating Technology**

See the Keystroke Guide: *Graph* for instructions on using a graphing calculator to graph functions.

Some properties of an exponential function can be seen from its graph.

HOW TO • 2 Graph $f(x) = 2^x$.

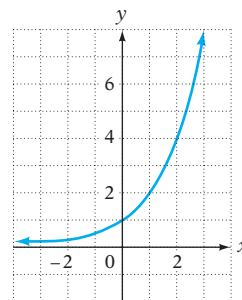
Think of this as the equation $y = 2^x$.

Choose values of x and find the corresponding values of y . The results can be recorded in a table.

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

x	$f(x) = y$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



Note that any vertical line would intersect the graph at only one point. Therefore, by the vertical-line test, the graph of $f(x) = 2^x$ is the graph of a function. Also note that any horizontal line would intersect the graph at only one point. Therefore, the graph of $f(x) = 2^x$ is the graph of a one-to-one function.

HOW TO • 3 Graph $f(x) = \left(\frac{1}{2}\right)^x$.

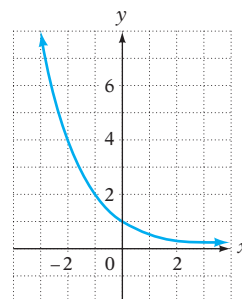
Think of this as the equation $y = \left(\frac{1}{2}\right)^x$.

Choose values of x and find the corresponding values of y .

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

x	$f(x) = y$
-3	$\left(\frac{1}{2}\right)^{-3} = 8$
-2	$\left(\frac{1}{2}\right)^{-2} = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$



Applying the vertical-line and horizontal-line tests reveals that the graph of $f(x) = \left(\frac{1}{2}\right)^x$ is also the graph of a one-to-one function.

HOW TO • 4 Graph $f(x) = 2^{-x}$.

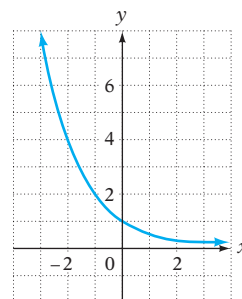
Think of this as the equation $y = 2^{-x}$.

Choose values of x and find the corresponding values of y .

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

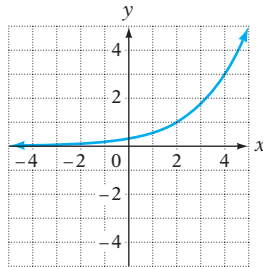
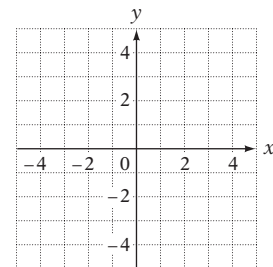
x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

**Take Note**

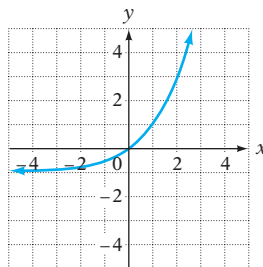
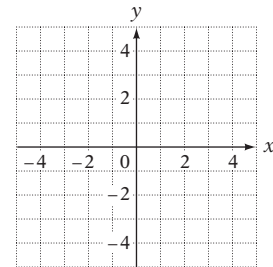
Note that because $2^{-x} = (2^{-1})^x = \left(\frac{1}{2}\right)^x$, the graphs of $f(x) = 2^{-x}$ and $f(x) = \left(\frac{1}{2}\right)^x$ are the same.

EXAMPLE • 4Graph: $f(x) = 3^{\frac{1}{2}x-1}$ **Solution**

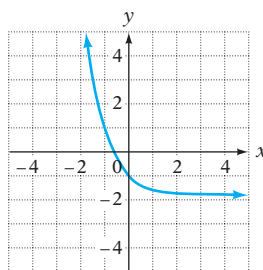
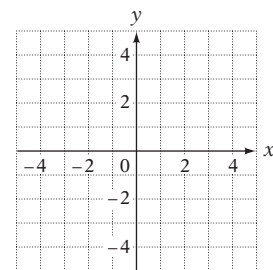
x	y
-2	$\frac{1}{9}$
0	$\frac{1}{3}$
2	1
4	3

**YOU TRY IT • 4**Graph: $f(x) = 2^{-\frac{1}{2}x}$ **Your solution****EXAMPLE • 5**Graph: $f(x) = 2^x - 1$ **Solution**

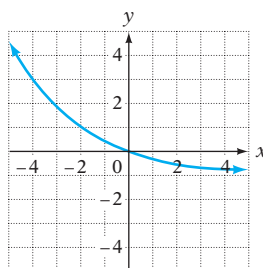
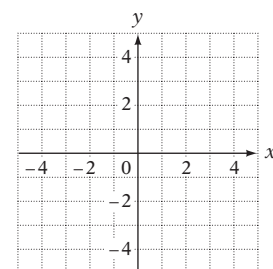
x	y
-2	$-\frac{3}{4}$
-1	$-\frac{1}{2}$
0	0
1	1
2	3
3	7

**YOU TRY IT • 5**Graph: $f(x) = 2^x + 1$ **Your solution****EXAMPLE • 6**Graph: $f(x) = \left(\frac{1}{3}\right)^x - 2$ **Solution**

x	y
-2	7
-1	1
0	-1
1	$-\frac{5}{3}$
2	$-\frac{17}{9}$

**YOU TRY IT • 6**Graph: $f(x) = 2^{-x} + 2$ **Your solution****EXAMPLE • 7**Graph: $f(x) = 2^{-\frac{1}{2}x} - 1$ **Solution**

x	y
-6	7
-4	3
-2	1
0	0
2	$-\frac{1}{2}$
4	$-\frac{3}{4}$

**YOU TRY IT • 7**Graph: $f(x) = \left(\frac{1}{2}\right)^{-\frac{1}{2}x} + 2$ **Your solution**

15. Given $F(x) = 2^{x^2}$, evaluate the following.

a. $F(2)$ b. $F(-2)$ c. $F\left(\frac{3}{4}\right)$

17. Given $f(x) = e^{-x^2/2}$, evaluate the following. Round to the nearest ten-thousandth.

a. $f(-2)$ b. $f(2)$ c. $f(-3)$

16. Given $Q(x) = 2^{-x^2}$, evaluate the following.

a. $Q(3)$ b. $Q(-1)$ c. $Q(-2)$

18. Given $f(x) = e^{-2x} + 1$, evaluate the following. Round to the nearest ten-thousandth.

a. $f(-1)$ b. $f(3)$ c. $f(-2)$



19. Suppose a and b are real numbers with $a < b$. If $f(x) = 2^{-x}$, then is $f(a) < f(b)$ or is $f(a) > f(b)$?



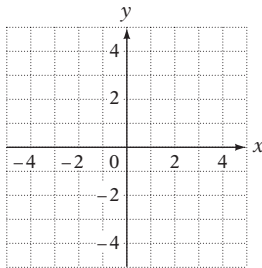
20. Suppose that u and v are real numbers with $u < v$ and that $f(x) = b^x$ ($b > 0$, $b \neq 1$). If $f(u) < f(v)$, then is $0 < b < 1$ or is $b > 1$?

OBJECTIVE B

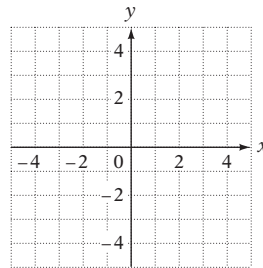
To graph an exponential function

Graph the functions in Exercises 21 to 32.

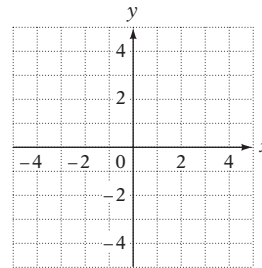
21. $f(x) = 3^x$



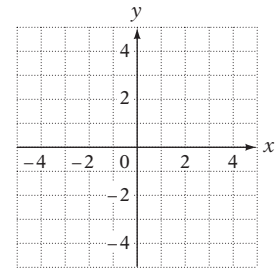
22. $f(x) = 3^{-x}$



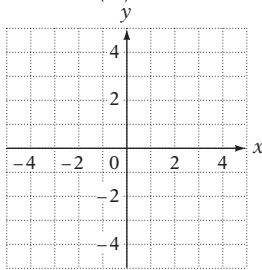
23. $f(x) = 2^{x+1}$



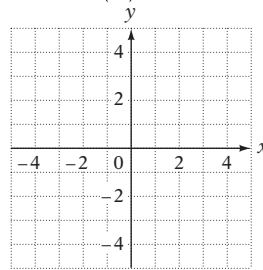
24. $f(x) = 2^{x-1}$



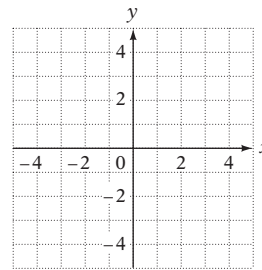
25. $f(x) = \left(\frac{1}{3}\right)^x$



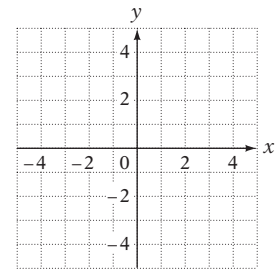
26. $f(x) = \left(\frac{2}{3}\right)^x$



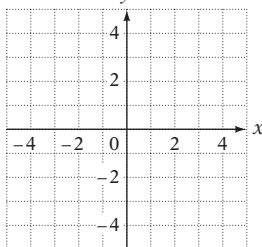
27. $f(x) = 2^{-x} + 1$



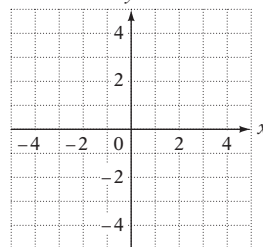
28. $f(x) = 2^x - 3$



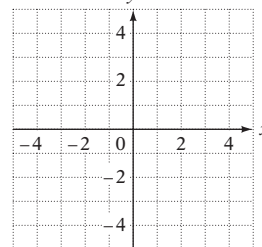
29. $f(x) = \left(\frac{1}{3}\right)^{-x}$



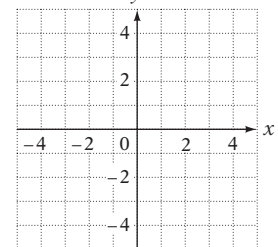
30. $f(x) = \left(\frac{3}{2}\right)^{-x}$



31. $f(x) = \left(\frac{1}{2}\right)^{-x} + 2$



32. $f(x) = \left(\frac{1}{2}\right)^x - 1$



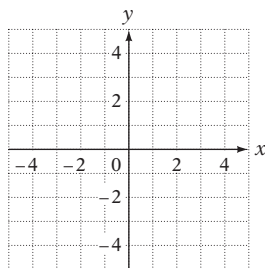
33. Graph $f(x) = 3^x$ and $f(x) = 3^{-x}$, and find the point of intersection of the two graphs.
34. Graph $f(x) = 2^{x+1}$ and $f(x) = 2^{-x+1}$, and find the point of intersection of the two graphs.
35. Graph $f(x) = \left(\frac{1}{3}\right)^x$. What are the x - and y -intercepts of the graph of the function?
36. Graph $f(x) = \left(\frac{1}{3}\right)^{-x}$. What are the x - and y -intercepts of the graph of the function?
37. Which of the following have the same graph?
 (i) $f(x) = 3^x$ (ii) $f(x) = \left(\frac{1}{3}\right)^x$ (iii) $f(x) = \left(\frac{1}{3}\right)^{-x}$ (iv) $f(x) = 3^{-x}$
38. Which of the following have the same graph?
 (i) $f(x) = \left(\frac{1}{4}\right)^{-x}$ (ii) $f(x) = 4^{-x}$ (iii) $f(x) = 4^x$ (iv) $f(x) = \left(\frac{1}{4}\right)^x$

Applying the Concepts

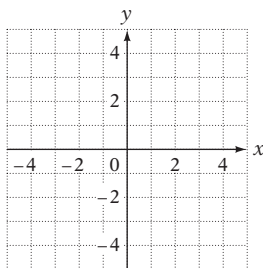


Use a graphing calculator to graph the functions in Exercises 39 to 41.

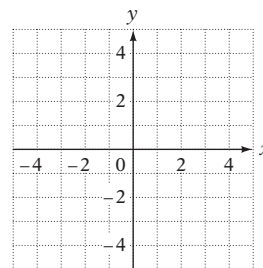
39. $P(x) = (\sqrt{3})^x$



40. $Q(x) = (\sqrt{3})^{-x}$



41. $f(x) = \pi^x$



42. Evaluate $\left(1 + \frac{1}{n}\right)^n$ for $n = 100$, 1000 , $10,000$, and $100,000$, and compare the results with the value of e , the base of the natural exponential function. On the basis of your evaluation, complete the following sentence:

As n increases, $\left(1 + \frac{1}{n}\right)^n$ becomes closer to _____.

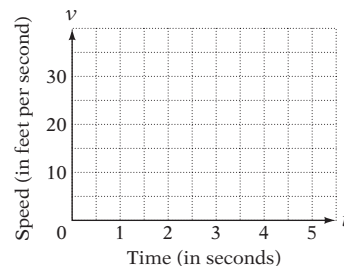
43. **Physics** If air resistance is ignored, the speed v , in feet per second, of an object t seconds after it has been dropped is given by $v = 32t$. However, if air resistance is considered, then the speed depends on the mass (and on other things). For a certain mass, the speed t seconds after an object has been dropped is given by $v = 32(1 - e^{-t})$.



a. Graph this equation. *Suggestion:* Use $X_{\min} = 0$, $X_{\max} = 5.5$, $Y_{\min} = 0$, $Y_{\max} = 40$, and $Y_{\text{sc1}} = 5$.



b. The point whose approximate coordinates are $(2, 27.7)$ is on this graph. Write a sentence that explains the meaning of these coordinates.



SECTION

10.2 Introduction to Logarithms

OBJECTIVE A

To find the logarithm of a number 

Suppose a bacteria colony that originally contained 1000 bacteria doubled in size every hour. Then the table at the right would show the number of bacteria in that colony after 1 h, 2 h, and 3 h.

Time (in hours)	Number of Bacteria
0	1000
1	2000
2	4000
3	8000

The exponential function $A = 1000(2^t)$, where A is the number of bacteria in the colony at time t , is a model of the growth of the colony. For instance, when $t = 3$ h, we have

$$A = 1000(2^t)$$

$$A = 1000(2^3) \quad \bullet \text{ Replace } t \text{ by } 3.$$

$$A = 1000(8) = 8000$$

After 3 h, there are 8000 bacteria in the colony.

Now we ask, “How long will it take for there to be 32,000 bacteria in the colony?” To answer the question, we must solve the *exponential equation* $32,000 = 1000(2^t)$. By trial and error, we find that when $t = 5$,

$$A = 1000(2^t)$$

$$A = 1000(2^5) \quad \bullet \text{ Replace } t \text{ by } 5.$$

$$A = 1000(32) = 32,000$$

After 5 h, there will be 32,000 bacteria in the colony.

Now suppose we want to know how long it will take before the colony reaches 50,000 bacteria. To answer that question, we must find t so that $50,000 = 1000(2^t)$. Using trial and error again, we find that

$$1000(2^5) = 32,000 \quad \text{and} \quad 1000(2^6) = 64,000$$

Because 50,000 is between 32,000 and 64,000, we conclude that t is between 5 h and 6 h. If we try $t = 5.5$ (halfway between 5 and 6), then

$$A = 1000(2^t)$$

$$A = 1000(2^{5.5}) \quad \bullet \text{ Replace } t \text{ by } 5.5.$$

$$A \approx 1000(45.25) = 45,250$$

In 5.5 h, there are approximately 45,250 bacteria in the colony. Because this is less than 50,000, the value of t must be a little greater than 5.5.

We could continue to use trial and error to find the correct value of t , but it would be more efficient if we could just solve the exponential equation $50,000 = 1000(2^t)$ for t . If we follow the procedures for solving equations that were discussed earlier in the text, we have

$$50,000 = 1000(2^t)$$

$$50 = 2^t$$

• Divide each side of the equation by 1000.

To proceed to the next step, it would be helpful to have a function that would find the power of 2 that produces 50.

Integrating Technology

Using a calculator, we can verify that $2^{5.644} \approx 50$. On a graphing calculator, press $2 \quad \wedge \quad 5.644$.

Take Note

Note that when we tried $t = 5.5$ earlier, we stated that the actual value of t must be greater than 5.5 and that 5.644 is a little greater than 5.5.

Around the mid-sixteenth century, mathematicians created such a function, which we now call a *logarithmic function*. We write the solution of $50 = 2^t$ as $t = \log_2 50$. This is read “ t equals the logarithm base 2 of 50” and it means “ t equals the power of 2 that produces 50.” When logarithms were first introduced, tables were used to find a numerical value of t . Today, a calculator is used. Using a calculator, we can approximate the value of t as 5.644. This means that $2^{5.644} \approx 50$.

The equivalence of the expressions $50 = 2^t$ and $t = \log_2 50$ are described in the following definition of **logarithm**.

Take Note

Read $\log_b x$ as “the logarithm of x , base b ” or “log base b of x .”

Definition of Logarithm

For $x > 0$, $b > 0$, $b \neq 1$, $y = \log_b x$ is equivalent to $x = b^y$.

Tips for Success

Be sure you can rewrite an exponential equation as a logarithmic equation and a logarithmic equation as an exponential equation. This relationship is very important.

The table at the right shows equivalent statements written in both exponential and logarithmic form.

Exponential Form	Logarithmic Form
$2^4 = 16$	$\log_2 16 = 4$
$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$	$\log_{2/3} \left(\frac{4}{9}\right) = 2$
$10^{-1} = 0.1$	$\log_{10} (0.1) = -1$

HOW TO • 1

Write $\log_3 81 = 4$ in exponential form.

$\log_3 81 = 4$ is equivalent to $3^4 = 81$.

HOW TO • 2

Write $10^{-2} = 0.01$ in logarithmic form.

$10^{-2} = 0.01$ is equivalent to $\log_{10} (0.01) = -2$.

It is important to note that the exponential function is a 1–1 function and thus has an inverse function. The inverse function of the exponential function is called a **logarithm**.

The 1–1 property of exponential functions can be used to evaluate some logarithms.

1–1 Property of Exponential Functions

For $b > 0$, $b \neq 1$, if $b^u = b^v$, then $u = v$.

HOW TO • 3

Evaluate $\log_2 8$.

$$\log_2 8 = x$$

$$8 = 2^x$$

$$2^3 = 2^x$$

$$3 = x$$

$$\log_2 8 = 3$$

- Write an equation.
- Write the equation in its equivalent exponential form.
- Write 8 as 2^3 .
- Use the 1–1 Property of Exponential Functions.

HOW TO • 4

Solve $\log_4 x = -2$ for x .

$$\log_4 x = -2$$

$$4^{-2} = x$$

$$\frac{1}{16} = x$$

- Write the equation in its equivalent exponential form.
- Simplify the negative exponent.

The solution is $\frac{1}{16}$.



Integrating Technology

The logarithms of most numbers are irrational numbers. Therefore, the value displayed by a calculator is an approximation.

Logarithms to the base 10 are called **common logarithms**. Usually the base, 10, is omitted when writing the common logarithm of a number. Therefore, $\log_{10} x$ is written $\log x$. To find the common logarithm of most numbers, a calculator is necessary. A calculator was used to find the value of $\log 384$, shown below.

$$\log 384 \approx 2.5843312$$

When e (the base of the natural exponential function) is used as the base of a logarithm, the logarithm is referred to as the **natural logarithm** and is abbreviated $\ln x$. This is read “el en x .” Use a calculator to approximate natural logarithms.

$$\ln 23 \approx 3.135494216$$

EXAMPLE • 1

Evaluate: $\log_3 \frac{1}{9}$

Solution

$$\log_3 \frac{1}{9} = x$$

$$\frac{1}{9} = 3^x$$

$$3^{-2} = 3^x$$

$$-2 = x$$

$$\log_3 \frac{1}{9} = -2$$

- Write an equation.
- Write the equivalent exponential form.
- $\frac{1}{9} = 3^{-2}$
- The bases are the same. The exponents are equal.

YOU TRY IT • 1

Evaluate: $\log_4 64$

Your solution

EXAMPLE • 2Solve for x : $\log_5 x = 2$ **Solution**

$$\log_5 x = 2$$

$$5^2 = x$$

$$25 = x$$

- Write the equivalent exponential form.

The solution is 25.

YOU TRY IT • 2Solve for x : $\log_2 x = -4$ **Your solution****EXAMPLE • 3**Solve $\log x = -1.5$ for x . Round to the nearest ten-thousandth.**Solution**

$$\log x = -1.5$$

$$10^{-1.5} = x$$

$$0.0316 \approx x$$

- Write the equivalent exponential form.
- Use a calculator.

YOU TRY IT • 3Solve $\ln x = 3$ for x . Round to the nearest ten-thousandth.**Your solution**

Solutions on p. S29

OBJECTIVE B**To use the Properties of Logarithms to simplify expressions containing logarithms** **Take Note**

The properties stated here can be proved using the definition of logarithm and the properties of exponents. For instance, to prove the Logarithm Property of 1, let $\log_b 1 = x$. Then, by the definition of logarithm, $b^x = 1$. Because $b^0 = 1$, we have $x = 0$.

From the definition of a logarithm, a number of properties of logarithms can be stated.

Logarithm Property of 1If $b > 0$, $b \neq 1$, then $\log_b 1 = 0$.

Here are some examples.

$$\log_9 1 = 0 \quad \log_2 1 = 0 \quad \log_{3/4} 1 = 0$$

Logarithm of the Base PropertyIf $b > 0$, $b \neq 1$, then $\log_b b = 1$.

Here are some examples.

$$\log_7 7 = 1 \quad \log 10 = 1 \quad \log_{1/2} \left(\frac{1}{2} \right) = 1$$

1–1 Property of LogarithmsSuppose that $b > 0$, $b \neq 1$, and x and y are positive real numbers. If $\log_b x = \log_b y$, then $x = y$.For instance, if $\log_b 7 = \log_b x$, then $x = 7$.

The next property is a direct result of the fact that logarithmic functions and exponential functions are inverses of one another.

Take Note

By the Inverse Property of Logarithms, $9^{\log_9 5x} = 5x$.

Inverse Property of LogarithmsIf $b > 0$, $b \neq 1$, and $x > 0$, then $\log_b b^x = x$ and $b^{\log_b x} = x$.

**Take Note**

Pay close attention to this property. Note, for instance, that this property states that $\log_3(4p) = \log_3 4 + \log_3 p$. It also states that $\log_5 9 + \log_5 z = \log_5(9z)$. It does *not* state any relationship regarding the expression $\log_b(x + y)$. **This expression cannot be simplified.**

Because a logarithm is related to an exponential expression, logarithms have properties that are similar to those of exponential expressions. The proofs of the next three properties can be found in the Appendix.

The following property relates to the logarithm of the product of two numbers.

The Logarithm Property of the Product of Two Numbers

For any positive real numbers x , y , and b , $b \neq 1$, $\log_b(xy) = \log_b x + \log_b y$.

In words, this property states that the logarithm of the product of two numbers equals the sum of the logarithms of the two numbers.

HOW TO • 5

Write $\log_b(6z)$ in expanded form.

$$\log_b(6z) = \log_b 6 + \log_b z \quad \bullet \text{ Use the Logarithm Property of Products.}$$

HOW TO • 6

Write $\log_b 12 + \log_b r$ as a single logarithm.

$$\log_b 12 + \log_b r = \log_b(12r) \quad \bullet \text{ Use the Logarithm Property of Products.}$$

The Logarithm Property of Products can be extended to include the logarithm of the product of more than two factors. For instance,

$$\log_b(7rt) = \log_b 7 + \log_b r + \log_b t$$

The next property relates to the logarithm of the quotient of two numbers.

**Take Note**

This property is used to rewrite expressions such as $\log_5\left(\frac{m}{8}\right) = \log_5 m - \log_5 8$. It does *not* state any relationship regarding the expression $\frac{\log_b x}{\log_b y}$. **This expression cannot be simplified.**

The Logarithm Property of the Quotient of Two Numbers

For any positive real numbers x , y , and b , $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.

In words, this property states that the logarithm of the quotient of two numbers equals the difference of the logarithms of the two numbers.

HOW TO • 7

Write $\log_b \frac{p}{8}$ in expanded form.

$$\log_b \frac{p}{8} = \log_b p - \log_b 8 \quad \bullet \text{ Use the Logarithm Property of Quotients.}$$

HOW TO • 8

Write $\log_b y - \log_b v$ as a single logarithm.

$$\log_b y - \log_b v = \log_b \frac{y}{v} \quad \bullet \text{ Use the Logarithm Property of Quotients.}$$

Another property of logarithms is used to simplify powers of a number.

The Logarithm Property of the Power of a Number

For any positive real numbers x and b , $b \neq 1$, and for any real number r , $\log_b x^r = r \log_b x$.

In words, this property states that the logarithm of the power of a number equals the power times the logarithm of the number.

HOW TO • 9

Rewrite $\log_b x^3$ in terms of $\log_b x$.

$$\log_b x^3 = 3 \log_b x \quad \bullet \text{ Use the Logarithm Property of Powers.}$$

Point of Interest

Logarithms were developed independently by Jobst Burgi (1552–1632) and John Napier (1550–1617) as a means of simplifying the calculations of astronomers. The idea was to devise a method by which two numbers could be multiplied by performing additions. Napier is usually given credit for logarithms because he published his results first.



John Napier

New York
The Granger Collection.

HOW TO • 10

Rewrite $\frac{2}{3} \log_b x$ with a coefficient of 1.

$$\frac{2}{3} \log_b x = \log_b x^{2/3}$$

- Use the Logarithm Property of Powers.

Here are some additional examples that combine some of the properties of logarithms we have discussed.

HOW TO • 11

Write $\log_b \frac{x^3}{y^2z}$ in expanded form.

$$\log_b \frac{x^3}{y^2z} = \log_b x^3 - \log_b y^2z$$

$$= \log_b x^3 - (\log_b y^2 + \log_b z)$$

$$= 3 \log_b x - (2 \log_b y + \log_b z)$$

$$= 3 \log_b x - 2 \log_b y - \log_b z$$

- Use the Logarithm Property of Quotients.
- Use the Logarithm Property of Products.
- Use the Logarithm Property of Powers.
- Write in simplest form.

HOW TO • 12

Write $2 \log_b x + 4 \log_b y - \log_b z$ as a single logarithm with a coefficient of 1.

$$2 \log_b x + 4 \log_b y - \log_b z = \log_b x^2 + \log_b y^4 - \log_b z$$

$$= \log_b (x^2 y^4) - \log_b z$$

$$= \log_b \frac{x^2 y^4}{z}$$

- Use the Logarithm Property of Powers.
- Use the Logarithm Property of Products.
- Use the Logarithm Property of Quotients.

EXAMPLE • 4

Write $\log \sqrt{x^3 y}$ in expanded form.

Solution

$$\begin{aligned} \log \sqrt{x^3 y} &= \log (x^3 y)^{1/2} = \frac{1}{2} \log (x^3 y) && \text{• Power Property} \\ &= \frac{1}{2} (\log x^3 + \log y) && \text{• Product Property} \\ &= \frac{1}{2} (3 \log x + \log y) && \text{• Power Property} \\ &= \frac{3}{2} \log x + \frac{1}{2} \log y && \text{• Distributive Property} \end{aligned}$$

EXAMPLE • 5

Write $\frac{1}{2} (\log_3 x - 3 \log_3 y + \log_3 z)$ as a single logarithm with a coefficient of 1.

Solution

$$\begin{aligned} &\frac{1}{2} (\log_3 x - 3 \log_3 y + \log_3 z) \\ &= \frac{1}{2} (\log_3 x - \log_3 y^3 + \log_3 z) \\ &= \frac{1}{2} \left(\log_3 \frac{x}{y^3} + \log_3 z \right) \\ &= \frac{1}{2} \left(\log_3 \frac{xz}{y^3} \right) = \log_3 \left(\frac{xz}{y^3} \right)^{1/2} = \log_3 \sqrt{\frac{xz}{y^3}} \end{aligned}$$

YOU TRY IT • 4

Write $\log_8 \sqrt[3]{xy^2}$ in expanded form.

Your solution

YOU TRY IT • 5

Write $\frac{1}{3} (\log_4 x - 2 \log_4 y + \log_4 z)$ as a single logarithm with a coefficient of 1.

Your solution

OBJECTIVE C**To use the Change-of-Base Formula****Integrating Technology**

To evaluate $\frac{\log 22}{\log 5}$ using a graphing calculator,

use the keystrokes

. The display should read 1.92057266.

If natural logarithms are used,

use the key rather

than the key. The result will be the same.

HOW TO • 13Evaluate $\log_5 22$.

$$\log_5 22 = x$$

$$5^x = 22$$

$$\log 5^x = \log 22$$

$$x \log 5 = \log 22$$

$$x = \frac{\log 22}{\log 5}$$

$$x \approx 1.92057$$

$$\log_5 22 \approx 1.92057$$

- Write an equation.
- Write the equation in its equivalent exponential form.
- Apply the common logarithm to each side of the equation.
- Use the Power Property of Logarithms.
- Exact answer
- Approximate answer

In the third step above, the natural logarithm, instead of the common logarithm, could have been applied to each side of the equation. The same result would have been obtained.

Using a procedure similar to the one used to evaluate $\log_5 22$, we can derive a formula for changing bases.

Change-of-Base Formula

$$\log_a N = \frac{\log_b N}{\log_b a}$$

HOW TO • 14Evaluate $\log_2 14$.

$$\begin{aligned}\log_2 14 &= \frac{\log 14}{\log 2} \\ &\approx 3.80735\end{aligned}$$

- Use the Change-of-Base Formula with $N = 14$, $a = 2$, $b = 10$.

In the last example, common logarithms were used. Here is the same example using natural logarithms. Note that the answers are the same.

$$\log_2 14 = \frac{\ln 14}{\ln 2} \approx 3.80735$$

EXAMPLE • 6

Evaluate $\log_8 0.137$ by using natural logarithms.

Solution

$$\log_8 0.137 = \frac{\ln 0.137}{\ln 8} \approx -0.95592$$

EXAMPLE • 7

Evaluate $\log_2 90.813$ by using common logarithms.

Solution

$$\log_2 90.813 = \frac{\log 90.813}{\log 2} \approx 6.50483$$

YOU TRY IT • 6

Evaluate $\log_3 0.834$ by using natural logarithms.

Your solution**YOU TRY IT • 7**

Evaluate $\log_7 6.45$ by using common logarithms.

Your solution

10.2 EXERCISES

OBJECTIVE A

To find the logarithm of a number



1. a. What is a common logarithm?
b. How is the common logarithm of $4z$ written?



2. a. What is a natural logarithm?
b. How is the natural logarithm of $3x$ written?

For Exercises 3 to 10, write the exponential expression in logarithmic form.

3. $5^2 = 25$	4. $10^3 = 1000$	5. $4^{-2} = \frac{1}{16}$	6. $3^{-3} = \frac{1}{27}$
7. $10^y = x$	8. $e^y = x$	9. $a^x = w$	10. $b^y = c$

For Exercises 11 to 18, write the logarithmic expression in exponential form.

11. $\log_3 9 = 2$	12. $\log_2 32 = 5$	13. $\log 0.01 = -2$	14. $\log_5 \frac{1}{5} = -1$
15. $\ln x = y$	16. $\log x = y$	17. $\log_b u = v$	18. $\log_c x = y$

Evaluate the expressions in Exercises 19 to 30.

19. $\log_3 81$	20. $\log_7 49$	21. $\log_2 128$	22. $\log_5 125$
23. $\log 100$	24. $\log 0.001$	25. $\ln e^3$	26. $\ln e^2$
27. $\log_8 1$	28. $\log_3 243$	29. $\log_5 625$	30. $\log_2 64$

For Exercises 31 to 38, solve for x .

31. $\log_3 x = 2$	32. $\log_5 x = 1$	33. $\log_4 x = 3$	34. $\log_2 x = 6$
35. $\log_7 x = -1$	36. $\log_8 x = -2$	37. $\log_6 x = 0$	38. $\log_4 x = 0$

For Exercises 39 to 46, solve for x . Round to the nearest hundredth.

39. $\log x = 2.5$

40. $\log x = 3.2$

41. $\log x = -1.75$

42. $\log x = -2.1$

43. $\ln x = 2$

44. $\ln x = 1.4$

45. $\ln x = -\frac{1}{2}$

46. $\ln x = -1.7$



47. If $\log_3 x > 0$, then $x > \underline{\hspace{1cm}}$.



48. Why is there no real number value for the expression $\log_5(-2)$?

OBJECTIVE B

To use the Properties of Logarithms to simplify expressions containing logarithms



49. What is the Product Property of Logarithms?



50. What is the Quotient Property of Logarithms?



51. True or false? Using the Product Property of Logarithms,
 $\log_5(-2) + \log_5(-3) = \log_5[(-2)(-3)] = \log_5 6$



52. For what values of x is the equation $\log_b x^2 = 2 \log_b x$ true?

For Exercises 53 to 64, evaluate the expression.

53. $\log_{12} 1$

54. $\ln 1$

55. $\ln e$

56. $\log_{10} 10$

57. $\log_3 3^x$

58. $8^{\log_8 p}$

59. $e^{\ln v}$

60. $\ln e^{3x}$

61. $2^{\log_2(x^2 + 1)}$

62. $\log_4 4^{3x + 1}$

63. $\log_5 5^{x^2 - x - 1}$

64. $8^{\log_8(3x - 7)}$

For Exercises 65 to 88, write the logarithm in expanded form.

65. $\log_8(xz)$

66. $\log_7(rt)$

67. $\log_3 x^5$

68. $\log_2 y^7$

69. $\log_b \frac{r}{s}$

70. $\log_c \frac{z}{4}$

71. $\log_3(x^2 y^6)$

72. $\log_4(t^4 u^2)$

73. $\log_7 \frac{u^3}{v^4}$

74. $\log_{10} \frac{s^5}{t^2}$

75. $\log_2 (rs)^2$

76. $\log_3 (x^2 y)^3$

77. $\ln (x^2 yz)$

78. $\ln (xy^2 z^3)$

79. $\log_5 \frac{xy^2}{z^4}$

80. $\log_b \frac{r^2 s}{t^3}$

81. $\log_8 \frac{x^2}{yz^2}$

82. $\log_9 \frac{x}{y^2 z^3}$

83. $\log_4 \sqrt{x^3 y}$

84. $\log_3 \sqrt{x^5 y^3}$

85. $\log_7 \sqrt{\frac{x^3}{y}}$

86. $\log_b \sqrt[3]{\frac{r^2}{t}}$

87. $\log_3 \frac{t}{\sqrt{x}}$

88. $\log_4 \frac{x}{\sqrt{y^2 z}}$

For Exercises 89 to 112, express as a single logarithm with a coefficient of 1.

89. $\log_3 x^3 + \log_3 y^2$

90. $\log_7 x + \log_7 z^2$

91. $\ln x^4 - \ln y^2$

92. $\ln x^2 - \ln y$

93. $3 \log_7 x$

94. $4 \log_8 y$

95. $3 \ln x + 4 \ln y$

96. $2 \ln x - 5 \ln y$

97. $2(\log_4 x + \log_4 y)$

98. $3(\log_5 r + \log_5 t)$

99. $2 \log_3 x - \log_3 y + 2 \log_3 z$

100. $4 \log_5 r - 3 \log_5 s + \log_5 t$

101. $\ln x - (2 \ln y + \ln z)$

102. $2 \log_b x - 3(\log_b y + \log_b z)$

103. $\frac{1}{2}(\log_6 x - \log_6 y)$

104. $\frac{1}{3}(\log_8 x - \log_8 y)$

105. $2(\log_4 s - 2 \log_4 t + \log_4 r)$

106. $3(\log_9 x + 2 \log_9 y - 2 \log_9 z)$

107. $\ln x - 2(\ln y + \ln z)$

108. $\ln t - 3(\ln u + \ln v)$

109. $\frac{1}{2}(3 \log_4 x - 2 \log_4 y + \log_4 z)$

110. $\frac{1}{3}(4 \log_5 t - 5 \log_5 u - 7 \log_5 v)$

111. $\frac{1}{2} \log_2 x - \frac{2}{3} \log_2 y + \frac{1}{2} \log_2 z$

112. $\frac{2}{3} \log_3 x + \frac{1}{3} \log_3 y - \frac{1}{2} \log_3 z$

OBJECTIVE C**To use the Change-of-Base Formula**

Evaluate the expressions in Exercises 113 to 128. Round to the nearest ten-thousandth.

113. $\log_8 6$

114. $\log_4 8$

115. $\log_5 30$

116. $\log_6 28$

117. $\log_3 0.5$

118. $\log_5 0.6$

119. $\log_7 1.7$

120. $\log_6 3.2$

121. $\log_5 15$

122. $\log_3 25$

123. $\log_{12} 120$

124. $\log_9 90$

125. $\log_4 2.55$

126. $\log_8 6.42$

127. $\log_5 67$

128. $\log_8 35$

129. Use the Change-of-Base Formula to write $\log_5 x$ in terms of $\log x$.130. Use the Change-of-Base Formula to write $\log_5 x$ in terms of $\ln x$.**Applying the Concepts**

131. For each of the following, answer True or False. Assume all variables represent positive numbers.

a. $\log_3(-9) = -2$

b. $x^y = z$ and $\log_x z = y$ are equivalent equations.

c. $\log(x^{-1}) = \frac{1}{\log x}$

d. $\log \frac{x}{y} = \log x - \log y$

e. $\log(x \cdot y) = \log x \cdot \log y$

f. If $\log x = \log y$, then $x = y$.132. Complete each statement. For part a, use the equation $\log_a b = c$.

a. $a^c = \underline{\hspace{2cm}}$

b. $\log_a(a^c) = \underline{\hspace{2cm}}$

SECTION

10.3 Graphs of Logarithmic Functions

OBJECTIVE A

To graph a logarithmic function



The graph of a logarithmic function can be drawn by using the relationship between the exponential and logarithmic functions.



Point of Interest

Although logarithms were originally developed to assist with computations, logarithmic functions have a much broader use today. These functions occur in geology, acoustics, chemistry, and economics, for example.

HOW TO 1

Graph: $f(x) = \log_2 x$

Think of $f(x) = \log_2 x$ as the equation $y = \log_2 x$.

Write the equivalent exponential equation.

Because the equation is solved for x in terms of y , it is easier to choose values of y and find the corresponding values of x . The results can be recorded in a table.

Graph the ordered pairs on a rectangular coordinate system.

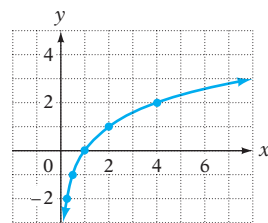
Connect the points with a smooth curve.

$$f(x) = \log_2 x$$

$$y = \log_2 x$$

$$x = 2^y$$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2



Applying the vertical-line and horizontal-line tests reveals that $f(x) = \log_2 x$ is a one-to-one function.

HOW TO 2

Graph: $f(x) = \log_2 x + 1$

Think of $f(x) = \log_2 x + 1$ as the equation $y = \log_2 x + 1$.

Solve for $\log_2 x$.

Write the equivalent exponential equation.

Choose values of y and find the corresponding values of x .

Graph the ordered pairs on a rectangular coordinate system.

Connect the points with a smooth curve.

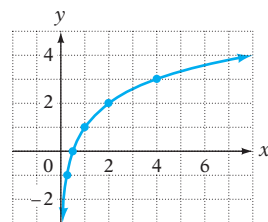
$$f(x) = \log_2 x + 1$$

$$y = \log_2 x + 1$$

$$y - 1 = \log_2 x$$

$$2^{y-1} = x$$

x	y
$\frac{1}{4}$	-1
$\frac{1}{2}$	0
1	1
2	2
4	3



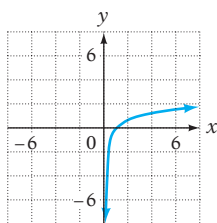
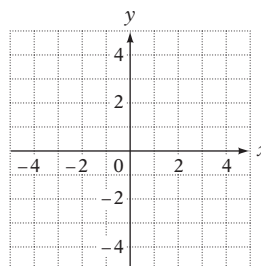
Integrating Technology

See the Projects and Group Activities at the end of this chapter for suggestions on graphing logarithmic functions using a graphing calculator.

EXAMPLE • 1Graph: $f(x) = \log_3 x$

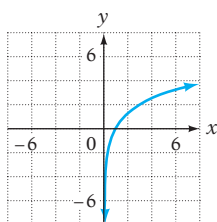
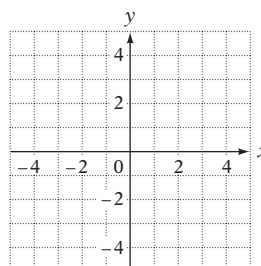
Solution $f(x) = \log_3 x$
 $y = \log_3 x$ • $f(x) = y$
 $3^y = x$ • Write the equivalent exponential equation.

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1

**YOU TRY IT • 1**Graph: $f(x) = \log_2(x - 1)$ **Your solution****EXAMPLE • 2**Graph: $f(x) = 2 \log_3 x$

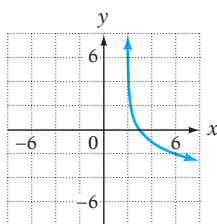
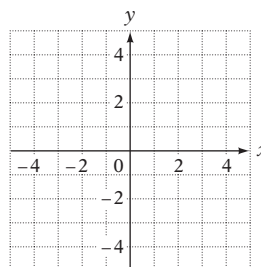
Solution $f(x) = 2 \log_3 x$
 $y = 2 \log_3 x$ • $f(x) = y$
 $\frac{y}{2} = \log_3 x$ • Divide both sides by 2.
 $3^{y/2} = x$ • Write the equivalent exponential equation.

x	y
$\frac{1}{9}$	-4
$\frac{1}{3}$	-2
1	0
3	2

**YOU TRY IT • 2**Graph: $f(x) = \log_3(2x)$ **Your solution****EXAMPLE • 3**Graph: $f(x) = -\log_2(x - 2)$

Solution $f(x) = -\log_2(x - 2)$
 $y = -\log_2(x - 2)$ • $f(x) = y$
 $-y = \log_2(x - 2)$ • Multiply both sides by -1.
 $2^{-y} = x - 2$ • Write the equivalent exponential equation.
 $2^{-y} + 2 = x$

x	y
6	-2
4	-1
3	0
$\frac{5}{2}$	1
$\frac{9}{4}$	2
$\frac{17}{8}$	3

**YOU TRY IT • 3**Graph: $f(x) = -\log_3(x + 1)$ **Your solution**

10.3 EXERCISES

OBJECTIVE A

To graph a logarithmic function



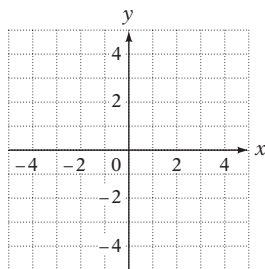
1. What is the relationship between the graphs of
- $x = 3^y$
- and
- $y = \log_3 x$
- ?



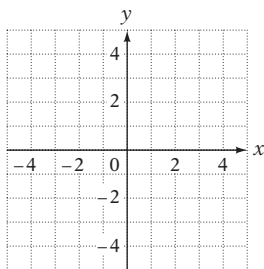
2. What is the relationship between the graphs of
- $y = 3^x$
- and
- $y = \log_3 x$
- ?

For Exercises 3 to 14, graph the function.

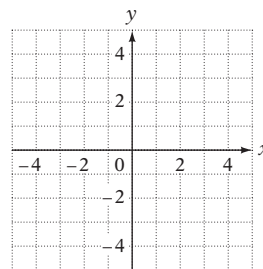
3. $f(x) = \log_4 x$



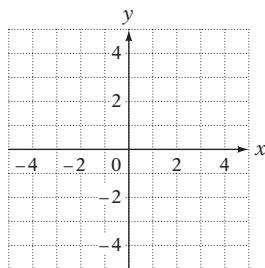
4. $f(x) = \log_2(x + 1)$



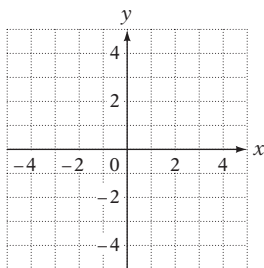
5. $f(x) = \log_3(2x - 1)$



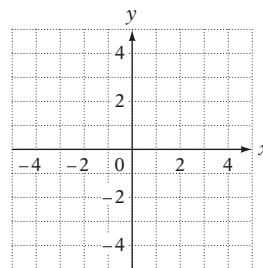
6. $f(x) = \log_2\left(\frac{1}{2}x\right)$



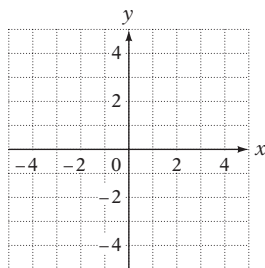
7. $f(x) = 3 \log_2 x$



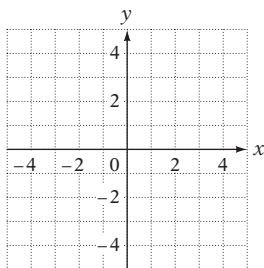
8. $f(x) = \frac{1}{2} \log_2 x$



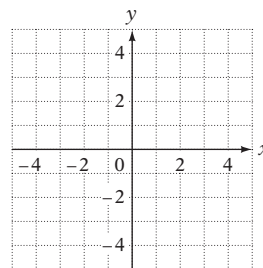
9. $f(x) = -\log_2 x$



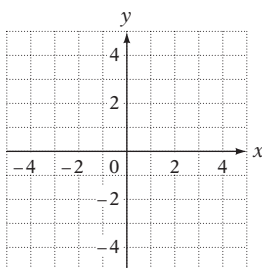
10. $f(x) = -\log_3 x$



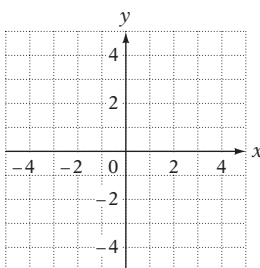
11. $f(x) = \log_2(x - 1)$



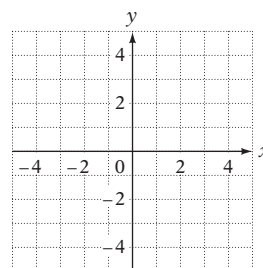
12. $f(x) = \log_3(2 - x)$



13. $f(x) = -\log_2(x - 1)$



14. $f(x) = -\log_2(1 - x)$

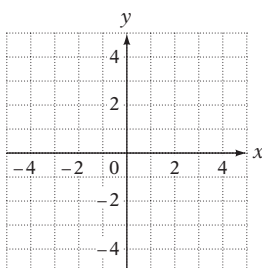
15. If $a \neq b$, at what point do the graphs of $y = \log_b x$ and $y = \log_a x$ intersect?16. If $a \neq b$, what is the difference between the y-coordinates of the graphs of $y = \log_b b$ and $y = \log_a a$?

Applying the Concepts

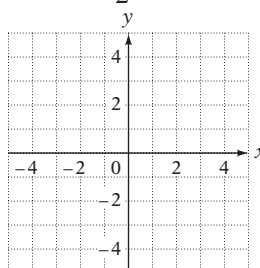


Use a graphing calculator to graph the functions in Exercises 17 to 22.

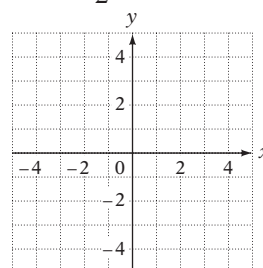
17. $f(x) = x - \log_2(1 - x)$



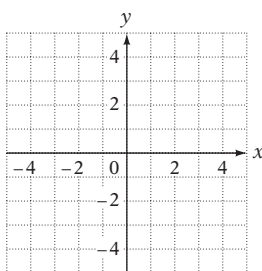
18. $f(x) = -\frac{1}{2} \log_2 x - 1$



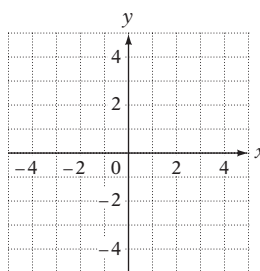
19. $f(x) = \frac{x}{2} - 2 \log_2(x + 1)$



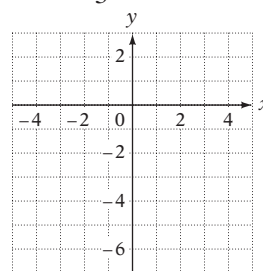
20. $f(x) = x + \log_3(2 - x)$



21. $f(x) = x^2 - 10 \ln(x - 1)$



22. $f(x) = \frac{x}{3} - 3 \log_2(x + 3)$



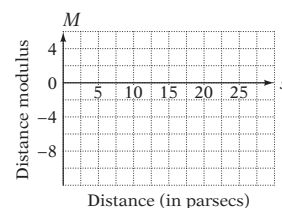
23. **Astronomy** Astronomers use the *distance modulus* of a star as a method of determining the star's distance from Earth. The formula is $M = 5 \log s - 5$, where M is the distance modulus and s is the star's distance from Earth in parsecs. (One parsec $\approx 1.9 \times 10^{13}$ mi)



a. Graph the equation.



b. The point with coordinates (25.1, 2) is on the graph. Write a sentence that describes the meaning of this ordered pair.



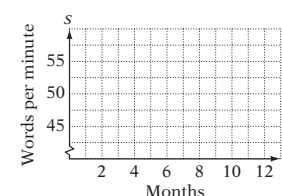
24. **Typing** Without practice, the proficiency of a typist decreases. The equation $s = 60 - 7 \ln(t + 1)$, where s is the typing speed in words per minute and t is the number of months without typing, approximates this decrease.



a. Graph the equation.



b. The point with coordinates (4, 49) is on the graph. Write a sentence that describes the meaning of this ordered pair.



SECTION

10.4

Solving Exponential and Logarithmic Equations

OBJECTIVE A

To solve an exponential equation

An **exponential equation** is one in which a variable occurs in an exponent. The equations at the right are exponential equations.

$$6^{2x+1} = 6^{3x-2}$$

$$4^x = 3$$

$$2^{x+1} = 7$$

An exponential equation in which each side of the equation can be expressed in terms of the same base can be solved by using the 1–1 Property of Exponential Functions. Recall that the 1–1 Property of Exponential Functions states that for $b > 0$, $b \neq 1$,

$$\text{if } b^x = b^y, \text{ then } x = y.$$

In the two examples below, this property is used in solving exponential equations.

HOW TO • 1

Solve: $10^{3x+5} = 10^{x-3}$

$$10^{3x+5} = 10^{x-3}$$

$$3x + 5 = x - 3$$

$$2x + 5 = -3$$

$$2x = -8$$

$$x = -4$$

• Use the 1–1 Property of Exponential Functions to equate the exponents.

• Solve the resulting equation.

Check:

$$\begin{array}{r|l} 10^{3x+5} = 10^{x-3} & \\ 10^{3(-4)+5} & 10^{-4-3} \\ 10^{-12+5} & 10^{-7} \\ 10^{-7} & 10^{-7} \end{array}$$

The solution is -4 .

 Take Note

The 1–1 Property of Exponential Functions requires that the bases be equal. For the problem at the right, we can write $9 = 3^2$ and $27 = 3^3$. After simplifying, the bases of the exponential expressions are equal.

HOW TO • 2

Solve: $9^{x+1} = 27^{x-1}$

$$9^{x+1} = 27^{x-1}$$

$$(3^2)^{x+1} = (3^3)^{x-1}$$

$$3^{2x+2} = 3^{3x-3}$$

$$2x + 2 = 3x - 3$$

$$2 = x - 3$$

$$5 = x$$

• $3^2 = 9$; $3^3 = 27$

• Use the 1–1 Property of Exponential Functions to equate the exponents.

• Solve for x .

Check:

$$\begin{array}{r|l} 9^{x+1} = 27^{x-1} & \\ 9^{5+1} & 27^{5-1} \\ 9^6 & 27^4 \\ 531,441 & 531,441 \end{array}$$

The solution is 5 .

**Integrating Technology**

To evaluate $\frac{\log 7}{\log 4}$ on a scientific calculator, use the keystrokes

7 **LOG** **÷** 4 **LOG**

ENTER

The display should read 1.4036775.

**Tips for Success**

Always check the solution of an equation, even when the solution is an approximation. For the equation at the right:

$$\begin{array}{r|l} 3^{x+1} = 5 & \\ 3^{0.4650+1} & 5 \\ 3^{1.4650} & 5 \\ 5.000145 \approx 5 & \end{array}$$

When both sides of an exponential equation cannot easily be expressed in terms of the same base, logarithms are used to solve the exponential equation.

HOW TO • 3Solve: $4^x = 7$

$$\begin{aligned} 4^x &= 7 \\ \log 4^x &= \log 7 \end{aligned}$$

$$x \log 4 = \log 7$$

$$x = \frac{\log 7}{\log 4} \approx 1.4037$$

The solution is 1.4037.

- Take the common logarithm of each side of the equation.
- Rewrite the equation using the Properties of Logarithms.
- Solve for x .
- Note that $\frac{\log 7}{\log 4} \neq \log 7 - \log 4$.

HOW TO • 4Solve: $3^{x+1} = 5$

$$\begin{aligned} 3^{x+1} &= 5 \\ \log 3^{x+1} &= \log 5 \\ (x+1)\log 3 &= \log 5 \end{aligned}$$

$$x+1 = \frac{\log 5}{\log 3}$$

$$\begin{aligned} x+1 &\approx 1.4650 \\ x &\approx 0.4650 \end{aligned}$$

The solution is 0.4650.

- Take the common logarithm of each side of the equation.
- Rewrite the equation using the Properties of Logarithms.
- Solve for x .

EXAMPLE • 1Solve for n : $(1.1)^n = 2$ **Solution**

$$\begin{aligned} (1.1)^n &= 2 \\ \log (1.1)^n &= \log 2 \\ n \log 1.1 &= \log 2 \\ n &= \frac{\log 2}{\log 1.1} \\ n &\approx 7.2725 \end{aligned}$$

The solution is 7.2725.

- Take the log of each side.
- Power Property
- Divide both sides by $\log 1.1$.

YOU TRY IT • 1Solve for n : $(1.06)^n = 1.5$ **Your solution****EXAMPLE • 2**Solve for x : $3^{2x} = 4$ **Solution**

$$\begin{aligned} 3^{2x} &= 4 \\ \log 3^{2x} &= \log 4 \\ 2x \log 3 &= \log 4 \\ 2x &= \frac{\log 4}{\log 3} \\ 2x &\approx 1.2619 \\ x &\approx 0.6310 \end{aligned}$$

The solution is 0.6310.

- Take the log of each side.
- Power Property
- Divide both sides by $\log 3$.
- Divide both sides by 2.

YOU TRY IT • 2Solve for x : $4^{3x} = 25$ **Your solution**

OBJECTIVE B**To solve a logarithmic equation**

The 1–1 Property of Logarithms from Section 10.2 can be used to solve some logarithmic equations.

**Take Note**

It is important to check a proposed solution of a logarithmic equation.

Consider the equation $\log(2x - 3) = \log(x - 2)$.

$$\log(2x - 3) = \log(x - 2)$$

$$2x - 3 = x - 2$$

$$x = 1$$

Substituting 1 into the original equation gives

$$\log(-1) = \log(-1).$$

However, logarithms of negative numbers are not defined. Therefore, the equation

$$\log(2x - 3) = \log(x - 2)$$

has no solution.

HOW TO • 5

Solve: $\log_5(3x - 1) = \log_5(7 - x)$

$$\log_5(3x - 1) = \log_5(7 - x)$$

$$3x - 1 = 7 - x$$

$$4x = 8$$

$$x = 2$$

• Use the 1–1 Property of Logarithms.

• Solve for x .

Because logarithms are defined only for positive numbers, we must check the solution.

Check:

$$\frac{\log_5(3x - 1) = \log_5(7 - x)}{\log_5[3(2) - 1] \quad \log_5(7 - 2)}$$

$$\log_5(5) = \log_5(5)$$

• Replace x by 2.

The solution checks. The solution is 2.

Solving some logarithmic equations may require using several of the properties of logarithms.

HOW TO • 6

Solve: $\log_3 6 - \log_3(2x + 3) = \log_3(x + 1)$

$$\log_3 6 - \log_3(2x + 3) = \log_3(x + 1)$$

$$\log_3 \frac{6}{2x + 3} = \log_3(x + 1)$$

$$\frac{6}{2x + 3} = x + 1$$

$$6 = (2x + 3)(x + 1)$$

$$6 = 2x^2 + 5x + 3$$

$$0 = 2x^2 + 5x - 3$$

$$0 = (2x - 1)(x + 3)$$

• Use the Quotient Property of Logarithms.

• Use the 1–1 Property of Logarithms.

• Write in standard form.

• Factor and use the Principle of Zero Products.

$$2x - 1 = 0$$

$$x + 3 = 0$$

$$x = \frac{1}{2}$$

$$x = -3$$

-3 does not check as a solution. The solution is $\frac{1}{2}$.

HOW TO • 7

Solve: $\log_9 x + \log_9(x - 8) = 1$

$$\log_9 x + \log_9(x - 8) = 1$$

$$\log_9[x(x - 8)] = 1$$

$$9^1 = x(x - 8)$$

$$9 = x^2 - 8x$$

$$0 = x^2 - 8x - 9$$

$$0 = (x - 9)(x + 1)$$

• Use the Product Property of Logarithms.

• Write the equation in exponential form.

• Write in standard form.

• Factor and use the Principle of Zero Products.

$$x - 9 = 0$$

$$x + 1 = 0$$

$$x = 9$$

$$x = -1$$

-1 does not check as a solution. The solution is 9.

EXAMPLE • 3Solve for x : $\log_3(2x - 1) = 2$ **Solution**

$$\begin{aligned}
 \log_3(2x - 1) &= 2 \\
 3^2 &= 2x - 1 && \bullet \text{ Write in exponential form.} \\
 9 &= 2x - 1 \\
 10 &= 2x \\
 5 &= x
 \end{aligned}$$

The solution is 5.

YOU TRY IT • 3Solve for x : $\log_4(x^2 - 3x) = 1$ **Your solution****EXAMPLE • 4**Solve for x :
 $\log_2(3x + 8) = \log_2(2x + 2) + \log_2(x - 2)$ **Solution**

$$\begin{aligned}
 \log_2(3x + 8) &= \log_2(2x + 2) + \log_2(x - 2) \\
 \log_2(3x + 8) &= \log_2[(2x + 2)(x - 2)] \\
 \log_2(3x + 8) &= \log_2(2x^2 - 2x - 4) \\
 3x + 8 &= 2x^2 - 2x - 4 && \bullet \text{ 1-1 Property of Logarithms} \\
 0 &= 2x^2 - 5x - 12 \\
 0 &= (2x + 3)(x - 4) && \bullet \text{ Solve by factoring.} \\
 2x + 3 = 0 && x - 4 = 0 \\
 x = -\frac{3}{2} && x = 4
 \end{aligned}$$

 $-\frac{3}{2}$ does not check as a solution. The solution is 4.**YOU TRY IT • 4**Solve for x : $\log_3 x + \log_3(x + 3) = \log_3 4$ **Your solution****EXAMPLE • 5**Solve for x : $\log_3(5x + 4) - \log_3(2x - 1) = 2$ **Solution**

$$\begin{aligned}
 \log_3(5x + 4) - \log_3(2x - 1) &= 2 \\
 \log_3 \frac{5x + 4}{2x - 1} &= 2 && \bullet \text{ Quotient Property} \\
 3^2 &= \frac{5x + 4}{2x - 1} && \bullet \text{ Definition of logarithm} \\
 9 &= \frac{5x + 4}{2x - 1} \\
 (2x - 1)9 &= (2x - 1) \left(\frac{5x + 4}{2x - 1} \right) \\
 18x - 9 &= 5x + 4 \\
 13x &= 13 \\
 x &= 1
 \end{aligned}$$

1 checks as a solution. The solution is 1.

YOU TRY IT • 5Solve for x : $\log_3 x + \log_3(x + 6) = 3$ **Your solution**

10.4 EXERCISES

OBJECTIVE A

To solve an exponential equation



1. What is an exponential equation?

2. a. What does the 1–1 Property of Exponential Functions state?
b. Provide an example of when you would use this property.3. Let $2^x = \frac{1}{3}$. Without solving the equation, is $x < 0$ or is $x > 0$?4. Without solving, determine which of the following equations have no solution.
(i) $5^{-x} = 6$ (ii) $5^x = -6$ (iii) $5^{-x} = -6$ (iv) $5^x = 6$ For Exercises 5 to 36, solve for x . If necessary, round to the nearest ten-thousandth.

5. $5^{4x-1} = 5^{x-2}$

6. $7^{4x-3} = 7^{2x+1}$

7. $8^{x-4} = 8^{5x+8}$

8. $10^{4x-5} = 10^{x+4}$

9. $9^x = 3^{x+1}$

10. $2^{x-1} = 4^x$

11. $8^{x+2} = 16^x$

12. $9^{3x} = 81^{x-4}$

13. $16^{2-x} = 32^{2x}$

14. $27^{2x-3} = 81^{4-x}$

15. $25^{3-x} = 125^{2x-1}$

16. $8^{4x-7} = 64^{x-3}$

17. $5^x = 6$

18. $7^x = 10$

19. $8^{x/4} = 0.4$

20. $5^{x/2} = 0.5$

21. $2^{3x} = 5$

22. $3^{6x} = 0.5$

23. $2^{-x} = 7$

24. $3^{-x} = 14$

25. $2^{x-1} = 6$

26. $4^{x+1} = 9$

27. $3^{2x-1} = 4$

28. $4^{-x+2} = 12$

29. $\left(\frac{1}{2}\right)^{x+1} = 3$

30. $\left(\frac{3}{5}\right)^{-2x} = 2$

31. $3 \cdot 2^x = 7$

32. $5 \cdot 3^{2-x} = 4$

33. $7 = 10\left(\frac{1}{2}\right)^{x/8}$

34. $8 = 15\left(\frac{1}{2}\right)^{x/22}$

35. $15 = 12e^{0.05x}$

36. $7 = 42e^{-3x}$

OBJECTIVE B

To solve a logarithmic equation

For Exercises 37 to 54, solve for x . If necessary, round to the nearest ten-thousandth.

37. $\log x = \log(1 - x)$

38. $\ln(3x - 2) = \ln(x + 1)$

39. $\ln(3x + 2) = \ln(5x + 4)$

40. $\log_3(x - 2) = \log_3(2x)$

41. $\log_2(8x) - \log_2(x^2 - 1) = \log_2 3$

42. $\log_5(3x) - \log_5(x^2 - 1) = \log_5 2$

43. $\log_9 x + \log_9(2x - 3) = \log_9 2$

44. $\log_6 x + \log_6(3x - 5) = \log_6 2$

45. $\log_2(2x - 3) = 3$

46. $\log_4(3x + 1) = 2$

47. $\ln(3x + 2) = 4$

48. $\ln(2x + 3) = -1$

49. $\log_2(x + 1) + \log_2(x + 3) = 3$

50. $\log_4(3x - 4) + \log_4(x + 6) = 2$

51. $\log_5(2x) - \log_5(x - 1) = 1$

52. $\log_3(3x) - \log_3(2x - 1) = 2$

53. $\log_8(6x) = \log_8 2 + \log_8(x - 4)$

54. $\log_7(5x) = \log_7 3 + \log_7(2x + 1)$



55. If u and v are positive numbers and $u < v$, then $\log u < \log v$. Use this fact to explain why the equation $\log(x - 2) - \log x = 3$ has no solution. Do not solve the equation.



56. Sometimes it is easy to represent one number as a power of another. For instance, 81 represented as a power of 3 is $3^4 = 81$. Although it is not as easy, show that we could represent 81 as a power of 7.

Applying the Concepts

For Exercises 57 to 60, solve for x . Round to the nearest ten-thousandth.

57. $3^{x+1} = 2^{x-2}$

58. $2^{2x} = 5^{x+1}$

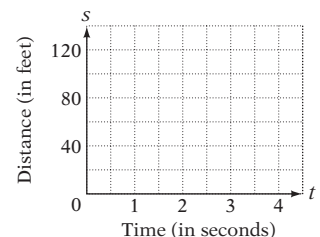
59. $7^{2x-1} = 3^{2x+3}$

60. $4^{3x+2} = 3^{2x-5}$

61. **Physics** A model for the distance s , in feet, that an object experiencing air resistance will fall in t seconds is given by $s = 312.5 \ln \frac{e^{0.32t} + e^{-0.32t}}{2}$.



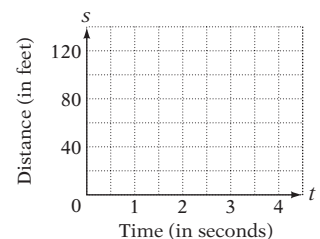
- a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 4.5, Ymin = 0, Ymax = 140, and Yscl = 20.
b. Determine, to the nearest hundredth of a second, the time it takes for the object to fall 100 ft.



62. **Physics** A model for the distance s , in feet, that an object experiencing air resistance will fall in t seconds is given by $s = 78 \ln \frac{e^{0.8t} + e^{-0.8t}}{2}$.



- a. Graph this equation. *Suggestion:* Use Xmin = 0, Xmax = 4.5, Ymin = 0, Ymax = 140, and Yscl = 20.
b. Determine, to the nearest hundredth of a second, the time it takes for the object to fall 125 ft.



SECTION

10.5

Applications of Exponential and Logarithmic Functions

OBJECTIVE A

To solve application problems



Point of Interest



Express Newspapers/Hulton Archive/Getty Images

C. Northcote Parkinson

Parkinson's Law, named after C. Northcote Parkinson, is sometimes stated as "A job will expand to fill the time allotted for the job." However, Parkinson actually said that in any new government administration, administrative employees will be added at the rate of about 5% to 6% per year. This is an example of exponential growth and means that a staff of 500 will grow to approximately 630 by the end of a 4-year term.

A biologist places one single-celled bacterium in a culture, and each hour that particular species of bacteria divides into two bacteria. After 1 h, there will be two bacteria. After 2 h, each of those two bacteria will divide and there will be four bacteria. After 3 h, each of the four bacteria will divide and there will be eight bacteria.

The table at the right shows the number of bacteria in the culture after various intervals of time t , in hours. Values in this table could also be found by using the exponential equation $N = 2^t$.

Time, t	Number of Bacteria, N
0	1
1	2
2	4
3	8
4	16

The equation $N = 2^t$ is an example of an **exponential growth equation**. In general, any equation that can be written in the form $A = A_0b^{kt}$, where A is the size at time t , A_0 is the initial size, $b > 1$, and k is a positive real number, is an exponential growth equation. These equations are important not only in population growth studies but also in physics, chemistry, psychology, and economics.

Recall that interest is the amount of money that one pays (or receives) when borrowing (or investing) money. **Compound interest** is interest that is computed not only on the original principal but also on the interest already earned. The compound interest formula is an exponential equation.

The **compound interest formula** is $A = P(1 + i)^n$, where P is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods.

HOW TO 1

An investment broker deposits \$1000 into an account that earns 8% annual interest compounded quarterly. What is the value of the investment after 3 years?

$$i = \frac{8\%}{4} = \frac{0.08}{4} = 0.02$$

$$n = 4 \cdot 3 = 12$$

$$A = P(1 + i)^n$$

$$A = 1000(1 + 0.02)^{12}$$

$$A \approx 1268$$

- Find i , the interest rate per quarter. The quarterly rate is the annual rate divided by 4, the number of quarters in 1 year.
- Find n , the number of compounding periods. The investment is compounded quarterly, 4 times a year, for 3 years.
- Use the compound interest formula.
- Replace P , i , and n by their values.
- Solve for A .

The value of the investment after 3 years is approximately \$1268.

Exponential decay offers another example of an exponential equation. A common illustration of exponential decay is the decay of a radioactive element. One form of the equation that is used to model radioactive decay is $A = A_0(0.5)^{t/k}$, where A is the amount of the substance remaining after a time period t , A_0 is the initial amount of the radioactive material, and k is the *half-life* of the material. The **half-life** of a radioactive substance is the time it takes for one-half of the material to disintegrate.

The table at the right indicates the amount of an initial 10-microgram sample of tritium that remains after various intervals of time t in years. Note from the table that after each 12-year period of time, the amount of tritium is reduced by one-half. The half-life of tritium is 12 years. The equation that models the decay of the 10-microgram sample is $A = 10(0.5)^{t/12}$, where t is in years and A is in micrograms.

Time, t	Amount, A
0	10
12	5
24	2.5
36	1.25

HOW TO 2

Yttrium-90 is a radioactive isotope that is used to treat some cancers. A sample that originally contained 5 mg of yttrium-90 was measured again after 20 h and found to have 4 mg of yttrium-90. What is the half-life of yttrium-90? Round to the nearest whole number.

$$A = A_0(0.5)^{t/k}$$

$$4 = 5(0.5)^{20/k} \quad \bullet \text{ Replace } A \text{ by } 4, A_0 \text{ by } 5, \text{ and } t \text{ by } 20.$$

$$\frac{4}{5} = (0.5)^{20/k} \quad \bullet \text{ Divide each side by } 5 \text{ to isolate the exponential expression.}$$

$$\log(0.8) = \log(0.5)^{20/k} \quad \bullet \text{ Take the common logarithm of each side.}$$

$$\log(0.8) = \frac{20}{k} \log(0.5) \quad \bullet \text{ Use the Power Property of Logarithms.}$$

$$k \log(0.8) = 20 \log(0.5) \quad \bullet \text{ Solve for } k.$$

$$k = \frac{20 \log(0.5)}{\log(0.8)} \quad \bullet \text{ Exact answer}$$

$$k \approx 62.13 \quad \bullet \text{ Approximate answer}$$

The half-life of yttrium-90 is approximately 62 h.



© Richard T. Nowitz/Corbis

A method by which an archaeologist can measure the age of a bone is called *carbon dating*. Carbon dating is based on a radioactive isotope of carbon called carbon-14, which has a half-life of approximately 5570 years. The exponential decay equation is given by $A = A_0(0.5)^{t/5570}$, where A_0 is the original amount of carbon-14 present in the bone, t is the age of the bone in years, and A is the amount of carbon-14 present after t years.

HOW TO • 3

A bone that originally contained 100 mg of carbon-14 now has 70 mg of carbon-14. What is the approximate age of the bone?

$$A = A_0(0.5)^{t/5570}$$

$$70 = 100(0.5)^{t/5570}$$

$$0.7 = (0.5)^{t/5570}$$

$$\log 0.7 = \log(0.5)^{t/5570}$$

$$\log 0.7 = \frac{t}{5570} \log 0.5$$

$$\frac{5570 \log 0.7}{\log 0.5} = t$$

$$2866 \approx t$$

- Use the exponential decay equation.
- Replace A by 70 and A_0 by 100, and solve for t .
- Divide each side by 100.
- Take the common logarithm of each side of the equation.
- Power Property
- Multiply by 5570, and divide by $\log 0.5$.

The bone is approximately 2866 years old.



Logarithmic functions are used to scale very large or very small numbers into numbers that are easier to comprehend. For instance, the *Richter scale magnitude* of an earthquake uses a logarithmic function to convert the intensity of shock waves I into a number M , which for most earthquakes is in the range of 0 to 10. The intensity I of an earthquake is often given in terms of the constant I_0 , where I_0 is the intensity of the smallest earthquake, called a *zero-level earthquake*, that can be measured on a seismograph near the earthquake's epicenter. An earthquake with an intensity I has a Richter scale magnitude of $M = \log \frac{I}{I_0}$, where I_0 is the measure of the intensity of a zero-level earthquake.

**Point of Interest**

© Bettmann/Corbis

Charles F. Richter

The Richter scale was created by seismologist Charles F. Richter in 1935. Note that a tenfold increase in the intensity level of an earthquake increases the Richter scale magnitude of the earthquake by only 1.

HOW TO • 4

The Richter magnitude of an earthquake that occurred on May, 2008, in Sichuan, China, was 7.9. Find the intensity of the earthquake in terms of I_0 . Round to the nearest thousand.

$$M = \log \frac{I}{I_0}$$

$$7.9 = \log \frac{I}{I_0}$$

$$\frac{I}{I_0} = 10^{7.9}$$

$$I = 10^{7.9} I_0$$

$$I \approx 79,432,823 I_0$$

- Replace M by 7.9.
- Write in exponential form.
- Solve for I .

The Sichuan earthquake had an intensity that was 79,433,000 times the intensity of a zero-level earthquake.

HOW TO 5

In April 2008, there was an earthquake near Wells, Nevada, that measured 6.0 on the Richter scale. In July 2008, an earthquake of magnitude 5.4 occurred near Los Angeles, California. How many times stronger was the Wells earthquake than the Los Angeles earthquake? Round to the nearest whole number.

Let I_1 be the intensity of the Wells earthquake, and let I_2 be the intensity of the Los Angeles earthquake. Use the Quotient Property of Logarithms to write $M = \log \frac{I}{I_0}$ as $M = \log I - \log I_0$. For the Wells earthquake, $6.0 = \log I_1 - \log I_0$. For the Los Angeles earthquake, $5.4 = \log I_2 - \log I_0$. Using these equations, we have

$$6.0 = \log I_1 - \log I_0$$

$$5.4 = \log I_2 - \log I_0$$

$$0.6 = \log I_1 - \log I_2$$

$$0.6 = \log \frac{I_1}{I_2}$$

$$\frac{I_1}{I_2} = 10^{0.6}$$

$$I_1 = 10^{0.6} I_2$$

$$I_1 \approx 3.981 I_2$$

• Subtract the equations.

• Use the Quotient Property of Logarithms.

• Write in exponential form.

• Solve for I_1 .

The Wells earthquake was approximately 4 times stronger than the Los Angeles earthquake.

**Point of Interest**

Søren Sørensen

The pH scale was created by the Danish biochemist Søren Sørensen in 1909 to measure the acidity of water used in the brewing of beer. pH is an abbreviation for *pondus hydrogenii*, which translates as “potential hydrogen.”

Edgar Fahs Smith Collection, University of Pennsylvania Collection



A chemist measures the acidity or alkalinity of a solution by measuring the concentration of hydrogen ions, H^+ , in the solution using the formula $pH = -\log(H^+)$. A neutral solution such as distilled water has a pH of 7, acids have a pH less than 7, and alkaline solutions (also called basic solutions) have a pH greater than 7.

HOW TO 6

Find the pH of orange juice that has a hydrogen ion concentration, H^+ , of 2.9×10^{-4} . Round to the nearest tenth.

$$pH = -\log(H^+)$$

$$= -\log(2.9 \times 10^{-4}) \quad \bullet H^+ = 2.9 \times 10^{-4}$$

$$\approx 3.5376$$

The pH of the orange juice is approximately 3.5.

EXAMPLE • 1

An investment of \$3000 is placed into an account that earns 12% annual interest compounded monthly. In approximately how many years will the investment be worth twice the original amount?

Strategy

To find the time, solve the compound interest formula for n . Use $A = 6000$,

$$P = 3000, \text{ and } i = \frac{12\%}{12} = \frac{0.12}{12} = 0.01.$$

Solution

$$A = P(1 + i)^n$$

$$6000 = 3000(1 + 0.01)^n$$

$$6000 = 3000(1.01)^n$$

$$2 = (1.01)^n$$

$$\log 2 = \log (1.01)^n$$

$$\log 2 = n \log 1.01$$

$$\frac{\log 2}{\log 1.01} = n$$

$$70 \approx n$$

$$70 \text{ months} \div 12 \approx 5.8 \text{ years}$$

In approximately 6 years, the investment will be worth \$6000.

- Divide by 3000.
- Take the log of each side.
- Power Property
- Divide both sides by log 1.01.

YOU TRY IT • 1

Find the hydrogen ion concentration, H^+ , of vinegar that has a pH of 2.9.

Your strategy**Your solution****EXAMPLE • 2**

Rhenium-186 is a radioactive isotope with a half-life of approximately 3.78 days. Rhenium-186 is sometimes used for pain management. If a patient receives a dose of 5 micrograms of rhenium-186, how long (in days) will it take for the rhenium-186 level to reach 2 micrograms?

Strategy

- Use the equation $A = A_0(0.5)^{t/k}$.
- Replace A with 2, A_0 with 5, and k with 3.78. Then solve for t .

Solution

$$A = A_0(0.5)^{t/k}$$

$$2 = 5(0.5)^{t/3.78}$$

$$\frac{2}{5} = (0.5)^{t/3.78}$$

$$\log (0.4) = \frac{t}{3.78} \log (0.5)$$

$$\frac{3.78 \log (0.4)}{\log (0.5)} = t$$

$$4.997 \approx t$$

In approximately 5 days, there will be 2 micrograms of rhenium-186 remaining in the patient's body.

YOU TRY IT • 2

The percent of light p , as a decimal, that passes through a substance of thickness d , in meters, is given by $\log p = -kd$. The value of k for some opaque glass is 6. How thick a piece of this glass is necessary so that 20% of the light passes through the glass?

Your strategy**Your solution**

10.5 EXERCISES

OBJECTIVE A

To solve application problems

Compound Interest For Exercises 1 to 4, use the compound interest formula $A = P(1 + i)^n$, where P is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods.

1. An investment broker deposits \$1000 into an account that earns 8% annual interest compounded quarterly. What is the value of the investment after 2 years? Round to the nearest dollar.
2. A financial advisor recommends that a client deposit \$2500 into a fund that earns 7.5% annual interest compounded monthly. What will be the value of the investment after 3 years? Round to the nearest cent.
3. To save for college tuition, the parents of a preschooler invest \$5000 in a bond fund that earns 6% annual interest compounded monthly. In approximately how many years will the investment be worth \$15,000?
4. A hospital administrator deposits \$10,000 into an account that earns 6% annual interest compounded monthly. In approximately how many years will the investment be worth \$15,000?

Radioactivity For Exercises 5 to 8, use the exponential decay equation $A = A_0(0.5)^{t/k}$, where A is the amount of a radioactive material present after time t , k is the half-life of the radioactive substance, and A_0 is the original amount of the radioactive substance. Round to the nearest tenth.

5. An isotope of technetium is used to prepare images of internal body organs. This isotope has a half-life of approximately 6 h. A patient is injected with 30 mg of this isotope.
 - a. What is the technetium level in the patient after 3 h?
 - b. How long (in hours) will it take for the technetium level to reach 20 mg?
6. Iodine-131 is an isotope that is used to study the functioning of the thyroid gland. This isotope has a half-life of approximately 8 days. A patient is given an injection that contains 8 micrograms of iodine-131.
 - a. What is the iodine level in the patient after 5 days?
 - b. How long (in days) will it take for the iodine level to reach 5 micrograms?
7. A sample of promethium-147 (used in some luminous paints) weighs 25 mg. One year later, the sample weighs 18.95 mg. What is the half-life of promethium-147, in years?



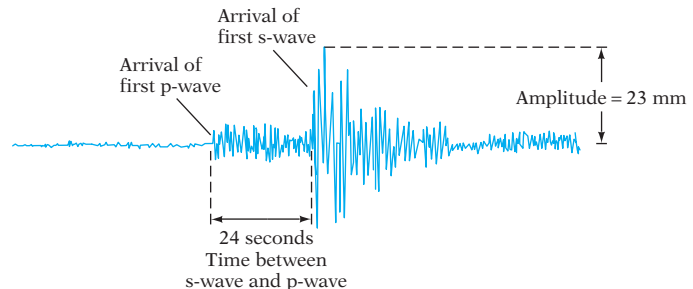
Mark Harmer/Stone/Getty Images

8. Francium-223 is a very rare radioactive isotope discovered in 1939 by Marguerite Percy. A 3-microgram sample of francium-223 decays to 2.54 micrograms in 5 min. What is the half-life of francium-223, in minutes?

Seismology For Exercises 9 to 12, use the Richter scale equation $M = \log \frac{I}{I_0}$, where M is the magnitude of an earthquake, I is the intensity of the shock waves, and I_0 is the measure of the intensity of a zero-level earthquake.

9. An earthquake in Japan on March 2, 1933, measured 8.9 on the Richter scale. Find the intensity of the earthquake in terms of I_0 . Round to the nearest whole number.
10. An earthquake that occurred in China in 1978 measured 8.2 on the Richter scale. Find the intensity of the earthquake in terms of I_0 . Round to the nearest whole number.
11. In 2008, a 6.9-magnitude earthquake occurred off the coast of Honshu, Japan. In the same year, a 6.4-magnitude earthquake occurred near Quetta, Pakistan. How many times stronger was the Honshu earthquake than the Quetta earthquake? Round to the nearest tenth.
12. In 2008, a 6.4-magnitude earthquake occurred off the coast of Oregon. In the same year, a 7.2-magnitude earthquake occurred near Xinjiang, China. How many times stronger was the Xinjiang earthquake than the one off the coast of Oregon? Round to the nearest tenth.

Seismology Shown at the right is a *seismogram*, which is used to measure the magnitude of an earthquake. The magnitude is determined by the amplitude A of a shock wave and the difference in time t , in seconds, between the occurrences of two types of waves called *primary waves* and *secondary waves*. As you can see on the graph, a primary wave is abbreviated *p-wave* and a secondary wave is abbreviated *s-wave*. The amplitude A of a wave is one-half the difference between its highest and lowest points. For this graph, A is 23 mm. The equation is $M = \log A + 3 \log 8t - 2.92$. Use this information for Exercises 13 to 15.



13. Determine the magnitude of the earthquake for the seismogram given in the figure. Round to the nearest tenth.
14. Find the magnitude of an earthquake that has a seismogram with an amplitude of 30 mm and for which t is 21 s.
15. Find the magnitude of an earthquake that has a seismogram with an amplitude of 28 mm and for which t is 28 s.

Chemistry For Exercises 16 to 19, use the equation $\text{pH} = -\log(\text{H}^+)$, where H^+ is the hydrogen ion concentration of a solution.

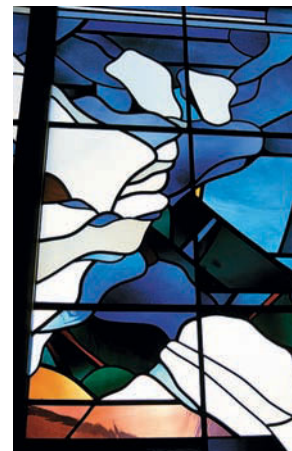
16. Find the pH of milk, which has a hydrogen ion concentration of 3.97×10^{-7} . Round to the nearest tenth.
17. Find the pH of a baking soda solution for which the hydrogen ion concentration is 3.98×10^{-9} . Round to the nearest tenth.
18. The pH of pure water is 7. What is the hydrogen ion concentration of pure water?
19. Peanuts grow best in soils that have a pH between 5.3 and 6.6. What is the range of hydrogen ion concentration for these soils?

Sound For Exercises 20 to 23, use the equation $D = 10(\log I + 16)$, where D is the number of decibels of a sound and I is the intensity of the sound in watts per square centimeter.

20. Find the number of decibels of normal conversation. The intensity of the sound of normal conversation is approximately 3.2×10^{-10} watts/cm². Round to the nearest whole number.
-  21. The loudest sound made by any animal is made by the blue whale and can be heard from more than 500 mi away. The intensity of the sound is 630 watts/cm². Find the number of decibels of sound emitted by the blue whale. Round to the nearest whole number.
22. Although pain thresholds for sound vary in humans, a decibel level of 125 decibels will produce pain (and even hearing loss) for most people. What is the intensity, in watts per square centimeter, of 125 decibels?
23. The purr of a cat is approximately 25 decibels. What is the intensity, in watts per square centimeter, of 25 decibels?

Light For Exercises 24 and 25, use the equation $\log P = -kd$, which gives the percent P , as a decimal, of light passing through a substance of thickness d , in meters.

24. The value of k for a swimming pool is approximately 0.05. At what depth, in meters, will the percent of light be 75% of the light at the surface of the pool?
25. The constant k for a piece of blue stained glass is 20. What percent of light will pass through a piece of this glass that is 0.005 m thick?



- 26. Earth Science** Earth's atmospheric pressure changes as you rise above its surface. At an altitude of h kilometers, where $0 < h < 80$, the pressure P in newtons per square centimeter is approximately modeled by the equation $P(h) = 10.13e^{-0.116h}$.
- What is the approximate pressure at 40 km above Earth's surface?
 - What is the approximate pressure on Earth's surface?
 - Does atmospheric pressure increase or decrease as you rise above Earth's surface?
- 27. Chemistry** The intensity I of an x-ray after it has passed through a material that is x centimeters thick is given by $I = I_0 e^{-kx}$, where I_0 is the initial intensity of the x-ray and k is a number that depends on the material. The constant k for copper is 3.2. Find the thickness of copper necessary so that the intensity of an x-ray after passing through the copper is 25% of the original intensity. Round to the nearest tenth.
- 28. Disk Storage** The cost per megabyte of disk storage in 1981 was \$700. In 2010, the cost per megabyte of disk storage was about \$.00002. Assuming that the cost per megabyte is decreasing exponentially, find an exponential model of the form $C(t) = Ae^{kt}$ that gives the cost per megabyte C in year t . Use 1981 for $t = 0$, and round k to the nearest thousandth. According to the model, in what year was the cost per megabyte of storage \$.10?
- 29. Steroid Use** When air resistance is considered, the height f , in feet, of a baseball x feet from home plate and hit at a certain angle can be approximated by $f(x) = \left(\frac{0.5774v + 155.3}{v}\right)x + 565.3 \ln\left(\frac{v - 0.2747x}{v}\right) + 3.5$, where v is the speed of the ball in feet per second when it leaves the bat.
- If $v = 160$ ft/s, show that a baseball will hit near the bottom of a 15-foot-high fence 375 ft from home plate.
 - If the speed in part (a) is increased by 4%, show that the baseball will clear the fence by approximately 9 ft.
-  **30. Steroid Use** In Exercise 29, a 4% increase in the speed of the ball as it leaves the bat gives $v = 166.4$ ft/s. Using f from Exercise 29, determine the greatest distance from home plate the 15-foot fence could be placed so that a ball hit with speed v would clear the fence.
-  **31. Ball Flight** Air resistance plays a large role in the flight of a ball. If air resistance is ignored, then the height h , in feet, of a baseball x feet from home plate and hit at the same angle as in Exercise 29 can be approximated by $h(x) = \frac{-21.33}{v^2}x^2 + 0.5774x + 3.5$. If the speed of a ball as it leaves the bat is 160 ft/s, how much farther would the ball travel before hitting the ground if air resistance were ignored? Round to the nearest foot.

In the News**Steroids Increase Number of Home Runs**

Steroids can help batters hit 50 percent more home runs by boosting their muscle mass by just 10 percent, a U.S. physicist said on Thursday. Calculations show that by putting on 10 percent more muscle mass, a batter can swing about 5 percent faster, increasing the ball's speed by 4 percent as it leaves the bat. According to Roger Tobin of Tufts University in Boston, depending on the ball's trajectory, this added speed could take it into home run territory 50 percent more often.

Source: www.reuters.com



32. Assuming that an exponential model is an appropriate model for each of the following, would the situation be modeled using exponential growth or exponential decay?
- The amount of a medication in a patient's system t hours after taking the medicine
 - The atmospheric pressure x meters above the surface of Earth
 - The temperature of a roast t minutes after it is put in a hot oven
 - The temperature of a hot cup of tea that is put in a refrigerator
 - The spread of a contagious disease

Applying the Concepts

33. **Uranium Dating** Read the article at the right. The carbon-14 dating method does not work on rocks, so methods based on different radioactive elements are used. One such method uses uranium-235, which has a half-life of approximately 713 million years. Use the uranium-235 dating equation

$$A = A_0 \left(\frac{1}{2} \right)^{t/713,000,000}$$
 to estimate the ratio of the current amount of uranium-235 in the Hudson Bay rock to the original amount of uranium-235 in the rock. (Hint: You are finding $\frac{A}{A_0}$ when $t = 4.28$ billion years.) Round to the nearest thousandth.

34. **Investments** The value of an investment in an account that earns an annual interest rate of 7% compounded daily grows according to the equation

$$A = A_0 \left(1 + \frac{0.07}{365} \right)^{365t}$$
, where A_0 is the original value of an investment and t is the time in years. Find the time for the investment to double in value. Round to the nearest year.

35. **Continuous Compounding** In some scenarios, economists will use continuous compounding of an investment. The equation for continuous compounding is $A = A_0 e^{rt}$, where A is the present value of an initial investment of A_0 at an annual interest rate r (as a decimal), and t is the time in years.

- Find the value of an investment of \$5000 after 3 years if interest is compounded continuously at an annual rate of 6%.
- If an investor wanted to grow a continuously compounded investment of \$1000 to \$1250 in 2 years, what interest rate must the investor receive? Round to the nearest tenth.

In the News

Scientists Discover Ancient Rocks

Scientists have found that a pinkish tract of bedrock on the eastern shore of Canada's Hudson Bay contains the oldest known rocks on Earth, formed 4.28 billion years ago, not long after Earth itself was formed.

Source: news.yahoo.com



Jonathan O'Neil

FOCUS ON PROBLEM SOLVING

Proof by Contradiction The four-step plan for solving problems that we have used before is restated here.

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review the solution.

One of the techniques that can be used in the second step is a method called *proof by contradiction*. In this method, you assume that the conditions of the problem you are trying to solve can be met and then show that your assumption leads to a condition you already know is not true.

To illustrate this method, suppose we try to prove that $\sqrt{2}$ is a rational number. We begin by recalling that a rational number is one that can be written as the quotient of integers. Therefore, let a and b be two integers with no common factors. If $\sqrt{2}$ is a rational number, then

$$\begin{aligned}\sqrt{2} &= \frac{a}{b} \\ (\sqrt{2})^2 &= \left(\frac{a}{b}\right)^2 && \bullet \text{ Square each side.} \\ 2 &= \frac{a^2}{b^2} \\ 2b^2 &= a^2 && \bullet \text{ Multiply each side by } b^2.\end{aligned}$$

From the last equation, a^2 is an even number. Because a^2 is an even number, a is an even number. Now divide each side of the last equation by 2.

$$\begin{aligned}2b^2 &= a^2 \\ 2b^2 &= a \cdot a \\ b^2 &= a \cdot x && \bullet x = \frac{a}{2}\end{aligned}$$

Because a is an even number, $a \cdot x$ is an even number. Because $a \cdot x$ is an even number, b^2 is an even number, and this in turn means that b is an even number. This result, however, contradicts the assumption that a and b are two integers with no common factors. Because this assumption is now known not to be true, we conclude that our original assumption, that $\sqrt{2}$ is a rational number, is false. This proves that $\sqrt{2}$ is an irrational number.

Try a proof by contradiction for the following problem: “Is it possible to write numbers using each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 exactly once such that the sum of the numbers is exactly 100?”* Here are some suggestions. First note that the sum of the 10 digits is 45. This means that some of the digits used must be tens digits. Let x be the sum of those digits.

- a. What is the sum of the remaining units digits?
- b. Express “the sum of the units digits and the tens digits equals 100” as an equation.
- c. Solve the equation for x .
- d. Explain why this result means that it is impossible to satisfy both conditions of the problem.

*Problem adapted from G. Polya, *How to Solve It: A New Aspect of Mathematical Method*. Copyright © 1947 by G. Polya. Reprinted by permission of Princeton University Press.

PROJECTS AND GROUP ACTIVITIES

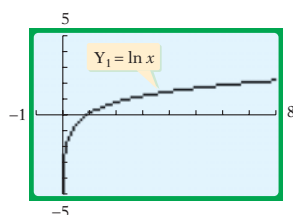
Solving Exponential and Logarithmic Equations Using a Graphing Calculator



A graphing calculator can be used to draw the graphs of logarithmic functions. Note that there are two logarithmic keys, **LOG** and **LN**, on a graphing calculator. The first key gives the values of common logarithms (base 10), and the second gives the values of natural logarithms (base e).

To graph $y = \ln x$, press the **Y=** key. Clear any equations already entered.

Press **LN** **X,T,θ,n** **)** **GRAPH**. The graph is shown at the left with a viewing window of $X_{\min} = -1$, $X_{\max} = 8$, $Y_{\min} = -5$, and $Y_{\max} = 5$.



Some exponential and logarithmic equations cannot be solved algebraically. In these cases, a graphical approach may be appropriate. Here is an example.

HOW TO 1

Solve $\ln(2x + 4) = x^2$ for x . Round to the nearest hundredth.

Rewrite the equation by subtracting x^2 from each side.

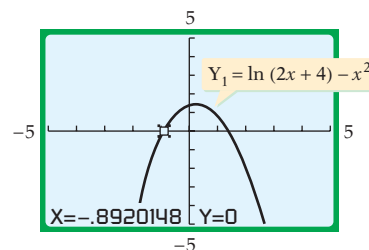
$$\begin{aligned}\ln(2x + 4) &= x^2 \\ \ln(2x + 4) - x^2 &= 0\end{aligned}$$

The zeros of $f(x) = \ln(2x + 4) - x^2$ are the solutions of $\ln(2x + 4) - x^2 = 0$.

Graph f and use the zero feature of the graphing calculator to estimate the solutions to the nearest hundredth.

The zeros are approximately -0.89 and 1.38 .

The solutions are -0.89 and 1.38 .



For Exercises 1 to 8, solve for x by graphing. Round to the nearest hundredth.

1. $2^x = 2x + 4$

2. $3^x = -x - 1$

3. $e^x = -2x - 2$

4. $e^x = 3x + 4$

5. $\log(2x - 1) = -x + 3$

6. $\log(x + 4) = -2x + 1$

7. $\ln(x + 2) = x^2 - 3$

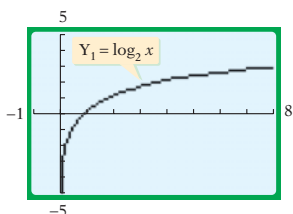
8. $\ln x = -x^2 + 1$

To graph a logarithmic function with a base other than base 10 or base e , the Change-of-Base Formula is used. Here is the Change-of-Base Formula, discussed earlier:

$$\log_a N = \frac{\log_b N}{\log_b a}$$

To graph $y = \log_2 x$, first change from base 2 logarithms to the equivalent base 10 logarithms or natural logarithms. Base 10 is shown here.

$$y = \log_2 x = \frac{\log_{10} x}{\log_{10} 2} \quad \bullet \ a = 2, \ b = 10, \ N = x$$



Then graph the equivalent equation $y = \frac{\log x}{\log 2}$. The graph is shown at the left with a viewing window of $X_{\min} = -1$, $X_{\max} = 8$, $Y_{\min} = -5$, and $Y_{\max} = 5$.

Credit Reports and FICO® Scores¹

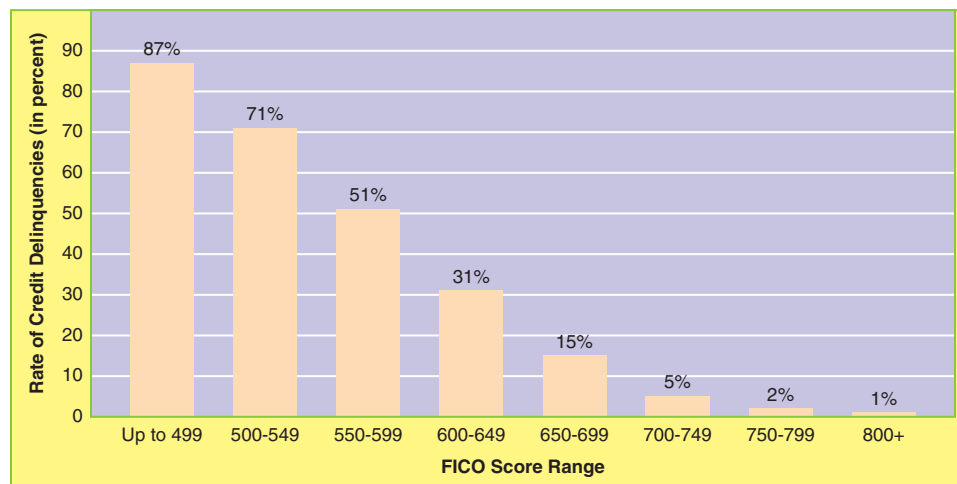


© David Young-Wolff/PhotoEdit, Inc.

When a consumer applies for a loan, the lender generally wants to know the consumer's credit history. For this, the lender turns to a credit reporting agency. These agencies maintain files on millions of borrowers. They provide the lender with a credit report, which lists information such as the types of credit the consumer uses, the length of time the consumer's credit accounts have been open, the amounts owed by the consumer, and whether the consumer has paid his or her bills on time. Along with the credit report, the lender can buy a credit score based on the information in the report. The credit score gives the lender a quick measure of the consumer's credit risk, or how likely the consumer is to repay a debt. It answers the lender's question: "If I lend this person money (or give this person a credit card), what is the probability that I will be paid back in a timely manner?"

The most widely used credit bureau score is the FICO® score, so named because scores are produced from software developed by Fair Isaac Corporation (FICO). FICO scores range from 300 to 850. The higher the score, the lower the predicted credit risk for lenders.

The following graph shows the delinquency rate, or credit risk, associated with ranges of FICO scores. For example, the delinquency rate of consumers in the 550–599 range of scores is 51%. This means that within the next 2 years, for every 100 borrowers in this range, approximately 51 will default on a loan, file for bankruptcy, or fail to pay a credit card bill within 90 days of the due date.



Because the delinquency rate depends on the FICO score, we can let the independent variable x represent the FICO score and the dependent variable y represent the delinquency rate. We will use the middle of each range for the values of x . The resulting ordered pairs are recorded in the table at the right.

x	y
400	87
525	71
575	51
625	31
675	15
725	5
775	2
825	1

¹Source: www.myfico.com



EDIT	CALC	TESTS
1: Edit		
2: L1	L2	L3
3: 525	71	
4: 575	51	
5: 625	31	
	15	
	5	
	2	
	1	
L2(8) = 1		

EDIT	CALC	TESTS
7: QuartReg		
8: LinReg(a+bx)		
9: LnReg		
0: LnReg		
A: $y = a + b \ln x$		
B: $a = 918.223447$		
C: $b = -137.4868338$		

1. Use your graphing calculator to find a logarithmic equation that approximates these data. Here are instructions for the TI-84:

Press **STAT** **ENTER**. This will bring up a table into which you can enter data. Enter the values of x in L1. Enter the values of y in L2.

Press **STAT** again, press the right arrow key to highlight **CALC**, and arrow down to **LnReg** (ln for the natural logarithm). Press **ENTER** twice. The values for a and b in the equation $y = a + b \ln x$ will appear on the screen.

2. Use your equation to predict the delinquency rate of a consumer with a score of 500. Round to the nearest whole number.
3. The equation pairs a delinquency rate of 36% with what score?
4. Use the graph on page 590 to determine the highest FICO score that will result in a delinquency rate of 71% or more.
5. Use the graph on page 590 to determine the lowest FICO score that will result in a delinquency rate of 5% or lower.
6. Use the Internet to find the name of at least one of the major credit reporting agencies.

CHAPTER 10

SUMMARY

KEY WORDS

A function of the form $f(x) = b^x$, where b is a positive real number not equal to 1, is an *exponential function*. The number b is the *base* of the exponential function. [10.1A, p. 550]

The function defined by $f(x) = e^x$ is called the *natural exponential function*. [10.1A, p. 551]

Because the exponential function is a 1–1 function, it has an inverse function that is called a *logarithm*. The definition of logarithm is: For $x > 0$, $b > 0$, $b \neq 1$, $y = \log_b x$ is equivalent to $x = b^y$. [10.2A, p. 558]

Logarithms with base 10 are called *common logarithms*. We usually omit the base, 10, when writing the common logarithm of a number. [10.2A, p. 559]

When e (the base of the natural exponential function) is used as a base of a logarithm, the logarithm is referred to as the *natural logarithm* and is abbreviated $\ln x$. [10.2A, p. 559]

EXAMPLES

$f(x) = 3^x$ is an exponential function. 3 is the base of the function.

$f(x) = 2e^{x-1}$ is a natural exponential function. e is an irrational number approximately equal to 2.71828183.

$\log_2 8 = 3$ is equivalent to $8 = 2^3$.

$\log_{10} 100 = 2$ is usually written $\log 100 = 2$.

$\log_e 100 \approx 4.61$ is usually written $\ln 100 \approx 4.61$.

An <i>exponential equation</i> is one in which a variable occurs in an exponent. [10.4A, p. 572]	$2^x = 12$ is an exponential equation.
An <i>exponential growth equation</i> is an equation that can be written in the form $A = A_0b^{kt}$, where A is the size at time t , A_0 is the initial size, $b > 1$, and k is a positive real number. In an <i>exponential decay equation</i> , the base is between 0 and 1. [10.5A, pp. 578–579]	$P = 1000(1.03)^n$ is an exponential growth equation. $A = 10(0.5)^x$ is an exponential decay equation.

ESSENTIAL RULES AND PROCEDURES

EXAMPLES

1–1 Property of Exponential Functions [10.2A, p. 559] For $b > 0$, $b \neq 1$, if $b^u = b^v$, then $u = v$.	If $b^x = b^5$, then $x = 5$.
Logarithm Property of 1 [10.2B, p. 560] For any positive real numbers x , y , and b , $b \neq 1$, $\log_b 1 = 0$.	$\log_6 1 = 0$
Inverse Property of Logarithms [10.2B, p. 560] For any positive real numbers x , y , and b , $b \neq 1$, $b^{\log_b x} = x$ and $\log_b b^x = x$.	$\log_3 3^4 = 4$
1–1 Property of Logarithms [10.2B, p. 560] For any positive real numbers x , y , and b , $b \neq 1$, if $\log_b x = \log_b y$, then $x = y$.	If $\log_5(x - 2) = \log_5 3$, then $x - 2 = 3$.
Logarithm Property of the Product of Two Numbers [10.2B, p. 561] For any positive real numbers x , y , and b , $b \neq 1$, $\log_b(xy) = \log_b x + \log_b y$.	$\log_b(3x) = \log_b 3 + \log_b x$
Logarithm Property of the Quotient of Two Numbers [10.2B, p. 561] For any positive real numbers x , y , and b , $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.	$\log_b \frac{x}{20} = \log_b x - \log_b 20$
Logarithm Property of the Power of a Number [10.2B, p. 561] For any positive real numbers x and b , $b \neq 1$, and for any real number r , $\log_b x^r = r \log_b x$.	$\log_b x^5 = 5 \log_b x$

Change-of-Base Formula [10.2C, p. 563]

$\log_a N = \frac{\log_b N}{\log_b a}$	$\log_3 12 = \frac{\log 12}{\log 3}$	$\log_6 16 = \frac{\ln 16}{\ln 6}$

CHAPTER 10**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. How do you know that a function is an exponential function?
2. Why are there limitations on the values of the base of an exponential function?
3. What kind of number is e , and what is its approximate value?
4. Why are there limitations on the values used in a logarithmic function?
5. What is the relationship between logarithmic functions and exponential functions?
6. Why is $\log_b(xy) = (\log_b x)(\log_b y)$ a false statement?
7. Why is $\log_b\left(\frac{x}{y}\right) = \frac{\log_b x}{\log_b y}$ a false statement?
8. What practical purpose does the Change-of-Base Formula have?
9. What is the domain of a logarithmic function?
10. What is the compound interest formula?
11. What is the relationship between earthquakes and logarithms?

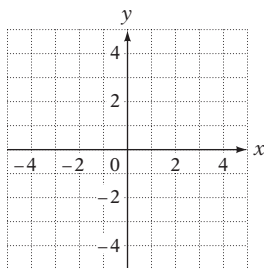
CHAPTER 10

REVIEW EXERCISES

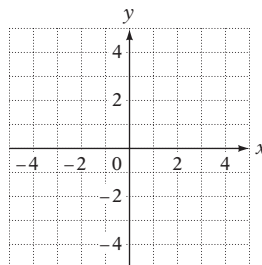
1. Evaluate $f(x) = e^{x-2}$ at $x = 2$.

2. Write $\log_5 25 = 2$ in exponential form.

3. Graph: $f(x) = 3^{-x} + 2$



4. Graph: $f(x) = \log_3(x - 1)$



5. Write $\log_3 \sqrt[5]{x^2 y^4}$ in expanded form.

6. Write $2 \log_3 x - 5 \log_3 y$ as a single logarithm with a coefficient of 1.

7. Solve: $27^{2x+4} = 81^{x-3}$

8. Solve: $\log_5 \frac{7x+2}{3x} = 1$

9. Find $\log_6 22$. Round to the nearest ten-thousandth.

10. Solve: $\log_2 x = 5$

11. Solve: $\log_3(x + 2) = 4$

12. Solve: $\log_{10} x = 3$

13. Write $\frac{1}{3}(\log_7 x + 4 \log_7 y)$ as a single logarithm with a coefficient of 1.

14. Write $\log_8 \sqrt{\frac{x^5}{y^3}}$ in expanded form.

15. Write $2^5 = 32$ in logarithmic form.

16. Find $\log_3 1.6$. Round to the nearest ten-thousandth.

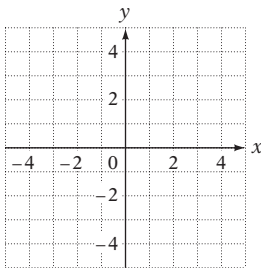
17. Solve $3^{x+2} = 5$. Round to the nearest thousandth.

18. Evaluate $f(x) = \left(\frac{2}{3}\right)^{x+2}$ at $x = -3$.

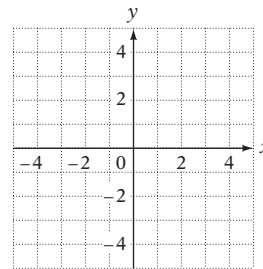
19. Solve: $\log_2(x + 3) - \log_2(x - 1) = 3$

20. Solve: $\log_3(2x + 3) + \log_3(x - 2) = 2$

21. Graph: $f(x) = \left(\frac{2}{3}\right)^{x+1}$



22. Graph: $f(x) = \log_2(2x - 1)$



23. Evaluate: $\log_6 36$

24. Write $\frac{1}{3}(\log_2 x - \log_2 y)$ as a single logarithm with a coefficient of 1.

25. Solve for x : $9^{2x} = 3^{x+3}$

26. Solve for x : $5 \cdot 3^{x/2} = 12$. Round to the nearest ten-thousandth.

27. Solve for x : $\log_5 x = -1$

28. Write $3^4 = 81$ in logarithmic form.

29. Solve for x : $\log x + \log(x - 2) = \log 15$

30. Write $\log_5 \sqrt[3]{x^2 y}$ in expanded form.

31. Solve for x : $6e^{-2x} = 17$. Round to the nearest ten-thousandth.

32. Evaluate $f(x) = 7^{x+2}$ at $x = -3$.

33. Evaluate: $\log_2 16$
34. Solve for x : $\log_6 x = \log_6 2 + \log_6 (2x - 3)$
35. Evaluate $\log_2 5$. Round to the nearest ten-thousandth.
36. Solve for x : $4^x = 8^{x-1}$
37. Solve for x : $\log_5 x = 4$
38. Write $3 \log_b x - 7 \log_b y$ as a single logarithm with a coefficient of 1.
39. Evaluate $f(x) = 5^{-x-1}$ at $x = -2$.
40. Solve $5^{x-2} = 7$ for x . Round to the nearest ten-thousandth.
41. **Investments** Use the compound interest formula $P = A(1 + i)^n$, where A is the original value of an investment, i is the interest rate per compounding period, n is the number of compounding periods, and P is the value of the investment after n periods, to find the value of an investment after 2 years. The amount of the investment is \$4000, and it is invested at 8% compounded monthly. Round to the nearest dollar.
42. **Seismology** An earthquake in Japan in September, 2003, had an intensity of $I = 199,526,232I_0$. Find the Richter scale magnitude of the earthquake. Use the Richter scale equation $M = \log \frac{I}{I_0}$, where M is the magnitude of an earthquake, I is the intensity of the shock waves, and I_0 is the measure of the intensity of a zero-level earthquake. Round to the nearest tenth.
43. **Radioactivity** Use the exponential decay equation $A = A_0(0.5)^{t/k}$, where A is the amount of a radioactive material present after time t , k is the half-life of the radioactive material, and A_0 is the original amount of radioactive material, to find the half-life of a material that decays from 25 mg to 15 mg in 20 days. Round to the nearest whole number.
44. **Sound** The number of decibels D of a sound can be given by the equation $D = 10(\log I + 16)$, where I is the intensity of the sound measured in watts per square centimeter. Find the number of decibels of sound emitted from a busy street corner for which the intensity of the sound is 5×10^{-6} watts per square centimeter.



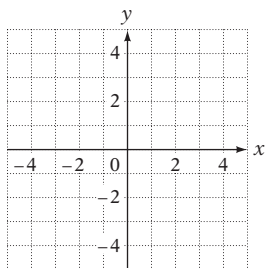
CHAPTER 10

TEST

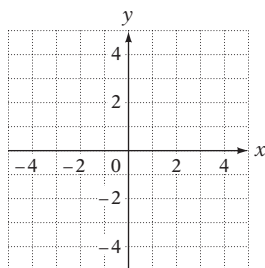
1. Evaluate $f(x) = \left(\frac{2}{3}\right)^x$ at $x = 0$.

2. Evaluate $f(x) = 3^{x+1}$ at $x = -2$.

3. Graph: $f(x) = 2^x - 3$



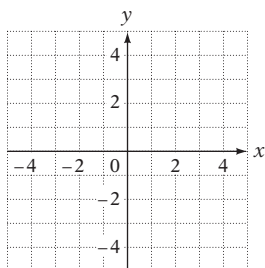
4. Graph: $f(x) = 2^x + 2$



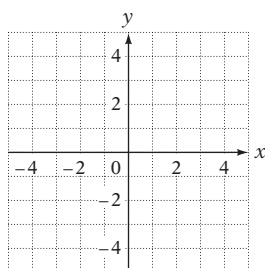
5. Evaluate: $\log_4 16$

6. Solve for x : $\log_3 x = -2$

7. Graph: $f(x) = \log_2(2x)$



8. Graph: $f(x) = \log_3(x + 1)$



9. Write $\log_6 \sqrt{xy^3}$ in expanded form.

10. Write $\frac{1}{2}(\log_3 x - \log_3 y)$ as a single logarithm with a coefficient of 1.

11. Write $\ln \frac{x}{\sqrt{z}}$ in expanded form.

12. Write $3 \ln x - \ln y - \frac{1}{2} \ln z$ as a single logarithm with a coefficient of 1.

13. Solve for x : $3^{7x+1} = 3^{4x-5}$

14. Solve for x : $8^x = 2^{x-6}$

15. Solve for x : $3^x = 17$

16. Solve for x : $\log x + \log(x - 4) = \log 12$

17. Solve for x : $\log_6 x + \log_6(x - 1) = 1$

18. Find $\log_5 9$.

19. Find $\log_3 19$.

20. Solve for x : $5^{2x-5} = 9$

21. Solve for x : $2e^{x/4} = 9$

22. Solve for x : $\log_5(30x) - \log_5(x + 1) = 2$

23. **Carbon Dating** A shard from a vase originally contained 250 mg of carbon-14 and now contains 170 mg of carbon-14. Use the equation $A = A_0(0.5)^{t/5570}$, where A_0 is the original amount of carbon-14 in the shard and A is the amount of carbon-14 in the shard t years later, to find the approximate age of the shard. Round to the nearest whole number.

24. **Sound** Use the decibel equation $D = 10(\log I + 16)$, where D is the decibel level and I is the intensity of a sound in watts per square centimeter, to find the intensity of a 75-decibel dial tone.

25. **Radioactivity** Use the exponential decay equation $A = A_0(0.5)^{t/k}$, where A is the amount of a radioactive material present after time t , k is the half-life of the material, and A_0 is the original amount of radioactive material, to find the half-life of a material that decays from 10 mg to 9 mg in 5 h. Round to the nearest whole number.

CUMULATIVE REVIEW EXERCISES

1. Solve: $4 - 2[x - 3(2 - 3x) - 4x] = 2x$

2. Find the equation of the line that contains the point $(2, -2)$ and is parallel to the line $2x - y = 5$.

3. Factor: $4x^{2n} + 7x^n + 3$

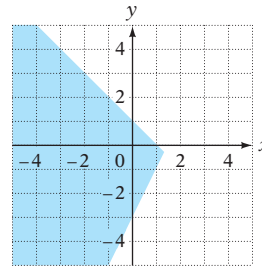
4. Simplify: $\frac{1 - \frac{5}{x} + \frac{6}{x^2}}{1 + \frac{1}{x} - \frac{6}{x^2}}$

5. Simplify: $\frac{\sqrt{xy}}{\sqrt{x} - \sqrt{y}}$

6. Solve by completing the square: $x^2 - 4x - 6 = 0$

7. Write a quadratic equation that has integer coefficients and has solutions $\frac{1}{3}$ and -3 .

8. Graph the solution set: $\begin{cases} 2x - y < 3 \\ x + y < 1 \end{cases}$



9. Solve by the addition method:

$$3x - y + z = 3$$

$$x + y + 4z = 7$$

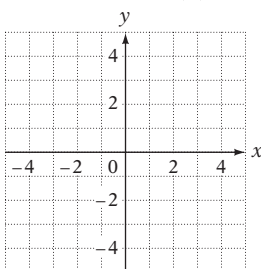
$$3x - 2y + 3z = 8$$

10. Subtract: $\frac{x - 4}{2 - x} - \frac{1 - 6x}{2x^2 - 7x + 6}$

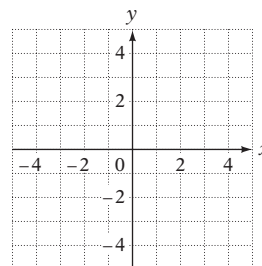
11. Solve: $x^2 + 4x - 5 \leq 0$
Write the solution set in set-builder notation.

12. Solve: $|2x - 5| \leq 3$

13. Graph: $f(x) = \left(\frac{1}{2}\right)^x + 1$



14. Graph: $f(x) = \log_2 x - 1$



15. Evaluate $f(x) = 2^{-x-1}$ at $x = -3$.
16. Solve for x : $\log_5 x = 3$
17. Write $3 \log_b x - 5 \log_b y$ as a single logarithm with a coefficient of 1.
18. Find $\log_3 7$.
19. Solve for x : $4^{5x-2} = 4^{3x+2}$
20. Solve for x : $\log x + \log(2x + 3) = \log 2$
21. **Banking** A bank offers two types of business checking accounts. One account has a charge of \$5 per month plus 2 cents per check. The second account has a charge of \$2 per month plus 8 cents per check. How many checks can a customer who has the second type of account write if it is to cost the customer less than the first type of checking account?
22. **Mixtures** Find the cost per pound of a mixture made from 16 lb of chocolate that costs \$4.00 per pound and 24 lb of chocolate that costs \$2.50 per pound.
23. **Uniform Motion** A plane can fly at a rate of 225 mph in calm air. Traveling with the wind, the plane flew 1000 mi in the same amount of time it took to fly 800 mi against the wind. Find the rate of the wind.
24. **Physics** The distance d that a spring stretches varies directly as the force f used to stretch the spring. If a force of 20 lb stretches a spring 6 in., how far will a force of 34 lb stretch the spring?
25. **Carpentry** A carpenter purchased 80 ft of redwood and 140 ft of fir for a total cost of \$67. A second purchase, at the same prices, included 140 ft of redwood and 100 ft of fir for a total cost of \$81. Find the cost of redwood and of fir.
26. **Investments** The compound interest formula is $A = P(1 + i)^n$, where P is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods. Use the compound interest formula to find how many years it will take for an investment of \$5000 to double in value. The investment earns 7% annual interest and is compounded semiannually.



Conic Sections

OBJECTIVES

SECTION 11.1

- A** To graph a parabola

SECTION 11.2

- A** To find the equation of a circle and then graph the circle
B To write the equation of a circle in standard form

SECTION 11.3

- A** To graph an ellipse with center at the origin
B To graph a hyperbola with center at the origin

SECTION 11.4

- A** To solve a nonlinear system of equations

SECTION 11.5

- A** To graph the solution set of a quadratic inequality in two variables
B To graph the solution set of a nonlinear system of inequalities

ARE YOU READY?

Take the Chapter 11 Prep Test to find out if you are ready to learn to:

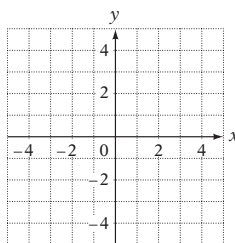
- Graph a parabola, a circle, an ellipse, and a hyperbola
- Solve a nonlinear system of equations
- Graph the solution set of a quadratic inequality in two variables
- Graph the solution set of a nonlinear system of inequalities

PREP TEST

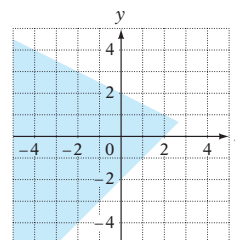
Do these exercises to prepare for Chapter 11.

- Find the distance between the points whose coordinates are $(-2, 3)$ and $(4, -1)$. Round to the nearest hundredth.
- Complete the square on $x^2 - 8x$. Write the resulting perfect-square trinomial as the square of a binomial.
- Solve $\frac{x^2}{16} + \frac{y^2}{9} = 1$ for x when $y = 3$ and when $y = 0$.
- Solve by the substitution method:
 $7x + 4y = 3$
 $y = x - 2$
- Solve by the addition method:
 $4x - y = 9$
 $2x + 3y = -13$
- Find the axis of symmetry and the vertex of the graph of $y = x^2 - 4x + 2$.

7. Graph: $f(x) = -2x^2 + 4x$



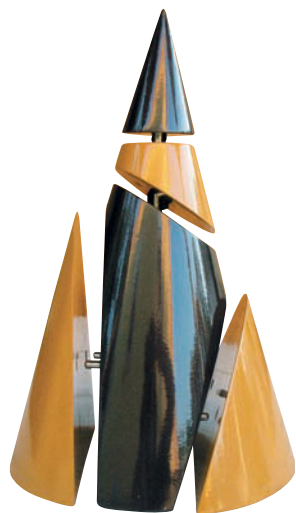
8. Graph the solution set:
 $x + 2y \leq 4$
 $x - y \leq 2$



SECTION

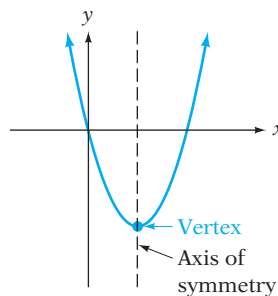
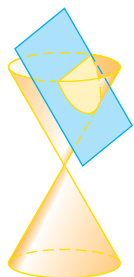
11.1 The Parabola

OBJECTIVE A

To graph a parabola 

Jennifer Waddell/Houghton Mifflin Company

The four conic sections (parabola, circle, ellipse, and hyperbola) are obtained by slicing a cone with planes of various orientations.



The **conic sections** are curves that can be constructed from the intersection of a plane and a right circular cone. The *parabola*, which was introduced earlier, is one of these curves. Here we will review some of that previous discussion and look at equations of parabolas that were not discussed before.

Every parabola has an axis of symmetry and a vertex that is on the axis of symmetry. To understand the axis of symmetry, think of folding the paper along that axis. The two halves of the curve will match up.

The graph of the equation $y = ax^2 + bx + c$, $a \neq 0$, is a parabola with the axis of symmetry parallel to the y -axis. The parabola opens up when $a > 0$ and opens down when $a < 0$. When the parabola opens up, the vertex is the lowest point on the parabola. When the parabola opens down, the vertex is the highest point on the parabola.

The coordinates of the vertex can be found by completing the square.

HOW TO 1

Find the coordinates of the vertex of the parabola whose equation is $y = x^2 - 4x + 5$.

$$y = x^2 - 4x + 5$$

$$y = (x^2 - 4x) + 5$$

$$y = (x^2 - 4x + 4) - 4 + 5$$

- **Group the terms involving x .**
- **Complete the square on $x^2 - 4x$. Note that 4 is added and subtracted. Because $4 - 4 = 0$, the equation is not changed.**
- **Factor the trinomial and combine like terms.**

$$y = (x - 2)^2 + 1$$

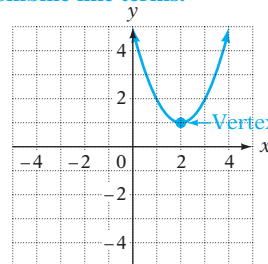
The coefficient of x^2 is positive, so the parabola opens up. The vertex is the lowest point on the parabola, or the point that has the least y -coordinate.

Because $(x - 2)^2 \geq 0$ for all x , the least y -coordinate occurs when $(x - 2)^2 = 0$, which occurs when $x = 2$. This means the x -coordinate of the vertex is 2.

To find the y -coordinate of the vertex, replace x in $y = (x - 2)^2 + 1$ by 2 and solve for y .

$$\begin{aligned} y &= (x - 2)^2 + 1 \\ &= (2 - 2)^2 + 1 = 1 \end{aligned}$$

The vertex is (2, 1).





Point of Interest



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Golden Gate Bridge

The suspension cables for some bridges, such as the Golden Gate bridge, hang in the shape of a parabola. Parabolic shapes are also used for mirrors in telescopes and in certain antenna designs.

By following the procedure of the last example and completing the square on the equation $y = ax^2 + bx + c$, we find that the **x -coordinate of the vertex** is $-\frac{b}{2a}$. The y -coordinate of the vertex can then be determined by substituting this value of x into $y = ax^2 + bx + c$ and solving for y .

Because the axis of symmetry is parallel to the y -axis and passes through the vertex, the equation of the axis of symmetry is $x = -\frac{b}{2a}$.

HOW TO • 2

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = -3x^2 + 6x + 1$. Then sketch its graph.

$$x\text{-coordinate: } -\frac{b}{2a} = -\frac{6}{2(-3)} = 1$$

• Find the x -coordinate of the vertex and the axis of symmetry. $a = -3, b = 6$

The x -coordinate of the vertex is 1.

The equation of the axis of symmetry is $x = 1$.

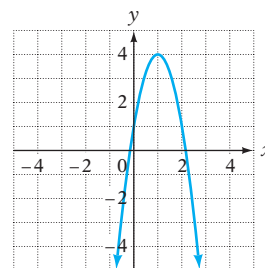
To find the y -coordinate of the vertex, replace x by 1 and solve for y .

$$\begin{aligned} y &= -3x^2 + 6x + 1 \\ &= -3(1)^2 + 6(1) + 1 = 4 \end{aligned}$$

The vertex is (1, 4).

Because a is negative, the parabola opens down.

Find a few ordered pairs and use symmetry to sketch the graph.



Integrating Technology

The instructions in the Keystroke Guide: *Min and Max* can be used to find the vertex of a parabola.

HOW TO • 3

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = x^2 - 2$. Then sketch its graph.

$$x\text{-coordinate: } -\frac{b}{2a} = -\frac{0}{2(1)} = 0$$

• Find the x -coordinate of the vertex and the axis of symmetry. $a = 1, b = 0$

The x -coordinate of the vertex is 0.

The equation of the axis of symmetry is $x = 0$.

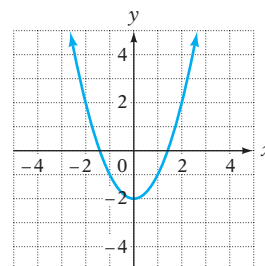
To find the y -coordinate of the vertex, replace x by 0 and solve for y .

$$\begin{aligned} y &= x^2 - 2 \\ &= 0^2 - 2 = -2 \end{aligned}$$

The vertex is (0, -2).

Because a is positive, the parabola opens up.

Find a few ordered pairs and use symmetry to sketch the graph.



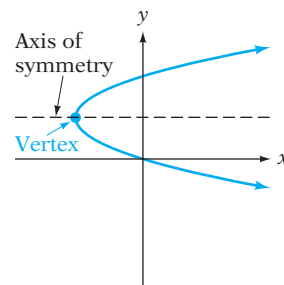


Tips for Success

Express in your own words the difference between the equation of a parabola that opens up or down and the equation of a parabola that opens right or left. Doing so will help you to remember the difference.

The graph of an equation of the form $x = ay^2 + by + c$, $a \neq 0$, is also a parabola. In this case, the parabola opens to the right when a is positive and opens to the left when a is negative.

For a parabola of this form, the **y-coordinate of the vertex** is $-\frac{b}{2a}$. The axis of symmetry is the line $y = -\frac{b}{2a}$.



Using the vertical-line test, the graph of a parabola of this form is not the graph of a function. The graph of $x = ay^2 + by + c$ is a relation.

HOW TO 4

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $x = 2y^2 - 8y + 5$. Then sketch its graph.

$$\text{y-coordinate: } -\frac{b}{2a} = -\frac{-8}{2(2)} = 2$$

• Find the y-coordinate of the vertex and the axis of symmetry. $a = 2$, $b = -8$

The y-coordinate of the vertex is 2.

The equation of the axis of symmetry is $y = 2$.

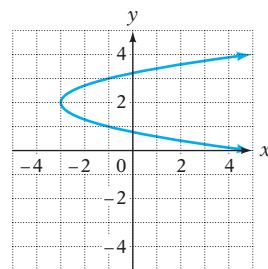
To find the x-coordinate of the vertex, replace y by 2 and solve for x .

$$\begin{aligned} x &= 2y^2 - 8y + 5 \\ &= 2(2)^2 - 8(2) + 5 = -3 \end{aligned}$$

The vertex is $(-3, 2)$.

Since a is positive, the parabola opens to the right.

Find a few ordered pairs and use symmetry to sketch the graph.



Point of Interest



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Hypatia

Hypatia (c. 340–415) is considered the first prominent woman mathematician. She lectured in mathematics and philosophy at the Museum in Alexandria, at that time the most distinguished place of learning in the world. One of the topics on which Hypatia lectured was conic sections. One historian has claimed that with the death (actually the murder) of Hypatia, “the long and glorious history of Greek mathematics was at an end.”

HOW TO 5

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $x = -2y^2 - 4y - 3$. Then sketch its graph.

$$\text{y-coordinate: } -\frac{b}{2a} = -\frac{-4}{2(-2)} = -1$$

• Find the y-coordinate of the vertex and the axis of symmetry. $a = -2$, $b = -4$

The y-coordinate of the vertex is -1 .

The equation of the axis of symmetry is $y = -1$.

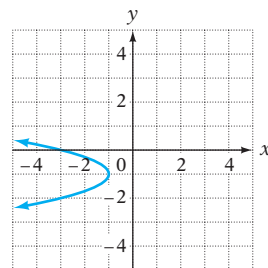
To find the x-coordinate of the vertex, replace y by -1 and solve for x .

$$\begin{aligned} x &= -2y^2 - 4y - 3 \\ &= -2(-1)^2 - 4(-1) - 3 = -1 \end{aligned}$$

The vertex is $(-1, -1)$.

Because a is negative, the parabola opens to the left.

Find a few ordered pairs and use symmetry to sketch the graph.



EXAMPLE • 1

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = x^2 - 4x + 3$. Then sketch its graph.

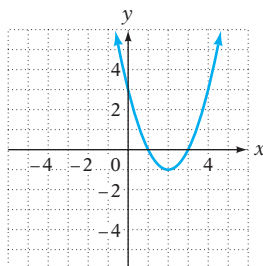
Solution

$$-\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

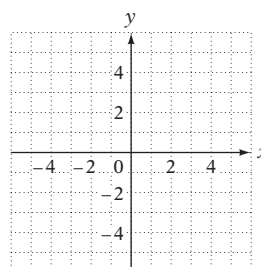
Axis of symmetry:
 $x = 2$

$$\begin{aligned} y &= x^2 - 4x + 3 \\ y &= 2^2 - 4(2) + 3 \\ &= -1 \end{aligned}$$

Vertex: $(2, -1)$

**YOU TRY IT • 1**

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = x^2 + 2x + 1$. Then sketch its graph.

Your solution**EXAMPLE • 2**

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $x = 2y^2 - 4y + 1$. Then sketch its graph.

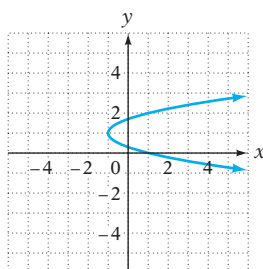
Solution

$$-\frac{b}{2a} = -\frac{-4}{2(2)} = 1$$

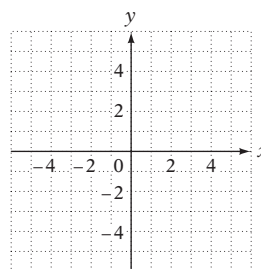
Axis of symmetry:
 $y = 1$

$$\begin{aligned} x &= 2y^2 - 4y + 1 \\ x &= 2(1)^2 - 4(1) + 1 \\ &= -1 \end{aligned}$$

Vertex: $(-1, 1)$

**YOU TRY IT • 2**

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $x = -y^2 - 2y + 2$. Then sketch its graph.

Your solution**EXAMPLE • 3**

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = x^2 + 1$. Then sketch its graph.

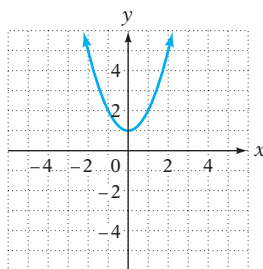
Solution

$$-\frac{b}{2a} = -\frac{0}{2(1)} = 0$$

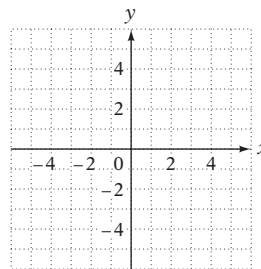
Axis of symmetry:
 $x = 0$

$$\begin{aligned} y &= x^2 + 1 \\ y &= 0^2 + 1 \\ &= 1 \end{aligned}$$

Vertex: $(0, 1)$

**YOU TRY IT • 3**

Find the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = x^2 - 2x - 1$. Then sketch its graph.

Your solution

11.1 EXERCISES

OBJECTIVE A

To graph a parabola

 For Exercises 1 to 6, **a.** state whether the axis of symmetry is a vertical or a horizontal line, and **b.** state the direction in which the parabola opens.

1. $y = 3x^2 - 4x + 7$

2. $y = -x^2 + 5x - 2$

3. $x = y^2 + 2y - 8$

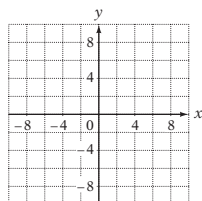
4. $x = -3y^2 - y + 9$

5. $x = -\frac{1}{2}y^2 - 4y - 7$

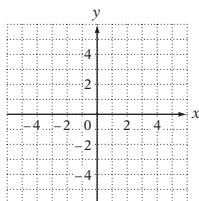
6. $y = \frac{1}{4}x^2 + 6x - 1$

For Exercises 7 to 24, find the equation of the axis of symmetry and the coordinates of the vertex of the parabola given by the equation. Then sketch its graph.

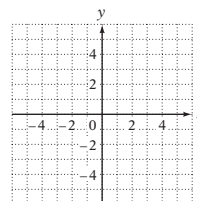
7. $x = y^2 - 3y - 4$



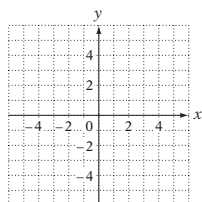
8. $y = x^2 - 2$



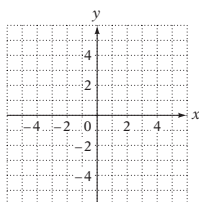
9. $y = x^2 + 2$



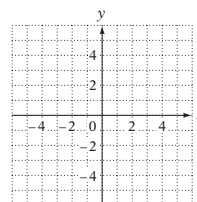
10. $x = -\frac{1}{2}y^2 + 4$



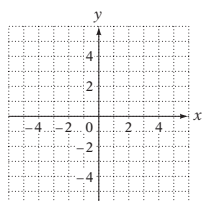
11. $x = -\frac{1}{4}y^2 - 1$



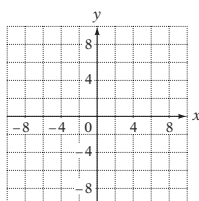
12. $x = \frac{1}{2}y^2 - y + 1$



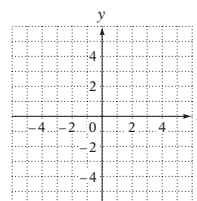
13. $x = -\frac{1}{2}y^2 + 2y - 3$



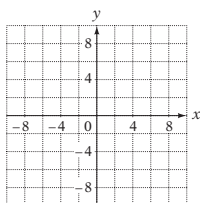
14. $y = -\frac{1}{2}x^2 + 2x + 6$



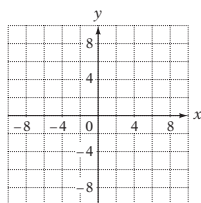
15. $y = \frac{1}{2}x^2 + x - 3$



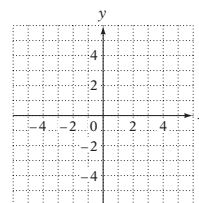
16. $x = y^2 + 6y + 5$



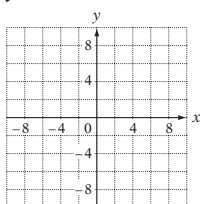
17. $x = y^2 - y - 6$



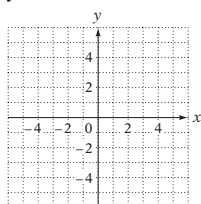
18. $x = -y^2 + 2y - 3$



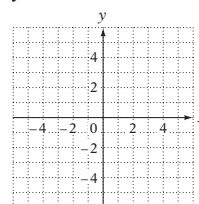
19. $y = 2x^2 + 4x - 5$



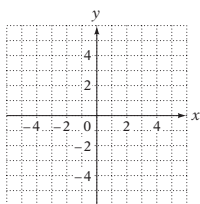
20. $y = 2x^2 - x - 5$



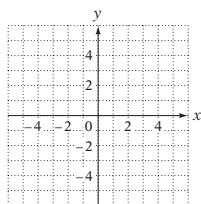
21. $y = 2x^2 - x - 3$



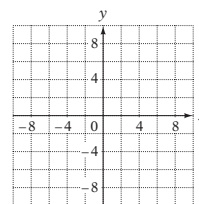
22. $y = x^2 - 5x + 4$



23. $y = x^2 + 5x + 6$



24. $x = y^2 - 2y - 5$

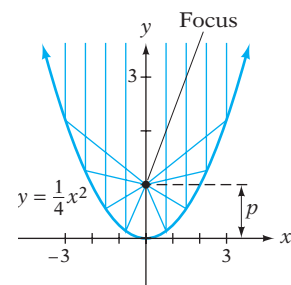


Applying the Concepts

Science Parabolas have a unique property that is important in the design of telescopes and antennas. If a parabola has a mirrored surface, then all light rays parallel to the axis of symmetry of the parabola are reflected to a single point called the **focus** of the parabola. The location of this point is p units from the vertex on the axis of symmetry. The value of p is given by $p = \frac{1}{4a}$, where $y = ax^2$ is the equation of a parabola with vertex at the origin. For the graph of $y = \frac{1}{4}x^2$ shown at the right, the coordinates of the focus are $(0, 1)$. For Exercises 25 and 26, find the coordinates of the focus of the parabola given by the equation.

25. $y = 2x^2$

26. $y = \frac{1}{10}x^2$



Parallel rays of light are reflected to the focus

$$a = \frac{1}{4}$$

$$p = \frac{1}{4a} = \frac{1}{4(1/4)} = 1$$

SECTION

11.2 The Circle

OBJECTIVE A

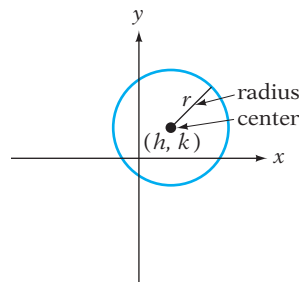
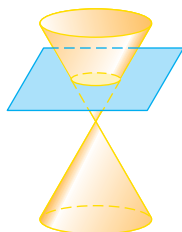
To find the equation of a circle and then graph the circle



A **circle** is a conic section formed by the intersection of a cone and a plane parallel to the base of the cone.

**Take Note**

As the angle of the plane that intersects the cone changes, different conic sections are formed. For a parabola, the plane was *parallel to the side of the cone*. For a circle, the plane is *parallel to the base of the cone*.



A circle can be defined as all points (x, y) in the plane that are a fixed distance from a given point (h, k) , called the **center**. The fixed distance is the **radius** of the circle.

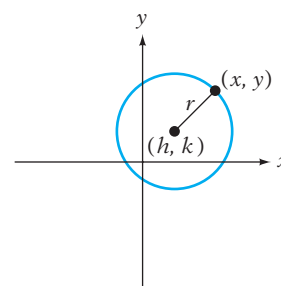
Using the distance formula, the equation of a circle can be determined.

Let (h, k) be the coordinates of the center of the circle, let r be the radius, and let (x, y) be any point on the circle. Then, by the distance formula,

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

Squaring each side of the equation gives the equation of a circle.

$$\begin{aligned} r^2 &= [\sqrt{(x - h)^2 + (y - k)^2}]^2 \\ r^2 &= (x - h)^2 + (y - k)^2 \end{aligned}$$

**The Standard Form of the Equation of a Circle**

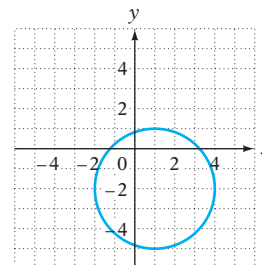
Let r be the radius of a circle and let (h, k) be the coordinates of the center of the circle. Then the equation of the circle is given by

$$(x - h)^2 + (y - k)^2 = r^2$$

HOW TO 1

Find the radius and the coordinates of the center of the circle whose equation is $(x - 1)^2 + (y + 2)^2 = 9$. Then sketch a graph of the circle.

To find the radius and the coordinates of the center, write the equation in standard form. The standard form is $(x - 1)^2 + [y - (-2)]^2 = 3^2$. The center is $(1, -2)$, and the radius is 3. The graph is shown at the right.

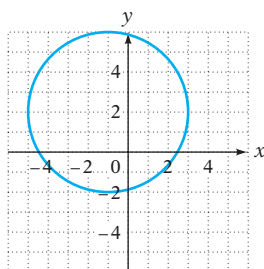


HOW TO • 2Find the equation of the circle with radius 4 and center $(-1, 2)$.

Then sketch its graph.

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2 \\ [x - (-1)]^2 + (y - 2)^2 &= 4^2 \\ (x + 1)^2 + (y - 2)^2 &= 16\end{aligned}$$

- Use the standard form of the equation of a circle.
- Replace h by -1 , k by 2 , and r by 4 .
- Sketch the graph by drawing a circle with center $(-1, 2)$ and radius 4.

**Take Note**

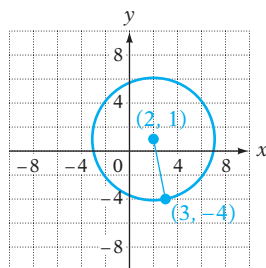
Applying the vertical-line test reveals that the graph of a circle is not the graph of a function. The graph of a circle is the graph of a relation.

HOW TO • 3Find the equation of the circle whose center is $(2, 1)$ and that passes through the point whose coordinates are $(3, -4)$.

The distance between the given point on the circle and the center is the radius. See the figure at the left. Use the distance formula, where (x_1, y_1) is the given point on the circle $(3, -4)$ and (x_2, y_2) is the center $(2, 1)$, to find the radius.

$$\begin{aligned}r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(3 - 2)^2 + (-4 - 1)^2} \\ &= \sqrt{1^2 + (-5)^2} \\ &= \sqrt{1 + 25} \\ &= \sqrt{26}\end{aligned}$$

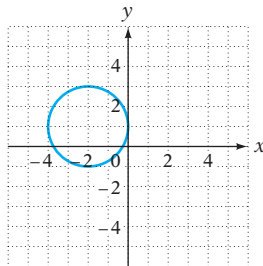
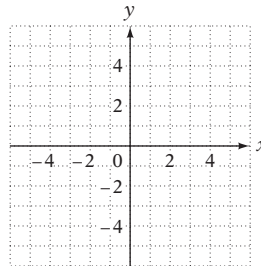
- Use the distance formula.
- $(x_1, y_1) = (3, -4)$; $(x_2, y_2) = (2, 1)$



$$(x - 2)^2 + (y - 1)^2 = 26$$

Use the equation of a circle in standard form with $(h, k) = (2, 1)$ and $r^2 = (\sqrt{26})^2 = 26$.

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2 \\ (x - 2)^2 + (y - 1)^2 &= 26\end{aligned}$$

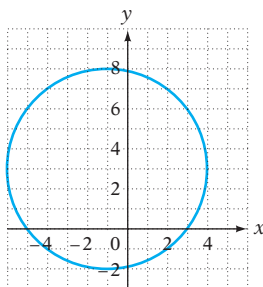
EXAMPLE • 1Sketch a graph of $(x + 2)^2 + (y - 1)^2 = 4$.**Solution**Center: $(-2, 1)$ Radius: $\sqrt{4} = 2$ **YOU TRY IT • 1**Sketch a graph of $(x - 2)^2 + (y + 3)^2 = 9$.**Your solution**

EXAMPLE • 2

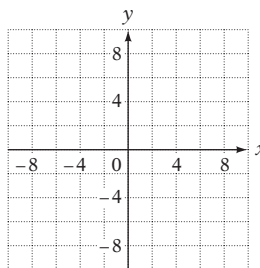
Find the equation of the circle with radius 5 and center $(-1, 3)$. Then sketch its graph.

Solution

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2 \\ [x - (-1)]^2 + (y - 3)^2 &= 5^2 \quad \bullet \quad h = -1, k = 3, r = 5 \\ (x + 1)^2 + (y - 3)^2 &= 25\end{aligned}$$

**YOU TRY IT • 2**

Find the equation of the circle with radius 4 and center $(2, -3)$. Then sketch its graph.

Your solution

Solution on p. S31

OBJECTIVE B**To write the equation of a circle in standard form****Tips for Success**

See Objective 8.2A to review the process of completing the square.

The equation of a circle can also be expressed as $x^2 + y^2 + ax + by + c = 0$. This is called the **general form of the equation of a circle**. An equation in general form can be written in standard form by completing the square.

HOW TO • 4

Find the center and the radius of the circle whose equation is given by $x^2 + y^2 + 4x + 2y + 1 = 0$. Then sketch its graph.

Write the equation in standard form by completing the square on x and y .

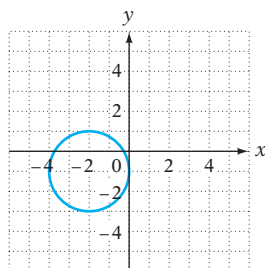
$$\begin{aligned}x^2 + y^2 + 4x + 2y + 1 &= 0 \\ x^2 + y^2 + 4x + 2y &= -1\end{aligned}$$

$$(x^2 + 4x) + (y^2 + 2y) = -1$$

$$(x^2 + 4x + 4) + (y^2 + 2y + 1) = -1 + 4 + 1$$

$$(x + 2)^2 + (y + 1)^2 = 4$$

The center is $(-2, -1)$; the radius is 2.



- Subtract the constant term from each side of the equation.
- Rewrite the equation by grouping x terms and y terms.
- Complete the square on $x^2 + 4x$ and $y^2 + 2y$. Add 4 and 1 to both sides.
- Factor the trinomials.

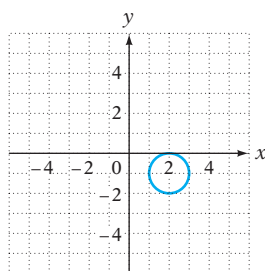
EXAMPLE • 3

Write the equation of the circle
 $x^2 + y^2 - 4x + 2y + 4 = 0$ in standard form.
 Then sketch its graph.

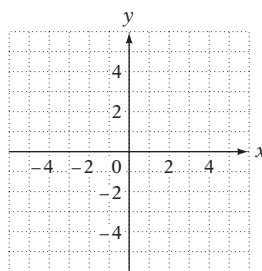
Solution

$$\begin{aligned} x^2 + y^2 - 4x + 2y + 4 &= 0 \\ (x^2 - 4x) + (y^2 + 2y) &= -4 \\ (x^2 - 4x + 4) + (y^2 + 2y + 1) &= -4 + 4 + 1 \\ (x - 2)^2 + (y + 1)^2 &= 1 \end{aligned}$$

Center: $(2, -1)$; Radius: 1

**YOU TRY IT • 3**

Write the equation of the circle
 $x^2 + y^2 - 2x - 15 = 0$ in standard form.
 Then sketch its graph.

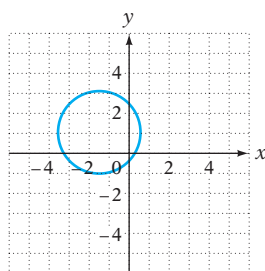
Your solution**EXAMPLE • 4**

Write the equation of the circle
 $x^2 + y^2 + 3x - 2y - 1 = 0$ in standard form.
 Then sketch its graph.

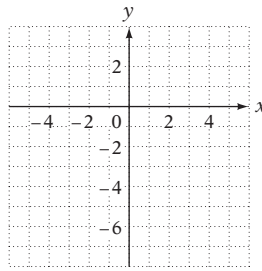
Solution

$$\begin{aligned} x^2 + y^2 + 3x - 2y - 1 &= 0 \\ x^2 + y^2 + 3x - 2y &= 1 \\ (x^2 + 3x) + (y^2 - 2y) &= 1 \\ \left(x^2 + 3x + \frac{9}{4}\right) + (y^2 - 2y + 1) &= 1 + \frac{9}{4} + 1 \\ \left(x + \frac{3}{2}\right)^2 + (y - 1)^2 &= \frac{17}{4} \end{aligned}$$

Center: $\left(-\frac{3}{2}, 1\right)$; Radius: $\sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2} \approx 2$

**YOU TRY IT • 4**

Write the equation of the circle
 $x^2 + y^2 - 4x + 8y + 15 = 0$ in standard form.
 Then sketch its graph.

Your solution

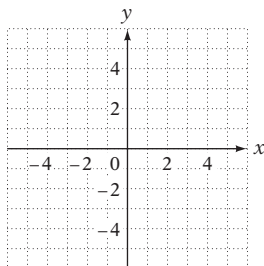
11.2 EXERCISES

OBJECTIVE A

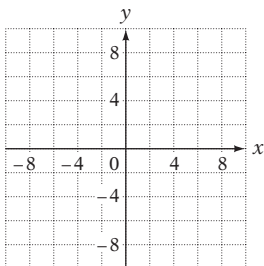
To find the equation of a circle
and then graph the circle

For Exercises 1 to 6, sketch a graph of the circle given by the equation.

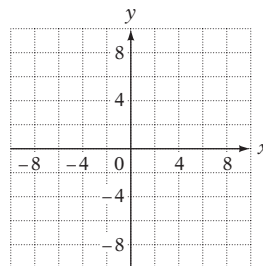
1. $(x - 2)^2 + (y + 2)^2 = 9$



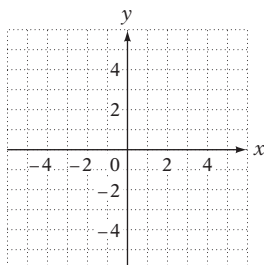
2. $(x + 2)^2 + (y - 3)^2 = 16$



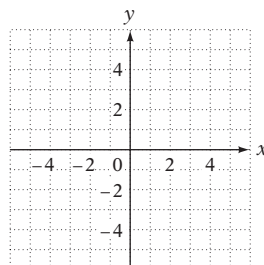
3. $(x + 3)^2 + (y - 1)^2 = 25$



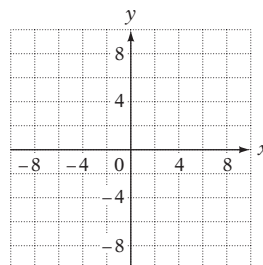
4. $(x - 2)^2 + (y + 3)^2 = 4$



5. $(x + 2)^2 + (y + 2)^2 = 4$



6. $(x - 1)^2 + (y - 2)^2 = 25$



7. Find the equation of the circle with radius 2 and center
- $(2, -1)$
- .

8. Find the equation of the circle with radius 3 and center
- $(-1, -2)$
- .

9. Find the equation of the circle with radius
- $\sqrt{7}$
- and center
- $(0, -4)$
- .

10. Find the equation of the circle with radius
- $\sqrt{13}$
- and center
- $(5, -2)$
- .

11. Find the equation of the circle with radius 9 and center
- $(-3, 0)$
- .

12. Find the equation of the circle with radius 4 and center
- $(0, -4)$
- .

13. Find the equation of the circle whose center is
- $(-1, 1)$
- and that passes through the point whose coordinates are
- $(1, 2)$
- .

14. Find the equation of the circle whose center is
- $(-2, 1)$
- and that passes through the point whose coordinates are
- $(-1, 3)$
- .

15. Find the equation of the circle whose center is
- $(-1, 0)$
- and that passes through the point whose coordinates are
- $(-3, -4)$
- .

16. Find the equation of the circle whose center is
- $(5, -4)$
- and that passes through the point whose coordinates are
- $(2, 0)$
- .

17. Find the equation of the circle whose center is
- $(0, 3)$
- and that passes through the point whose coordinates are
- $(1, -5)$
- .

18. Find the equation of the circle whose center is
- $(0, 0)$
- and that passes through the point whose coordinates are
- $(4, -2)$
- .



19. What is the equation of a circle whose center is the origin and whose radius is
- r
- ?

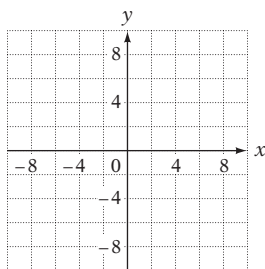


20. Describe the graph of
- $(x - 3)^2 + (y + 2)^2 = 0$
- .

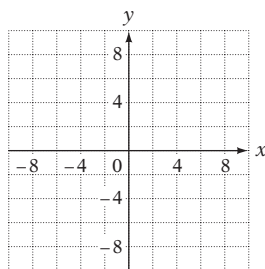
OBJECTIVE B**To write the equation of a circle in standard form**

For Exercises 21 to 23, write the equation of the circle in standard form. Then sketch its graph.

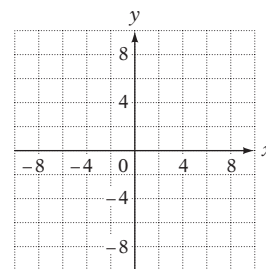
21. $x^2 + y^2 - 2x + 4y - 20 = 0$



22. $x^2 + y^2 - 4x + 8y + 4 = 0$



23. $x^2 + y^2 + 6x + 8y + 9 = 0$



For Exercises 24 to 33, write the equation of the circle in standard form.

24. $x^2 + y^2 + 6x - 4y - 12 = 0$

25. $x^2 + y^2 - 10x - 8y - 9 = 0$

26. $x^2 + y^2 + 6x - 16 = 0$

27. $x^2 + y^2 - 12x + 12 = 0$

28. $x^2 + y^2 - 12y - 13 = 0$

29. $x^2 + y^2 + 14y + 13 = 0$

30. $x^2 + y^2 + 5x - 4y - 10 = 0$

31. $x^2 + y^2 - 4x - 7y - 5 = 0$

32. $x^2 + y^2 + 3x - y - 1 = 0$

33. $x^2 + y^2 + 5x + 5y - 8 = 0$

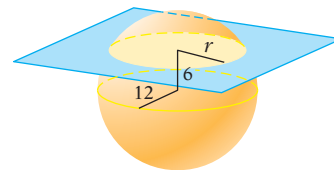


34. Is $x^2 + y^2 + 4x + 8y + 24 = 0$ the equation of a circle? Why or why not?

Applying the Concepts

35. Find the equation of the circle that has a diameter with endpoints $(-2, 4)$ and $(2, -2)$.

36. **Geometry** The radius of a sphere is 12 in. What is the radius of the circle that is formed by the intersection of the sphere and a plane that is 6 in. from the center of the sphere?



37. Find the equation of the circle with center at $(3, 3)$ if the circle is tangent to the x -axis.

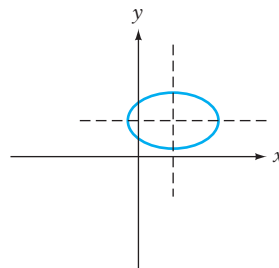
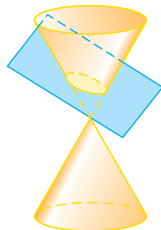
SECTION

11.3 The Ellipse and the Hyperbola

OBJECTIVE A

To graph an ellipse with center at the origin 

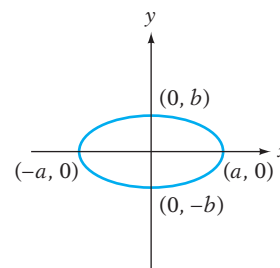
The orbits of the planets around the sun are “oval” shaped. This oval shape can be described as an **ellipse**, which is another of the conic sections.



There are two axes of symmetry for an ellipse. The intersection of these two axes is the **center** of the ellipse.

An ellipse with center at the origin is shown at the right. Note that there are two x -intercepts and two y -intercepts.

Using the vertical-line test, we find that the graph of an ellipse is not the graph of a function. The graph of an ellipse is the graph of a relation.


 Point of Interest

The word *ellipse* comes from the Greek word *ellipsis*, which means “deficient.” The method by which the early Greeks analyzed the conics caused a certain area in the construction of the ellipse to be less than another area (deficient). The word *ellipsis* in English, meaning “omission,” has the same Greek root as the word *ellipse*.

The Standard Form of the Equation of an Ellipse with Center at the Origin

The equation of an ellipse with center at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The x -intercepts are $(a, 0)$ and $(-a, 0)$. The y -intercepts are $(0, b)$ and $(0, -b)$.

By finding the x - and y -intercepts of an ellipse and using the fact that the ellipse is oval-shaped, we can sketch a graph of the ellipse.

HOW TO 1

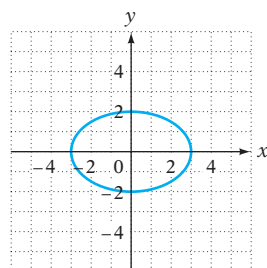
Find the coordinates of the intercepts of the ellipse whose equation is $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Then sketch its graph.

The x -intercepts are $(3, 0)$ and $(-3, 0)$.

The y -intercepts are $(0, 2)$ and $(0, -2)$.

• $a^2 = 9, b^2 = 4$

• Use the intercepts and symmetry to sketch the graph of the ellipse.


 Integrating Technology

See the Projects and Group Activities at the end of this chapter for instructions on graphing conic sections using a graphing calculator.

**Point of Interest**

For a circle, $a = b$ and thus $\frac{a}{b} = 1$. Early Greek astronomers thought that each planet had a circular orbit. Today we know that the planets have elliptical orbits. However, in most cases, the ellipse is very nearly a circle. For Earth, $\frac{a}{b} \approx 1.00014$.

HOW TO • 2

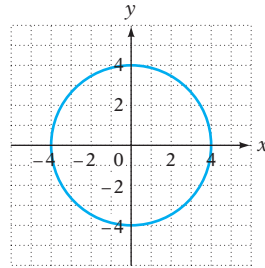
Find the coordinates of the intercepts of the ellipse whose equation is $\frac{x^2}{16} + \frac{y^2}{16} = 1$. Then sketch its graph.

The x -intercepts are $(4, 0)$ and $(-4, 0)$.

The y -intercepts are $(0, 4)$ and $(0, -4)$.

• $a^2 = 16, b^2 = 16$

• Use the intercepts and symmetry to sketch the graph of the ellipse.



The graph in this last example is the graph of a circle. A circle is a special case of an ellipse. It occurs when $a^2 = b^2$ in the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

EXAMPLE • 1

Find the coordinates of the intercepts of the ellipse whose equation is $\frac{x^2}{9} + \frac{y^2}{16} = 1$. Then sketch its graph.

Solution

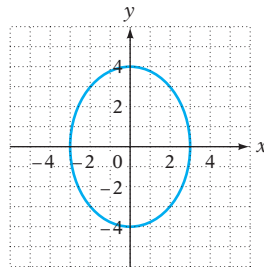
$a^2 = 9, b^2 = 16$

x -intercepts:

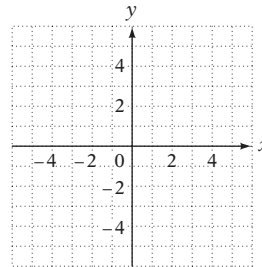
$(3, 0)$ and $(-3, 0)$

y -intercepts:

$(0, 4)$ and $(0, -4)$

**YOU TRY IT • 1**

Find the coordinates of the intercepts of the ellipse whose equation is $\frac{x^2}{4} + \frac{y^2}{25} = 1$. Then sketch its graph.

Your solution**EXAMPLE • 2**

Find the coordinates of the intercepts of the ellipse whose equation is $\frac{x^2}{16} + \frac{y^2}{12} = 1$. Then sketch its graph.

Solution

$a^2 = 16, b^2 = 12$

x -intercepts:

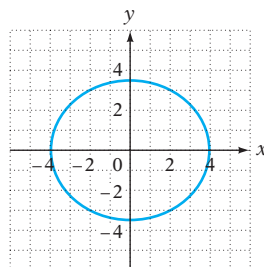
$(4, 0)$ and $(-4, 0)$

y -intercepts:

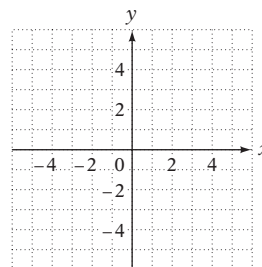
$(0, 2\sqrt{3})$

and $(0, -2\sqrt{3})$

$(2\sqrt{3} \approx 3.5)$

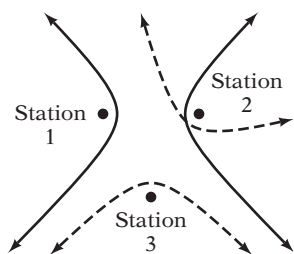
**YOU TRY IT • 2**

Find the coordinates of the intercepts of the ellipse whose equation is $\frac{x^2}{18} + \frac{y^2}{9} = 1$. Then sketch its graph.

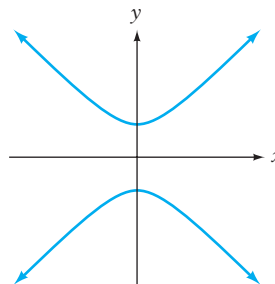
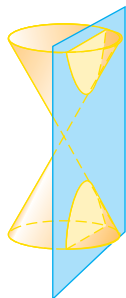
Your solution

OBJECTIVE B**To graph a hyperbola with center at the origin****Point of Interest**

Hyperbolas are used in LORAN (Long Range Navigation) as a method by which a ship's navigator can determine the position of the ship. A minimal system includes three stations that transmit pulses at precisely timed intervals. A user gets location information by measuring the very small difference in arrival times of the pulses. You can find more information on LORAN on the Internet.



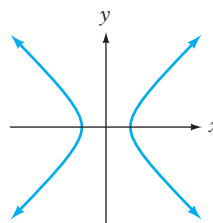
A **hyperbola** is a conic section that can be formed by the intersection of a cone and a plane perpendicular to the base of the cone.



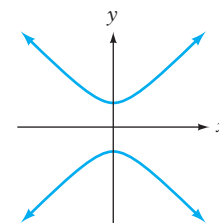
The hyperbola has two **vertices** and an **axis of symmetry** that passes through the vertices. The **center** of a hyperbola is the point halfway between the vertices.

The graphs at the right show two possible graphs of a hyperbola with center at the origin.

In the first graph, an axis of symmetry is the x -axis, and the vertices are x -intercepts.

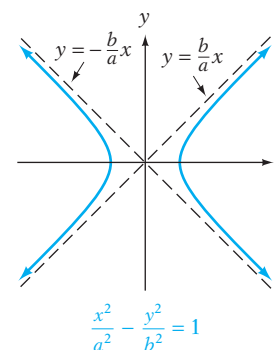


In the second graph, an axis of symmetry is the y -axis, and the vertices are y -intercepts.



Note that in either case, the graph of a hyperbola is not the graph of a function. The graph of a hyperbola is the graph of a relation.

To sketch a hyperbola, it is helpful to draw two lines that are “approached” by the hyperbola. These two lines are called **asymptotes**. As a point on the hyperbola gets farther from the origin, the hyperbola “gets closer to” the asymptotes.



Because the asymptotes are straight lines, their equations are linear equations. The equations of the asymptotes for a hyperbola with center at the origin are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

**Point of Interest**

The word *hyperbola* comes from the Greek word *yperboli*, which means “exceeding.” The method by which the early Greeks analyzed the conics caused a certain area in the construction of the hyperbola to be greater than (to exceed) another area. The word *hyperbole* in English, meaning “exaggeration,” has the same Greek root as the word *hyperbola*.

The word *asymptote* comes from the Greek word *asymptotos*, which means “not capable of meeting.”

The Standard Form of the Equation of a Hyperbola with Center at the Origin

The equation of a hyperbola for which an axis of symmetry is the x -axis is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

The coordinates of the vertices are $(a, 0)$ and $(-a, 0)$.

The equation of a hyperbola for which an axis of symmetry is the y -axis is $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

The coordinates of the vertices are $(0, b)$ and $(0, -b)$.

For each equation, the equations of the asymptotes are $y = -\frac{b}{a}x$ and $y = \frac{b}{a}x$.

**Tips for Success**

You now have completed the lessons on the four conic sections. You need to be able to recognize the equation of each. To test yourself, try the Chapter 11 Review Exercises.

HOW TO • 3

Find the coordinates of the vertices and the equations of the

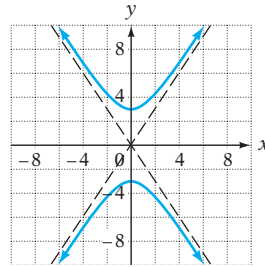
asymptotes of the hyperbola whose equation is $\frac{y^2}{9} - \frac{x^2}{4} = 1$. Then sketch its graph.

An axis of symmetry is the y-axis.

The coordinates of the vertices are (0, 3) and (0, -3).

• $b^2 = 9, a^2 = 4$

The equations of the asymptotes are $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$.



- **Locate the vertices. Sketch the asymptotes. Use symmetry and the fact that the hyperbola will approach the asymptotes to sketch the graph.**

EXAMPLE • 3

Find the coordinates of the vertices and the equations of the asymptotes of the hyperbola whose equation is $\frac{x^2}{16} - \frac{y^2}{4} = 1$. Then sketch its graph.

Solution

$a^2 = 16, b^2 = 4$

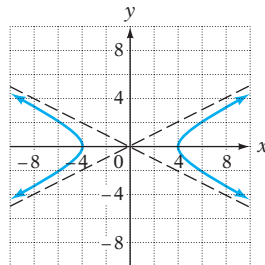
Axis of symmetry:
x-axis

Vertices:

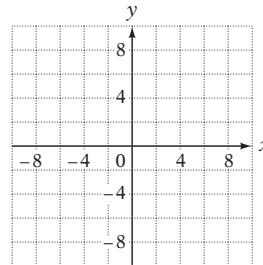
(4, 0) and (-4, 0)

Asymptotes:

$y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$

**YOU TRY IT • 3**

Find the coordinates of the vertices and the equations of the asymptotes of the hyperbola whose equation is $\frac{x^2}{9} - \frac{y^2}{25} = 1$. Then sketch its graph.

Your solution**EXAMPLE • 4**

Find the coordinates of the vertices and the equations of the asymptotes of the hyperbola whose equation is $\frac{y^2}{16} - \frac{x^2}{25} = 1$. Then sketch its graph.

Solution

$a^2 = 25, b^2 = 16$

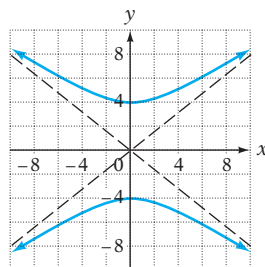
Axis of symmetry:
y-axis

Vertices:

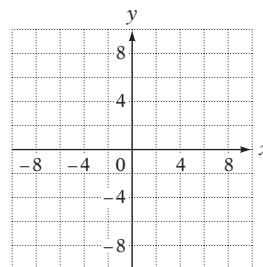
(0, 4) and (0, -4)

Asymptotes:

$y = \frac{4}{5}x$ and $y = -\frac{4}{5}x$

**YOU TRY IT • 4**

Find the coordinates of the vertices and the equations of the asymptotes of the hyperbola whose equation is $\frac{y^2}{9} - \frac{x^2}{9} = 1$. Then sketch its graph.

Your solution

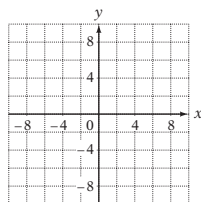
11.3 EXERCISES

OBJECTIVE A

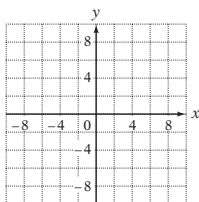
To graph an ellipse with center at the origin

For Exercises 1 to 9, find the coordinates of the intercepts of the ellipse given by the equation. Then sketch its graph.

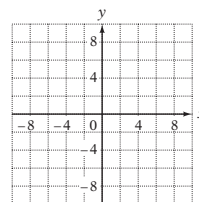
1. $\frac{x^2}{4} + \frac{y^2}{9} = 1$



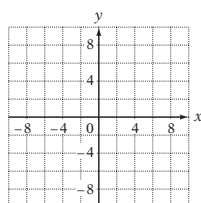
2. $\frac{x^2}{25} + \frac{y^2}{16} = 1$



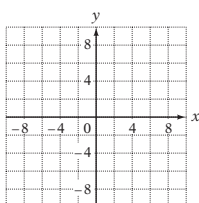
3. $\frac{x^2}{25} + \frac{y^2}{9} = 1$



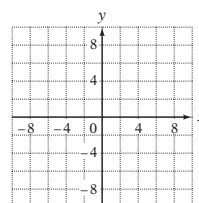
4. $\frac{x^2}{16} + \frac{y^2}{9} = 1$



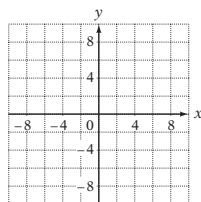
5. $\frac{x^2}{36} + \frac{y^2}{16} = 1$



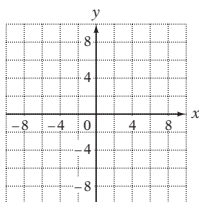
6. $\frac{x^2}{49} + \frac{y^2}{64} = 1$



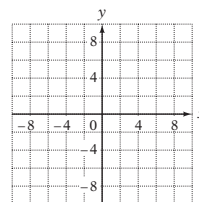
7. $\frac{x^2}{16} + \frac{y^2}{49} = 1$



8. $\frac{x^2}{25} + \frac{y^2}{36} = 1$



9. $\frac{x^2}{4} + \frac{y^2}{25} = 1$



10. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. If b remains fixed and a increases, does the graph of the ellipse get flatter or rounder?



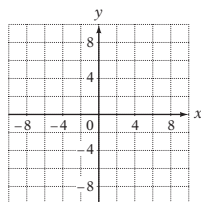
11. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. If a remains fixed and the values of b get closer to a , does the graph of the ellipse get flatter or rounder?

OBJECTIVE B

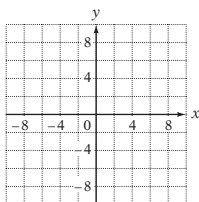
To graph a hyperbola with center at the origin

For Exercises 12 to 23, find the coordinates of the vertices and the equations of the asymptotes of the hyperbola given by the equation. Then sketch its graph.

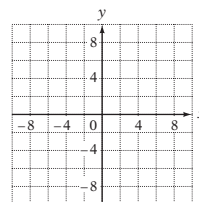
12. $\frac{x^2}{9} - \frac{y^2}{16} = 1$



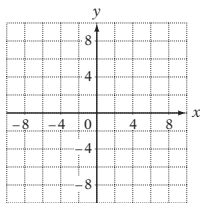
13. $\frac{x^2}{25} - \frac{y^2}{4} = 1$



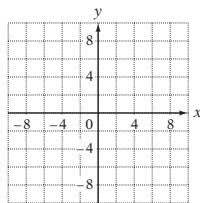
14. $\frac{y^2}{16} - \frac{x^2}{9} = 1$



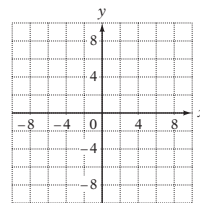
15. $\frac{y^2}{16} - \frac{x^2}{25} = 1$



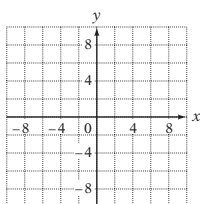
16. $\frac{x^2}{16} - \frac{y^2}{4} = 1$



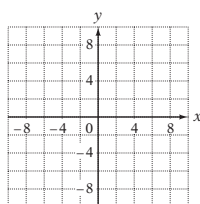
17. $\frac{x^2}{9} - \frac{y^2}{49} = 1$



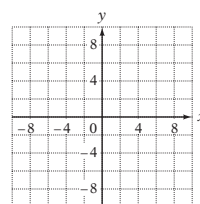
18. $\frac{y^2}{25} - \frac{x^2}{9} = 1$



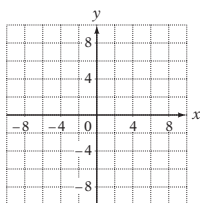
19. $\frac{y^2}{4} - \frac{x^2}{16} = 1$



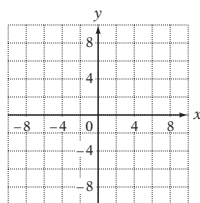
20. $\frac{x^2}{4} - \frac{y^2}{25} = 1$



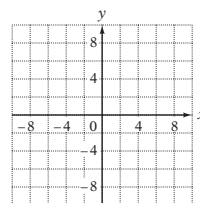
21. $\frac{x^2}{36} - \frac{y^2}{9} = 1$



22. $\frac{y^2}{9} - \frac{x^2}{36} = 1$



23. $\frac{y^2}{25} - \frac{x^2}{4} = 1$

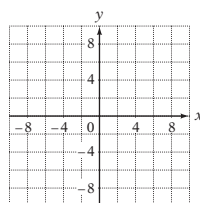


24. Are the vertices of $\frac{y^2}{4} - \frac{x^2}{9} = -1$ on the x -axis or the y -axis?

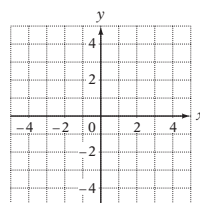
Applying the Concepts

For Exercises 25 to 30, write the equation in standard form and identify the graph. Then graph the equation.

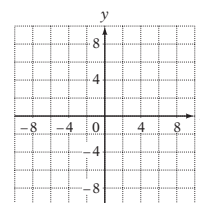
25. $16x^2 + 25y^2 = 400$



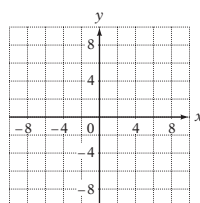
26. $4x^2 + 9y^2 = 36$



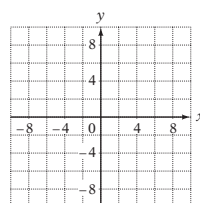
27. $25y^2 - 4x^2 = -100$



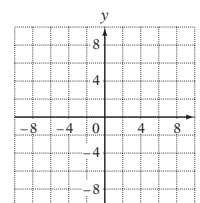
28. $9x^2 - 25y^2 = 225$



29. $4y^2 - x^2 = 36$



30. $9y^2 - 16x^2 = 144$



SECTION

11.4

Solving Nonlinear Systems of Equations

OBJECTIVE A

To solve a nonlinear system of equations



A **nonlinear system of equations** is a system in which one or more of the equations are not linear equations. A nonlinear system of equations can be solved by using either the substitution method or the addition method.

HOW TO 1

$$\begin{aligned}\text{Solve: } 2x - y &= 4 & (1) \\ y^2 &= 4x & (2)\end{aligned}$$

When a nonlinear system of equations contains a linear equation, the substitution method is used.

Solve Equation (1) for y .

$$\begin{aligned}2x - y &= 4 \\ -y &= -2x + 4 \\ y &= 2x - 4\end{aligned}$$

Substitute $2x - 4$ for y in equation (2).

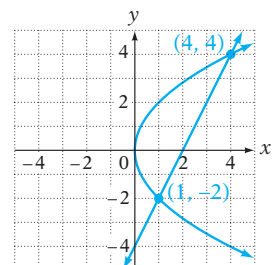
$$\begin{aligned}y^2 &= 4x \\ (2x - 4)^2 &= 4x & \bullet y = 2x - 4 \\ 4x^2 - 16x + 16 &= 4x & \bullet \text{Solve for } x. \\ 4x^2 - 20x + 16 &= 0 \\ 4(x^2 - 5x + 4) &= 0 \\ 4(x - 4)(x - 1) &= 0 \\ x - 4 = 0 & \quad x - 1 = 0 \\ x = 4 & \quad x = 1\end{aligned}$$

Substitute the values of x into the equation $y = 2x - 4$ and solve for y .

$$\begin{aligned}y &= 2x - 4 & y &= 2x - 4 \\ y &= 2(4) - 4 & y &= 2(1) - 4 & \bullet x = 4 & \bullet x = 1 \\ y &= 4 & y &= -2\end{aligned}$$

The solutions are $(4, 4)$ and $(1, -2)$.

The graph of this system of equations is shown at the right. The graph of the line intersects the parabola at two points. The coordinates of these points are the solutions of the system of equations.



HOW TO 2

$$\begin{aligned}\text{Solve: } x^2 + y^2 &= 4 \quad (1) \\ y &= x + 3 \quad (2)\end{aligned}$$

Use the substitution method.

Substitute the expression for y into Equation (1).

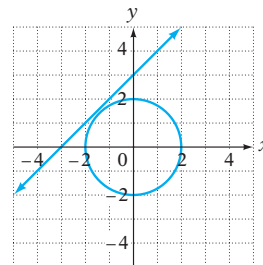
$$\begin{aligned}x^2 + y^2 &= 4 \\ x^2 + (x + 3)^2 &= 4 && \bullet y = x + 3 \\ x^2 + x^2 + 6x + 9 &= 4 && \bullet \text{Solve for } x. \\ 2x^2 + 6x + 5 &= 0\end{aligned}$$

Consider the discriminant of $2x^2 + 6x + 5 = 0$.

$$\begin{aligned}b^2 - 4ac \\ 6^2 - 4 \cdot 2 \cdot 5 &= 36 - 40 = -4 && \bullet a = 2, b = 6, c = 5\end{aligned}$$

Because the discriminant is less than zero, the equation $2x^2 + 6x + 5 = 0$ has two complex number solutions. Therefore, the system of equations has no real number solutions.

The graphs of the equations of this system are shown at the right. Note that the two graphs do not intersect.

**HOW TO 3**

$$\begin{aligned}\text{Solve: } 4x^2 + y^2 &= 16 \quad (1) \\ x^2 + y^2 &= 4 \quad (2)\end{aligned}$$

Use the addition method.

$$\begin{aligned}4x^2 + y^2 &= 16 \\ -x^2 - y^2 &= -4 \\ \hline 3x^2 &= 12 && \bullet \text{Multiply Equation (2) by } -1. \\ &&& \bullet \text{Add the equations.}\end{aligned}$$

Solve for x .

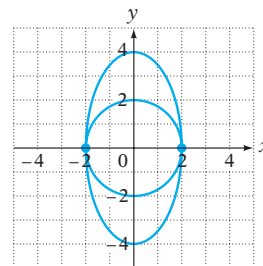
$$\begin{aligned}x^2 &= 4 \\ x &= \pm 2\end{aligned}$$

Substitute the values of x into Equation (2) and solve for y .

$$\begin{aligned}x^2 + y^2 &= 4 && x^2 + y^2 = 4 \\ 2^2 + y^2 &= 4 && (-2)^2 + y^2 = 4 && \bullet x = 2 && \bullet x = -2 \\ y^2 &= 0 && y^2 = 0 \\ y &= 0 && y = 0\end{aligned}$$

The solutions are $(2, 0)$ and $(-2, 0)$.

The graphs of the equations in this system are shown at the right. Note that the graphs intersect at the points whose coordinates are $(2, 0)$ and $(-2, 0)$.



HOW TO 4

$$\begin{aligned}\text{Solve: } 2x^2 - y^2 &= 1 & (1) \\ x^2 + 2y^2 &= 18 & (2)\end{aligned}$$

Use the addition method.

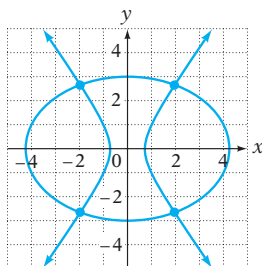
$$\begin{array}{rcl} 4x^2 - 2y^2 & = & 2 \\ x^2 + 2y^2 & = & 18 \\ \hline 5x^2 & = & 20 \\ x^2 & = & 4 \\ x & = & \pm 2 \end{array}$$

- **Multiply Equation (1) by 2.**
- **Add the equations.**
- **Solve for x .**

Substitute the values of x into one of the equations and solve for y . Equation (2) is used here.

$$\begin{array}{rcl} x^2 + 2y^2 & = & 18 \\ 2^2 + 2y^2 & = & 18 & \bullet \ x = 2 \\ 2y^2 & = & 14 \\ y^2 & = & 7 \\ y & = & \pm\sqrt{7} \end{array} \qquad \begin{array}{rcl} x^2 + 2y^2 & = & 18 \\ (-2)^2 + 2y^2 & = & 18 & \bullet \ x = -2 \\ 2y^2 & = & 14 \\ y^2 & = & 7 \\ y & = & \pm\sqrt{7} \end{array}$$

The solutions are $(2, \sqrt{7})$, $(2, -\sqrt{7})$, $(-2, \sqrt{7})$, and $(-2, -\sqrt{7})$.

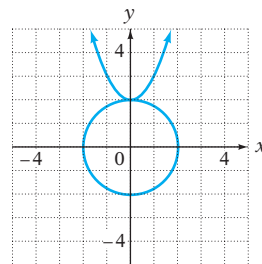


The graphs of the equations in this system are shown at the left. Note that there are four points of intersection.

Note from the preceding three examples that the number of points at which the graphs of the equations of the system intersect is the same as the number of real number solutions of the system of equations. Here are three more examples.

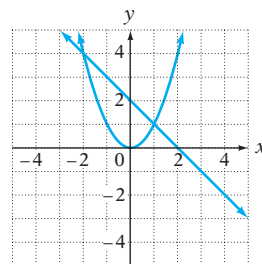
$$\begin{aligned}x^2 + y^2 &= 4 \\ y &= x^2 + 2\end{aligned}$$

The graphs intersect at one point.
The system of equations has one solution.



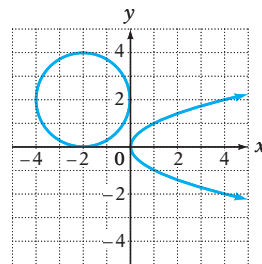
$$\begin{aligned}y &= x^2 \\ y &= -x + 2\end{aligned}$$

The graphs intersect at two points.
The system of equations has two solutions.



$$\begin{aligned}(x + 2)^2 + (y - 2)^2 &= 4 \\ x &= y^2\end{aligned}$$

The graphs do not intersect.
The system of equations has no solution.



EXAMPLE • 1

Solve: (1) $y = 2x^2 - 3x - 1$
 (2) $y = x^2 - 2x + 5$

Solution

Use the substitution method.

$$\begin{aligned} y &= x^2 - 2x + 5 \\ 2x^2 - 3x - 1 &= x^2 - 2x + 5 \\ x^2 - x - 6 &= 0 \\ (x - 3)(x + 2) &= 0 \\ x - 3 &= 0 & x + 2 &= 0 \\ x &= 3 & x &= -2 \end{aligned}$$

Substitute each value of x into Equation (1).

$$\begin{aligned} y &= 2x^2 - 3x - 1 \\ y &= 2(3)^2 - 3(3) - 1 \\ y &= 18 - 9 - 1 \\ y &= 8 \\ y &= 2x^2 - 3x - 1 \\ y &= 2(-2)^2 - 3(-2) - 1 \\ y &= 8 + 6 - 1 \\ y &= 13 \end{aligned}$$

The solutions are $(3, 8)$ and $(-2, 13)$.

YOU TRY IT • 1

Solve: $y = 2x^2 + x - 3$
 $y = 2x^2 - 2x + 9$

Your solution**EXAMPLE • 2**

Solve: (1) $3x^2 - 2y^2 = 26$
 (2) $x^2 - y^2 = 5$

Solution

Use the addition method. Multiply Equation (2) by -2 .

$$\begin{array}{r} -2x^2 + 2y^2 = -10 \\ 3x^2 - 2y^2 = 26 \\ \hline x^2 = 16 \\ x = \pm\sqrt{16} = \pm 4 \end{array}$$

Substitute each value of x into Equation (2).

$$\begin{array}{ll} x^2 - y^2 = 5 & x^2 - y^2 = 5 \\ 4^2 - y^2 = 5 & (-4)^2 - y^2 = 5 \\ 16 - y^2 = 5 & 16 - y^2 = 5 \\ -y^2 = -11 & -y^2 = -11 \\ y^2 = 11 & y^2 = 11 \\ y = \pm\sqrt{11} & y = \pm\sqrt{11} \end{array}$$

The solutions are $(4, \sqrt{11})$, $(4, -\sqrt{11})$, $(-4, \sqrt{11})$, and $(-4, -\sqrt{11})$.

YOU TRY IT • 2

Solve: $x^2 - y^2 = 10$
 $x^2 + y^2 = 8$

Your solution

11.4 EXERCISES

OBJECTIVE A

To solve a nonlinear system of equations

For Exercises 1 to 33, solve.

$$\begin{aligned} 1. \quad & y = x^2 - x - 1 \\ & y = 2x + 9 \end{aligned}$$

$$\begin{aligned} 2. \quad & y = x^2 - 3x + 1 \\ & y = x + 6 \end{aligned}$$

$$\begin{aligned} 3. \quad & y^2 = -x + 3 \\ & x - y = 1 \end{aligned}$$

$$\begin{aligned} 4. \quad & y^2 = 4x \\ & x - y = -1 \end{aligned}$$

$$\begin{aligned} 5. \quad & y^2 = 2x \\ & x + 2y = -2 \end{aligned}$$

$$\begin{aligned} 6. \quad & y^2 = 2x \\ & x - y = 4 \end{aligned}$$

$$\begin{aligned} 7. \quad & x^2 + 2y^2 = 12 \\ & 2x - y = 2 \end{aligned}$$

$$\begin{aligned} 8. \quad & x^2 + 4y^2 = 37 \\ & x - y = -4 \end{aligned}$$

$$\begin{aligned} 9. \quad & x^2 + y^2 = 13 \\ & x + y = 5 \end{aligned}$$

$$\begin{aligned} 10. \quad & x^2 + y^2 = 16 \\ & x - 2y = -4 \end{aligned}$$

$$\begin{aligned} 11. \quad & 4x^2 + y^2 = 12 \\ & y = 4x^2 \end{aligned}$$

$$\begin{aligned} 12. \quad & 2x^2 + y^2 = 6 \\ & y = 2x^2 \end{aligned}$$

$$\begin{aligned} 13. \quad & y = x^2 - 2x - 3 \\ & y = x - 6 \end{aligned}$$

$$\begin{aligned} 14. \quad & y = x^2 + 4x + 5 \\ & y = -x - 3 \end{aligned}$$

$$\begin{aligned} 15. \quad & 3x^2 - y^2 = -1 \\ & x^2 + 4y^2 = 17 \end{aligned}$$

$$\begin{aligned} 16. \quad & x^2 + y^2 = 10 \\ & x^2 + 9y^2 = 18 \end{aligned}$$

$$\begin{aligned} 17. \quad & 2x^2 + 3y^2 = 30 \\ & x^2 + y^2 = 13 \end{aligned}$$

$$\begin{aligned} 18. \quad & x^2 + y^2 = 61 \\ & x^2 - y^2 = 11 \end{aligned}$$

$$\begin{aligned} 19. \quad & y = 2x^2 - x + 1 \\ & y = x^2 - x + 5 \end{aligned}$$

$$\begin{aligned} 20. \quad & y = -x^2 + x - 1 \\ & y = x^2 + 2x - 2 \end{aligned}$$

$$\begin{aligned} 21. \quad & 2x^2 + 3y^2 = 24 \\ & x^2 - y^2 = 7 \end{aligned}$$

22.
$$\begin{aligned} 2x^2 + 3y^2 &= 21 \\ x^2 + 2y^2 &= 12 \end{aligned}$$

23.
$$\begin{aligned} x^2 + y^2 &= 36 \\ 4x^2 + 9y^2 &= 36 \end{aligned}$$

24.
$$\begin{aligned} 2x^2 + 3y^2 &= 12 \\ x^2 - y^2 &= 25 \end{aligned}$$

25.
$$\begin{aligned} 11x^2 - 2y^2 &= 4 \\ 3x^2 + y^2 &= 15 \end{aligned}$$

26.
$$\begin{aligned} x^2 + 4y^2 &= 25 \\ x^2 - y^2 &= 5 \end{aligned}$$

27.
$$\begin{aligned} 2x^2 - y^2 &= 7 \\ 2x - y &= 5 \end{aligned}$$

28.
$$\begin{aligned} 3x^2 + 4y^2 &= 7 \\ x - 2y &= -3 \end{aligned}$$

29.
$$\begin{aligned} y &= 3x^2 + x - 4 \\ y &= 3x^2 - 8x + 5 \end{aligned}$$

30.
$$\begin{aligned} y &= 2x^2 + 3x + 1 \\ y &= 2x^2 + 9x + 7 \end{aligned}$$

31.
$$\begin{aligned} x &= y + 3 \\ x^2 + y^2 &= 5 \end{aligned}$$

32.
$$\begin{aligned} x - y &= -6 \\ x^2 + y^2 &= 4 \end{aligned}$$

33.
$$\begin{aligned} y &= x^2 + 4x + 4 \\ x + 2y &= 4 \end{aligned}$$



34. Give the maximum number of points at which the following pairs of graphs can intersect.
- A line and an ellipse
 - A line and a parabola
 - Two ellipses with centers at the origin
 - Two different circles with centers at the origin
 - Two hyperbolas with centers at the origin

Applying the Concepts



For Exercises 35 to 40, solve by graphing. Round the solutions of the system to the nearest thousandth.

35.
$$\begin{aligned} y &= 2^x \\ x + y &= 3 \end{aligned}$$

36.
$$\begin{aligned} y &= 3^{-x} \\ x^2 + y^2 &= 9 \end{aligned}$$

37.
$$\begin{aligned} y &= \log_2 x \\ \frac{x^2}{9} + \frac{y^2}{1} &= 1 \end{aligned}$$

38.
$$\begin{aligned} y &= \log_3 x \\ x^2 + y^2 &= 4 \end{aligned}$$

39.
$$\begin{aligned} y &= -\log_3 x \\ x + y &= 4 \end{aligned}$$

40.
$$\begin{aligned} y &= \left(\frac{1}{2}\right)^x \\ \frac{x^2}{9} + \frac{y^2}{4} &= 1 \end{aligned}$$

SECTION

11.5

Quadratic Inequalities and Systems of Inequalities

OBJECTIVE A

To graph the solution set of a quadratic inequality in two variables



The **graph of a quadratic inequality in two variables** is a region of the plane that is bounded by one of the conic sections (parabola, circle, ellipse, or hyperbola). When graphing an inequality of this type, first replace the inequality symbol with an equals sign. Graph the resulting conic using a dashed curve when the original inequality is less than ($<$) or greater than ($>$). Use a solid curve when the original inequality is $\leq$ or $\geq$. Use the point $(0, 0)$ to determine which region of the plane to shade. If $(0, 0)$ is a solution of the inequality, then shade the region of the plane containing $(0, 0)$. If not, shade the other portion of the plane.

HOW TO 1

Graph the solution set: $x^2 + y^2 > 9$

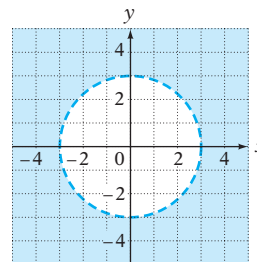
Change the inequality to an equality.

$$x^2 + y^2 = 9$$

This is the equation of a circle with center $(0, 0)$ and radius 3.

The original inequality is $>$, so the graph is drawn as a **dashed circle**.

Substitute the point $(0, 0)$ in the inequality. Because $0^2 + 0^2 > 9$ is not true, the point $(0, 0)$ should not be in the shaded region.



Not all inequalities have a solution set that can be graphed by shading a region of the plane. For example, the inequality

$$\frac{x^2}{9} + \frac{y^2}{4} < -1$$

has no ordered-pair solutions. The solution set is the empty set, and no region of the plane is shaded.

 Take Note

Any point that is not on the graph of the parabola can be used as a test point. For example, the point $(2, 1)$ can be used.

$$\begin{aligned} y &\leq x^2 + 2x + 2 \\ 1 &\leq 2^2 + 2(2) + 2 \\ 1 &\leq 10 \end{aligned}$$

This is a true inequality. The point $(2, 1)$ should be in the shaded region.

HOW TO 2

Graph the solution set: $y \leq x^2 + 2x + 2$

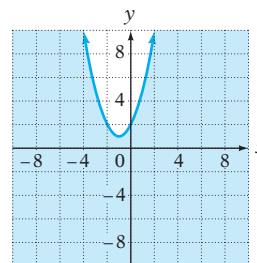
Change the inequality to an equality.

$$y = x^2 + 2x + 2$$

This is the equation of a parabola that opens up. The vertex is $(-1, 1)$ and the axis of symmetry is the line $x = -1$.

The original inequality is $\leq$, so the graph is drawn as a **solid curve**.

Substitute the point $(0, 0)$ into the inequality. Because $0 \leq 0^2 + 2(0) + 2$ is true, the point $(0, 0)$ should be in the shaded region.



HOW TO • 3Graph the solution set: $\frac{y^2}{9} - \frac{x^2}{4} \geq 1$

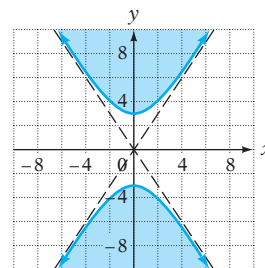
Write the inequality as an equality.

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

This is the equation of a hyperbola. The vertices are $(0, -3)$ and $(0, 3)$. The equations of the asymptotes are $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$.

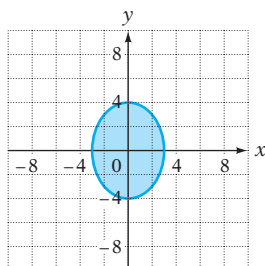
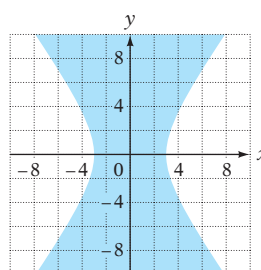
The original inequality is $\geq$, so the graph is drawn as a **solid curve**.

Substitute the point $(0, 0)$ into the inequality. Because $\frac{0^2}{9} - \frac{0^2}{4} \geq 1$ is not true, the point $(0, 0)$ should not be in the shaded region.

**EXAMPLE • 1**Graph the solution set: $\frac{x^2}{9} + \frac{y^2}{16} \leq 1$ **Solution**

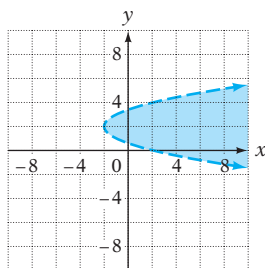
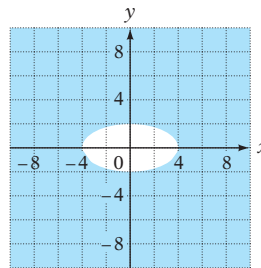
Graph the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ as a solid curve.

Shade the region of the plane that includes $(0, 0)$.

**YOU TRY IT • 1**Graph the solution set: $\frac{x^2}{9} - \frac{y^2}{16} \leq 1$ **Your solution****EXAMPLE • 2**Graph the solution set: $x > y^2 - 4y + 2$ **Solution**

Graph the parabola $x = y^2 - 4y + 2$ as a dashed curve.

Shade the region that does not include $(0, 0)$.

**YOU TRY IT • 2**Graph the solution set: $\frac{x^2}{16} + \frac{y^2}{4} > 1$ **Your solution**

OBJECTIVE B**To graph the solution set of a nonlinear system of inequalities**

Recall that the solution set of a system of inequalities is the intersection of the solution sets of the individual inequalities. To graph the solution set of a system of inequalities, first graph the solution set of each inequality. The solution set of the system of inequalities is the region of the plane represented by the intersection of the two shaded regions.

HOW TO • 4

Graph the solution set: $x^2 + y^2 \leq 16$
 $y \geq x^2$

Graph the solution set of each inequality.

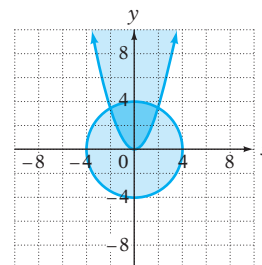
$x^2 + y^2 = 16$ is the equation of a circle with center at the origin and a radius of 4. Because the original inequality is $\leq$, use a solid curve.

$y = x^2$ is the equation of a parabola that opens up. The vertex is $(0, 0)$. Because the original inequality is $\geq$, use a solid curve.

Substitute the point $(0, 0)$ into the inequality $x^2 + y^2 \leq 16$. $0^2 + 0^2 \leq 16$ is a true statement. Shade inside the circle.

Substitute $(2, 0)$ into the inequality $y \geq x^2$. [We cannot use $(0, 0)$, a point on the parabola.] $0 \geq 4$ is a false statement. Shade inside the parabola.

The solution is the intersection of the two shaded regions.

**Take Note**

Other points can be used as test points. For example, the point $(6, 2)$ can be used for the circle.

$$x^2 + y^2 \leq 16$$

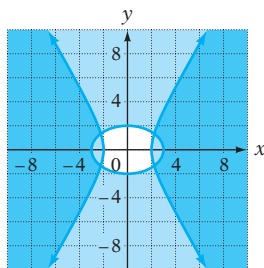
$$6^2 + 2^2 \leq 16$$

$$40 \leq 16$$

This is false. The point $(6, 2)$ should not be in the shaded region.

EXAMPLE • 3

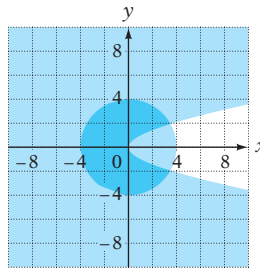
Graph the solution set: $\frac{x^2}{9} + \frac{y^2}{4} \geq 1$
 $\frac{x^2}{4} - \frac{y^2}{9} \geq 1$

Solution

- Draw the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ as a solid curve. Shade outside the ellipse.
- Draw the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ as a solid curve. $(0, 0)$ should not be in the shaded region.

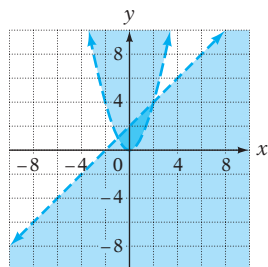
YOU TRY IT • 3

Graph the solution set: $x^2 + y^2 < 16$
 $y^2 > x$

Your solution

EXAMPLE 4

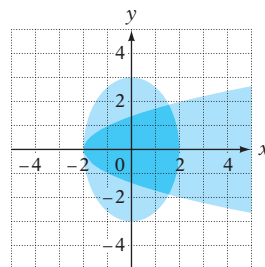
Graph the solution set: $y > x^2$
 $y < x + 2$

Solution

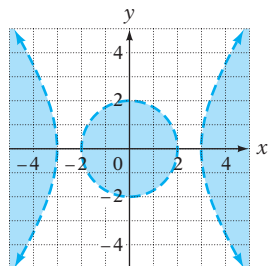
- Draw the parabola $y = x^2$ as a dashed curve. Shade inside the parabola.
- Draw the line $y = x + 2$ as a dashed line. Shade below the line.

YOU TRY IT 4

Graph the solution set: $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$
 $x > y^2 - 2$

Your solution**EXAMPLE 5**

Graph the solution set: $\frac{x^2}{9} - \frac{y^2}{16} > 1$
 $x^2 + y^2 < 4$

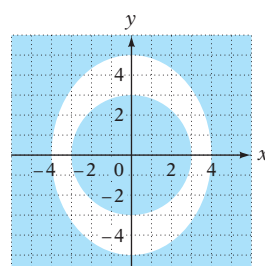
Solution

- Draw the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ as a dashed curve. $(0, 0)$ should not be in the shaded region.
- Draw the circle $x^2 + y^2 = 4$ as a dashed curve. Shade inside the circle.

The solution sets of the two inequalities do not intersect. This system of inequalities has no real number solution.

YOU TRY IT 5

Graph the solution set: $\frac{x^2}{16} + \frac{y^2}{25} \geq 1$
 $x^2 + y^2 < 9$

Your solution

11.5 EXERCISES

OBJECTIVE A

To graph the solution set of a quadratic inequality in two variables



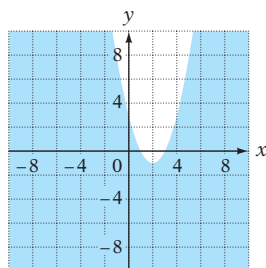
1. How is the graph of the solution set of a nonlinear inequality that uses $\leq$ or $\geq$ different from the graph of the solution set of a nonlinear inequality that uses $<$ or $>$?



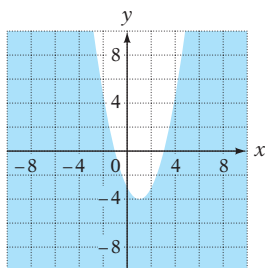
2. How can you determine which region to shade when graphing the solution set of a nonlinear inequality?

For Exercises 3 to 26, graph the solution set.

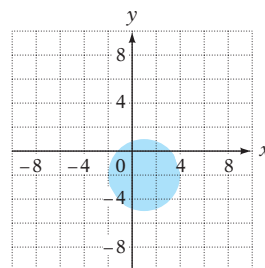
3. $y \leq x^2 - 4x + 3$



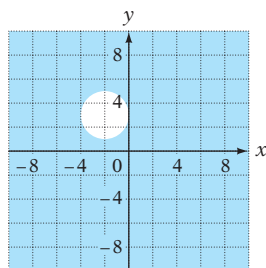
4. $y < x^2 - 2x - 3$



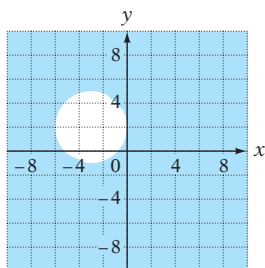
5. $(x - 1)^2 + (y + 2)^2 \leq 9$



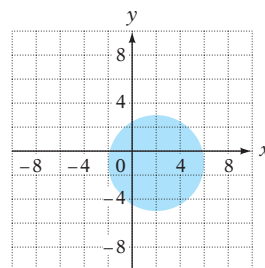
6. $(x + 2)^2 + (y - 3)^2 > 4$



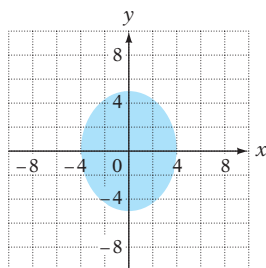
7. $(x + 3)^2 + (y - 2)^2 \geq 9$



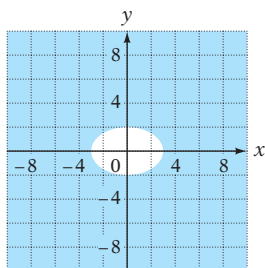
8. $(x - 2)^2 + (y + 1)^2 \leq 16$



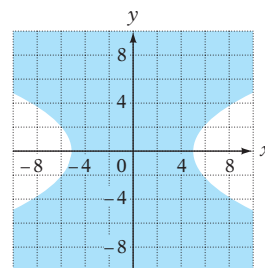
9. $\frac{x^2}{16} + \frac{y^2}{25} < 1$



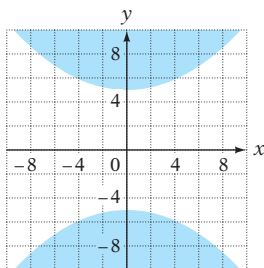
10. $\frac{x^2}{9} + \frac{y^2}{4} \geq 1$



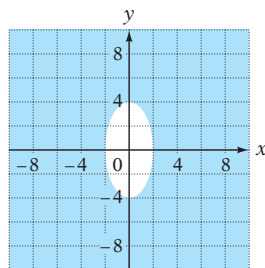
11. $\frac{x^2}{25} - \frac{y^2}{9} \leq 1$



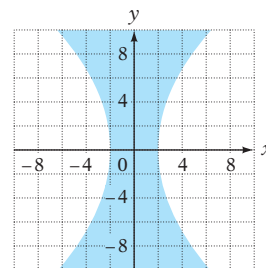
12. $\frac{y^2}{25} - \frac{x^2}{36} > 1$



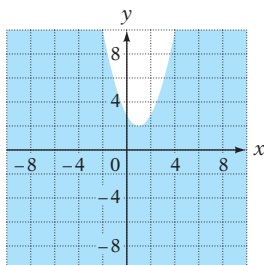
13. $\frac{x^2}{4} + \frac{y^2}{16} \geq 1$



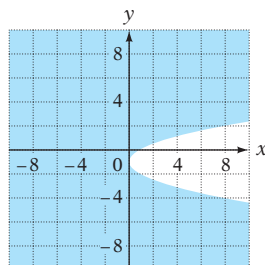
14. $\frac{x^2}{4} - \frac{y^2}{16} \leq 1$



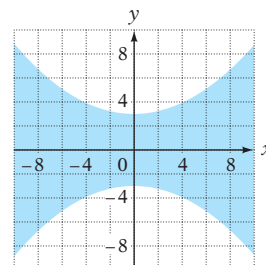
15. $y \leq x^2 - 2x + 3$



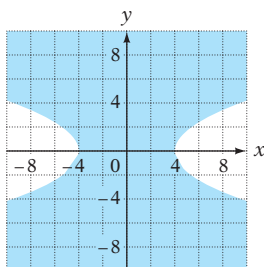
16. $x \leq y^2 + 2y + 1$



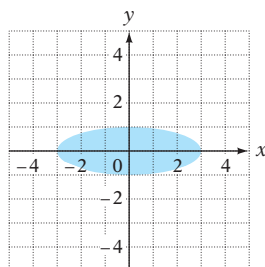
17. $\frac{y^2}{9} - \frac{x^2}{16} \leq 1$



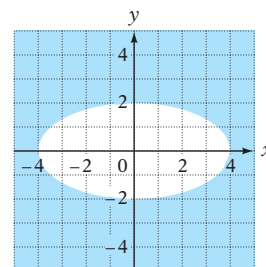
18. $\frac{x^2}{16} - \frac{y^2}{4} < 1$



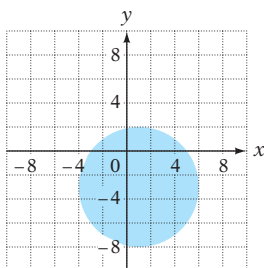
19. $\frac{x^2}{9} + \frac{y^2}{1} \leq 1$



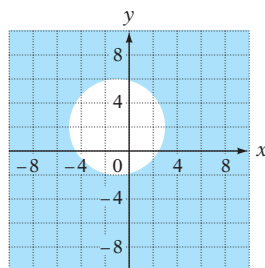
20. $\frac{x^2}{16} + \frac{y^2}{4} > 1$



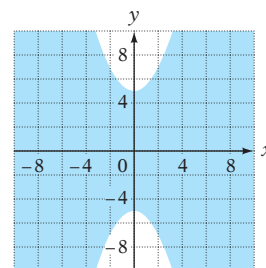
21. $(x - 1)^2 + (y + 3)^2 \leq 25$



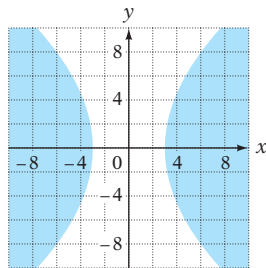
22. $(x + 1)^2 + (y - 2)^2 \geq 16$



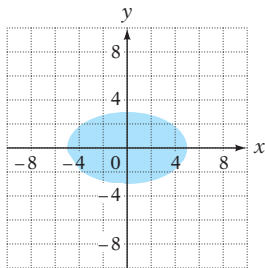
23. $\frac{y^2}{25} - \frac{x^2}{4} \leq 1$



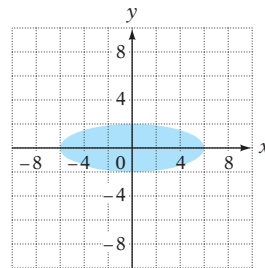
24. $\frac{x^2}{9} - \frac{y^2}{25} \geq 1$



25. $\frac{x^2}{25} + \frac{y^2}{9} \leq 1$



26. $\frac{x^2}{36} + \frac{y^2}{4} \leq 1$

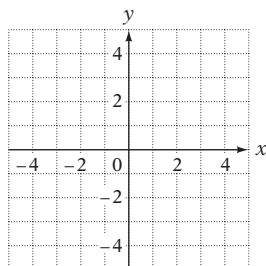


27. Can the point $(0, 0)$ be used to determine whether to shade inside or outside the graph of $y \leq x^2 - 4x$?

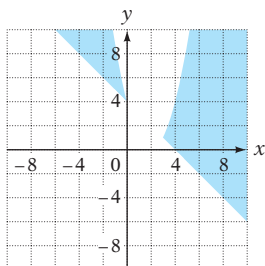
OBJECTIVE B**To graph the solution set of a nonlinear system of inequalities**

For Exercises 28 to 39, graph the solution set.

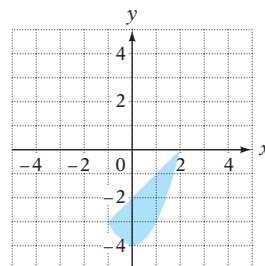
28. $x^2 + y^2 < 1$
 $x + y \geq 4$



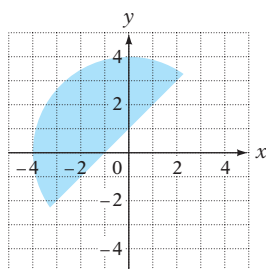
29. $y \leq (x - 2)^2$
 $y + x > 4$



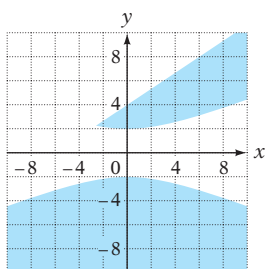
30. $y > x^2 - 4$
 $y < x - 2$



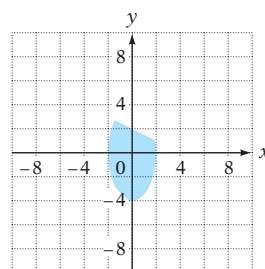
31. $x^2 + y^2 < 16$
 $y > x + 1$



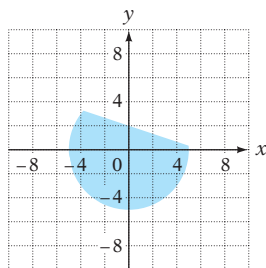
32. $\frac{y^2}{4} - \frac{x^2}{25} \geq 1$
 $y \leq \frac{2}{3}x + 4$



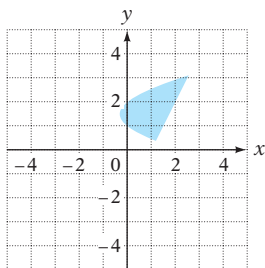
33. $\frac{x^2}{4} + \frac{y^2}{16} \leq 1$
 $y \leq -\frac{1}{2}x + 2$



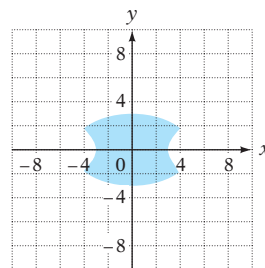
34. $x^2 + y^2 \leq 25$
 $y \leq -\frac{1}{3}x + 2$



35. $x \geq y^2 - 3y + 2$
 $y \geq 2x - 2$

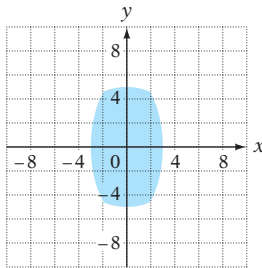


36. $\frac{x^2}{9} - \frac{y^2}{4} < 1$
 $\frac{x^2}{25} + \frac{y^2}{9} < 1$

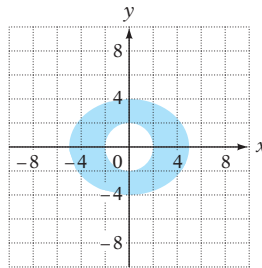


37. $x^2 + y^2 < 25$

$$\frac{x^2}{9} + \frac{y^2}{36} < 1$$

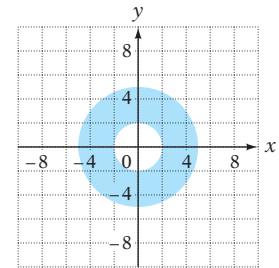


38. $\frac{x^2}{25} + \frac{y^2}{16} \leq 1$
 $\frac{x^2}{4} + \frac{y^2}{4} \geq 1$



39. $x^2 + y^2 > 4$

$$x^2 + y^2 < 25$$

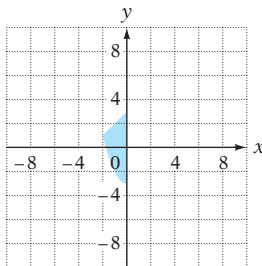


40. Let a and b be positive numbers with $a < b$. Describe the solution set of $\begin{matrix} x^2 + y^2 < a \\ x^2 + y^2 > b \end{matrix}$.

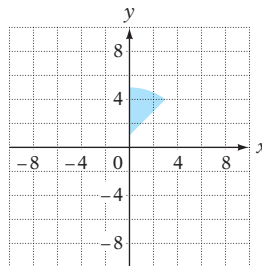
Applying the Concepts

For Exercises 41 to 49, graph the solution set.

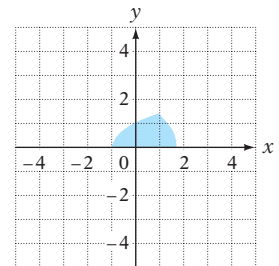
41. $y > x^2 - 3$
 $y < x + 3$
 $x \leq 0$



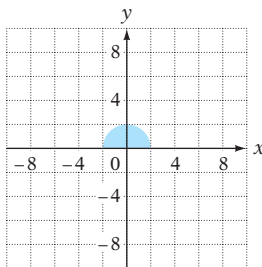
42. $x^2 + y^2 \leq 25$
 $y > x + 1$
 $x \geq 0$



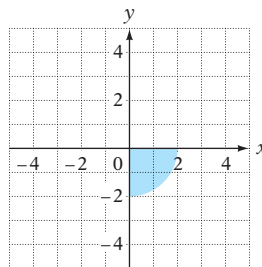
43. $x^2 + y^2 < 3$
 $x > y^2 - 1$
 $y \geq 0$



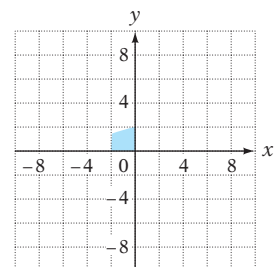
44. $\frac{x^2}{4} - \frac{y^2}{25} \leq 1$
 $\frac{x^2}{4} + \frac{y^2}{4} \leq 1$
 $y \geq 0$



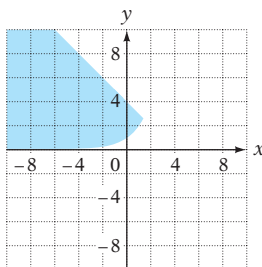
45. $\frac{x^2}{16} + \frac{y^2}{4} \leq 1$
 $x^2 + y^2 \leq 4$
 $x \geq 0$
 $y \leq 0$



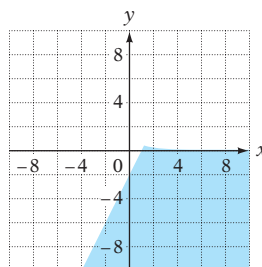
46. $\frac{x^2}{4} + \frac{y^2}{25} \leq 1$
 $x > y^2 - 4$
 $x \leq 0$
 $y \geq 0$



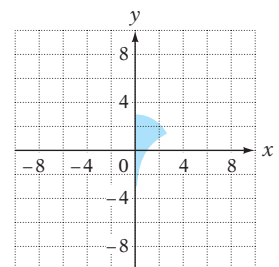
47. $y > 2^x$
 $x + y < 4$



48. $y < \left(\frac{1}{2}\right)^x$
 $2x - y \geq 2$



49. $y \geq \log_2 x$
 $x^2 + y^2 < 9$



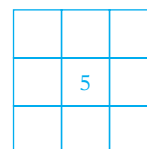
FOCUS ON PROBLEM SOLVING

Using a Variety of Problem-Solving Techniques

We have examined several problem-solving strategies throughout the text. See if you can apply those techniques to Exercises 1 to 6.

1. Eight coins look exactly alike, but one is lighter than the others. Explain how the lighter coin can be found in two weighings on a balance scale.
2. For the sequence of numbers 1, 1, 2, 3, 5, 8, 13, $\dots$, identify a possible pattern and then use that pattern to determine the next number in the sequence.

3. Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the squares at the right such that the sum of any row, column, or diagonal is 15. (*Suggestion:* Note that 1, 5, 9; 2, 5, 8; 3, 5, 7; and 4, 5, 6 all add to 15. Because 5 is part of each sum, this suggests that 5 should be placed in the center of the squares.)



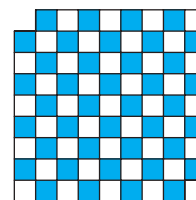
4. A restaurant charges \$12.00 for a pizza that has a diameter of 9 in. Determine the selling price of a pizza with a diameter of 18 in. such that the selling price per square inch is the same as for the 9-inch pizza.



David Toase/Photodisc/Getty Images

5. You have a balance scale and weights of 1 g, 4 g, 8 g, and 16 g. Using only these weights, can you weigh something that weighs each of the following?
 - a. 3 g
 - b. 7 g
 - c. 11 g
 - d. 13 g
 - e. 19 g

6. Can the checkerboard at the right be covered with dominos (which look like ) such that every square on the board is covered by a domino? Why or why not? (*Note:* The dominos cannot overlap.)



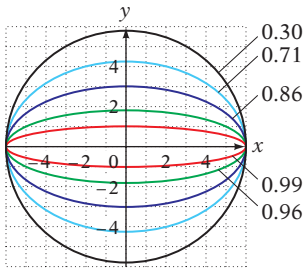
PROJECTS AND GROUP ACTIVITIES

The Eccentricity and Foci of an Ellipse

The graph of an ellipse can be long and thin, or it can have a shape that is very close to a circle. The **eccentricity** e of an ellipse is a measure of its “roundness.”

The shapes of ellipses with various eccentricities are shown below.

Planet	Eccentricity
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.049
Saturn	0.051
Uranus	0.046
Neptune	0.005



1. Based on the eccentricities of the ellipses shown above, complete the sentence. “As the eccentricity of an ellipse gets closer to 1, the ellipse gets flatter/rounder.”
2. The planets travel around the sun in elliptical orbits. The eccentricities of the orbits of the planets are shown in the table at the left. Which planet has the most nearly circular orbit?

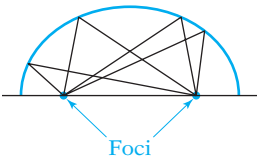
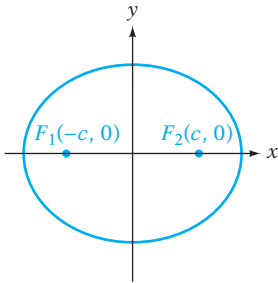
For an ellipse given by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$, a formula for eccentricity is $e = \frac{\sqrt{a^2 - b^2}}{a}$. Use this formula to find the eccentricity of the ellipses in Exercises 3 and 4. If necessary, round to the nearest hundredth.

3. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ 4. $\frac{x^2}{9} + \frac{y^2}{4} = 1$



Joseph Sohm/Terra/Corbis

Ellipses have a reflective property that has been used in the design of some buildings. The **foci** of an ellipse are two points on the longer axis, called the **major axis**, of the ellipse.

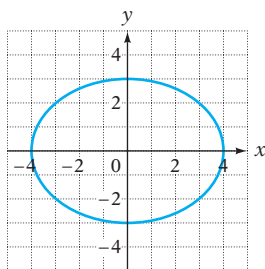


If light or sound emanates from one focus, it is reflected to the other focus. This phenomenon results in what are called “whispering galleries.” The rotunda of the Capitol Building in Washington, D.C. is a whispering gallery. A whisper spoken by a person at one focus can be heard clearly by a person at the other focus.

The foci are c units from the center of an ellipse, where $c = \sqrt{a^2 - b^2}$ for an ellipse whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$.

5. Find the foci for an ellipse whose equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

Graphing Conic Sections Using a Graphing Calculator



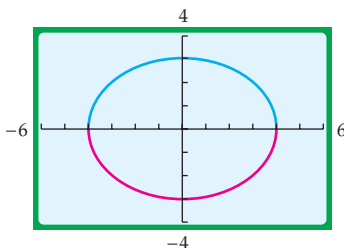
Consider the graph of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ shown at the left. Because a vertical line can intersect the graph at more than one point, the graph is not the graph of a function. And because the graph is not the graph of a function, the equation does not represent a function. Consequently, the equation cannot be entered into a graphing calculator to be graphed. However, by solving the equation for y , we have

$$\begin{aligned}\frac{y^2}{9} &= 1 - \frac{x^2}{16} \\ y^2 &= 9\left(1 - \frac{x^2}{16}\right) \\ y &= \pm 3\sqrt{1 - \frac{x^2}{16}}\end{aligned}$$

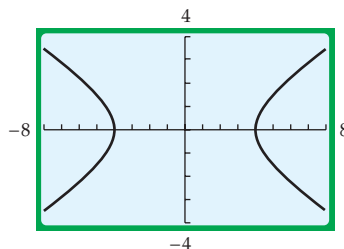
There are two solutions for y , which can be written

$$y_1 = 3\sqrt{1 - \frac{x^2}{16}} \quad \text{and} \quad y_2 = -3\sqrt{1 - \frac{x^2}{16}}$$

Each of these equations is the equation of a function and therefore can be entered into a graphing calculator. The graph of $y_1 = 3\sqrt{1 - \frac{x^2}{16}}$ is shown in blue below, and the graph of $y_2 = -3\sqrt{1 - \frac{x^2}{16}}$ is shown in red. Note that together, the graphs are the graph of an ellipse.



A similar technique can be used to graph a hyperbola. To graph $\frac{x^2}{16} - \frac{y^2}{4} = 1$, solve for y . Then graph each equation, $y_1 = 2\sqrt{\frac{x^2}{16} - 1}$ and $y_2 = -2\sqrt{\frac{x^2}{16} - 1}$.



For Exercises 1 to 4, solve each equation for y . Then graph using a graphing calculator.

1. $\frac{x^2}{25} + \frac{y^2}{49} = 1$
2. $\frac{x^2}{4} + \frac{y^2}{64} = 1$
3. $\frac{x^2}{16} - \frac{y^2}{4} = 1$
4. $\frac{x^2}{9} - \frac{y^2}{36} = 1$

CHAPTER 11

SUMMARY

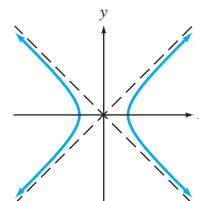
KEY WORDS

Conic sections are curves that can be constructed from the intersection of a plane and a right circular cone. [11.1A, p. 602]

The *asymptotes* of a hyperbola are the two straight lines that are “approached” by the hyperbola. As the graph of the hyperbola gets farther from the origin, the hyperbola “gets closer to” the asymptotes. [11.3B, p. 616]

EXAMPLES

The four conic sections presented in this chapter are the parabola, the circle, the ellipse, and the hyperbola.

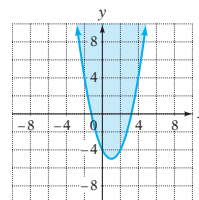


A *nonlinear system of equations* is a system in which one or more of the equations are not linear equations. [11.4A, p. 620]

$$\begin{aligned}x^2 + y^2 &= 16 \\ y &= x^2 - 2\end{aligned}$$

The *graph of a quadratic inequality in two variables* is a region of the plane that is bounded by one of the conic sections. [11.5A, p. 626]

$$y \geq x^2 - 2x - 4$$



ESSENTIAL RULES AND PROCEDURES

Equations of a Parabola [11.1A, pp. 602–604]

$$y = ax^2 + bx + c, a \neq 0$$

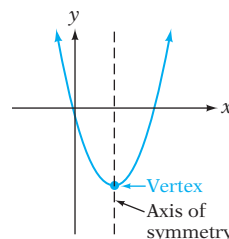
When $a > 0$, the parabola opens up.

When $a < 0$, the parabola opens down.

The x -coordinate of the vertex is $-\frac{b}{2a}$.

The axis of symmetry is the line $x = -\frac{b}{2a}$.

EXAMPLES



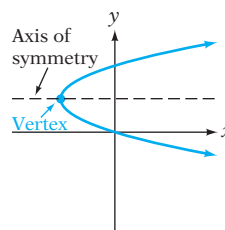
$$x = ay^2 + by + c, a \neq 0$$

When $a > 0$, the parabola opens to the right.

When $a < 0$, the parabola opens to the left.

The y -coordinate of the vertex is $-\frac{b}{2a}$.

The axis of symmetry is the line $y = -\frac{b}{2a}$.



Standard Form of the Equation of a Circle [11.2A, p. 608]

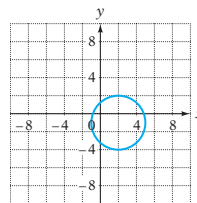
$$(x - h)^2 + (y - k)^2 = r^2$$

The center of the circle is (h, k) , and the radius is r .

$$(x - 2)^2 + (y + 1)^2 = 9$$

$$(h, k) = (2, -1)$$

$$r = 3$$

**General Form of the Equation of a Circle** [11.2B, p. 610]

The equation of a circle can also be expressed as

$$x^2 + y^2 + ax + by + c = 0.$$

$x^2 + y^2 + 4x + 2y + 1 = 0$ is the equation of a circle in general form.

Equation of an Ellipse [11.3A, p. 614]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The x -intercepts are $(a, 0)$ and $(-a, 0)$.

The y -intercepts are $(0, b)$ and $(0, -b)$.

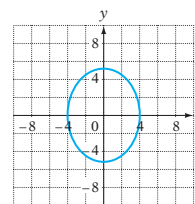
$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

x -intercepts:

$$(4, 0) \text{ and } (-4, 0)$$

y -intercepts:

$$(0, 5) \text{ and } (0, -5)$$

**Equation of a Hyperbola** [11.3B, p. 616]

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

An axis of symmetry is the x -axis.

The vertices are $(a, 0)$ and $(-a, 0)$.

The equations of the asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

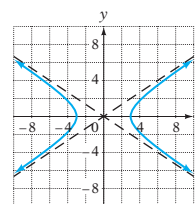
$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Vertices:

$$(3, 0) \text{ and } (-3, 0)$$

Asymptotes:

$$y = \frac{2}{3}x \text{ and } y = -\frac{2}{3}x$$



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

An axis of symmetry is the y -axis.

The vertices are $(0, b)$ and $(0, -b)$.

The equations of the asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

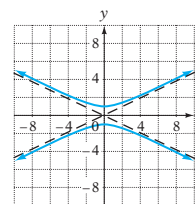
$$\frac{y^2}{1} - \frac{x^2}{4} = 1$$

Vertices:

$$(0, 1) \text{ and } (0, -1)$$

Asymptotes:

$$y = \frac{1}{2}x \text{ and } y = -\frac{1}{2}x$$



CHAPTER 11**CONCEPT REVIEW**

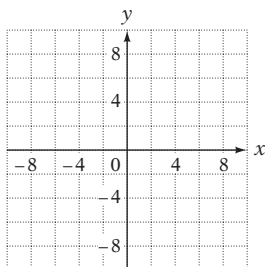
Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. What are the four conic sections described in this section, and how are they formed?
2. Do parabolas only open up or down? Why or why not?
3. What is the standard form of the equation of a circle?
4. What is the general form of the equation of a circle?
5. What does the distance formula have to do with the radius of a circle?
6. What is the difference between a circle and an ellipse with center at the origin?
7. What is the standard form of the equation of an ellipse with center at the origin?
8. What is the standard form of the equation of a hyperbola with center at the origin?
9. What are the asymptotes of a hyperbola, and how do you find them?
10. True or false? A nonlinear system of equations is a system in which no equation is a linear equation.
11. True or false? A nonlinear system of equations always has a solution set.
12. For the graph of the solution set of a quadratic inequality, when are dashed lines used and when are solid lines used?
13. When graphing the solution set of a nonlinear system of inequalities, why is it helpful to use different kinds of shading for each region?

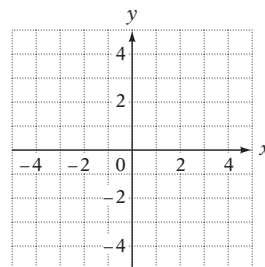
CHAPTER 11

REVIEW EXERCISES

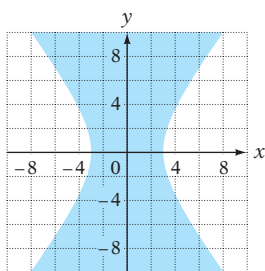
1. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $y = -2x^2 + x - 2$. Then sketch its graph.



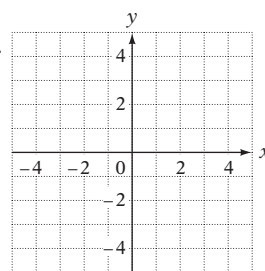
2. Find the vertices of the graph of $\frac{x^2}{1} + \frac{y^2}{9} = 1$. Then sketch the graph.



3. Graph the solution set:
 $\frac{x^2}{9} - \frac{y^2}{16} < 1$



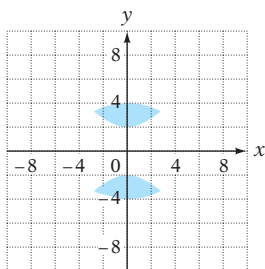
4. Sketch a graph of $(x + 3)^2 + (y + 1)^2 = 1$.



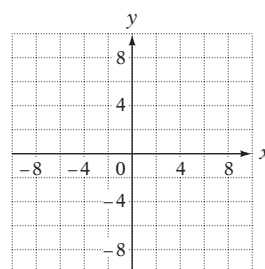
5. Find the equation of the axis of symmetry and the coordinates of the vertex of $y = x^2 - 4x + 8$.

6. Solve: $y^2 = 2x^2 - 3x + 6$
 $y^2 = 2x^2 + 5x - 2$

7. Graph the solution set:
 $\frac{x^2}{25} + \frac{y^2}{16} \leq 1$
 $\frac{y^2}{4} - \frac{x^2}{4} \geq 1$



8. Sketch a graph of $\frac{x^2}{25} - \frac{y^2}{1} = 1$.



9. Find the equation of the circle whose center is $(-1, 2)$ and that passes through the point whose coordinates are $(2, -1)$.

10. Find the equation of the axis of symmetry and the coordinates of the vertex of $y = -x^2 + 7x - 8$.

11. Solve: $x = 2y^2 - 3y + 1$
 $3x - 2y = 0$

12. Find the equation of the circle whose center is $(0, -3)$ and that passes through the point whose coordinates are $(4, 6)$.

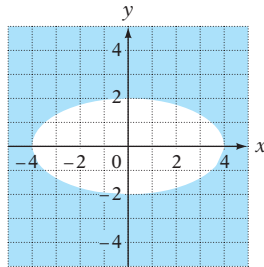
13. Find the equation of the circle with radius 6 and center $(-1, 5)$.

14. Solve: $2x^2 + y^2 = 19$
 $3x^2 - y^2 = 6$

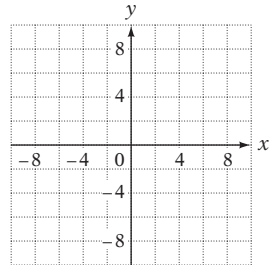
15. Write the equation $x^2 + y^2 + 4x - 2y = 4$ in standard form.

16. Solve: $y = x^2 + 5x - 6$
 $y = x - 10$

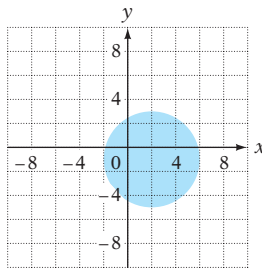
17. Graph the solution set:
 $\frac{x^2}{16} + \frac{y^2}{4} > 1$



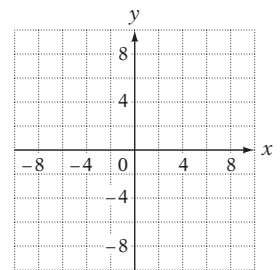
18. Find the coordinates of the vertices and the equations of the asymptotes of the graph of $\frac{y^2}{16} - \frac{x^2}{9} = 1$. Then sketch its graph.



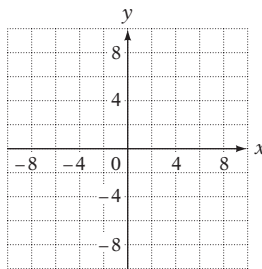
19. Graph the solution set:
 $(x - 2)^2 + (y + 1)^2 \leq 16$



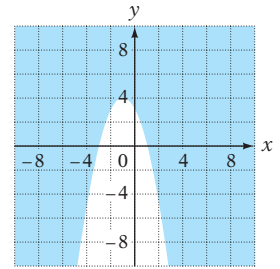
20. Sketch a graph of
 $x^2 + (y - 2)^2 = 9$.



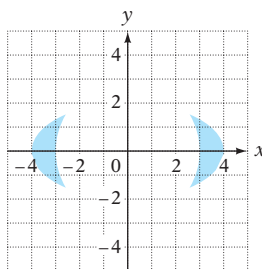
21. Find the vertices of the graph of $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Then sketch its graph.



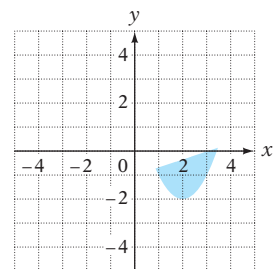
22. Graph the solution set:
 $y \geq -x^2 - 2x + 3$



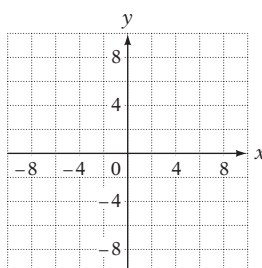
23. Graph the solution set:
 $\frac{x^2}{16} + \frac{y^2}{4} < 1$
 $x^2 + y^2 > 9$



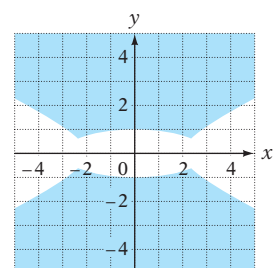
24. Graph the solution set:
 $y \geq x^2 - 4x + 2$
 $y \leq \frac{1}{3}x - 1$



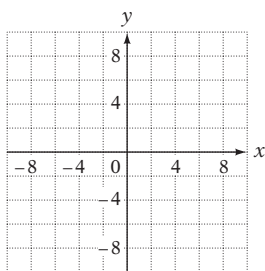
25. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $x = 2y^2 - 6y + 5$. Then sketch its graph.



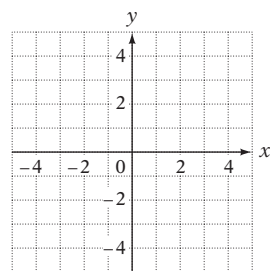
26. Graph the solution set:
 $\frac{x^2}{9} + \frac{y^2}{1} \geq 1$
 $\frac{x^2}{4} - \frac{y^2}{1} \leq 1$



27. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $y = x^2 - 4x - 1$. Then sketch the graph.



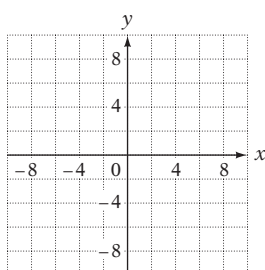
28. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $x = y^2 - 1$. Then sketch the graph.



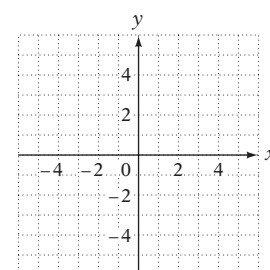
29. Find the equation of the circle with radius 5 and center $(3, -4)$.

30. Write the equation of the circle $x^2 + y^2 - 6x + 4y - 23 = 0$ in standard form.

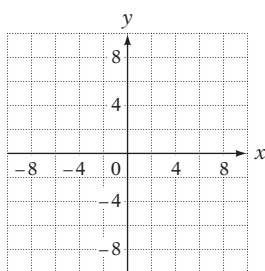
31. Find the coordinates of the center and the radius of the graph of $(x + 1)^2 + (y - 4)^2 = 36$. Then sketch the graph.



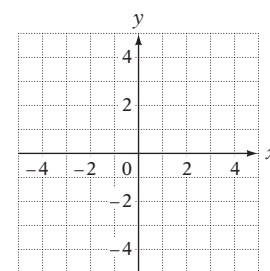
32. Find the coordinates of the center and the radius of the graph of $x^2 + y^2 - 8x + 4y + 16 = 0$. Then sketch the graph.



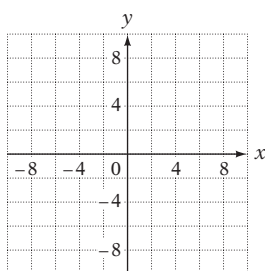
33. Find the vertices of the graph of $\frac{x^2}{36} + \frac{y^2}{16} = 1$. Then sketch the graph.



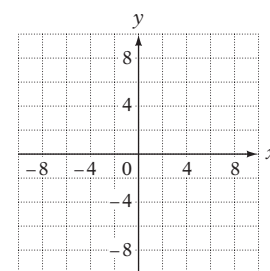
34. Find the vertices of the graph of $\frac{x^2}{1} + \frac{y^2}{25} = 1$. Then sketch the graph.



35. Find the coordinates of the vertices and the equations of the asymptotes of the graph of $\frac{x^2}{9} - \frac{y^2}{36} = 1$. Then sketch the graph.

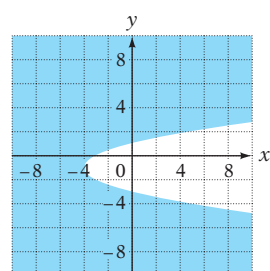


36. Find the coordinates of the vertices and the equations of the asymptotes of the graph of $\frac{y^2}{36} - \frac{x^2}{4} = 1$. Then sketch the graph.



37. Solve: $y = x^2 - 3x - 4$
 $y = x + 1$

38. Graph the solution set: $x \leq y^2 + 2y - 3$

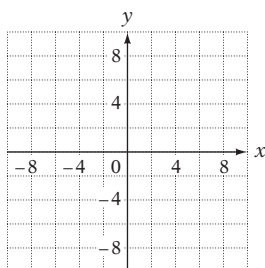


CHAPTER 11

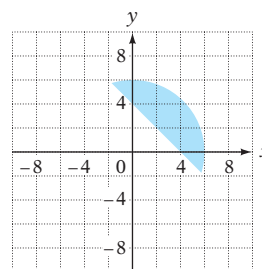
TEST

- Find the equation of the circle with radius 4 and center $(-3, -3)$.
- Find the equation of the circle whose center is $(-2, 1)$ and that passes through the point whose coordinates are $(2, 5)$.

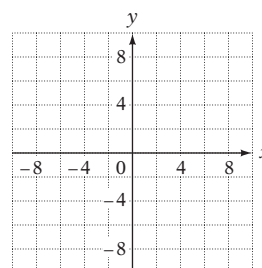
- Sketch a graph of $\frac{y^2}{25} - \frac{x^2}{16} = 1$.



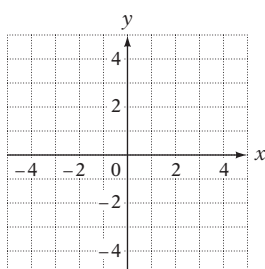
- Graph the solution set: $x^2 + y^2 < 36$
 $x + y > 4$



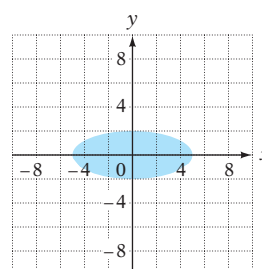
- Find the equation of the axis of symmetry of the parabola $y = -x^2 + 6x - 5$.
- Solve: $x^2 - y^2 = 24$
 $2x^2 + 5y^2 = 55$
- Find the coordinates of the vertex of the parabola $y = -x^2 + 3x - 2$.
- Sketch a graph of $x = y^2 - y - 2$.



- Sketch a graph of $\frac{x^2}{16} + \frac{y^2}{4} = 1$.



- Graph the solution set: $\frac{x^2}{25} + \frac{y^2}{4} \leq 1$



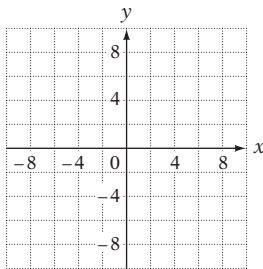
11. Find the equation of the circle with radius 3 and center $(-2, 4)$.

12. Solve: $x = 3y^2 + 2y - 4$
 $x = y^2 - 5y$

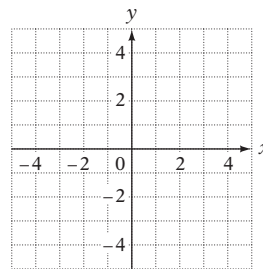
13. Solve: $x^2 + 2y^2 = 4$
 $x + y = 2$

14. Find the equation of the circle whose center is $(-1, -3)$ and that passes through the point whose coordinates are $(2, 4)$.

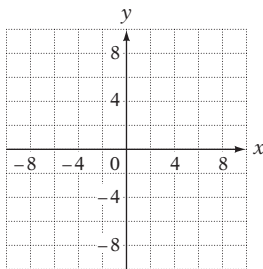
15. Graph the solution set: $\frac{x^2}{25} - \frac{y^2}{16} \geq 1$
 $x^2 + y^2 \leq 9$



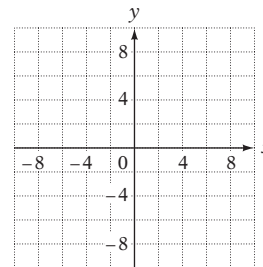
16. Write the equation $x^2 + y^2 - 4x + 2y + 1 = 0$ in standard form, and then sketch its graph.



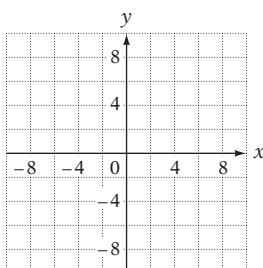
17. Sketch a graph of $y = \frac{1}{2}x^2 + x - 4$.



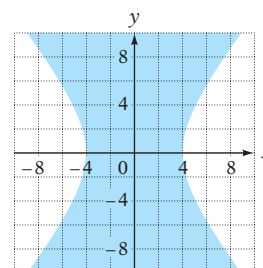
18. Sketch a graph of $(x - 2)^2 + (y + 1)^2 = 9$.



19. Sketch a graph of $\frac{x^2}{9} - \frac{y^2}{4} = 1$.



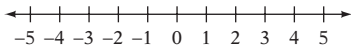
20. Graph the solution set: $\frac{x^2}{16} - \frac{y^2}{25} < 1$



CUMULATIVE REVIEW EXERCISES

1. Graph the solution set.

$$\{x|x < 4\} \cap \{x|x > 2\}$$



3. Solve: $4 + |3x + 2| < 6$

2. Solve: $\frac{5x - 2}{3} - \frac{1 - x}{5} = \frac{x + 4}{10}$

4. Find the equation of the line that contains the point $(2, -3)$ and has slope $-\frac{3}{2}$.

5. Find the equation of the line that contains the point $(4, -2)$ and is perpendicular to the line $y = -x + 5$.

6. Multiply: $3a^2b(4a - 3b^2)$

7. Factor: $(x - 1)^3 - y^3$

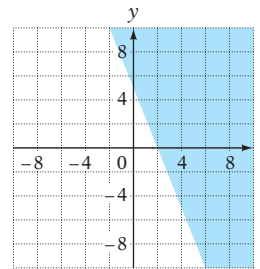
8. Solve: $\frac{3x - 2}{x + 4} \leq 1$

9. Simplify: $\frac{ax - bx}{ax + ay - bx - by}$

10. Subtract: $\frac{x - 4}{3x - 2} - \frac{1 + x}{3x^2 + x - 2}$

11. Solve: $\frac{6x}{2x - 3} - \frac{1}{2x - 3} = 7$

12. Graph the solution set: $5x + 2y > 10$



13. Simplify: $\left(\frac{12a^2b^2}{a^{-3}b^{-4}}\right)^{-1} \left(\frac{ab}{4^{-1}a^{-2}b^4}\right)^2$

14. Write $2\sqrt[4]{x^3}$ as an exponential expression.

15. Multiply: $(3 + 5i)(4 - 6i)$

16. Solve: $2x^2 + 2x - 3 = 0$

17. Solve: $x^2 + y^2 = 20$
 $x^2 - y^2 = 12$

18. Solve: $x - \sqrt{2x - 3} = 3$

19. Evaluate $f(x) = -x^2 + 3x - 2$ at $x = -3$.

20. Find the inverse of $f(x) = 4x + 8$.

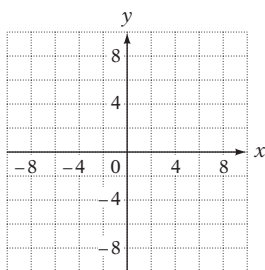
21. Find the maximum value of $f(x) = -2x^2 + 4x - 2$.

22. Find the maximum product of two numbers whose sum is 40.

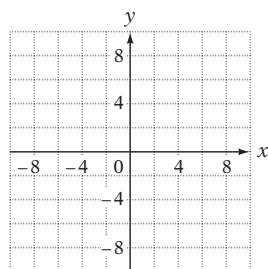
23. Find the distance between the points whose coordinates are $(2, 4)$ and $(-1, 0)$.

24. Find the equation of the circle whose center is $(-1, 2)$ and that passes through the point whose coordinates are $(3, 1)$.

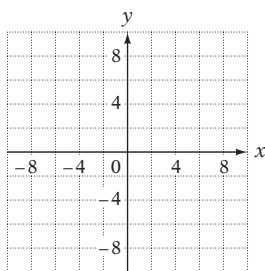
25. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $x = y^2 - 2y + 3$. Then sketch the graph.



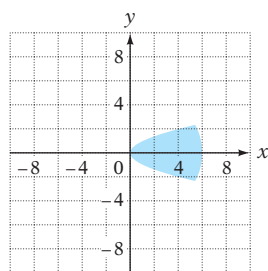
26. Find the coordinates of the vertices of the graph of $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Then sketch the graph.



27. Find the vertices and the equations of the asymptotes of the graph of $\frac{y^2}{4} - \frac{x^2}{25} = 1$. Then sketch the graph.



28. Graph the solution set:
 $(x - 1)^2 + y^2 \leq 25$
 $y^2 < x$



29. **The Arts** Tickets for a school play sold for \$12.00 for each adult and \$4.50 for each child. The total receipts for the 192 tickets sold were \$1479. Find the number of adult tickets sold.

30. **Uniform Motion** A motorcycle travels 180 mi in the same amount of time that a car travels 144 mi. The rate of the motorcycle is 12 mph greater than the rate of the car. Find the rate of the motorcycle.

31. **Sports** The rate of a river's current is 1.5 mph. A rowing crew can row 12 mi downstream and 12 mi back in 6 h. Find the rowing rate of the crew in calm water.

32. **Mechanics** The speed v of a gear varies inversely as the number of teeth t . If a gear that has 36 teeth makes 30 revolutions per minute, how many revolutions per minute will a gear that has 60 teeth make?



Sequences and Series

OBJECTIVES

SECTION 12.1

- A** To write the terms of a sequence
- B** To find the sum of a series

SECTION 12.2

- A** To find the n th term of an arithmetic sequence
- B** To find the sum of an arithmetic series
- C** To solve application problems

SECTION 12.3

- A** To find the n th term of a geometric sequence
- B** To find the sum of a finite geometric series
- C** To find the sum of an infinite geometric series
- D** To solve application problems

SECTION 12.4

- A** To expand $(a + b)^n$

ARE YOU READY?

Take the Chapter 12 Prep Test to find out if you are ready to learn to:

- Find the n th term of an arithmetic sequence and of a geometric sequence
- Find the sum of an arithmetic series
- Find the sum of a finite geometric series and of an infinite geometric series
- Expand $(a + b)^n$

PREP TEST

Do these exercises to prepare for Chapter 12.

1. Simplify:
 $[3(1) - 2] + [3(2) - 2] + [3(3) - 2]$
2. Evaluate $f(n) = \frac{n}{n+2}$ for
 $n = 6$.
3. Evaluate $a_1 + (n - 1)d$ for
 $a_1 = 2$, $n = 5$, and $d = 4$.
4. Evaluate $a_1 r^{n-1}$ for
 $a_1 = -3$, $r = -2$, and
 $n = 6$.
5. Evaluate $\frac{a_1(1 - r^n)}{1 - r}$ for
 $a_1 = -2$, $r = -4$, and $n = 5$.
6. Simplify: $\frac{\frac{4}{10}}{1 - \frac{1}{10}}$
7. Expand: $(x + y)^2$
8. Expand: $(x + y)^3$

SECTION

12.1 Introduction to Sequences and Series

OBJECTIVE A

To write the terms of a sequence

An investor deposits \$100 in an account that earns 10% interest compounded annually. The amount of interest earned each year can be determined by using the compound interest formula.

The amount of interest earned in each of the first 4 years of the investment is shown at the right.

Year	1	2	3	4
Interest Earned	\$10	\$11	\$12.10	\$13.31

The list of numbers 10, 11, 12.10, 13.31 is called a *sequence*. A **sequence** is an ordered list of numbers. The list 10, 11, 12.10, 13.31 is ordered because the position of a number in this list indicates the year in which that amount of interest was earned.

Each of the numbers of a sequence is called a **term** of the sequence.

For the sequence at the right, the first term is 2, the second term is 4, the third term is 6, and the fourth term is 8. 2, 4, 6, 8, ...

Examples of other sequences are shown at the right. These sequences are separated into two groups. A **finite sequence** contains a finite number of terms. An **infinite sequence** contains an infinite number of terms.

1, 1, 2, 3, 5, 8	} Finite Sequences
1, 2, 3, 4, 5, 6, 7, 8	
1, -1, 1, -1	
1, 3, 5, 7, ...	} Infinite Sequences
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$	
1, 1, 2, 3, 5, 8, ...	

A general sequence is shown at the right. The first term is a_1 , the second term is a_2 , the third term is a_3 , and the n th term, also called the **general term** of the sequence, is a_n . Note that each term of the sequence is paired with a natural number. $a_1, a_2, a_3, \dots, a_n, \dots$

Frequently a sequence has a definite pattern that can be expressed by a formula.

Each term of the sequence shown at the right is paired with a natural number by the formula $a_n = 3n$. The first term, a_1 , is 3. The second term, a_2 , is 6. The third term, a_3 , is 9. The n th term, a_n , is $3n$.

$$\begin{aligned}
 a_n &= 3n \\
 a_1, \quad a_2, \quad a_3, \quad &\dots, a_n, \dots \\
 3(1), 3(2), 3(3), \dots, &3(n), \dots \\
 3, \quad 6, \quad 9, \quad &\dots, 3n, \dots
 \end{aligned}$$

 Take Note

Note that a sequence is a function whose domain is the set of natural numbers.

HOW TO 1

Write the first three terms of the sequence whose n th term is given by the formula $a_n = 4n$.

$$a_n = 4n$$

$$a_1 = 4(1) = 4$$

• Replace n by 1.

$$a_2 = 4(2) = 8$$

• Replace n by 2.

$$a_3 = 4(3) = 12$$

• Replace n by 3.

The first term is 4, the second term is 8, and the third term is 12.

HOW TO • 2

Find the fifth and seventh terms of the sequence whose n th term is given by the formula $a_n = \frac{1}{n^2}$.

$$a_n = \frac{1}{n^2}$$

$$a_5 = \frac{1}{5^2} = \frac{1}{25}$$

• Replace n by 5.

$$a_7 = \frac{1}{7^2} = \frac{1}{49}$$

• Replace n by 7.

The fifth term is $\frac{1}{25}$ and the seventh term is $\frac{1}{49}$.

EXAMPLE • 1

Write the first three terms of the sequence whose n th term is given by the formula $a_n = 2n - 1$.

Solution

$$a_n = 2n - 1$$

$$a_1 = 2(1) - 1 = 1 \quad \text{The first term is 1.}$$

$$a_2 = 2(2) - 1 = 3 \quad \text{The second term is 3.}$$

$$a_3 = 2(3) - 1 = 5 \quad \text{The third term is 5.}$$

YOU TRY IT • 1

Write the first four terms of the sequence whose n th term is given by the formula $a_n = n(n + 1)$.

Your solution**EXAMPLE • 2**

Find the eighth and tenth terms of the sequence whose n th term is given by the formula $a_n = \frac{n}{n+1}$.

Solution

$$a_n = \frac{n}{n+1}$$

$$a_8 = \frac{8}{8+1} = \frac{8}{9} \quad \text{The eighth term is } \frac{8}{9}.$$

$$a_{10} = \frac{10}{10+1} = \frac{10}{11} \quad \text{The tenth term is } \frac{10}{11}.$$

YOU TRY IT • 2

Find the sixth and ninth terms of the sequence whose n th term is given by the formula $a_n = \frac{1}{n(n+2)}$.

Your solution

Solutions on p. S33

OBJECTIVE B**To find the sum of a series**

In the last objective, the sequence 10, 11, 12.10, 13.31 was shown to represent the amount of interest earned in each of 4 years of an investment.

The sum of the terms of this sequence represents the total interest earned by the investment over the four-year period.

10, 11, 12.10, 13.31

$$10 + 11 + 12.10 + 13.31 = 46.41$$

The total interest earned over the four-year period is \$46.41.

The indicated sum of the terms of a sequence is called a **series**. Given the sequence 10, 11, 12.10, 13.31, we can write the series $10 + 11 + 12.10 + 13.31$.

S_n is used to designate the sum of the first n terms of a sequence.

For the preceding example, the sums of the series S_1 , S_2 , S_3 , and S_4 represent the total interest earned for 1, 2, 3, and 4 years, respectively.

$$\begin{aligned} S_1 &= 10 & &= 10 \\ S_2 &= 10 + 11 & &= 21 \\ S_3 &= 10 + 11 + 12.10 & &= 33.10 \\ S_4 &= 10 + 11 + 12.10 + 13.31 & &= 46.41 \end{aligned}$$

For the general sequence a_1 , a_2 , a_3 , ..., a_n , the series S_1 , S_2 , S_3 , and S_n are shown at the right.

$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= a_1 + a_2 + a_3 \\ S_n &= a_1 + a_2 + a_3 + \cdots + a_n \end{aligned}$$

It is convenient to represent a series in a compact form called **summation notation**, or **sigma notation**. The Greek letter sigma, Σ , is used to indicate a sum.

The first four terms of the sequence whose n th term is given by the formula $a_n = 2n$ are 2, 4, 6, and 8. The corresponding series is shown at the right written in summation notation; it is read “the sum from 1 to 4 of $2n$.” The letter n is called the **index** of the summation.

$$\sum_{n=1}^4 2n$$

To write the terms of the series, replace n by the consecutive integers from 1 to 4.

$$\sum_{n=1}^4 2n = 2(1) + 2(2) + 2(3) + 2(4)$$

The series is $2 + 4 + 6 + 8$.

$$= 2 + 4 + 6 + 8$$

The sum of the series is 20.

$$= 20$$

HOW TO 3 Evaluate the series $\sum_{n=1}^6 n^2$.

$$\begin{aligned} \sum_{n=1}^6 n^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 \\ &= 1 + 4 + 9 + 16 + 25 + 36 \\ &= 91 \end{aligned}$$

- Replace n by 1, 2, 3, 4, 5, and 6.
- Simplify.

Take Note

The placement of the parentheses in this example is important. For instance,

$$\begin{aligned} \sum_{i=1}^3 2i - 1 &= 2(1) + 2(2) + 2(3) - 1 \\ &= 2 + 4 + 6 - 1 = 11 \end{aligned}$$

This is *not* the same result as the sum of $\sum_{i=1}^3 (2i - 1)$.

HOW TO 4 Evaluate the series $\sum_{i=1}^3 (2i - 1)$.

$$\begin{aligned} \sum_{i=1}^3 (2i - 1) &= [2(1) - 1] + [2(2) - 1] + [2(3) - 1] \\ &= 1 + 3 + 5 \\ &= 9 \end{aligned}$$

- Replace i by 1, 2, and 3.
- Simplify.

HOW TO • 5Evaluate the series $\sum_{n=3}^6 \frac{1}{2}n$.

$$\begin{aligned}\sum_{n=3}^6 \frac{1}{2}n &= \frac{1}{2}(3) + \frac{1}{2}(4) + \frac{1}{2}(5) + \frac{1}{2}(6) && \bullet \text{ Replace } n \text{ by } 3, 4, 5, \text{ and } 6. \\ &= \frac{3}{2} + 2 + \frac{5}{2} + 3 && \bullet \text{ Simplify.} \\ &= 9\end{aligned}$$

HOW TO • 6Write $\sum_{i=1}^5 x^i$ in expanded form.

This is a variable series.

$$\sum_{i=1}^5 x^i = x + x^2 + x^3 + x^4 + x^5 \quad \bullet \text{ Replace } i \text{ by } 1, 2, 3, 4, \text{ and } 5.$$

EXAMPLE • 3Evaluate the series $\sum_{n=1}^4 (7 - n)$.**Solution**

$$\begin{aligned}\sum_{n=1}^4 (7 - n) &= (7 - 1) + (7 - 2) + (7 - 3) + (7 - 4) \\ &= 6 + 5 + 4 + 3 \\ &= 18\end{aligned}$$

YOU TRY IT • 3Evaluate the series $\sum_{i=3}^6 (i^2 - 2)$.**Your solution****EXAMPLE • 4**Write $\sum_{i=1}^5 ix^i$ in expanded form.**Solution**

$$\begin{aligned}\sum_{i=1}^5 ix^i &= 1 \cdot x + 2 \cdot x^2 + 3 \cdot x^3 + 4 \cdot x^4 + 5 \cdot x^5 \\ &= x + 2x^2 + 3x^3 + 4x^4 + 5x^5\end{aligned}$$

YOU TRY IT • 4Write $\sum_{n=1}^4 \frac{x^n}{n}$ in expanded form.**Your solution**

12.1 EXERCISES

OBJECTIVE A

To write the terms of a sequence



1. What is a sequence?



2. What is the difference between a finite sequence and an infinite sequence?

3. Name the third term in the sequence
2, 5, 8, 11, 14, ...4. Name the fourth term in the sequence
1, 2, 4, 8, 16, 32, ...

For Exercises 5 to 16, write the first four terms of the sequence whose n th term is given by the formula.

5. $a_n = n + 1$

6. $a_n = n - 1$

7. $a_n = 2n + 1$

8. $a_n = 3n - 1$

9. $a_n = 2 - 2n$

10. $a_n = 1 - 2n$

11. $a_n = 2^n$

12. $a_n = 3^n$

13. $a_n = n^2 + 1$

14. $a_n = n - \frac{1}{n}$

15. $a_n = n^2 - \frac{1}{n}$

16. $a_n = (-1)^{n+1}n$

For Exercises 17 to 28, find the indicated term of the sequence whose n th term is given by the formula.

17. $a_n = 3n + 4; a_{12}$

18. $a_n = 2n - 5; a_{10}$

19. $a_n = n(n - 1); a_{11}$

20. $a_n = \frac{n}{n+1}; a_{12}$

21. $a_n = (-1)^{n-1}n^2; a_{15}$

22. $a_n = (-1)^{n-1}(n - 1); a_{25}$

23. $a_n = \left(\frac{1}{2}\right)^n; a_8$

24. $a_n = \left(\frac{2}{3}\right)^n; a_5$

25. $a_n = (n + 2)(n + 3); a_{17}$

26. $a_n = (n + 4)(n + 1); a_7$

27. $a_n = \frac{(-1)^{2n-1}}{n^2}; a_6$

28. $a_n = \frac{(-1)^{2n}}{n + 4}; a_{16}$



29. Write a formula for the sequence of the squares of the natural numbers.



30. Write a formula for the sequence of the reciprocals of the natural numbers.

OBJECTIVE B**To find the sum of a series**

For Exercises 31 to 42, find the sum of the series.

31. $\sum_{n=1}^5 (2n + 3)$

32. $\sum_{i=1}^7 (i + 2)$

33. $\sum_{i=1}^4 2i$

34. $\sum_{n=1}^7 n$

35. $\sum_{i=1}^6 i^2$

36. $\sum_{i=1}^5 (i^2 + 1)$

37. $\sum_{n=1}^6 (-1)^n$

38. $\sum_{n=1}^4 \frac{1}{2n}$

39. $\sum_{i=3}^6 i^3$

40. $\sum_{i=1}^4 (-1)^{i-1}(i + 1)$

41. $\sum_{n=3}^5 \frac{(-1)^{n-1}}{n - 2}$

42. $\sum_{n=4}^7 \frac{(-1)^{n-1}}{n - 3}$

For Exercises 43 to 51, write the series in expanded form.

43. $\sum_{n=1}^5 2x^n$

44. $\sum_{n=1}^4 \frac{2n}{x^n}$

45. $\sum_{i=1}^5 \frac{x^i}{i}$

46. $\sum_{i=2}^4 \frac{x^i}{2i - 1}$

47. $\sum_{n=1}^5 x^{2n}$

48. $\sum_{n=1}^4 x^{2n-1}$

49. $\sum_{i=1}^4 \frac{x^i}{i^2}$

50. $\sum_{n=1}^4 nx^{n-1}$

51. $\sum_{i=1}^5 x^{-i}$



52. Write the sum of the squares of the first n natural numbers using sigma notation.



53. Write the square of the sum of the first n natural numbers using sigma notation.

Applying the Concepts

54. A **recursive sequence** is one in which successive terms are defined as some combination of previous terms. For instance, suppose $a_1 = 1$ and $a_n = na_{n-1}$. Then

$$\begin{aligned} a_n &= na_{n-1} \\ a_2 &= 2a_{2-1} = 2a_1 = 2 \cdot 1 = 2 & \bullet \ n = 2, a_1 = 1 \\ a_3 &= 3a_{3-1} = 3a_2 = 3 \cdot 2 = 6 & \bullet \ n = 3, a_2 = 2 \\ a_4 &= 4a_{4-1} = 4a_3 = 4 \cdot 6 = 24 & \bullet \ n = 4, a_3 = 6 \end{aligned}$$

Find the next two terms of this recursive sequence.



55. The Fibonacci Sequence is a recursive sequence. (See Exercise 54.) Write a paragraph explaining this sequence, and give the first six terms of the sequence.



Fibonacci

SECTION

12.2 Arithmetic Sequences and Series

OBJECTIVE A

To find the n th term of an arithmetic sequence

A company's expenses for training a new employee are quite high. To encourage employees to continue their employment with the company, a company that has a six-month training program offers a starting salary of \$2400 per month and then a \$200-per-month pay increase each month during the training period.

The sequence at the right shows the employee's monthly salaries during the training period. Each term of the sequence is found by adding \$200 to the preceding term.

Month	1	2	3	4	5	6
Salary	2400	2600	2800	3000	3200	3400

Take Note

Arithmetic sequences are a special type of sequence, one in which the difference between any two successive terms is the same constant. For instance, 5, 10, 15, 20, 25, ... is an arithmetic sequence. The difference between any two successive terms is 5. The sequence 1, 4, 9, 16, ... is not an arithmetic sequence because $4 - 1 \neq 9 - 4$.

The sequence 2400, 2600, 2800, 3000, 3200, 3400 is called an *arithmetic sequence*. An **arithmetic sequence**, or **arithmetic progression**, is one in which the difference between any two consecutive terms is constant. The difference between consecutive terms is called the **common difference** of the sequence.

Each of the sequences shown at the right is an arithmetic sequence. To find the common difference of an arithmetic sequence, subtract the first term from the second term.

$$2, 7, 12, 17, 22, \dots$$

$$\text{Common difference: } 5$$

$$3, 1, -1, -3, -5, \dots$$

$$\text{Common difference: } -2$$

$$1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, \dots$$

$$\text{Common difference: } \frac{1}{2}$$

Consider an arithmetic sequence in which the first term is a_1 and the common difference is d . Adding the common difference to each successive term of the arithmetic sequence yields a formula for the n th term.

The first term is a_1 .

$$a_1 = a_1$$

To find the second term, add the common difference d to the first term.

$$a_2 = a_1 + d$$

To find the third term, add the common difference d to the second term.

$$\begin{aligned} a_3 &= a_2 + d = (a_1 + d) + d \\ a_3 &= a_1 + 2d \end{aligned}$$

To find the fourth term, add the common difference d to the third term.

$$\begin{aligned} a_4 &= a_3 + d = (a_1 + 2d) + d \\ a_4 &= a_1 + 3d \end{aligned}$$

Note the relationship between the term number and the number that multiplies d . The multiplier of d is 1 less than the term number.

$$a_n = a_1 + (n - 1)d$$



Tips for Success

For your reference, all the formulas presented in this chapter are in the Chapter 12 Summary at the end of this chapter.

The Formula for the n th Term of an Arithmetic Sequence

The n th term of an arithmetic sequence with a common difference of d is given by

$$a_n = a_1 + (n - 1)d.$$

HOW TO • 1Find the 27th term of the arithmetic sequence $-4, -1, 2, 5, 8, \dots$.Find the common difference, d . This is the difference between any two successive terms of the arithmetic sequence. We will use terms 1 and 2.

$$\begin{aligned} d &= a_2 - a_1 && \bullet \text{ Find the common difference.} \\ &= -1 - (-4) = 3 && \bullet a_1 = -4, a_2 = -1 \end{aligned}$$

To find the 27th term, use the Formula for the n th Term of an Arithmetic Sequence.

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ a_{27} &= -4 + (27 - 1)(3) && \bullet a_1 = -4, n = 27, d = 3 \\ a_{27} &= -4 + 26(3) = -4 + 78 \\ a_{27} &= 74 \end{aligned}$$

HOW TO • 2Find the formula for the n th term of the arithmetic sequence $2, -3, -8, -13, \dots$.

$$\begin{aligned} d &= a_2 - a_1 && \bullet \text{ Find the common difference.} \\ &= -3 - 2 = -5 && \bullet a_2 = -3, a_1 = 2 \end{aligned}$$

Use the Formula for the n th Term of an Arithmetic Sequence.

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ a_n &= 2 + (n - 1)(-5) && \bullet a_1 = 2, d = -5 \\ a_n &= 2 - 5n + 5 \\ a_n &= -5n + 7 \end{aligned}$$

HOW TO • 3Find the number of terms in the finite arithmetic sequence $2, 5, 8, 11, \dots, 44$.

$$d = a_2 - a_1 = 5 - 2 = 3 \quad \bullet \text{ Find the common difference. } a_2 = 5, a_1 = 2$$

Solve the Formula for the n th Term of the Arithmetic Sequence for n .

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ 44 &= 2 + (n - 1)(3) && \bullet a_n = 44, a_1 = 2, d = 3 \\ 44 &= 2 + 3n - 3 && \bullet \text{ Solve for } n. \\ 44 &= 3n - 1 \\ 45 &= 3n \\ 15 &= n \end{aligned}$$

The sequence has 15 terms.

EXAMPLE • 1Find the 10th term of the arithmetic sequence $1, 6, 11, 16, \dots$.**Solution**

$$\begin{aligned} d &= a_2 - a_1 = 6 - 1 = 5 && \bullet \text{ Find } d. \\ a_n &= a_1 + (n - 1)d \\ a_{10} &= 1 + (10 - 1)5 && \bullet \text{ Find } a_{10}. \\ &= 1 + (9)5 \\ &= 1 + 45 = 46 \end{aligned}$$

YOU TRY IT • 1Find the number of terms in the finite arithmetic sequence $7, 9, 11, \dots, 59$.**Your solution**

OBJECTIVE B**To find the sum of an arithmetic series**

The indicated sum of the terms of an arithmetic sequence is called an **arithmetic series**. The sum of an arithmetic series can be found by using a formula. A proof of this formula appears in the Appendix.

**Point of Interest**

This formula was written in *Aryabhatiya*, which was written by Aryabhata around 499. The book is the earliest known Indian mathematical work by an identifiable author. Although the proof of the formula appears in that text, the formula was known before Aryabhata's time.

The Formula for the Sum of n Terms of an Arithmetic Series

Let a_1 be the first term of a finite arithmetic sequence, let n be the number of terms of the sequence, and let a_n be the last term of the sequence.

Then the sum of the series S_n is given by $S_n = \frac{n}{2}(a_1 + a_n)$.

HOW TO • 4

Find the sum of the first 10 terms of the arithmetic sequence

$$2, 4, 6, 8, \dots$$

$$d = a_2 - a_1 = 4 - 2 = 2$$

$$a_n = a_1 + (n - 1)d$$

$$a_{10} = 2 + (10 - 1)2$$

$$a_{10} = 2 + (9)2 = 20$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{10} = \frac{10}{2}(2 + 20) = 5(22) = 110$$

- Find the common difference.
- Find the 10th term using the Formula for the n th Term of an Arithmetic Sequence.
 $n = 10, a_1 = 2, d = 2$
- Use the Formula for the Sum of n Terms of an Arithmetic Series. $n = 10, a_1 = 2, a_n = 20$

HOW TO • 5

Find the sum of the arithmetic series $\sum_{n=1}^{25} (3n + 1)$.

When written in sigma notation, a_n is the expression following the summation symbol, Σ .

$$a_n = 3n + 1$$

$$a_1 = 3(1) + 1 = 4$$

$$a_{25} = 3(25) + 1 = 76$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{25} = \frac{25}{2}(4 + 76) = \frac{25}{2}(80) = 1000$$

- Find the 1st term.
- Find the 25th term.
- Use the Formula for the Sum of n Terms of an Arithmetic Series. $n = 25, a_1 = 4, a_n = 76$

EXAMPLE • 2

Find the sum of the first 20 terms of the arithmetic sequence 3, 8, 13, 18, ...

Solution

$$d = a_2 - a_1 = 8 - 3 = 5$$

- Find d .

$$a_n = a_1 + (n - 1)d$$

$$a_{20} = 3 + (20 - 1)5 = 3 + (19)5$$

- Find a_{20} .

$$= 3 + 95 = 98$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{20} = \frac{20}{2}(3 + 98) = 10(101)$$

- Use the Formula for the Sum of n Terms of an Arithmetic Sequence.

$$= 1010$$

YOU TRY IT • 2

Find the sum of the first 25 terms of the arithmetic sequence $-4, -2, 0, 2, 4, \dots$

Your solution

EXAMPLE • 3

Evaluate the arithmetic series $\sum_{n=1}^{15} (2n + 3)$.

Solution

$$a_n = 2n + 3$$

$$a_1 = 2(1) + 3 = 5$$

$$a_{15} = 2(15) + 3 = 33$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\begin{aligned} S_{15} &= \frac{15}{2}(5 + 33) = \frac{15}{2}(38) \\ &= 285 \end{aligned}$$

- Find a_1 .
- Find a_{15} .
- Use the Formula for the Sum of n Terms of an Arithmetic Series.

YOU TRY IT • 3

Evaluate the arithmetic series $\sum_{n=1}^{18} (3n - 2)$.

Your solution

Solution on p. S33

OBJECTIVE C

To solve application problems

**EXAMPLE • 4**

The distance a ball rolls down a ramp each second is given by an arithmetic sequence. The distance in feet traveled by the ball during the n th second is given by $2n - 1$. Find the distance the ball will travel during the 10th second.

Strategy

To find the distance:

- Find the 1st and 2nd terms.
- Find the common difference of the arithmetic sequence.
- Use the Formula for the n th Term of an Arithmetic Sequence.

Solution

$$a_n = 2n - 1$$

$$a_1 = 2(1) - 1 = 1$$

$$a_2 = 2(2) - 1 = 3$$

$$d = a_2 - a_1 = 3 - 1 = 2$$

$$a_n = a_1 + (n - 1)d$$

$$\begin{aligned} a_{10} &= 1 + (10 - 1)2 = 1 + (9)2 \\ &= 1 + 18 = 19 \end{aligned}$$

- The given sequence
- Find a_1 .
- Find a_2 .
- Find d .
- Use the formula.

The ball will travel 19 ft during the 10th second.

YOU TRY IT • 4

A contest offers 20 prizes. The first prize is \$10,000, and each successive prize is \$300 less than the preceding prize. What is the value of the 20th-place prize? What is the total amount of prize money being awarded?

Your strategy**Your solution**

Solution on pp. S33–S34

12.2 EXERCISES**OBJECTIVE A****To find the n th term of an arithmetic sequence**

For Exercises 1 to 9, find the indicated term of the arithmetic sequence.

1. $1, 11, 21, \dots; a_{15}$

2. $3, 8, 13, \dots; a_{20}$

3. $-6, -2, 2, \dots; a_{15}$

4. $-7, -2, 3, \dots; a_{14}$

5. $2, \frac{5}{2}, 3, \dots; a_{31}$

6. $1, \frac{5}{4}, \frac{3}{2}, \dots; a_{17}$

7. $-4, -\frac{5}{2}, -1, \dots; a_{12}$

8. $-\frac{5}{3}, -1, -\frac{1}{3}, \dots; a_{22}$

9. $8, 5, 2, \dots; a_{40}$

For Exercises 10 to 15, find the formula for the n th term of the arithmetic sequence.

10. $1, 2, 3, \dots$

11. $1, 4, 7, \dots$

12. $6, 2, -2, \dots$

13. $3, 0, -3, \dots$

14. $2, \frac{7}{2}, 5, \dots$

15. $7, 4.5, 2, \dots$

For Exercises 16 to 24, find the number of terms in the finite arithmetic sequence.

16. $1, 5, 9, \dots, 81$

17. $3, 8, 13, \dots, 98$

18. $2, 0, -2, \dots, -56$

19. $1, -3, -7, \dots, -75$

20. $\frac{5}{2}, 3, \frac{7}{2}, \dots, 13$

21. $\frac{7}{3}, \frac{13}{3}, \frac{19}{3}, \dots, \frac{79}{3}$

22. $1, 0.75, 0.50, \dots, -4$

23. $3.5, 2, 0.5, \dots, -25$

24. $-3.4, -2.8, -2.2, \dots, 11$

 In Exercises 25 to 28, state whether the general term of the sequence produces an arithmetic sequence.

25. $a_n = n^2$

26. $a_n = 2n$

27. $a_n = 3 - 5n$

28. $a_n = \frac{1}{n}$

OBJECTIVE B**To find the sum of an arithmetic series**

For Exercises 29 to 34, find the sum of the indicated number of terms of the arithmetic sequence.

29. $1, 3, 5, \dots; n = 50$

30. $2, 4, 6, \dots; n = 25$

31. $20, 18, 16, \dots; n = 40$

32. $25, 20, 15, \dots; n = 22$

33. $\frac{1}{2}, 1, \frac{3}{2}, \dots; n = 27$

34. $2, \frac{11}{4}, \frac{7}{2}, \dots; n = 10$

For Exercises 35 to 40, evaluate the arithmetic series.

35. $\sum_{i=1}^{15} (3i - 1)$

36. $\sum_{i=1}^{15} (3i + 4)$

37. $\sum_{n=1}^{17} \left(\frac{1}{2}n + 1 \right)$

38. $\sum_{n=1}^{10} (1 - 4n)$

39. $\sum_{i=1}^{15} (4 - 2i)$

40. $\sum_{n=1}^{10} (5 - n)$



41. Give a formula for the sum of the first n odd natural numbers.



42. Give a formula for the sum of the first n even natural numbers.

OBJECTIVE C

To solve application problems

43. **Exercise** An exercise program calls for walking 10 min each day for a week. Each week thereafter, the amount of time spent walking increases by 5 min per day. In how many weeks will a person on the program be walking 60 min each day?
44. **Product Displays** A display of cans in a grocery store consists of 24 cans in the bottom row, 21 cans in the next row, and so on in an arithmetic sequence. The top row has 3 cans. Find the number of cans in the display.
45. **The Arts** The loge seating section in a concert hall consists of 27 rows of chairs. There are 73 seats in the first row, 79 seats in the second row, 85 seats in the third row, and so on in an arithmetic sequence. How many seats are in the loge seating section?



Exercise Exercise physiologists suggest that reducing a person's weight by 1 lb requires burning approximately 3500 calories. Use this information and the article at the right for Exercises 46 and 47.

46. Suppose the initial weight of a person is 200 lb. Write a formula for the weight of this person after n hours of jogging at 5 mph.

(Hint: $\frac{728 \text{ calories}}{1 \text{ h}} \cdot \frac{1 \text{ lb}}{3500 \text{ calories}} = 0.208 \text{ lb/h}$)

47. Suppose the initial weight of a person is 200 lb. Find the number of hours of running it will take for this person to reach a weight of 186 lb.



48. A culture of bacteria is doubling every hour. Can the number of bacteria in the culture after each hour be represented by an arithmetic sequence?

Applying the Concepts

49. Write $\sum_{i=1}^2 \log 2i$ as a single logarithm with a coefficient of 1.

50. State whether the following statements are true or false.

a. If a_n is the general term of an arithmetic sequence, then $\sum_{i=1}^6 a_i < \sum_{i=1}^{10} a_i$.

b. Each successive term in an arithmetic series is found by adding a fixed number.

In the News

Exercise Essential for Weight Loss

Being active—either through casual physical activity or through a formal exercise program—is an essential component of a weight-loss program. When you're active, your body uses energy (calories) to do work, helping to burn the calories you take in from the food you eat. The table below shows the number of calories burned per hour for a 200-pound person.

Exercise	Calories burned per hour
Walking, 3.5 mph	346
Jogging, 5 mph	728
Running, 8 mph	1225

Source: www.mayoclinic.com

SECTION

12.3 Geometric Sequences and Series

OBJECTIVE A

To find the n th term of a geometric sequence 

An ore sample contains 20 mg of a radioactive material with a half-life of 1 week. The amount of the radioactive material the sample contains at the beginning of each week can be determined by using an exponential decay equation.

The sequence at the right shows the amount in the sample at the beginning of each week. Each term of the sequence is found by multiplying the preceding term by $\frac{1}{2}$.

Week	1	2	3	4	5
Amount	20	10	5	2.5	1.25

The sequence 20, 10, 5, 2.5, 1.25 is called a *geometric sequence*. A **geometric sequence**, or **geometric progression**, is one in which each successive term of the sequence is the same nonzero constant multiple of the preceding term. The common multiple is called the **common ratio** of the sequence.

**Take Note**

Geometric sequences are different from arithmetic sequences. For a geometric sequence, every two successive terms have the same *ratio*. For an arithmetic sequence, every two successive terms have the same *difference*.

Each sequence shown at the right is a geometric sequence. To find the common ratio of a geometric sequence, divide the second term of the sequence by the first term.

3, 6, 12, 24, 48, ...

Common ratio: 2

4, -12, 36, -108, 324, ...

Common ratio: -3

6, 4, $\frac{8}{3}$, $\frac{16}{9}$, $\frac{32}{27}$, ...Common ratio: $\frac{2}{3}$

Consider a geometric sequence in which the first term is a_1 and the common ratio is r . Multiplying each successive term of the geometric sequence by the common ratio yields a formula for the n th term.

The first term is a_1 .

$$a_1 = a_1$$

To find the second term, multiply the first term by the common ratio r .

$$a_2 = a_1 r$$

To find the third term, multiply the second term by the common ratio r .

$$a_3 = (a_1 r) r$$

$$a_3 = a_1 r^2$$

To find the fourth term, multiply the third term by the common ratio r .

$$a_4 = (a_1 r^2) r$$

$$a_4 = a_1 r^3$$

Note the relationship between the term number and the number that is the exponent on r . The exponent on r is 1 less than the term number.

$$a_n = a_1 r^{n-1}$$

The Formula for the n th Term of a Geometric Sequence

The n th term of a geometric sequence with first term a_1 and common ratio r is given by

$$a_n = a_1 r^{n-1}.$$

**Integrating Technology**

See the Keystroke Guide: *Sequences and Series* for instructions on using a graphing calculator to display the terms of a sequence.

HOW TO • 1

Find the 6th term of the geometric sequence 3, 6, 12, ...

Find the common ratio r . This is the quotient of any two successive terms of the geometric sequence. We will use terms 1 and 2.

$$r = \frac{a_2}{a_1} = \frac{6}{3} = 2 \quad \bullet \ a_1 = 3, a_2 = 6$$

To find the 6th term, use the Formula for the n th Term of a Geometric Sequence.

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ a_6 &= 3(2)^{6-1} = 3(2)^5 \quad \bullet \ a_1 = 3, n = 6, r = 2 \\ a_6 &= 96 \end{aligned}$$

HOW TO • 2

Find a_3 for the geometric sequence 8, a_2 , a_3 , -27 , ...

To find the common ratio r , use the Formula for the n th Term of a Geometric Sequence. The first term, 8, and the fourth term, -27 , are known; $n = 4$.

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ a_4 &= a_1 r^{4-1} \quad \bullet \ n = 4 \\ -27 &= 8r^3 \quad \bullet \ a_1 = 8, a_4 = -27 \\ -\frac{27}{8} &= r^3 \quad \bullet \ \text{Solve for } r^3. \\ -\frac{3}{2} &= r \quad \bullet \ \text{Take the cube root of each side.} \end{aligned}$$

To find the 3rd term, use the formula for the n th Term of a Geometric Sequence.

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ a_3 &= 8 \left(-\frac{3}{2} \right)^{3-1} = 8 \left(-\frac{3}{2} \right)^2 = 8 \left(\frac{9}{4} \right) \quad \bullet \ a_1 = 8, r = -\frac{3}{2}, n = 3 \\ a_3 &= 18 \end{aligned}$$

EXAMPLE • 1

Find the 7th term of the geometric sequence 3, -6 , 12, ...

Solution

Find the common ratio r .

$$r = \frac{a_2}{a_1} = \frac{-6}{3} = -2 \quad \bullet \ a_1 = 3, a_2 = -6$$

To find the 7th term, use the Formula for the n th Term of a Geometric Sequence.

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ a_7 &= 3(-2)^{7-1} \quad \bullet \ a_1 = 3, n = 7, r = -2 \\ a_7 &= 3(-2)^6 = 3(64) \\ a_7 &= 192 \end{aligned}$$

YOU TRY IT • 1

Find the 5th term of the geometric sequence 5, 2, $\frac{4}{5}$, ...

Your solution

EXAMPLE • 2

Find a_2 for the geometric sequence 2, a_2 , a_3 , 54, ...

Solution

Find the common ratio r .

$$a_n = a_1 r^{n-1}$$

$$a_4 = a_1 r^{4-1} \quad \bullet \quad n = 4$$

$$54 = 2r^3 \quad \bullet \quad a_1 = 2, a_4 = 54$$

$$27 = r^3 \quad \bullet \quad \text{Solve for } r^3.$$

$$3 = r \quad \bullet \quad \text{The common ratio is 3.}$$

To find the 2nd term, use the Formula for the n th Term of a Geometric Sequence.

$$a_n = a_1 r^{n-1}$$

$$a_2 = 2(3)^{2-1} = 2(3) \quad \bullet \quad a_1 = 2, r = 3, n = 2$$

$$a_2 = 6$$

YOU TRY IT • 2

Find a_3 for the geometric sequence 3, a_2 , a_3 , -192, ...

Your solution

Solution on p. S34

OBJECTIVE B**To find the sum of a finite geometric series**

The indicated sum of the terms of a geometric sequence is called a **geometric series**. The sum of a geometric series can be found by a formula. A proof of this formula appears in the Appendix.

**Point of Interest**

Geometric series are used extensively in the mathematics of finance. Finite geometric series are used to calculate loan balances and monthly payments for amortized loans.

The Formula for the Sum of n Terms of a Finite Geometric Series

Let a_1 be the first term of a finite geometric sequence, let n be the number of terms of the

sequence, and let r be the common ratio. Then the sum of the series S_n is given by $S_n = \frac{a_1(1 - r^n)}{1 - r}$.

HOW TO • 3

Find the sum of the terms of the geometric sequence 2, 8, 32, 128, 512.

$$r = \frac{a_2}{a_1} = \frac{8}{2} = 4$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_5 = \frac{2(1 - 4^5)}{1 - 4} = \frac{2(1 - 1024)}{-3}$$

$$S_5 = \frac{2(-1023)}{-3} = \frac{-2046}{-3}$$

$$S_5 = 682$$

• Find the common ratio.

• Use the Formula for the Sum of n Terms of a Finite Geometric Series. $n = 5$, $a_1 = 2$, $r = 4$



Integrating Technology

See the Keystroke Guide: *Sequences and Series* for instructions on using a graphing calculator to find the sum of a series.

HOW TO • 4

Find the sum of the geometric series $\sum_{n=1}^{10} 3(4^{n-1})$.

When a series is written using sigma notation, the expression following Σ is a_n , the general term of the sequence. Therefore, $a_n = 3(4^{n-1})$. Use the Formula for the n th Term of a Geometric Sequence to find r , the common ratio.

$$a_n = 3(4^{n-1})$$

$$a_1 = 3(4^{1-1}) = 3(4^0) = 3 \cdot 1 = 3$$

• Find the 1st term.

$$a_2 = 3(4^{2-1}) = 3(4^1) = 3 \cdot 4 = 12$$

• Find the 2nd term.

$$r = \frac{a_2}{a_1} = \frac{12}{3} = 4$$

• Find the common ratio r .

Now use the Formula for the Sum of n Terms of a Finite Geometric Series.

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_{10} = \frac{3(1 - 4^{10})}{1 - 4}$$

• $n = 10, a_1 = 3, r = 4$

$$S_{10} = \frac{3(1 - 1,048,576)}{-3} = \frac{3(-1,048,575)}{-3}$$

$$S_{10} = 1,048,575$$

EXAMPLE • 3

Find the sum of the terms of the geometric sequence 3, 6, 12, 24, 48, 96.

Solution

$$r = \frac{a_2}{a_1} = \frac{6}{3} = 2$$

• Find r . $a_2 = 6, a_1 = 3$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

• Find the sum.

$$S_6 = \frac{3(1 - 2^6)}{1 - 2} = \frac{3(1 - 64)}{-1} = -3(-63) = 189$$

YOU TRY IT • 3

Find the sum of the terms of the geometric sequence $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$.

Your solution

EXAMPLE • 4

Find the sum of the geometric series $\sum_{n=1}^4 5^n$.

Solution

$$a_n = 5^n$$

$$a_1 = 5^1 = 5$$

• Find a_1 .

$$a_2 = 5^2 = 25$$

• Find a_2 .

$$r = \frac{a_2}{a_1} = \frac{25}{5} = 5$$

• Find r .

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

• Find the sum.

$$S_4 = \frac{5(1 - 5^4)}{1 - 5} = \frac{5(1 - 625)}{-4} = \frac{5(-624)}{-4} = 780$$

YOU TRY IT • 4

Find the sum of the geometric series $\sum_{n=1}^5 \left(\frac{1}{2}\right)^n$.

Your solution

OBJECTIVE C

To find the sum of an infinite geometric series



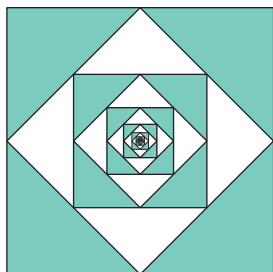
Point of Interest

The midpoints of the sides of each square in the figure below are connected to form the next smaller square. By the Pythagorean Theorem, if a side of one square has length s , then a side of the next smaller square has

length $\frac{\sqrt{2}}{2}s$. Therefore, the

sides of the consecutively smaller and smaller squares form a sequence in which

$$s_n = \frac{\sqrt{2}}{2} s_{n-1}.$$



It is from applications such as this that *geometric* sequences got their name.

When the absolute value of the common ratio of a geometric sequence is less than 1, $|r| < 1$, then as n becomes larger, r^n becomes closer to zero.

Examples of geometric sequences for which $|r| < 1$ are shown at the right. Note that as the number of terms increases, the value of the last term listed gets closer to zero.

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}, \dots$$

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \dots$$

The indicated sum of the terms of an infinite geometric sequence is called an **infinite geometric series**.

An example of an infinite geometric series is shown at the right. The first term is 1. The common ratio is $\frac{1}{3}$.

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \dots$$

The sums of the first 5, 7, 12, and 15 terms, along with the values of r^n , are shown at the right. Note that as n increases, the sum of the terms gets closer to 1.5 and the value of r^n gets closer to zero.

n	S_n	r^n
5	1.4938272	0.0041152
7	1.4993141	0.0004572
12	1.4999972	0.0000019
15	1.4999999	0.0000001

Using the Formula for the Sum of n Terms of a Finite Geometric Series and the fact that r^n approaches zero when $|r| < 1$ and n increases, we can find a formula for the sum of an infinite geometric series.

The sum of the first n terms of a geometric series is shown at the right. If $|r| < 1$, then r^n can be made very close to zero by using larger and larger values of n . Therefore, the sum of the first n terms is approximately $\frac{a_1}{1-r}$.

$$S_n = \frac{a_1(1 - \overset{\text{approximately zero}}{r^n})}{1 - r}$$

$$S_n \approx \frac{a_1(1 - 0)}{1 - r} = \frac{a_1}{1 - r}$$

The Formula for the Sum of an Infinite Geometric Series

The sum of an infinite geometric series in which $|r| < 1$ and a_1 is the first term is given

$$\text{by } S = \frac{a_1}{1 - r}.$$

When $|r| > 1$, the infinite geometric series does not have a sum. For example, the sum of the infinite geometric series $1 + 2 + 4 + 8 + \dots$ increases without limit.

HOW TO • 5

Find the sum of the terms of the infinite geometric sequence

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots$$

The common ratio is $-\frac{1}{2}$. $\left|-\frac{1}{2}\right| < 1$.

$$S = \frac{a_1}{1-r} = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

• Use the Formula for the Sum of an Infinite Geometric Series.

The Formula for the Sum of an Infinite Geometric Series can be used to find an equivalent fraction for a repeating decimal.

Consider the repeating decimal $0.3333\dots$. This decimal can be written as the sum of an infinite geometric sequence.

$$0.333\dots = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots, r = \frac{\frac{3}{100}}{\frac{3}{10}} = \frac{1}{10}$$

$$S = \frac{a_1}{1-r} = \frac{\frac{3}{10}}{1-\frac{1}{10}} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{3}{9} = \frac{1}{3}$$

$$0.333\dots = \frac{1}{3}$$

Here is an example in which the repeating digits do not begin immediately.

HOW TO • 6Find an equivalent fraction for $0.47272\dots$

$$0.4\overline{7272} = 0.4 + 0.072 + 0.00072 + \dots$$

$$= \frac{4}{10} + \frac{72}{1000} + \frac{72}{100,000} + \dots, r = \frac{\frac{72}{100,000}}{\frac{72}{1000}} = \frac{1}{100}$$

Use the Formula for the Sum of an Infinite Geometric Series.

$$S = \frac{a_1}{1-r} = \frac{\frac{72}{1000}}{1-\frac{1}{100}} = \frac{\frac{72}{1000}}{\frac{99}{100}} = \frac{72}{990} = \frac{4}{55}$$

$$0.4\overline{7272} = \frac{4}{10} + \frac{4}{55} = \frac{26}{55}$$

An equivalent fraction is $\frac{26}{55}$.

EXAMPLE • 5

Find the sum of the terms of the infinite geometric sequence $2, \frac{1}{2}, \frac{1}{8}, \dots$

Solution

$$r = \frac{a_2}{a_1} = \frac{\frac{1}{2}}{2} = \frac{1}{4} \quad \bullet \text{ Find } r. a_2 = \frac{1}{2}, a_1 = 2$$

$$S = \frac{a_1}{1 - r} = \frac{2}{1 - \frac{1}{4}} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$$

YOU TRY IT • 5

Find the sum of the terms of the infinite geometric sequence $3, -2, \frac{4}{3}, -\frac{8}{9}, \dots$

Your solution**EXAMPLE • 6**

Find an equivalent fraction for $0.3\overline{636}$.

Solution

$$0.3\overline{636} = 0.36 + 0.0036 + 0.000036 + \dots$$

$$= \frac{36}{100} + \frac{36}{10,000} + \frac{36}{1,000,000} + \dots$$

$$S = \frac{a_1}{1 - r} = \frac{\frac{36}{100}}{1 - \frac{1}{100}} = \frac{\frac{36}{100}}{\frac{99}{100}} = \frac{36}{99} = \frac{4}{11}$$

An equivalent fraction is $\frac{4}{11}$.

YOU TRY IT • 6

Find an equivalent fraction for $0.6\overline{6}$.

Your solution**EXAMPLE • 7**

Find an equivalent fraction for $0.23\overline{45}$.

Solution

$$0.23\overline{45} = 0.23 + 0.0045 + 0.000045 + \dots$$

$$= \frac{23}{100} + \frac{45}{10,000} + \frac{45}{1,000,000} + \dots$$

$$S = \frac{a_1}{1 - r} = \frac{\frac{45}{10,000}}{1 - \frac{1}{100}} = \frac{\frac{45}{10,000}}{\frac{99}{100}} = \frac{45}{9900}$$

$$0.23\overline{45} = \frac{23}{100} + \frac{45}{9900} = \frac{2277}{9900} + \frac{45}{9900}$$

$$= \frac{2322}{9900} = \frac{129}{550}$$

An equivalent fraction is $\frac{129}{550}$.

YOU TRY IT • 7

Find an equivalent fraction for $0.34\overline{21}$.

Your solution

OBJECTIVE D**To solve application problems****EXAMPLE • 8**

On the first swing, the length of the arc through which a pendulum swings is 16 in. The length of each successive swing is $\frac{7}{8}$ the length of the preceding swing. Find the length of the arc on the fifth swing. Round to the nearest tenth.

Strategy

To find the length of the arc on the fifth swing, use the Formula for the n th Term of a Geometric Sequence.

Solution

$$a_n = a_1 r^{n-1} \quad \bullet \ n = 5, a_1 = 16, r = \frac{7}{8}$$

$$a_5 = 16 \left(\frac{7}{8} \right)^{5-1} = 16 \left(\frac{7}{8} \right)^4 = 16 \left(\frac{2401}{4096} \right) \approx 9.4$$

The length of the arc on the fifth swing is 9.4 in.

YOU TRY IT • 8

You start a chain letter and send it to three friends. Each of the three friends sends the letter to three other friends, and the sequence is repeated. Assuming that no one breaks the chain, find how many letters will have been mailed from the first through the sixth mailings.

Your strategy**Your solution****EXAMPLE • 9**

The value of a mutual fund increased from \$10,000 to \$25,000 in 12 years. Find the average annual percent increase for the mutual fund. Round to the nearest tenth.

Strategy

The geometric sequence is 10,000, $10,000(1 + p)$, $10,000(1 + p)^2$, ..., 25,000, where p is the average annual percent increase as a decimal. To find the average percent increase, solve the Formula for the n th Term of a Geometric Sequence for the common ratio $r = 1 + p$.

Solution

$$a_1 = 10,000, n = 12, a_{12} = 25,000$$

$$a_n = a_1 r^{n-1}$$

$$a_{12} = a_1 r^{12-1} \quad \bullet \ n = 12$$

$$25,000 = 10,000 r^{11} \quad \bullet \ a_1 = 10,000, a_{12} = 25,000$$

$$2.5 = r^{11} \quad \bullet \ \text{Divide by 10,000.}$$

$$2.5^{1/11} = (r^{11})^{1/11} \quad \bullet \ \text{Solve for } r \text{ by taking the 11th root.}$$

$$1.087 \approx r$$

$$r = 1 + p, \text{ so } 1.087 = 1 + p \text{ or } p = 0.087.$$

The average annual percent increase was 8.7%.

YOU TRY IT • 9

Assume that the value of a mutual fund will increase at a rate of 8% per year. If \$20,000 is invested on January 1, 2008, find the value of the mutual fund on December 31, 2017. Round to the nearest cent.

Your strategy**Your solution**

12.3 EXERCISES

OBJECTIVE A

To find the n th term of a geometric sequence

1. Explain the difference between an arithmetic sequence and a geometric sequence.



2. Explain how to find the common ratio of a geometric sequence.

For Exercises 3 to 8, find the indicated term of the geometric sequence.

3. $2, 8, 32, \dots; a_9$

4. $4, 3, \frac{9}{4}, \dots; a_8$

5. $6, -4, \frac{8}{3}, \dots; a_7$

6. $-5, 15, -45, \dots; a_7$

7. $-\frac{1}{16}, \frac{1}{8}, -\frac{1}{4}, \dots; a_{10}$

8. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \dots; a_7$

For Exercises 9 to 14, find a_2 and a_3 for the geometric sequence.

9. $9, a_2, a_3, \frac{8}{3}, \dots$

10. $8, a_2, a_3, \frac{27}{8}, \dots$

11. $3, a_2, a_3, -\frac{8}{9}, \dots$

12. $6, a_2, a_3, -48, \dots$

13. $-3, a_2, a_3, 192, \dots$

14. $5, a_2, a_3, 625, \dots$

For Exercises 15 to 18, determine whether the formula defines a geometric sequence.

15. $a_n = n^2$

16. $a_n = \frac{1}{n}$

17. $a_n = 3^n$

18. $a_n = 2^{-n}$

OBJECTIVE B

To find the sum of a finite geometric series

For Exercises 19 to 24, find the sum of the indicated number of terms of the geometric sequence.

19. $2, 6, 18, \dots; n = 7$

20. $5, 2, \frac{4}{5}, \dots; n = 6$

21. $3, -2, \frac{4}{3}, \dots; n = 5$

22. $-4, 12, -36, \dots; n = 7$

23. $12, 9, \frac{27}{4}, \dots; n = 5$

24. $-2, \frac{3}{2}, -\frac{9}{8}, \dots; n = 6$

For Exercises 25 to 28, find the sum of the geometric series.

25. $\sum_{i=1}^5 (2)^i$

26. $\sum_{n=1}^6 \left(\frac{3}{2}\right)^n$

27. $\sum_{i=1}^5 \left(\frac{1}{3}\right)^i$

28. $\sum_{n=1}^6 \left(\frac{2}{3}\right)^n$

For Exercises 29 and 30, suppose $a_1, a_2, a_3, \dots$ is a geometric sequence in which $a_1 > 0$. For the given condition on r , is the sum of n terms of the sequence positive, negative, or either positive or negative depending on the value of n ?

29. $-1 < r < 0$

30. $r < -1$

OBJECTIVE C

To find the sum of an infinite geometric series

For Exercises 31 to 34, find the sum of the infinite geometric series.

31. $3 + 2 + \frac{4}{3} + \dots$

32. $2 - \frac{1}{4} + \frac{1}{32} + \dots$

33. $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \cdots$

34. $\frac{5}{100} + \frac{5}{10,000} + \frac{5}{1,000,000} + \cdots$

For Exercises 35 to 42, find an equivalent fraction for the repeating decimal.

35. $0.88\overline{8}$

36. $0.55\overline{5}$

37. $0.22\overline{2}$

38. $0.99\overline{9}$

39. $0.454\overline{5}$

40. $0.181\overline{8}$

41. $0.166\overline{6}$

42. $0.833\overline{3}$



For Exercises 43 to 46, determine whether the infinite geometric sequence has a finite sum.

43. $9, 3, 1, \dots$

44. $0.1, -0.01, 0.001, \dots$

45. $1, 1.5, 2.25, \dots$

46. $1, -1.2, 1.44, \dots$

OBJECTIVE D**To solve application problems**

47. **Sports** To test the “bounce” of a tennis ball, the ball is dropped from a height of 10 ft. The ball bounces to 75% of its previous height with each bounce. How high does the ball bounce on the sixth bounce? Round to the nearest tenth of a foot.

48. **Pendulum** On the first swing, the length of the arc through which a pendulum swings is 20 in. The length of each successive swing is $\frac{4}{5}$ the length of the preceding swing. What is the total distance the pendulum has traveled during four swings? Round to the nearest tenth of an inch.

Recycling For Exercises 49 to 51, use the information in the article at the right.

49. What was the average annual percent increase in waste generated on Long Island from 2001 to 2006? Round to the nearest tenth.
50. The article was written in 2008, two years after the most recent data from the Waste Reduction and Management Institute were published. If the total waste generated continued to grow at the same average annual rate, how many tons of waste were generated in 2008? Round to the nearest hundredth.
51. How many tons of waste were generated from 2001 to 2006? Round to the nearest tenth.



52. Suppose you have two job offers, each consisting of 1 month (30 days) of work. One job will pay you \$5 million at the end of 30 days. The other offer pays you 1¢ the first day, 2¢ the second day, 4¢ the third day, and so on in a geometric sequence. Which job would you choose if you were trying to select the job that paid the most? Explain.

In the News**A Long Way from Green**

Just over a quarter of Long Island's garbage is recycled. Roughly 43 percent is turned into electricity through incineration. The remainder—roughly 1.1 million tons, or 30 percent of the total waste stream—is hauled off of Long Island to out-of-state landfills and waste facilities. According to the Waste Reduction and Management Institute's most recent data, the total waste generated on Long Island increased from 2.85 million tons in 2001 to 3.7 million tons in 2006.

Applying the Concepts

53. State whether the following statements are true or false.
- The sum of an infinite geometric series is infinity.
 - The sum of a geometric series always increases in value.
 - $1, 8, 27, \dots, k^3, \dots$ is a geometric sequence.
 - $\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \dots, \sqrt{n}, \dots$ is a geometric sequence.

SECTION

12.4 Binomial Expansions

OBJECTIVE A

To expand $(a + b)^n$ 

By carefully observing the series for each expansion of the binomial $(a + b)^n$ shown below, it is possible to identify some interesting patterns.

$$(a + b)^1 = a + b$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

 Point of Interest


The Granger Collection, New York

Blaise Pascal

Blaise Pascal (1623–1662) is given credit for this triangle, which he first published in 1654. In that publication, the triangle looked like

```

1 2 3 4 5...
1 1 1 1 1 1...
2 1 2 3 4...
3 1 3 6...
4 1 4...
5 1

```

Thus the triangle was rotated 45° from the way it is shown today.

The first European publication of the triangle is attributed to Peter Apianus in 1527. However, there are versions of it in a Chinese text by Yang Hui that dates from 1250. In that text, Hui demonstrated how to find the third, fourth, fifth, and sixth roots of a number by using the triangle.

PATTERNS FOR THE VARIABLE PART

1. The first term is a^n . The exponent on a decreases by 1 for each successive term.
2. The exponent on b increases by 1 for each successive term. The last term is b^n .
3. The degree of each term is n .

HOW TO • 1

Write the variable parts of the terms in the expansion of $(a + b)^6$.

$$a^6, a^5b, a^4b^2, a^3b^3, a^2b^4, ab^5, b^6$$

- The first term is a^6 . For each successive term, the exponent on a decreases by 1, and the exponent on b increases by 1. The last term is b^6 .

The variable parts in the general expansion of $(a + b)^n$ are

$$a^n, a^{n-1}b, a^{n-2}b^2, \dots, a^{n-r}b^r, \dots, ab^{n-1}, b^n$$

A pattern for the coefficients of the terms of the expanded binomial can be found by writing the coefficients in a triangular array known as **Pascal's Triangle**.

Each row begins and ends with the number 1. Any other number in a row is the sum of the two closest numbers above it. For example, $4 + 6 = 10$.

For $(a + b)^1$: $1 \quad 1$

For $(a + b)^2$: $1 \quad 2 \quad 1$

For $(a + b)^3$: $1 \quad 3 \quad 3 \quad 1$

For $(a + b)^4$: $1 \quad 4 \quad 6 \quad 4 \quad 1$

For $(a + b)^5$: $1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

HOW TO • 2

Write the sixth row of Pascal's Triangle.

To write the sixth row, continue from the numbers of the fifth row shown above on the right. The first and last numbers of the sixth row are 1. Each of the other numbers of the sixth row can be obtained by finding the sum of the two closest numbers above it in the fifth row.

$$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$$

These numbers will be the coefficients of the terms in the expansion of $(a + b)^6$.

Point of Interest

The exclamation point was first used as a notation for factorial in 1808 in the book *Eléments d'Arithmétique Universelle*, for the convenience of the printer of the book. Some English textbooks suggested using the phrase “ n -admiration” for $n!$.

Using the numbers of the sixth row of Pascal’s Triangle for the coefficients and using the pattern for the variable part of each term, we can write the expanded form of $(a + b)^6$ as follows:

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Although Pascal’s Triangle can be used to find the coefficients for the expanded form of the power of any binomial, this method is inconvenient when the power of the binomial is large. An alternative method for determining these coefficients is based on the concept of **n factorial**.

n Factorial

$n!$ (which is read “ n factorial”) is the product of the first n consecutive natural numbers. $0!$ is defined to be 1.

$$n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

$1!$, $5!$, and $7!$ are shown at the right.

$$1! = 1$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

HOW TO 3 Evaluate: $6!$

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

• Write the factorial as a product. Then simplify.

HOW TO 4 Evaluate: $\frac{7!}{4!3!}$

$$\frac{7!}{4!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} = 35$$

• Write each factorial as a product. Then simplify.

The coefficients in a binomial expansion can be given in terms of factorials.

Note that in the expansion of $(a + b)^5$, the coefficient of a^2b^3 can be given by $\frac{5!}{2!3!}$. The numerator is the factorial of the power of the binomial. The denominator is the product of the factorials of the exponents on a and b .

$$\begin{aligned} (a + b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \\ \frac{5!}{2!3!} &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)} = 10 \end{aligned}$$

Point of Interest



The Granger Collection, New York

Leonhard Euler

Leonhard Euler (1707–1783)

used the notations $\left(\frac{n}{r}\right)$ and $\left[\frac{n}{r}\right]$ for binomial

coefficients around 1784. The notation $\binom{n}{r}$ appeared in the late 1820s.

Binomial Coefficients

The coefficients of $a^{n-r}b^r$ in the expansion of $(a + b)^n$ are given by $\frac{n!}{(n-r)!r!}$. The symbol $\binom{n}{r}$ is used to express this quotient.

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

HOW TO 5 Evaluate: $\binom{8}{5}$

$$\begin{aligned} \binom{8}{5} &= \frac{8!}{(8-5)!5!} = \frac{8!}{3!5!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 56 \end{aligned}$$

• Write the quotient of the factorials.

• Simplify.

HOW TO • 6Evaluate: $\binom{7}{0}$

$$\begin{aligned}\binom{7}{0} &= \frac{7!}{(7-0)!0!} = \frac{7!}{7!0!} \\ &= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(1)} = 1\end{aligned}$$

• Write the quotient of the factorials.

• Recall that $0! = 1$.

Using factorials and the pattern for the variable part of each term, we can write a formula for any natural number power of a binomial.

The Binomial Expansion Formula

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + \binom{n}{n}b^n$$

The Binomial Expansion Formula is used below to expand $(a + b)^5$.

$$\begin{aligned}(a + b)^5 &= \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + \binom{5}{5}b^5 \\ &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5\end{aligned}$$

HOW TO • 7Write $(x - 2)^4$ in expanded form.

Use the Binomial Expansion Formula. Then simplify each term.

$$\begin{aligned}(x - 2)^4 &= \binom{4}{0}x^4 + \binom{4}{1}x^3(-2) + \binom{4}{2}x^2(-2)^2 + \binom{4}{3}x(-2)^3 + \binom{4}{4}(-2)^4 \\ &= 1x^4 + 4x^3(-2) + 6x^2(4) + 4x(-8) + 1(16) \\ &= x^4 - 8x^3 + 24x^2 - 32x + 16\end{aligned}$$

HOW TO • 8Write $(4x + 3y)^3$ in expanded form.

Use the Binomial Expansion Formula. Then simplify each term.

$$\begin{aligned}(4x + 3y)^3 &= \binom{3}{0}(4x)^3 + \binom{3}{1}(4x)^2(3y) + \binom{3}{2}(4x)(3y)^2 + \binom{3}{3}(3y)^3 \\ &= 1(64x^3) + 3(16x^2)(3y) + 3(4x)(9y^2) + 1(27y^3) \\ &= 64x^3 + 144x^2y + 108xy^2 + 27y^3\end{aligned}$$

The Binomial Expansion Formula can also be used to write any term of a binomial expansion. The formula for the r th term is given below. First note that in the expansion of $(a + b)^5$, the exponent on b is 1 less than the term number.

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

The Formula for the r th Term in a Binomial Expansion

The r th term of $(a + b)^n$ is $\binom{n}{r-1}a^{n-r+1}b^{r-1}$.

HOW TO • 9 Find the 4th term in the expansion of $(x + 3)^7$.Use the Formula for the r th Term in a Binomial Expansion. $r = 4$

$$\begin{aligned} & \binom{n}{r-1} a^{n-r+1} b^{r-1} \\ & \binom{7}{4-1} x^{7-4+1} (3)^{4-1} = \binom{7}{3} x^4 (3)^3 = 35x^4(27) \\ & = 945x^4 \end{aligned}$$

HOW TO • 10 Find the 12th term in the expansion of $(2x - 1)^{14}$.Use the Formula for the r th Term in a Binomial Expansion. $r = 12$

$$\begin{aligned} & \binom{14}{12-1} (2x)^{14-12+1} (-1)^{12-1} = \binom{14}{11} (2x)^3 (-1)^{11} = 364(8x^3)(-1) \\ & = -2912x^3 \end{aligned}$$

EXAMPLE • 1Write $(2x + y)^3$ in expanded form.**Solution**

$$\begin{aligned} & (2x + y)^3 \\ & = \binom{3}{0} (2x)^3 + \binom{3}{1} (2x)^2(y) + \binom{3}{2} (2x)(y)^2 + \binom{3}{3} (y)^3 \\ & = 1(8x^3) + 3(4x^2)(y) + 3(2x)(y^2) + 1(y^3) \\ & = 8x^3 + 12x^2y + 6xy^2 + y^3 \end{aligned}$$

YOU TRY IT • 1Write $(3m - n)^4$ in expanded form.**Your solution****EXAMPLE • 2**Find the first three terms in the expansion of $(x + 3)^{15}$.**Solution**

$$\begin{aligned} & (x + 3)^{15} \\ & = \binom{15}{0} x^{15} + \binom{15}{1} x^{14}(3) + \binom{15}{2} x^{13}(3)^2 + \cdots \\ & = 1(x^{15}) + 15x^{14}(3) + 105x^{13}(9) + \cdots \\ & = x^{15} + 45x^{14} + 945x^{13} + \cdots \end{aligned}$$

YOU TRY IT • 2Find the first three terms in the expansion of $(y - 2)^{10}$.**Your solution****EXAMPLE • 3**Find the 4th term in the expansion of $(5x - y)^6$.**Solution**

$$\begin{aligned} & \binom{n}{r-1} a^{n-r+1} b^{r-1} \\ & \binom{6}{4-1} (5x)^{6-4+1} (-y)^{4-1} \quad \bullet \quad n = 6, a = 5x, b = -y, r = 4 \\ & = \binom{6}{3} (5x)^3 (-y)^3 \\ & = 20(125x^3)(-y^3) \\ & = -2500x^3y^3 \end{aligned}$$

YOU TRY IT • 3Find the 3rd term in the expansion of $(t - 2s)^7$.**Your solution**

12.4 EXERCISES**OBJECTIVE A****To expand $(a + b)^n$**

Evaluate the expressions in Exercises 1 to 24.

1. $3!$

2. $4!$

3. $8!$

4. $9!$

5. $0!$

6. $1!$

7. $\frac{5!}{2!3!}$

8. $\frac{8!}{5!3!}$

9. $\frac{6!}{6!0!}$

10. $\frac{10!}{10!0!}$

11. $\frac{9!}{6!3!}$

12. $\frac{10!}{2!8!}$

13. $\binom{7}{2}$

14. $\binom{8}{6}$

15. $\binom{10}{2}$

16. $\binom{9}{6}$

17. $\binom{9}{0}$

18. $\binom{10}{10}$

19. $\binom{6}{3}$

20. $\binom{7}{6}$

21. $\binom{11}{1}$

22. $\binom{13}{1}$

23. $\binom{4}{2}$

24. $\binom{8}{4}$

Write the expressions in Exercises 25 to 32 in expanded form.

25. $(x + y)^4$

26. $(r - s)^3$

27. $(x - y)^5$

28. $(y - 3)^4$

29. $(2m + 1)^4$

30. $(2x + 3y)^3$

31. $(2r - 3)^5$

32. $(x + 3y)^4$

For Exercises 33 to 42, find the first three terms in the expansion.

33. $(a + b)^{10}$

34. $(a + b)^9$

35. $(a - b)^{11}$

36. $(a - b)^{12}$

37. $(2x + y)^8$

38. $(x + 3y)^9$

39. $(4x - 3y)^8$

40. $(2x - 5)^7$

41. $\left(x + \frac{1}{x}\right)^7$

42. $\left(x - \frac{1}{x}\right)^8$

For Exercises 43 to 50, find the indicated term in the expansion.

43. $(2x - 1)^7$; 4th term

44. $(x + 4)^5$; 3rd term

45. $(x^2 - y^2)^6$; 2nd term

46. $(x^2 + y^2)^7$; 6th term

47. $(y - 1)^9$; 5th term

48. $(x - 2)^8$; 8th term

49. $\left(n + \frac{1}{n}\right)^5$; 2nd term

50. $\left(x + \frac{1}{2}\right)^6$; 3rd term



51. True or false? $\frac{(2n)!}{n!} = 2$



52. Is the 13th term in the expansion of $(x - 5)^{21}$ positive or negative?

Applying the Concepts

53. State whether the following statements are true or false.

a. $0! \cdot 4! = 0$

b. $\frac{4!}{0!}$ is undefined.

c. In the expansion of $(a + b)^8$, the exponent on a for the fifth term is 5.d. $n!$ is the sum of the first n positive numbers.e. There are n terms in the expansion of $(a + b)^n$.f. The sum of the exponents in each term of the expansion of $(a + b)^n$ is n .

54. Write a summary of the history of the development of Pascal's Triangle. Include some of the properties of the triangle.

FOCUS ON PROBLEM SOLVING

Forming Negations A **statement** is a declarative sentence that is either true or false but not both true and false. Here are a few examples of statements.

Denver is the capital of Colorado.

The word *dog* has four letters.

Florida is a state in the United States.

In the year 2020, the President of the United States will be a woman.

Problem solving sometimes requires that you form the **negation** of a statement. If a statement is true, its negation must be false. If a statement is false, its negation must be true. For instance,

“Henry Aaron broke Babe Ruth’s home run record” is a true sentence. Accordingly, the sentence “Henry Aaron did not break Babe Ruth’s home run record” is false.

“Dogs have twelve paws” is a false sentence, so the sentence “Dogs do not have twelve paws” is a true sentence.

Forming the negation of a statement requires a precise understanding of the statement being negated. Here are some phrases that demand special attention.

<i>Statement</i>	<i>Meaning</i>
<i>At least seven</i>	Greater than or equal to seven
<i>More than seven</i>	More than seven (does not include 7)
<i>At most seven</i>	Less than or equal to seven
<i>Less than (or fewer than) seven</i>	Less than seven (does not include 7)

Consider the sentence “At least three people scored over 90 on the last test.” The negation of that sentence is “Fewer than three people scored over 90 on the last test.”

For Exercises 1 to 4, write the negation of the given sentence.

1. There was at least one error on the test.
2. The temperature was less than 30 degrees.
3. The mountain is more than 5000 ft tall.
4. There are at most five vacancies for the field trip to New York.

When the word *all*, *no* (or *none*), or *some* occurs in a sentence, forming the negation requires careful thought. Consider the sentence “All roads are paved with cement.” This sentence is not true because there are some roads—dirt roads, for example—that are not paved with cement. Because the sentence is false, its negation must be true. You might be tempted to write “All roads are not paved with cement” as the negation, but that sentence is not true because some roads are paved with cement. The correct negation is “*Some* roads are *not* paved with cement.”

Now consider the sentence “Some fish live in aquariums.” Because this sentence is true, the negation must be false. Writing “Some fish do not live in aquariums” as the negation is not correct because that sentence is true. The correct negation is “*All* fish do *not* live in aquariums.”

The sentence “No houses have basements” is false because there is at least one house with a basement. Because this sentence is false, its negation must be true. The negation is “Some houses do have basements.”

<i>Statement</i>	<i>Negation</i>
All A are B .	Some A are not B .
No A are B .	Some A are B .
Some A are B .	No A are B .
Some A are not B .	All A are B .

Write the negation of each sentence in Exercises 5 to 12.

5. All trees are tall.
6. All cats chase mice.
7. Some flowers have red blooms.
8. No golfers like tennis.
9. Some students do not like math.
10. No honest people are politicians.
11. All cars have power steering.
12. Some televisions are black and white.

PROJECTS AND GROUP ACTIVITIES

ISBN and UPC Numbers



Every book that is cataloged for the Library of Congress must have an ISBN (International Standard Book Number). An ISBN is a 10-digit number of the form $a_1a_2a_3a_4a_5a_6a_7a_8a_9c$. For instance, the ISBN for the fourth edition of the *American Heritage Dictionary* is 0-395-82517-2. The first number, 0, indicates that the book is written in English. The next three numbers, 395, indicate the publisher (Houghton Mifflin Company). The next five numbers, 82517, identify the book (*American Heritage Dictionary*), and the last digit, c , is called a *check digit*. This digit is chosen such that the following sum is divisible by 11.

$$a_1(10) + a_2(9) + a_3(8) + a_4(7) + a_5(6) + a_6(5) + a_7(4) + a_8(3) + a_9(2) + c$$

For the *American Heritage Dictionary*,

$$0(10) + 3(9) + 9(8) + 5(7) + 8(6) + 2(5) + 5(4) + 1(3) + 7(2) + c = 229 + c$$

The last digit of the ISBN is chosen as 2 because $229 + 2 = 231$ and $231 \div 11 = 21$ remainder 0. The value of c could be any number from 0 to 10. The number 10 is coded as an X.

ISBN 0-395-82517-2



9 780395 825174

One purpose of the ISBN method of coding books is to ensure that orders placed for books are accurately filled. For instance, suppose a clerk sends an order for the *American Heritage Dictionary* and inadvertently enters the number 0-395-82571-2 (the 7 and the 1 have been transposed). Now

$$0(10) + 3(9) + 9(8) + 5(7) + 8(6) + 2(5) + 5(4) + 7(3) + 1(2) + 2 = 237$$

and if we divide 237 by 11, the remainder is not zero. Thus an error has been made in the ISBN.

1. Determine the ISBN check digit for the book *A Brief History of Time* by Stephen Hawking. The first nine digits of the ISBN are 0-553-05340-?
2. Is 0-395-12370-4 a possible ISBN?
3. Check the ISBN of this intermediate algebra textbook.

Another coding scheme that is closely related to the ISBN is the UPC (Universal Product Code). This number is placed on many items and is particularly useful in grocery stores. A check-out clerk passes the number by a scanner, which reads the number and records the price on the cash register.

The UPC is a 12-digit number. The first six digits identify the manufacturer. The next five digits are a product identification number given by the manufacturer to identify the product. The final digit is a check digit. The check digit is chosen such that the following sum is divisible by 10.

$$a_1(3) + a_2 + a_3(3) + a_4 + a_5(3) + a_6 + a_7(3) + a_8 + a_9(3) + a_{10} + a_{11}(3) + c$$

The UPC for the TI-83 Plus calculator is 0-33317-19865-8. Using the expression above, we find that

$$0(3) + 3 + 3(3) + 3 + 1(3) + 7 + 1(3) + 9 + 8(3) + 6 + 5(3) + c = 82 + c$$

Because $82 + 8$ is a multiple of 10, the check digit is 8.

4. A new product that a company has developed has been assigned the UPC 0-21443-32912-?. Determine the check digit for the UPC.
5. Is 1-32342-65933-9 a valid UPC?
6. Check the UPC of a product in your home.



CHAPTER 12

SUMMARY

KEY WORDS

A *sequence* is an ordered list of numbers. Each of the numbers of a sequence is called a *term* of the sequence. For a general sequence, the first term is a_1 , the second term is a_2 , the third term is a_3 , and the n th term, also called the *general term* of the sequence, is a_n . [12.1A, p. 648]

EXAMPLES

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ is a sequence.

The terms are $1, \frac{1}{3}, \frac{1}{9}$, and $\frac{1}{27}$.

A *finite sequence* contains a finite number of terms.
[12.1A, p. 648]

2, 4, 6, 8 is a finite sequence.

An *infinite sequence* contains an infinite number of terms.
[12.1A, p. 648]

1, 3, 5, 7, ... is an infinite sequence.

The indicated sum of the terms of a sequence is called a *series*. S_n is used to designate the sum of the first n terms of a sequence. [12.1B, p. 650]

Given the sequence 1, 5, 9, 13, the series is $1 + 5 + 9 + 13$.

Summation notation, or *sigma notation*, is used to represent a series in a compact form. The Greek capital letter sigma, Σ , is used to indicate the sum. [12.1B, p. 650]

$$\begin{aligned}\sum_{n=1}^4 2n &= 2(1) + 2(2) + 2(3) + 2(4) \\ &= 20\end{aligned}$$

An *arithmetic sequence*, or *arithmetic progression*, is one in which the difference between any two consecutive terms is the same constant. The difference between consecutive terms is called the *common difference* of the sequence. [12.2A, p. 654]

3, 9, 15, 21, ... is an arithmetic sequence.
 $9 - 3 = 6$; 6 is the common difference.

The indicated sum of the terms of an arithmetic sequence is an *arithmetic series*. [12.2B, p. 656]

1, 2, 3, 4, 5 is an arithmetic sequence. The indicated sum is $1 + 2 + 3 + 4 + 5$.

A *geometric sequence*, or *geometric progression*, is one in which each successive term of the sequence is the same nonzero constant multiple of the preceding term. The common multiple is called the *common ratio* of the sequence. [12.3A, p. 600]

$9, 3, 1, \frac{1}{3}, \dots$ is a geometric sequence.
 $\frac{3}{9} = \frac{1}{3}$; $\frac{1}{3}$ is the common ratio.

The indicated sum of the terms of a geometric sequence is a *geometric series*. [12.3B, p. 662]

5, 10, 20, 40 is a geometric sequence.
The indicated sum is $5 + 10 + 20 + 40$.

n factorial, written $n!$, is the product of the first n consecutive natural numbers. $0!$ is defined to be 1. [12.4A, p. 671]

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

ESSENTIAL RULES AND PROCEDURES

Formula for the n th Term of an Arithmetic Sequence [12.2A, p. 654]

The n th term of an arithmetic sequence with a common difference of d is given by $a_n = a_1 + (n - 1)d$.

EXAMPLES

$$\begin{aligned}4, 7, 10, 13, \dots \\ d &= 7 - 4 = 3; a_1 = 4 \\ a_{10} &= 4 + (10 - 1)3 = 4 + 27 = 31\end{aligned}$$

Formula for the Sum of n Terms of an Arithmetic Series

[12.2B, p. 656]

Let a_1 be the first term of a finite arithmetic sequence, n be the number of terms, and a_n be the last term of the sequence. Then the sum of the series S_n is given by $S_n = \frac{n}{2}(a_1 + a_n)$.

$$5, 8, 11, 14, \dots \quad d = 3, a_{12} = 38$$

$$S_{12} = \frac{12}{2}(5 + 38) = 258$$

Formula for the n th Term of a Geometric Sequence

[12.3A, p. 660]

The n th term of a geometric sequence with first term a_1 and common ratio r is given by $a_n = a_1 r^{n-1}$.

$$2, 6, 18, 54, \dots \quad a_1 = 2, r = 3$$

$$a_{10} = 2(3)^{10-1} = 2(3)^9 = 39,366$$

Formula for the Sum of n Terms of a Finite Geometric Series [12.3B, p. 662]

Let a_1 be the first term of a finite geometric sequence, n be the number of terms, and r be the common ratio. Then the sum of the series S_n is given by $S_n = \frac{a_1(1 - r^n)}{1 - r}$.

$$1, 4, 16, 64, \dots \quad a_1 = 1, r = 4$$

$$S_8 = \frac{1(1 - 4^8)}{1 - 4} = \frac{1 - 65,536}{-3} = 21,845$$

Formula for the Sum of an Infinite Geometric Series

[12.3C, p. 664]

The sum of an infinite geometric series in which $|r| < 1$ and a_1 is the first term is given by $S = \frac{a_1}{1 - r}$.

$$2, 1, \frac{1}{2}, \frac{1}{4}, \dots \quad a_1 = 2, r = \frac{1}{2}$$

$$S = \frac{2}{1 - \frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$$

Binomial Coefficients [12.4A, p. 671]

The coefficients of $a^{n-r}b^r$ in the expansion of $(a + b)^n$ are given by $\binom{n}{r}$, where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

$$\binom{6}{2} = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = 15$$

Binomial Expansion Formula [12.4A, p. 672]

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

$$\begin{aligned} (x + y)^4 &= \binom{4}{0}x^4 + \binom{4}{1}x^3y + \binom{4}{2}x^2y^2 \\ &\quad + \binom{4}{3}xy^3 + \binom{4}{4}y^4 \\ &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \end{aligned}$$

Formula for the r th Term in a Binomial Expansion

[12.4A, p. 672]

The r th term of $(a + b)^n$ is $\binom{n}{r-1}a^{n-r+1}b^{r-1}$.

The sixth term of $(2x - y)^9$ is $\binom{9}{5}(2x)^{9-6+1}(-y)^{6-1} = -2016x^4y^5$.

CHAPTER 12**CONCEPT REVIEW**

Test your knowledge of the concepts presented in this chapter. Answer each question. Then check your answers against the ones provided in the Answer Section.

1. What is the general term of a sequence?
2. How is a sequence different from a series?
3. What is the index of a summation?
4. How do you find the common difference of an arithmetic sequence?
5. What is the Formula for the n th Term of an Arithmetic Sequence?
6. What is the Formula for the Sum of n Terms of an Arithmetic Series?
7. How do you find the common ratio of a geometric sequence?
8. What is the Formula for the n th Term of a Geometric Sequence?
9. What is the Formula for the Sum of n Terms of a Finite Geometric Series?
10. How is it possible to find the sum of an infinite geometric series?
11. Recreate the first six rows of Pascal's Triangle.
12. Expand and simplify $7!$.
13. Evaluate $\binom{6}{4}$.

CHAPTER 12

REVIEW EXERCISES

1. Write $\sum_{i=1}^4 2x^{i-1}$ in expanded form.
2. Write the 10th term of the sequence whose n th term is given by the formula $a_n = 3n - 2$.
3. Find an equivalent fraction for $0.63\overline{33}$.
4. Find the sum of the infinite geometric series $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots$.
5. Find the sum of the arithmetic series $\sum_{i=1}^{30} (4i - 1)$.
6. Find the sum of the series $\sum_{n=1}^5 (3n - 2)$.
7. Find an equivalent fraction for $0.23\overline{23}$.
8. Evaluate: $\binom{9}{3}$
9. Find the sum of the geometric series $\sum_{n=1}^5 2(3)^n$.
10. Find the number of terms in the finite arithmetic sequence $8, 2, -4, \dots, -118$.
11. Find the sum of the geometric series $\sum_{n=1}^8 \left(\frac{1}{2}\right)^n$. Round to the nearest thousandth.
12. Find the 5th term in the expansion of $(3x - y)^8$.
13. Write the 5th term of the sequence whose n th term is given by the formula $a_n = \frac{(-1)^{2n-1}n}{n^2 + 2}$.
14. Find the sum of the first seven terms of the geometric sequence $5, 5\sqrt{5}, 25, \dots$. Round to the nearest whole number.
15. Find the formula for the n th term of the arithmetic sequence $-7, -2, 3, \dots$.
16. Find the 8th term in the expansion of $(x - 2y)^{11}$.

17. Find the 12th term of the geometric sequence $1, \sqrt{3}, 3, \dots$
18. Find the sum of the first 40 terms of the arithmetic sequence $11, 13, 15, \dots$
19. Evaluate: $\binom{12}{9}$
20. Find the sum of the series $\sum_{n=1}^4 \frac{(-1)^{n-1}n}{n+1}$.
21. Write $\sum_{i=1}^5 \frac{(2x)^i}{i}$ in expanded form.
22. Find the 8th term of the geometric sequence $3, 1, \frac{1}{3}, \dots$
23. Write $(x - 3y^2)^5$ in expanded form.
24. Find the sum of the infinite geometric series $4 - 1 + \frac{1}{4} - \dots$.
25. Find the sum of the infinite geometric series $2 + \frac{4}{3} + \frac{8}{9} + \dots$.
26. Evaluate: $\frac{12!}{5!8!}$
27. Find the 20th term of the arithmetic sequence $1, 5, 9, \dots$
28. Write $\sum_{i=1}^6 4x^i$ in expanded form.
29. Find the number of terms in the finite arithmetic sequence $-3, -7, -11, \dots, -59$.
30. Find the 7th term of the geometric sequence $5, 5\sqrt{3}, 15, \dots$
31. Find the sum of the terms of the infinite geometric sequence $3, 2, \frac{4}{3}, \dots$
32. Write the 17th term of the sequence whose n th term is given by the formula $a_n = \frac{10}{n+3}$.
33. Find the 10th term of the arithmetic sequence $-10, -4, 2, \dots$
34. Find the sum of the first 18 terms of the arithmetic sequence $-19, -12, -5, \dots$

35. Find the sum of the first six terms of the geometric sequence $8, -16, 32, \dots$
36. Evaluate: $\frac{7!}{5!2!}$
37. Find the 7th term in the expansion of $(3x + y)^9$.
38. Find the sum of the series $\sum_{n=1}^5 (4n - 3)$.
39. Write the 8th term of the sequence whose n th term is given by the formula $a_n = \frac{n+2}{2n}$.
40. Find the formula for the n th term of the arithmetic sequence $-4, 0, 4, \dots$
41. Find the 5th term of the geometric sequence $10, -2, \frac{2}{5}, \dots$
42. Find an equivalent fraction for $0.36\overline{66}$.
43. Find the 37th term of the arithmetic sequence $-37, -32, -27, \dots$
44. Find the sum of the first five terms of the geometric sequence $1, 4, 16, \dots$
45. **Exercise** An exercise program calls for walking 15 min each day for a week. Each week thereafter, the amount of time spent walking increases by 3 min per day. In how many weeks will a person on this program be walking 60 min each day?
46. **Temperature** The temperature of a hot water spa is 102°F . Each hour the temperature is 5% lower than during the preceding hour. Find the temperature of the spa after 8 h. Round to the nearest tenth.
47. **Compensation** The salary schedule for an apprentice electrician is \$2400 for the first month and an \$80-per-month salary increase for the next eight months. Find the total salary for the nine-month period.
48. **Radioactivity** A laboratory radioactive sample contains 200 mg of a material with a half-life of 1 h. Find the amount of radioactive material in the sample at the beginning of the seventh hour.

CHAPTER 12**TEST**

1. Find the sum of the series $\sum_{n=1}^4 (3n + 1)$.
2. Evaluate: $\binom{9}{6}$
3. Find the 7th term of the geometric sequence $4, 4\sqrt{2}, 8, \dots$.
4. Write the 14th term of the sequence whose n th term is given by the formula $a_n = \frac{8}{n+2}$.
5. Write in expanded form: $\sum_{i=1}^4 3x^i$
6. Find the sum of the first 18 terms of the arithmetic sequence $-25, -19, -13, \dots$.
7. Find the sum of the terms of the infinite geometric sequence $4, 3, \frac{9}{4}, \dots$.
8. Find the number of terms in the finite arithmetic sequence $-5, -8, -11, \dots, -50$.
9. Find an equivalent fraction for $0.233\overline{3}$.
10. Evaluate: $\frac{8!}{4!4!}$
11. Find the 5th term of the geometric sequence $6, 2, \frac{2}{3}, \dots$.
12. Find the 4th term in the expansion of $(x - 2y)^7$.

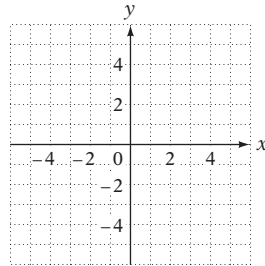
13. Find the 35th term of the arithmetic sequence $-13, -16, -19, \dots$
14. Find the formula for the n th term of the arithmetic sequence $12, 9, 6, \dots$
15. Find the sum of the first five terms of the geometric sequence $-6, 12, -24, \dots$
16. Find the 6th and 7th terms of the sequence whose n th term is given by the formula
$$a_n = \frac{n+1}{n}.$$
17. Find the sum of the first six terms of the geometric sequence $1, \frac{3}{2}, \frac{9}{4}, \dots$
18. Find the sum of the first 21 terms of the arithmetic sequence $5, 12, 19, \dots$
19. **Inventory** An inventory of supplies for a yarn manufacturer indicated that 7500 skeins of yarn were in stock on January 1. On February 1, and on the first of the month for each successive month, the manufacturer sent 550 skeins of yarn to retail outlets. How many skeins were in stock after the shipment on October 1?
20. **Radioactivity** An ore sample contains 320 mg of a radioactive substance with a half-life of 1 day. Find the amount of radioactive material in the sample at the beginning of the fifth day.



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CUMULATIVE REVIEW EXERCISES

1. Graph: $3x - 2y = -4$



2. Factor: $2x^6 + 16$

3. Subtract: $\frac{4x^2}{x^2 + x - 2} - \frac{3x - 2}{x + 2}$

4. Given $f(x) = 2x^2 - 3x$, find $f(-2)$.

5. Simplify: $\sqrt{2y}(\sqrt{8xy} - \sqrt{y})$

6. Solve by using the quadratic formula:
 $2x^2 - x + 7 = 0$

7. Solve: $5 - \sqrt{x} = \sqrt{x + 5}$

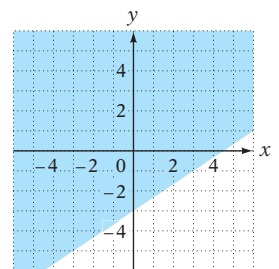
8. Find the equation of the circle whose center is $(-1, -1)$ and that passes through the point whose coordinates are $(4, 2)$.

9. Solve by the addition method:
 $3x - 3y = 2$
 $6x - 4y = 5$

10. Evaluate the determinant:
 $\begin{vmatrix} -3 & 1 \\ 4 & 2 \end{vmatrix}$

11. Solve: $2x - 1 > 3$ or $1 - 3x > 7$
Write the solution set in set-builder notation.

12. Graph: $2x - 3y < 9$



13. Write $\log_5 \sqrt{\frac{x}{y}}$ in expanded form.

14. Solve for x : $4^x = 8^{x-1}$

15. Write the 5th and 6th terms of the sequence whose n th term is given by the formula $a_n = n(n - 1)$.

16. Find the sum of the series $\sum_{n=1}^7 (-1)^{n-1}(n + 2)$.

17. Find the 33rd term of the arithmetic sequence $-7, -10, -13, \dots$
18. Find the sum of the terms of the infinite geometric sequence $3, -2, \frac{4}{3}, \dots$
19. Find an equivalent fraction for $0.4\overline{6}$.
20. Find the 6th term in the expansion of $(2x + y)^6$.
21. **Mixtures** How many ounces of pure water must be added to 200 oz of an 8% salt solution to make a 5% salt solution?
22. **Computers** A new computer can complete a payroll in 16 min less time than it takes an older computer to complete the same payroll. Working together, the computers can complete the payroll in 15 min. How long would it take each computer, working alone, to complete the payroll?
23. **Uniform Motion** A boat traveling with the current went 15 mi in 2 h. Against the current, it took 3 h to travel the same distance. Find the rate of the boat in calm water and the rate of the current.
24. **Radioactivity** An 80-milligram sample of a radioactive material decays to 55 mg in 30 days. Use the exponential decay equation $A = A_0\left(\frac{1}{2}\right)^{t/k}$, where A is the amount of radioactive material present after time t , k is the half-life of the material, and A_0 is the original amount of radioactive material, to find the half-life of the 80-milligram sample. Round to the nearest whole number.
25. **The Arts** A theater-in-the-round has 62 seats in the first row, 74 seats in the second row, 86 seats in the third row, and so on in an arithmetic sequence. Find the total number of seats in the theater if there are 12 rows of seats.
26. **Sports** To test the “bounce” of a ball, the ball is dropped from a height of 8 ft. The ball bounces to 80% of its previous height with each bounce. How high does the ball bounce on the fifth bounce? Round to the nearest tenth.



FINAL EXAM

1. Simplify: $12 - 8[3 - (-2)]^2 \div 5 - 3$

2. Evaluate $\frac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.

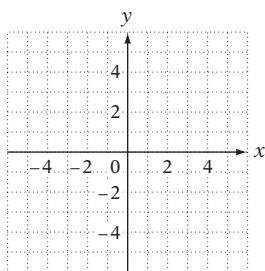
3. Given: $f(x) = 3x - 7$ and $t(x) = x^2 - 4x$, find $(f \circ t)(3)$.

4. Solve: $\frac{3}{4}x - 2 = 4$

5. Solve: $\frac{2 - 4x}{3} - \frac{x - 6}{12} = \frac{5x - 2}{6}$

6. Solve: $8 - |5 - 3x| = 1$

7. Graph $2x - 3y = 9$ using the x - and y -intercepts.



8. Find the equation of the line containing the points whose coordinates are $(3, -2)$ and $(1, 4)$.

9. Find the equation of the line that contains the point whose coordinates are $(-2, 1)$ and is perpendicular to the graph of $3x - 2y = 6$.

10. Simplify: $2a[5 - a(2 - 3a) - 2a] + 3a^2$

11. Factor: $8 - x^3y^3$

12. Factor: $x - y - x^3 + x^2y$

13. Divide: $(2x^3 - 7x^2 + 4) \div (2x - 3)$

14. Divide: $\frac{x^2 - 3x}{2x^2 - 3x - 5} \div \frac{4x - 12}{4x^2 - 4}$

15. Subtract: $\frac{x - 2}{x + 2} - \frac{x + 3}{x - 3}$

16. Simplify: $\frac{\frac{3}{x} + \frac{1}{x + 4}}{\frac{1}{x} + \frac{3}{x + 4}}$

17. Solve: $\frac{5}{x - 2} - \frac{5}{x^2 - 4} = \frac{1}{x + 2}$

18. Solve $a_n = a_1 + (n - 1)d$ for d .

19. Simplify: $\left(\frac{4x^2y^{-1}}{3x^{-1}y}\right)^{-2} \left(\frac{2x^{-1}y^2}{9x^{-2}y^2}\right)^3$

20. Simplify: $\left(\frac{3x^{2/3}y^{1/2}}{6x^2y^{4/3}}\right)^6$

21. Simplify: $x\sqrt{18x^2y^3} - y\sqrt{50x^4y}$

22. Simplify: $\frac{\sqrt{16x^5y^4}}{\sqrt{32xy^7}}$

23. Divide: $\frac{5 - 2i}{3 + 4i}$

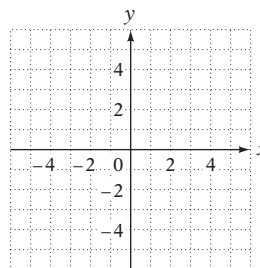
24. Write a quadratic equation that has integer coefficients and has solutions $-\frac{1}{2}$ and 2.

25. Solve by using the quadratic formula:
 $2x^2 - 3x - 1 = 0$

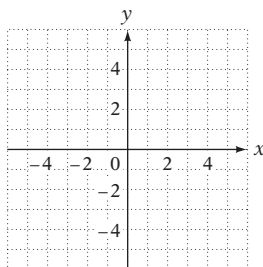
26. Solve: $x^{2/3} - x^{1/3} - 6 = 0$

27. Solve: $\frac{2}{x} - \frac{2}{2x + 3} = 1$

28. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $x = y^2 - 6y + 6$. Then sketch the graph.



29. Find the vertices of the graph of $\frac{x^2}{16} + \frac{y^2}{4} = 1$. Then sketch the graph.



30. Find the inverse function of $f(x) = \frac{2}{3}x - 4$.

31. Solve by the addition method:
 $3x - 2y = 1$
 $5x - 3y = 3$

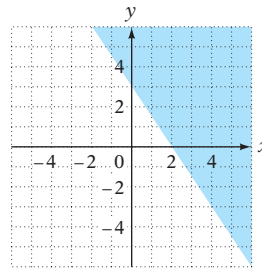
32. Evaluate the determinant:
 $\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix}$

33. Solve: $x^2 - y^2 = 4$
 $x + y = 1$

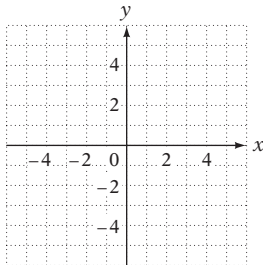
34. Solve: $2 - 3x < 6$ and $2x + 1 > 4$
 Write the solution set in interval notation.

35. Solve: $|2x + 5| < 3$

36. Graph the solution set: $3x + 2y > 6$



37. Graph: $f(x) = \log_2(x + 1)$



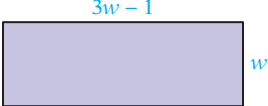
38. Write $2(\log_2 a - \log_2 b)$ as a single logarithm with a coefficient of 1.

39. Solve for x : $\log_3 x - \log_3(x - 3) = 2$

40. Evaluate the arithmetic series $\sum_{i=1}^{25} (2i - 1)$.

41. Find an equivalent fraction for $0.\overline{55}$.

42. Find the 3rd term in the expansion of $(x - 2y)^9$.

43. **Education** An average score of 70 to 79 in a history class receives a C grade. A student has scores of 64, 58, 82, and 77 on four history tests. Find the range of scores on the fifth test that will give the student a C grade for the course.
44. **Uniform Motion** A jogger and a cyclist set out at 8 A.M. from the same point headed in the same direction. The average speed of the cyclist is two and a half times the average speed of the jogger. In 2 h, the cyclist is 24 mi ahead of the jogger. How far did the cyclist ride in that time?
45. **Investments** You have a total of \$12,000 invested in two simple interest accounts. On one account, a money market fund, the annual simple interest rate is 8.5%. On the other account, a tax free bond fund, the annual simple interest rate is 6.4%. The total annual interest earned by the two accounts is \$936. How much do you have invested in each account?
46. **Geometry** The length of a rectangle is 1 ft less than three times the width. The area of the rectangle is 140 ft^2 . Find the length and width of the rectangle.
- 
47. **The Stock Market** Three hundred shares of a utility stock earn a yearly dividend of \$486. How many additional shares of the utility stock would give a total dividend income of \$810?
48. **Uniform Motion** An account executive traveled 45 mi by car and then an additional 1050 mi by plane. The rate of the plane was seven times the rate of the car. The total time for the trip was $3\frac{1}{4}$ h. Find the rate of the plane.

49. **Physics** An object is dropped from the top of a building. Find the distance the object has fallen when its speed reaches 75 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance in feet. Round to the nearest whole number.
50. **Uniform Motion** A small plane made a trip of 660 mi in 5 h. The plane traveled the first 360 mi at a constant rate before increasing its speed by 30 mph. Then it traveled another 300 mi at the increased speed. Find the rate of the plane for the first 360 mi.
51. **Light** The intensity L of a light source is inversely proportional to the square of the distance d from the source. If the intensity is 8 foot-candles at a distance of 20 ft, what is the intensity at a distance of 4 ft?
52. **Uniform Motion** A motorboat traveling with the current can go 30 mi in 2 h. Against the current, it takes 3 h to go the same distance. Find the rate of the motorboat in calm water and the rate of the current.
53. **Investments** An investor deposits \$4000 into an account that earns 9% annual interest compounded monthly. Use the compound interest formula $P = A(1 + i)^n$, where A is the original value of the investment, i is the interest rate per compounding period, n is the total number of compounding periods, and P is the value of the investment after n periods, to find the value of the investment after 2 years. Round to the nearest cent.
54. **Sports** When a golf ball is dropped and hits the floor, it rebounds to a height that is 80% of the height from which it was dropped. If the golf ball is originally dropped from a height of 5 ft, how high will it rebound on the fifth bounce? Round to the nearest hundredth.

Appendix A

Keystroke Guide for the TI-84 Plus

Basic Operations

Numerical calculations are performed on the **home screen**. You can always return to the home screen by pressing **2ND** **QUIT**. Pressing **CLEAR** erases the home screen.

To evaluate the expression $-2(3 + 5) - 8 \div 4$, use the following keystrokes.

(-) 2 **(** 3 **+** 5 **)** **-** 8 **÷** 4 **ENTER**



Take Note

The descriptions in the margins (for example, Basic Operations and Complex Numbers) are the same as those used in the text and are arranged alphabetically.

Note: There is a difference between the key to enter a negative number, **(-)**, and the key for subtraction, **-**. You cannot use these keys interchangeably.

The **2ND** key is used to access the commands in gold writing above a key. For instance, to evaluate $\sqrt{49}$, press **2ND** **√** 49 **)** **ENTER**.

The **ALPHA** key is used to place a letter on the screen. One reason to do this is to store a value of a variable. The following keystrokes give A the value of 5.

5 **STO** **ALPHA** **A** **ENTER**

This value is now available in calculations. For instance, we can find the value of $3a^2$ by using the following keystrokes: 3 **ALPHA** **A** **x²**. To display the value of the variable on the screen, press **2ND** **RCL** **ALPHA** **A**.

Note: When you use the **ALPHA** key, only capital letters are available on the TI-83 calculator.

Complex Numbers

To perform operations on complex numbers, first press **MODE** and then use the arrow keys to select **a+bi**. Then press **ENTER** **2ND** **QUIT**.

Addition of complex numbers To add $(3 + 4i) + (2 - 7i)$, use the keystrokes

(3 **+** 4 **2ND** **i** **)** **+**
(2 **-** 7 **2ND** **i** **)** **ENTER**.

Division of complex numbers To divide $\frac{26 + 2i}{2 + 4i}$, use the keystrokes **(** 26 **+** 2 **2ND** **i** **)** **÷**

(2 **+** 4 **2ND** **i** **)** **ENTER**.

Note: Operations for subtraction and multiplication are similar.

$$\frac{-2(3+5)-8}{4} = -18$$

$$\sqrt{49} = 7$$

$$5 \rightarrow A = 5$$

$$3A^2 = 75$$

Normal	Sci	Eng
Float	0123456789	
Radian	Degree	
Func	Par	Pol
Connected	Dot	
Sequential	Simul	
Real	a+bi	re^θi
Full	Horiz	G-T

$$(3+4i)+(2-7i) = 5-3i$$

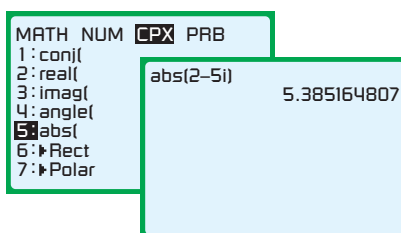
$$(26+2i)/(2+4i) = 3-5i$$

Additional operations on complex numbers can be found by selecting **CPX** under the **MATH** key.

To find the absolute value of $2 - 5i$, press

MATH (scroll to **CPX**) (scroll to **abs**)

ENTER (**(** 2 **-** 5 **2ND** **i** **)** **ENTER**.

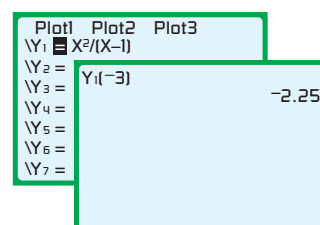


Evaluating Functions

There are various methods of evaluating a function but all methods require that the expression be entered as one of the ten functions Y_1 to Y_0 . To evaluate $f(x) = \frac{x^2}{x-1}$ when $x = -3$, enter the expression into,

for instance, Y_1 , and then press **VAR** **◀** 11

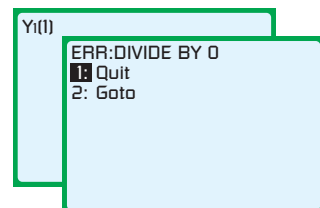
(**(-)** 3 **)** **ENTER**.



Take Note

Use the down arrow key to scroll past Y_7 to see Y_8 , Y_9 , and Y_0 .

Note: If you try to evaluate a function at a number that is not in the domain of the function, you will get an error message. For instance, 1 is not in the domain of $f(x) = \frac{x^2}{x-1}$. If we try to evaluate the function at 1, the error screen at the right appears.

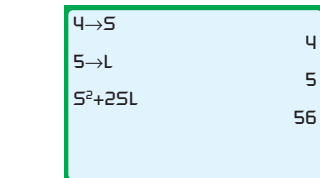


Evaluating Variable Expressions

To evaluate a variable expression, first store the values of each variable. Then enter the variable expression. For instance, to evaluate $s^2 + 2sl$ when $s = 4$ and $l = 5$, use the following keystrokes.

4 **STO** **▶** **ALPHA** **S** **ENTER** 5 **STO** **▶** **ALPHA** **L** **ENTER** **ALPHA** **S**

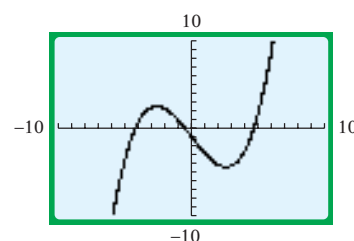
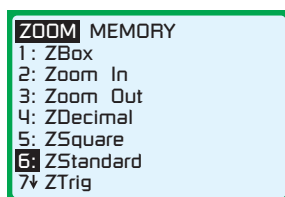
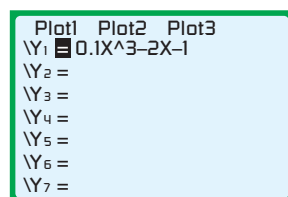
x² **+** 2 **ALPHA** **S** **ALPHA** **L** **ENTER**



Graph

To graph a function, use the **Y=** key to enter the expression for the function, select a suitable viewing window, and then press **GRAPH**. For instance, to graph $f(x) = 0.1x^3 - 2x - 1$ in the standard viewing window, use the following keystrokes.

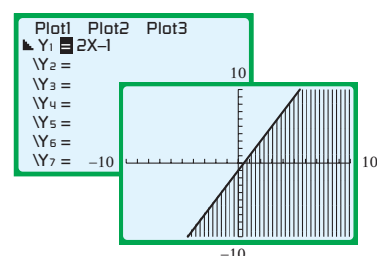
Y= 0.1 **X,T,θ,n** **^** 3 **-** 2 **X,T,θ,n** **-** 1 **ZOOM** (scroll to 6) **ENTER**



Note: For the keystrokes above, you do not have to scroll to 6. Alternatively, use **ZOOM** 6. This will select the standard viewing window and automatically start the graph. Use the **WINDOW** key to create a custom window for a graph.

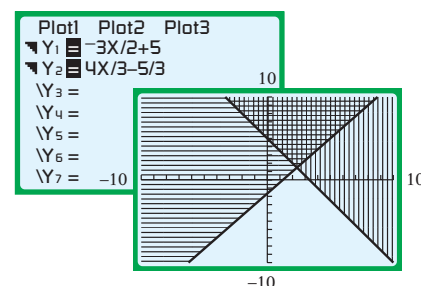
Graphing Inequalities

To illustrate this feature, we will graph $y \leq 2x - 1$. Enter $2x - 1$ into Y_1 . Because $y \leq 2x - 1$, we want to shade below the graph. Move the cursor to the left of Y_1 and press **ENTER** three times. Press **GRAPH**.



Note: To shade above the graph, move the cursor to the left of Y_1 and press **ENTER** two times. An inequality with the symbol $\leq$ or $\geq$ should be graphed with a solid line, and an inequality with the symbol $<$ or $>$ should be graphed with a dashed line. However, a graphing calculator does not distinguish between a solid line and a dashed line.

To graph the solution set of a system of inequalities, solve each inequality for y and graph each inequality. The solution set is the intersection of the two inequalities. The solution set of $3x + 2y > 10$ and $4x - 3y \leq 5$ is shown at the right.

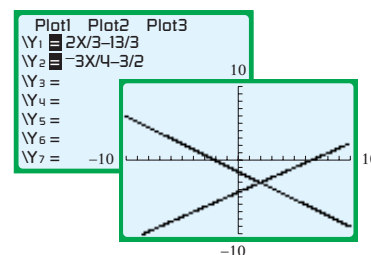


Intersect The INTERSECT feature is used to solve a system of equations. To illustrate this feature, we will use the system of equations $2x - 3y = 13$ and $3x + 4y = -6$.

Note: Some equations can be solved by this method. See the section “Solve an equation” on the next page. Also, this method is used to find a number in the domain of a function for a given number in the range. See the section “Find a domain element” on the next page.

Solve each of the equations in the system of equations for y . In this case, we have $y = \frac{2}{3}x - \frac{13}{3}$ and $y = -\frac{3}{4}x - \frac{3}{2}$.

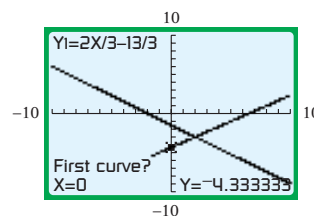
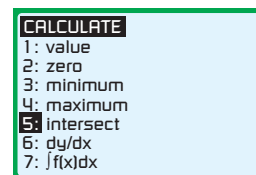
Use the Y-editor to enter $\frac{2}{3}x - \frac{13}{3}$ into Y_1 and $-\frac{3}{4}x - \frac{3}{2}$ into Y_2 . Graph the two functions in the standard viewing window. (If the window does not show the point of intersection of the two graphs, adjust the window until you can see the point of intersection.)



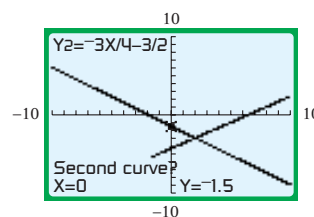
Press **2ND** **CALC** (scroll to 5, intersect) **ENTER**.

Alternatively, you can just press **2ND** **CALC** 5.

First curve? is shown at the bottom of the screen and identifies one of the two graphs on the screen. Press **ENTER**.

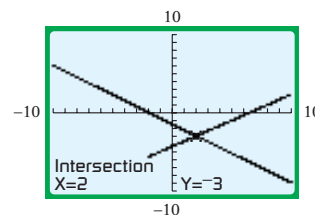


Second curve? is shown at the bottom of the screen and identifies the second of the two graphs on the screen. Press **ENTER**.



Guess? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the approximate location of the point of intersection. (If there are two or more points of intersection, it does not matter which one you choose first.) Press **ENTER**.

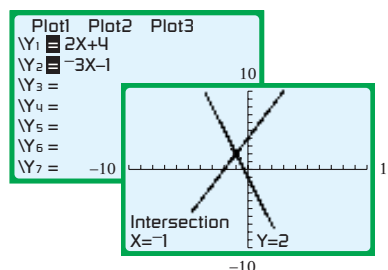
The solution of the system of equations is $(2, -3)$.



Solve an equation To illustrate the steps involved, we will solve the equation $2x + 4 = -3x - 1$. The idea is to write the equation as the system of equations $y = 2x + 4$ and $y = -3x - 1$ and then use the steps for solving a system of equations.

Use the Y-editor to enter the left and right sides of the equation into Y_1 and Y_2 . Graph the two functions and then follow the steps for **Intersect**.

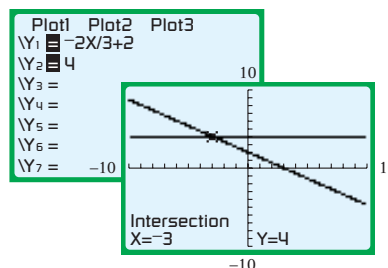
The solution is -1 , the x -coordinate of the point of intersection.



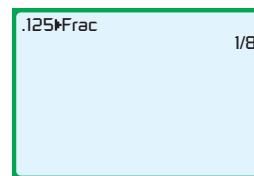
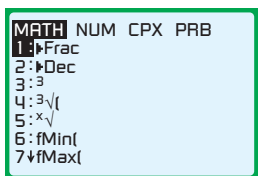
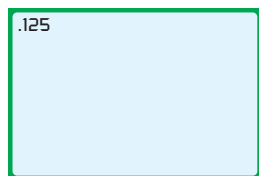
Find a domain element For this example, we will find a number in the domain of $f(x) = -\frac{2}{3}x + 2$ that corresponds to 4 in the range of the function. This is like solving the system of equations $y = -\frac{2}{3}x + 2$ and $y = 4$.

Use the Y = editor to enter the expression for the function in Y_1 and the desired output, 4, in Y_2 . Graph the two functions and then follow the steps for **Intersect**.

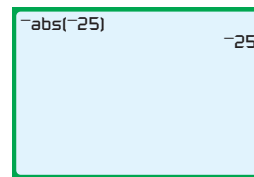
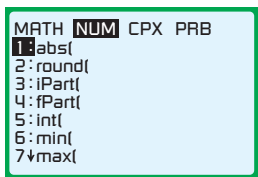
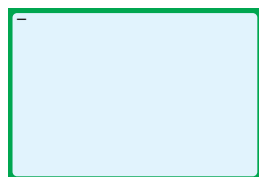
The point of intersection is $(-3, 4)$. The number -3 in the domain of f produces an output of 4 in the range of f .



Math Pressing **MATH** gives you access to many built-in functions. The following keystrokes will convert 0.125 to a fraction: $.125$ **MATH** 1 **ENTER**.



Additional built-in functions under **MATH** can be found by pressing **MATH** $\diamond$. For instance, to evaluate $-|-25|$, press $(-)$ **MATH** $\diamond$ 1 $(-)$ 25 **)** **ENTER**.

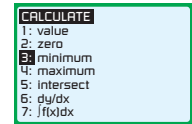


See your owner's manual for assistance with other functions under the **MATH** key.

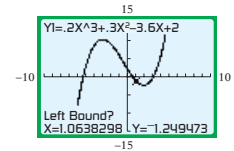
Min and Max The local minimum and the local maximum values of a function are calculated by accessing the CALC menu. For this demonstration, we will find the minimum value and the maximum value of $f(x) = 0.2x^3 + 0.3x^2 - 3.6x + 2$.

Enter the function into Y1. Press **2ND** **CALC** (scroll to 3 for **minimum** of the function) **ENTER**.

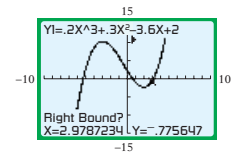
Alternatively, you can just press **2ND** **CALC** 3.



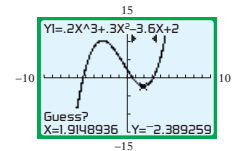
Left Bound? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the *left* of the minimum. Press **ENTER**.



Right Bound? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the *right* of the minimum. Press **ENTER**.

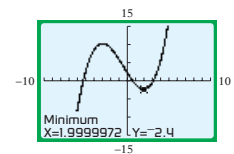


Guess? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the *approximate* location of the minimum. Press **ENTER**.

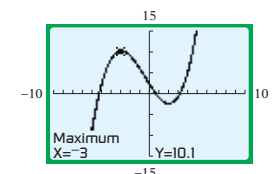
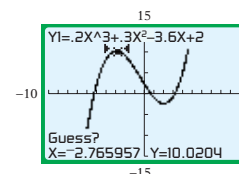
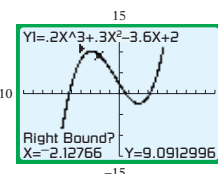
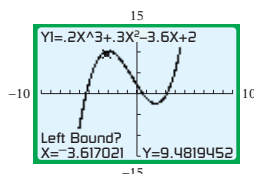
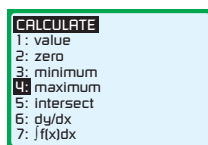


The minimum value of the function is the y-coordinate. For this example, the minimum value of the function is -2.4 .

The x-coordinate for the minimum is 2. However, because of rounding errors in the calculation, it is shown as a number close to 2.



To find the maximum value of the function, follow the same steps as above except select **maximum** under the CALC menu. The screens for this calculation are shown below.



The maximum value of the function is 10.1.

Radical Expressions To evaluate a square-root expression, press **2ND** $\sqrt{}$.

For instance, to evaluate $0.15\sqrt{p^2 + 4p + 10}$ when $p = 100,000$, first store 100,000 in P. Then press 0.15

2ND $\sqrt{}$ **ALPHA** **P** x^2 **+** 4 **ALPHA** **P** **+** 10 **)** **ENTER**.

100000→P	100000
0.15√(P²+4P+10)	15000.3

To evaluate a radical expression other than a square root, access $\sqrt[n]{}$ by pressing **MATH**. For instance, to evaluate $\sqrt[4]{67}$, press 4 (the index of the radical) **MATH** (scroll to 5) **ENTER** 67 **ENTER**.

MATH	NUM	CPX	PRB
1: Frac			
2: Dec			
3: 3			
4: 3√			
5: √	4√67		2.861005553
6: fMin			
7: fMax			

Scientific Notation

To enter a number in scientific notation, use **2ND** **EE**. For instance, to find $\frac{3.45 \times 10^{-12}}{1.5 \times 10^{25}}$, press 3.45 **2ND** **EE** **(-)** 12 **÷** 1.5 **2ND** **EE** 25 **ENTER**. The answer is 2.3×10^{-37} .

3.45E-12/1.5E25
2.3E-37

Sequences and Series

The terms of a sequence and the sum of a series can be calculated by using the **2ND** **LIST** feature.

Store a sequence A sequence is stored in one of the lists L1 through L6. For instance, to store the sequence 1, 3, 5, 7, 9 in L1, use the following keystrokes.

2ND **{** 1 **,** 3 **,** 5 **,** 7 **,** 9 **2ND** **}** **STO** **→** **2ND** **L1** **ENTER**

(1,3,5,7,9)→L1
(1,3,5,7,9)

Display the terms of a sequence The terms of a sequence are displayed by using the function **seq(expression, variable, begin, end, increment)**. For instance, to display the 3rd through 8th terms of the sequence given by $a_n = n^2 + 6$, enter the following keystrokes.

2ND **LIST** **(** (scroll to 5)
ENTER **X,T,θ,n** **x²** **+** 6
, **X,T,θ,n** **,** 3 **,** 8
, 1 **ENTER** **STO** **→** **2ND** **L1** **ENTER**

NAMES OPS MATH
1: SortA(
2: SortD(
3: dim(
4: Fill(
5: seq(
6: cumSum(
7: ΔList(
seq(X²+6,X,3,8,1)
(15 22 31 42 55...)

The keystrokes **STO** **→** **2ND** **L1** **ENTER** store the terms of the sequence in L1. This is not necessary but is sometimes helpful if additional work will be done with that sequence.

Find a sequence of partial sums To find a sequence of partial sums, use the **cumSum()** function. For instance, to find the sequence of partial sums for 2, 4, 6, 8, 10, use the following keystrokes.

2ND **LIST** **(** (scroll to 6)
ENTER **2ND** **{** 2 **,** 4 **,** 6
, 8 **,** 10 **2ND** **}** **)** **ENTER**

NAMES OPS MATH
1: SortA(
2: SortD(
3: dim(
4: Fill(
5: seq(
6: cumSum(
7: ΔList(
cumSum({2,4,6,8,10})
(2 6 12 20 30)

If a sequence is stored as a list in L1, then the sequence of partial sums can be calculated by pressing **2ND** **LIST** **(** (scroll to 6 [or press 6]) **ENTER** **2ND** **L1** **)** **ENTER**.

Find the sum of a series The sum of a series is calculated using **sum<list, start, end>**. For instance, to find $\sum_{n=3}^6 (n^2 + 2)$, enter the following keystrokes.

2ND **LIST** **(** (scroll to 5)
ENTER **2ND** **LIST** **(** (scroll to 5 [or press 5])
ENTER **X,T,θ,n** **x²** **+** 2 **,**
X,T,θ,n **,** 3 **,** 6 **,** 1 **)** **ENTER**

NAMES OPS MATH
1: min(
2: max(
3: mean(
4: median(
5: sum(
6: prod(
7: stdDev(
sum(seq(X²+2,X,3,6,1))
94

Table There are three steps in creating an input/output table for a function. First use the Y= editor to input the function. The second step is setting up the table, and the third step is displaying the table.

To set up the table, press 2ND TBLSET . **TblStart** is the first value of the independent variable in the input/output table. ΔTbl is the difference between successive values. Setting this to 1 means that, for this table, the input values are $-2, -1, 0, 1, 2, \dots$. If $\Delta\text{Tbl} = 0.5$, then the input values are $-2, -1.5, -1, -0.5, 0, 0.5, \dots$

TABLE SETUP		
TblStart=-2		
Indpnt:	Auto	Ask
Depend:	Auto	Ask

Indpnt is the independent variable. When this is set to **Auto**, values of the independent variable are automatically entered into the table. **Depend** is the dependent variable. When this is set to **Auto**, values of the dependent variable are automatically entered into the table.

To display the table, press 2ND TABLE . An input/output table for $f(x) = x^2 - 1$ is shown at the right.

Plot1 Plot2 Plot3		
Y1 =	X ² -1	
Y2 =		
Y3 =		
Y4 =		
Y5 =		
Y6 =		
Y7 =		

X	Y1
-2	3
-1	0
0	-1
1	0
2	3
3	8
4	15

X=-2

Once the table is on the screen, the up and down arrow keys can be used to display more values in the table. For the table at the right, we used the up arrow key to move to $x = -7$.

An input/output table for any given input can be created by selecting **Ask** for the independent variable. The table at the right shows an input/output table for $f(x) = \frac{4x}{x-2}$ for selected values of x . Note the word **ERROR** when 2 was entered. This occurred because f is not defined when $x = 2$.

Plot1 Plot2 Plot3		
Y1 =	4X/(X-2)	
Y2 =		
Y3 =		
Y4 =		
Y5 =		
Y6 =		
Y7 =		

TABLE SETUP		
TblStart=-2		
ΔTbl=1		
Indpnt:	Auto	Ask
Depend:	Auto	Ask

X	Y1
3	12
-5	2.8571
0	0
4	8
2	ERROR
-3	2.4

X=

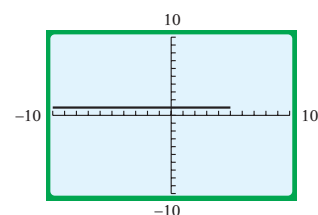
Note: Using the table feature in **Ask** mode is the same as evaluating a function for given values of the independent variable. For instance, from the table at the right, we have $f(4) = 8$.

Test The TEST feature has many uses, one of which is to graph the solution set of a linear inequality in one variable. To illustrate this feature, we will graph the solution set of $x - 1 < 4$. Press Y= X,T,θ,n $-$ 1 2ND TEST (scroll to 5) ENTER 4 GRAPH .

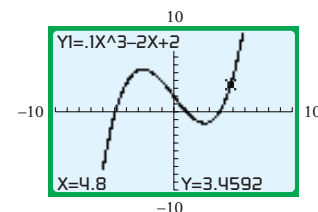
Plot1 Plot2 Plot3
Y1 = X-1
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =

TEST LOGIC
1: =
2: ≠
3: >
4: ≥
5: <
6: ≤

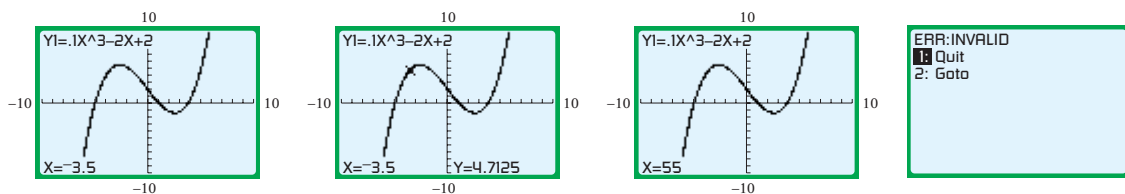
Plot1 Plot2 Plot3
Y1 = X-1<4
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =



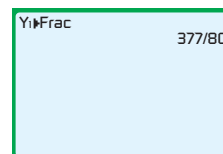
Trace Once a graph is drawn, pressing TRACE will place a cursor on the screen, and the coordinates of the point below the cursor are shown at the bottom of the screen. Use the left and right arrow keys to move the cursor along the graph. For the graph at the right, we have $f(4.8) = 3.4592$, where $f(x) = 0.1x^3 - 2x + 2$ is shown at the top left of the screen.



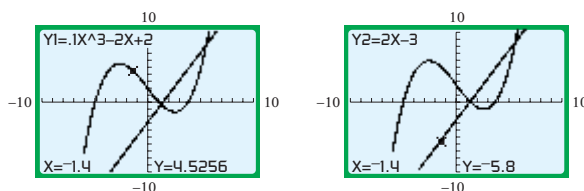
In TRACE mode, you can evaluate a function at any value of the independent variable that is within Xmin and Xmax. To do this, first graph the function. Now press **TRACE** (the value of x) **ENTER**. For the graph at the left below, we used $x = -3.5$. If a value of x is chosen outside the window, an error message is displayed.



In the example above in which we entered -3.5 for x , the value of the function was calculated as 4.7125 . This means that $f(-3.5) = 4.7125$. The keystrokes **2ND** **QUIT** **VAR** **1** **MATH** **1** **ENTER** will convert the decimal value to a fraction.

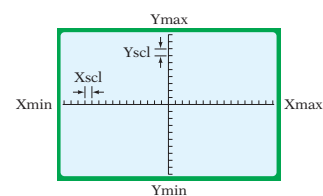


When the TRACE feature is used with two or more graphs, the up and down arrow keys are used to move between the graphs. The graphs below are for the functions $f(x) = 0.1x^3 - 2x + 2$ and $g(x) = 2x - 3$. By using the up and down arrows, we can place the cursor on either graph. The right and left arrows are used to move along the graph.



Window

The viewing window for a graph is controlled by pressing **WINDOW**. Xmin and Xmax are the minimum value and maximum value, respectively, of the independent variable shown on the graph. Xscl is the distance between tic marks on the x -axis. Ymin and Ymax are the minimum value and maximum value, respectively, of the dependent variable shown on the graph. Yscl is the distance between tic marks on the y -axis. Leave Xres as 1.

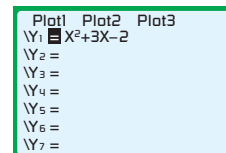


Note: In the standard viewing window, the distance between tic marks on the x -axis is different from the distance between tic marks on the y -axis. This will distort a graph. A more accurate picture of a graph can be created by using a square viewing window. See **ZOOM**.

Y=

The **Y=** editor is used to enter the expression for a function. There are ten possible functions, labeled Y_1 to Y_0 , that can be active at any one time. For instance, to enter $f(x) = x^2 + 3x - 2$ as Y_1 , use the following keystrokes.

Y= **X,T,θ,n** **x²** **+** **3** **X,T,θ,n** **-** **2**



Note: If an expression is already entered for Y_1 , place the cursor anywhere on that expression and press **CLEAR**.

To enter $s = \frac{2v-1}{v^3-3}$ into Y_2 , place the cursor to the right of the equals sign for Y_2 . Then press **(** **2** **X,T,θ,n** **-** **1** **)** **÷** **(** **X,T,θ,n** **^** **3** **-** **3** **)**.

```
Plot1 Plot2 Plot3
Y1 = X^2+3X-2
Y2 = (2X-1)/(X^3-3)
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =
```

Note: When we enter an equation, the independent variable, v in the expression above, is entered using **X,T,θ,n**. The dependent variable, s in the expression above, is one of Y_1 to Y_0 . Also note the use of parentheses to ensure the correct order of operations.

Observe the black rectangle that covers the equals sign for the two examples we have shown. This rectangle means that the function is “active.” If we were to press **GRAPH**, then the graph of both functions would appear. You can make a function inactive by using the arrow keys to move the cursor over the equals sign of that function and then pressing **ENTER**. This will remove the black rectangle. We have done that for Y_2 , as shown at the right. Now if **GRAPH** is pressed, only Y_1 will be graphed.

```
Plot1 Plot2 Plot3
Y1 = X^2+3X-2
Y2 = (2X-1)/(X^3-3)
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =
```

It is also possible to control the appearance of the graph by moving the cursor on the **Y=** screen to the left of any Y . With the cursor in this position, pressing **ENTER** will change the appearance of the graph. The options are shown at the right.

```
Plot1 Plot2 Plot3
Y1 = Default graph line
Y2 = Bold graph line
Y3 = Shade above graph
Y4 = Shade below graph
Y5 = Draw path of graph
Y6 = Travel path of graph
Y7 = Dashed graph line
```

Zero The ZERO feature of a graphing calculator is used for various calculations: to find the x -intercepts of a function, to solve some equations, and to find the zero of a function.

x -intercepts To illustrate the procedure for finding x -intercepts, we will use $f(x) = x^2 + x - 2$.

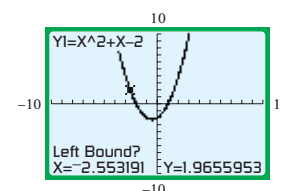
First, use the Y-editor to enter the expression for the function and then graph the function in the standard viewing window. (It may be necessary to adjust this window so that the intercepts are visible.) Once the graph is displayed, use the keystrokes below to find the x -intercepts of the graph of the function.

Press **2ND** **CALC** (scroll to 2 for **zero** of the function) **ENTER**.

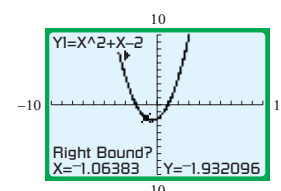
Alternatively, you can just press **2ND** **CALC** 2.

```
CALCULATE
1: value
2: zero
3: minimum
4: maximum
5: intersect
6: dy/dx
7: ∫f(x)dx
```

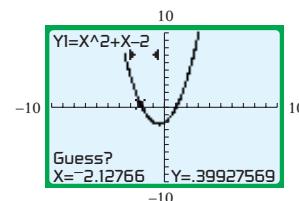
Left Bound? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the *left* of the desired x -intercept. Press **ENTER**.



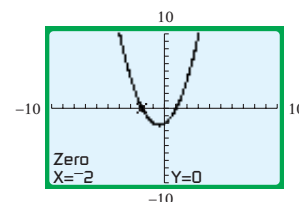
Right Bound? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the *right* of the desired x -intercept. Press **ENTER**.



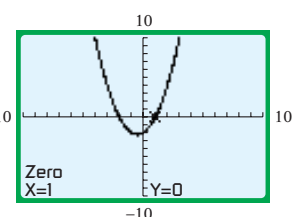
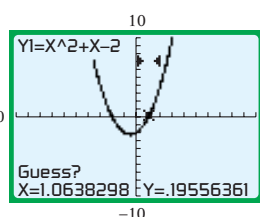
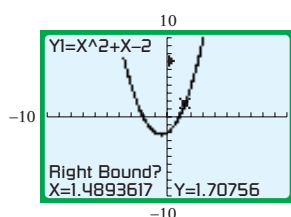
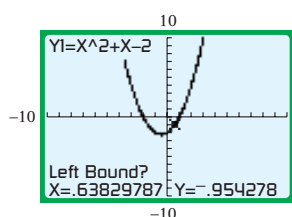
Guess? shown at the bottom of the screen asks you to use the left or right arrow key to move the cursor to the *approximate* location of the desired x -intercept. Press **ENTER**.



The x -coordinate of an x -intercept is -2 . Therefore, an x -intercept is $(-2, 0)$.



To find the other x -intercept, follow the same steps as above. The screens for this calculation are shown below.



A second x -intercept is $(1, 0)$.

Solve an equation To use the ZERO feature to solve an equation, first rewrite the equation with all terms on one side. For instance, one way to solve the equation $x^3 - x + 1 = -2x + 3$ is first to rewrite it as $x^3 + x - 2 = 0$. Enter $x^3 + x - 2$ into Y_1 and then follow the steps for finding x -intercepts.

Find the real zeros of a function To find the real zeros of a function, follow the steps for finding x -intercepts.

Zoom Pressing **ZOOM** allows you to select some preset viewing windows. This key also gives you access to **ZBox**, **Zoom In**, and **Zoom Out**. These functions enable you to redraw a selected portion of a graph in a new window. Some windows used frequently in this text are shown below.

ZOOM MEMORY	
1: ZBox	
2: Zoom In	
3: Zoom Out	
4: ZDecimal	
5: ZSquare	
6: ZStandard	
7: ZTrig	

WINDOW	
Xmin = -4.7	
Xmax = 4.7	
Xscl = 1	
Ymin = -3.1	
Ymax = 3.1	
Yscl = 1	
Xres = 1	

ZOOM MEMORY	
1: ZBox	
2: Zoom In	
3: Zoom Out	
4: ZDecimal	
5: ZSquare	
6: ZStandard	
7: ZTrig	

WINDOW	
Xmin = -15.16129...	
Xmax = 15.161290...	
Xscl = 1	
Ymin = -10	
Ymax = 10	
Yscl = 1	
Xres = 1	

ZOOM MEMORY	
1: ZBox	
2: Zoom In	
3: Zoom Out	
4: ZDecimal	
5: ZSquare	
6: ZStandard	
7: ZTrig	

WINDOW	
Xmin = -10	
Xmax = 10	
Xscl = 1	
Ymin = -10	
Ymax = 10	
Yscl = 1	
Xres = 1	

ZOOM MEMORY	
4: ZDecimal	
5: ZSquare	
6: ZStandard	
7: ZTrig	
8: ZInteger	
9: ZoomStat	
0: ZoomFit	

WINDOW	
Xmin = -47	
Xmax = 47	
Xscl = 10	
Ymin = -31	
Ymax = 31	
Yscl = 10	
Xres = 1	

Appendix B

Proofs and Tables

Proofs of Logarithmic Properties

In each of the following proofs of logarithmic properties, it is assumed that the Properties of Exponents are true for all real number exponents.

The Logarithm Property of the Product of Two Numbers

For any positive real numbers x , y , and b , $b \neq 1$, $\log_b xy = \log_b x + \log_b y$.

Proof: Let $\log_b x = m$ and $\log_b y = n$.

Write each equation in its equivalent exponential form.

Use substitution and the Properties of Exponents.

Write the equation in its equivalent logarithmic form.

Substitute $\log_b x$ for m and $\log_b y$ for n .

$$x = b^m \quad y = b^n$$

$$xy = b^m b^n$$

$$xy = b^{m+n}$$

$$\log_b xy = m + n$$

$$\log_b xy = \log_b x + \log_b y$$

The Logarithm Property of the Quotient of Two Numbers

For any positive real numbers x , y , and b , $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.

Proof: Let $\log_b x = m$ and $\log_b y = n$.

Write each equation in its equivalent exponential form.

Use substitution and the Properties of Exponents.

Write the equation in its equivalent logarithmic form.

Substitute $\log_b x$ for m and $\log_b y$ for n .

$$x = b^m \quad y = b^n$$

$$\frac{x}{y} = \frac{b^m}{b^n}$$

$$\frac{x}{y} = b^{m-n}$$

$$\log_b \frac{x}{y} = m - n$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

The Logarithm Property of the Power of a Number

For any real numbers x , r , and b , $b \neq 1$, $\log_b x^r = r \log_b x$.

Proof: Let $\log_b x = m$.

Write the equation in its equivalent exponential form.

Raise both sides to the r power.

Write the equation in its equivalent logarithmic form.

Substitute $\log_b x$ for m .

$$x = b^m$$

$$x^r = (b^m)^r$$

$$x^r = b^{mr}$$

$$\log_b x^r = mr$$

$$\log_b x^r = r \log_b x$$

Proof of the Formula for the Sum of n Terms of a Geometric Series

Theorem: The sum of the first n terms of a geometric sequence whose n th term is

$$ar^{n-1} \text{ is given by } S_n = \frac{a(1 - r^n)}{1 - r}.$$

Proof: Let S_n represent the sum of n terms of the sequence.

Multiply each side of the equation by r .

Subtract the two equations.

Assuming $r \neq 1$, solve for S_n .

$$S_n = a + ar + ar^2 + \cdots + ar^{n-2} + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$(1 - r)S_n = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Proof of the Formula for the Sum of n Terms of an Arithmetic Series

Each term of the arithmetic sequence shown at the right was found by adding 3 to the previous term. 2, 5, 8, ..., 17, 20

Each term of the reverse arithmetic sequence can be found by subtracting 3 from the previous term. 20, 17, 14, ..., 5, 2

This idea is used in the following proof.

Theorem: The sum of the first n terms of an arithmetic sequence for which a_1 is the first term, n is the number of terms, a_n is the last term, and d is the common difference is given by $S_n = \frac{n}{2}(a_1 + a_n)$.

Proof: Let S_n represent the sum of the sequence. $S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + a_n$
Write the terms of the sum of the sequence in reverse order. The sum will be the same. $S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + a_1$

Add the two equations. $2S_n = (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n)$

Simplify the right side of the equation by using the fact that there are n terms in the sequence. $2S_n = n(a_1 + a_n)$

Solve for S_n . $S_n = \frac{n}{2}(a_1 + a_n)$

Table of Symbols

+	add	<	is less than
−	subtract	≤	is less than or equal to
·, ×, (a)(b)	multiply	>	is greater than
$\frac{a}{b}$, ÷	divide	≥	is greater than or equal to
()	parentheses, a grouping symbol	(a, b)	an ordered pair whose first component is a and whose second component is b
[]	brackets, a grouping symbol	°	degree (for angles)
π	pi, a number approximately equal to $\frac{22}{7}$ or 3.14	$\sqrt{a}$	the principal square root of a
$-a$	the opposite, or additive inverse, of a	$\emptyset, \{ \}$	the empty set
$\frac{1}{a}$	the reciprocal, or multiplicative inverse, of a	$ a $	the absolute value of a
=	is equal to	$\cup$	union of two sets
≈	is approximately equal to	$\cap$	intersection of two sets
≠	is not equal to	$\in$	is an element of (for sets)
		$\notin$	is not an element of (for sets)

Table of Measurement Abbreviations

U.S. Customary System							
Length		Capacity		Weight		Area	
in.	inches	oz	fluid ounces	oz	ounces	in ²	square inches
ft	feet	c	cups	lb	pounds	ft ²	square feet
yd	yards	qt	quarts				
mi	miles	gal	gallons				
Metric System							
Length		Capacity		Weight/Mass		Area	
mm	millimeter (0.001 m)	ml	milliliter (0.001 L)	mg	milligram (0.001 g)	cm ²	square centimeters
cm	centimeter (0.01 m)	cl	centiliter (0.01 L)	cg	centigram (0.01 g)	m ²	square meters
dm	decimeter (0.1 m)	dl	deciliter (0.1 L)	dg	decigram (0.1 g)		
m	meter	L	liter	g	gram		
dam	decameter (10 m)	dal	decaliter (10 L)	dag	decagram (10 g)		
hm	hectometer (100 m)	hl	hectoliter (100 L)	hg	hectogram (100 g)		
km	kilometer (1000 m)	kl	kiloliter (1000 L)	kg	kilogram (1000 g)		
Time							
h	hours	min	minutes	s	seconds		

Table of Properties**Properties of Real Numbers****The Associative Property of Addition**

If a , b , and c are real numbers, then
 $(a + b) + c = a + (b + c)$.

The Commutative Property of Addition

If a and b are real numbers, then
 $a + b = b + a$.

The Addition Property of Zero

If a is a real number, then
 $a + 0 = 0 + a = a$.

The Multiplication Property of Zero

If a is a real number, then
 $a \cdot 0 = 0 \cdot a = 0$.

The Inverse Property of Addition

If a is a real number, then
 $a + (-a) = (-a) + a = 0$.

The Associative Property of Multiplication

If a , b , and c are real numbers, then
 $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

The Commutative Property of Multiplication

If a and b are real numbers, then
 $a \cdot b = b \cdot a$.

The Multiplication Property of One

If a is a real number, then
 $a \cdot 1 = 1 \cdot a = a$.

The Inverse Property of Multiplication

If a is a real number and $a \neq 0$, then
 $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

Distributive Property

If a , b , and c are real numbers, then
 $a(b + c) = ab + ac$.

Properties of Equations**Addition Property of Equations**

If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations

If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.

Properties of Inequalities**Addition Property of Inequalities**

If $a > b$, then $a + c > b + c$.
 If $a < b$, then $a + c < b + c$.

Multiplication Property of Inequalities

If $a > b$ and $c > 0$, then $ac > bc$.
 If $a < b$ and $c > 0$, then $ac < bc$.
 If $a > b$ and $c < 0$, then $ac < bc$.
 If $a < b$ and $c < 0$, then $ac > bc$.

Properties of Exponents

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.
 If m and n are integers, then $(x^m)^n = x^{mn}$.

If m , n , and p are integers, then $(x^m \cdot y^n)^p = x^{mp} y^{np}$.
 If n is a positive integer and $x \neq 0$, then

If $x \neq 0$, then $x^0 = 1$.

$$x^{-n} = \frac{1}{x^n} \text{ and } \frac{1}{x^{-n}} = x^n.$$

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

If m , n , and p are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

Principle of Zero Products

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

Properties of Radical Expressions

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$. If a and b are positive real numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

Properties of Logarithms

If x , y , and b are positive real numbers and $b \neq 1$, then
 $\log_b(xy) = \log_b x + \log_b y$.
 If x , y , and b are positive real numbers and $b \neq 1$, then
 $\log_b \frac{x}{y} = \log_b x - \log_b y$.

If x and b are positive real numbers, $b \neq 1$, and r is any real number, then $\log_b x^r = r \log_b x$.
 If x and b are positive real numbers and $b \neq 1$, then
 $\log_b b^x = x$.

Table of Algebraic and Geometric Formulas

Slope of a Line

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$$

Point-slope Formula for a Line

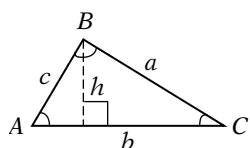
$$y - y_1 = m(x - x_1)$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{discriminant} = b^2 - 4ac$$

Perimeter and Area of a Triangle, and Sum of the Measures of the Angles

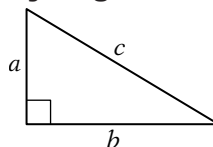


$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

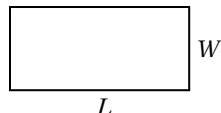
$$A + B + C = 180^\circ$$

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

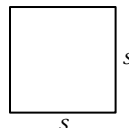
Perimeter and Area of a Rectangle



$$P = 2L + 2W$$

$$A = LW$$

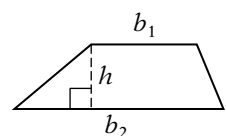
Perimeter and Area of a Square



$$P = 4s$$

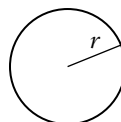
$$A = s^2$$

Area of a Trapezoid



$$A = \frac{1}{2}h(b_1 + b_2)$$

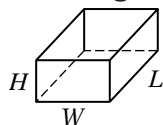
Circumference and Area of a Circle



$$C = 2\pi r$$

$$A = \pi r^2$$

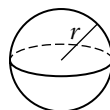
Volume and Surface Area of a Rectangular Solid



$$V = LWH$$

$$SA = 2LW + 2LH + 2WH$$

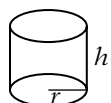
Volume and Surface Area of a Sphere



$$V = \frac{4}{3}\pi r^3$$

$$SA = 4\pi r^2$$

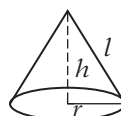
Volume and Surface Area of a Right Circular Cylinder



$$V = \pi r^2 h$$

$$SA = 2\pi r^2 + 2\pi rh$$

Volume and Surface Area of a Right Circular Cone



$$V = \frac{1}{3}\pi r^2 h$$

$$SA = \pi r^2 + \pi rl$$

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Solutions to “You Try It”

SOLUTIONS TO CHAPTER 1 “YOU TRY IT”

SECTION 1.1

You Try It 1 Replace z by each of the elements of the set and determine whether the inequality is true.

$$\begin{aligned} z &> -5 \\ -10 &> -5 \text{ False} \\ -5 &> -5 \text{ False} \\ 6 &> -5 \text{ True} \end{aligned}$$

The inequality is true for 6.

You Try It 2 a. Replace d in $-d$ by each element of the set and determine the value of the expression.

$$\begin{aligned} -d \\ -(-11) &= 11 \\ -(0) &= 0 \\ -(8) &= -8 \end{aligned}$$

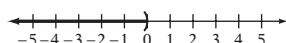
b. Replace d in $|d|$ by each element of the set and determine the value of the expression.

$$\begin{aligned} |d| \\ |-11| &= 11 \\ |0| &= 0 \\ |8| &= 8 \end{aligned}$$

You Try It 3 $\{-5, -4, -3, -2, -1\}$

You Try It 4 $\{x | x \geq 15, x \in \text{whole numbers}\}$

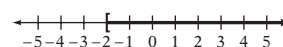
You Try It 5 Draw a parenthesis at 0 to indicate that 0 is not in the set. Draw a solid line to the left of 0.



You Try It 6 a. The set $\{x | x < -1\}$ is the numbers less than -1 . In interval notation, this is written $(-\infty, -1)$.
b. The set $\{x | -2 \leq x < 4\}$ is the numbers greater than or equal to -2 and less than 4. In interval notation, this is written $[-2, 4)$.

You Try It 7 a. The ∞ symbol indicates that the interval extends forever. Therefore, $(3, \infty)$ is the numbers greater than 3. In set-builder notation, this is written $\{x | x > 3\}$.
b. $(-4, 1]$ is the numbers greater than -4 and less than or equal to 1. In set-builder notation, this is written $\{x | -4 < x \leq 1\}$.

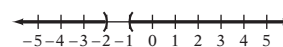
You Try It 8 Draw a bracket at -2 to show that it is in the set. The ∞ symbol indicates that the set extends forever. Draw a solid line to the right of -2 .



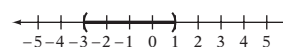
You Try It 9 $C \cup D = \{1, 3, 5, 7, 9, 11, 13, 17\}$

You Try It 10 No integer is both even and odd. Therefore, $E \cap F = \emptyset$.

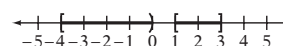
You Try It 11 This is the set of real numbers less than -2 or greater than -1 .



You Try It 12 This is the set of real numbers less than 1 and greater than -3 .



You Try It 13 The graph is the set of real numbers that belong to one or the other of the two intervals.



SECTION 1.2

You Try It 1 $6 - (-8) - 10$

$$= 6 + 8 + (-10)$$

$$= 14 + (-10)$$

$$= 4$$

• Rewrite subtraction as addition of the opposite. Then add.

You Try It 2 $12(-3)|-6|$

$$= 12(-3)(6)$$

$$= (-36)(6)$$

$$= -216$$

• Find the absolute value of -6 . Then multiply.

You Try It 3 $-\frac{28}{|-14|} = -\frac{28}{14}$

$$= -2$$

You Try It 4

$$\frac{5}{6} - \frac{3}{8} - \frac{7}{9} = \frac{5}{6} + \frac{-3}{8} + \frac{-7}{9}$$

$$= \frac{5}{6} \cdot \frac{12}{12} + \frac{-3}{8} \cdot \frac{9}{9} + \frac{-7}{9} \cdot \frac{8}{8}$$

$$= \frac{60}{72} + \frac{-27}{72} + \frac{-56}{72}$$

$$= \frac{60 + (-27) + (-56)}{72}$$

$$= \frac{-23}{72} = -\frac{23}{72}$$

You Try It 5 $\frac{5}{8} \div \left(-\frac{15}{40}\right) = -\frac{5}{8} \cdot \frac{40}{15}$

$$= -\frac{5 \cdot 40}{8 \cdot 15} = -\frac{5}{3}$$

You Try It 6 $-2^5 = -(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = -32$

$$(-5)^2 = (-5)(-5) = 25$$

You Try It 7 $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3}$$

$$= \frac{16}{81}$$

You Try It 8

$$-2\left[3 - 3^3 + \frac{16 + 2}{1 - 10}\right]$$

$$= -2\left[3 - 3^3 + \frac{18}{-9}\right]$$

$$= -2[3 - 3^3 + (-2)]$$

$$= -2[3 - 27 + (-2)]$$

$$= -2[-24 + (-2)]$$

$$= -2[-26]$$

$$= 52$$

• Inside brackets

• Multiplication

You Try It 9

$$\frac{1}{3} + \frac{5}{8} \div \frac{15}{16} - \frac{7}{12} = \frac{1}{3} + \frac{5}{8} \cdot \frac{16}{15} - \frac{7}{12}$$

$$= \frac{1}{3} + \frac{2}{3} - \frac{7}{12}$$

$$= 1 - \frac{7}{12}$$

$$= \frac{5}{12}$$

• Division and multiplication

• Addition and subtraction

You Try It 10

$$\frac{11}{12} - \frac{\frac{5}{4}}{2 - \frac{7}{2}} \cdot \frac{3}{4} = \frac{11}{12} - \frac{\frac{5}{4}}{-\frac{3}{2}} \cdot \frac{3}{4}$$

$$= \frac{11}{12} - \left[\frac{5}{4} \cdot \left(-\frac{2}{3}\right)\right] \cdot \frac{3}{4}$$

$$= \frac{11}{12} - \left(-\frac{5}{6}\right) \cdot \frac{3}{4}$$

$$= \frac{11}{12} - \left(-\frac{5}{8}\right)$$

$$= \frac{37}{24}$$

• Simplify the complex fraction.

• Multiplication

• Subtraction

SECTION 1.3

You Try It 1 $3x + (-3x) = 0$

You Try It 2 The Associative Property of Addition

You Try It 3

$$-2x^2 + 3(4xy - z)$$

$$-2(-3)^2 + 3[4(-3)(-1) - 2]$$

$$= -2(-3)^2 + 3[12 - 2]$$

$$= -2(-3)^2 + 3(10)$$

$$= -2(9) + 3(10)$$

$$= -18 + 30$$

$$= 12$$

You Try It 4

$$\begin{aligned}
 2x - y|4x^2 - y^2| \\
 2(-2) - (-6)|4(-2)^2 - (-6)^2| \\
 = 2(-2) - (-6)|4 \cdot 4 - 36| \\
 = 2(-2) - (-6)|16 - 36| \\
 = 2(-2) - (-6)|-20| \\
 = 2(-2) - (-6)(20) \\
 = -4 - (-120) \\
 = -4 + 120 \\
 = 116
 \end{aligned}$$

You Try It 5

$$\begin{aligned}
 9 - 2(3y - 4) + 5y \\
 = 9 - 6y + 8 + 5y \\
 = -y + 17
 \end{aligned}$$

- **The Order of Operations Agreement requires multiplication before subtraction.**

You Try It 6

$$\begin{aligned}
 6z - 3(5 - 3z) + 5(2z - 3) \\
 = 6z - 15 + 9z + 10z - 15 \\
 = 25z - 30
 \end{aligned}$$

- **Distributive Property**
- **Combine like terms.**

SECTION 1.4

You Try It 1 twice x divided by the difference between x and 7

$$\frac{2x}{x - 7}$$

You Try It 2 the product of negative 3 and the square of d

$$-3d^2$$

You Try It 3 The smaller number is x .
The larger number is $16 - x$.
the difference between twice the smaller number and the larger number

$$2x - (16 - x) = 2x - 16 + x$$

$$= 3x - 16$$

You Try It 4 Let the number be x .
the difference between 14 and the sum of a number and 7

$$14 - (x + 7) = 14 - x - 7$$

$$= -x + 7$$

You Try It 5 The height of the Empire State Building: h
The length of the *Destiny*: $h + 56$

You Try It 6 The depth of the shallow end: d
The depth of the deep end: $2d + 2$

You Try It 7 Amount placed in certificate of deposit:
 $5000 - x$

SOLUTIONS TO CHAPTER 2 "YOU TRY IT"**SECTION 2.1**

You Try It 1 $x + 4 = -3$ • **Subtract 4 from each side.**

$$x + 4 - 4 = -3 - 4$$

$$x = -7$$

The solution is -7 .

You Try It 2 $-3x = 18$ • **Divide each side by -3 .**

$$\frac{-3x}{-3} = \frac{18}{-3}$$

$$x = -6$$

The solution is -6 .

You Try It 3

$$4x - 9 = 5 - 3x$$

$$4x - 9 + 3x = 5 - 3x + 3x$$
 • **Add $3x$ to each side.**

$$7x - 9 = 5$$

$$7x - 9 + 9 = 5 + 9$$
 • **Add 9 to each side.**

$$7x = 14$$

$$\frac{7x}{7} = \frac{14}{7}$$
 • **Divide each side by 7.**

$$x = 2$$

The solution is 2.

You Try It 4 $6(5 - x) - 12 = 2x - 3(4 + x)$

$$30 - 6x - 12 = 2x - 12 - 3x$$

$$18 - 6x = -x - 12$$

$$18 - 6x + x = -x - 12 + x$$

$$18 - 5x = -12$$

$$18 - 5x - 18 = -12 - 18$$

$$-5x = -30$$

$$\frac{-5x}{-5} = \frac{-30}{-5}$$

$$x = 6$$

The solution is 6.

You Try It 5

$$S = C - rC$$
 • **Subtract C from each side.**

$$S - C = -rC$$
 • **Divide each side by $-C$.**

$$\frac{S - C}{-C} = r$$

$$r = -\frac{S - C}{C} = \frac{C - S}{C}$$

SECTION 2.2

You Try It 1

Strategy

- The first number: n
The second number: $2n$
The third number: $4n - 3$
- The sum of the numbers is 81.
 $n + 2n + (4n - 3) = 81$

Solution

$$\begin{aligned} n + 2n + (4n - 3) &= 81 \\ 7n - 3 &= 81 && \bullet \text{ Combine like terms.} \\ 7n &= 84 && \bullet \text{ Add 3 to each side.} \\ n &= 12 && \bullet \text{ Divide each side by 7.} \end{aligned}$$

$$\begin{aligned} 2n &= 2(12) = 24 \\ 4n - 3 &= 4(12) - 3 = 48 - 3 = 45 \end{aligned}$$

The numbers are 12, 24, and 45.

You Try It 2

Strategy

- First odd integer: n
Second odd integer: $n + 2$
Third odd integer: $n + 4$
- Three times the sum of the first two integers is ten more than the product of the third integer and four.
 $3[n + (n + 2)] = (n + 4)4 + 10$

Solution

$$\begin{aligned} 3[n + (n + 2)] &= (n + 4)4 + 10 \\ 3[2n + 2] &= 4n + 16 + 10 && \bullet \text{ Add like terms and use the Distributive Property.} \\ 6n + 6 &= 4n + 26 && \bullet \text{ Use the Distributive Property and add like terms.} \\ 2n + 6 &= 26 && \bullet \text{ Subtract } 4n \text{ from each side.} \\ 2n &= 20 && \bullet \text{ Subtract 6 from each side.} \\ n &= 10 && \bullet \text{ Divide each side by 2.} \end{aligned}$$

Because 10 is not an odd integer, there is no solution.

You Try It 3

Strategy

- Number of 3¢ stamps: x
Number of 10¢ stamps: $2x + 2$
Number of 15¢ stamps: $3x$

Stamp	Number	Value	Total Value
3¢	x	3	$3x$
10¢	$2x + 2$	10	$10(2x + 2)$
15¢	$3x$	15	$45x$

- The sum of the total values of each type of stamp equals the total value of all the stamps (156 cents).
 $3x + 10(2x + 2) + 45x = 156$

Solution

$$\begin{aligned} 3x + 10(2x + 2) + 45x &= 156 \\ 3x + 20x + 20 + 45x &= 156 && \bullet \text{ Distributive Property} \\ 68x + 20 &= 156 && \bullet \text{ Combine like terms.} \\ 68x &= 136 && \bullet \text{ Subtract 20 from each side.} \\ x &= 2 && \bullet \text{ Divide each side by 68.} \end{aligned}$$

$$3x = 3(2) = 6$$

There are six 15¢ stamps in the collection.

SECTION 2.3

You Try It 1

Strategy

- Ounces of \$320 gold alloy: x
Ounces of \$100 gold alloy: 100
Ounces of \$160 mixture: $x + 100$

	Amount	Cost	Value
\$320 alloy	x	320	$320x$
\$100 alloy	100	100	$100(100)$
Mixture	$x + 100$	160	$160(x + 100)$

- The sum of the values before mixing equals the value after mixing.
 $320x + 100(100) = 160(x + 100)$

Solution

$$\begin{aligned} 320x + 100(100) &= 160(x + 100) \\ 320x + 10,000 &= 160x + 16,000 \\ 160x + 10,000 &= 16,000 \\ 160x &= 6000 \\ x &= 37.5 \end{aligned}$$

The mixture must contain 37.5 oz of the \$320 gold alloy.

You Try It 2**Strategy**

- Pounds of 22% hamburger: x
Pounds of 12% hamburger: $80 - x$
Pounds of 18% hamburger: 80

	Amount	Percent	Quantity
22%	x	0.22	$0.22x$
12%	$80 - x$	0.12	$0.12(80 - x)$
18%	80	0.18	$0.18(80)$

- The sum of the quantities before mixing is equal to the quantity after mixing.
 $0.22x + 0.12(80 - x) = 0.18(80)$

Solution

$$0.22x + 0.12(80 - x) = 0.18(80)$$

$$0.22x + 9.6 - 0.12x = 14.4$$

$$0.10x + 9.6 = 14.4$$

$$0.10x = 4.8$$

$$x = 48$$

$$80 - x = 80 - 48 = 32$$

The butcher needs 48 lb of the hamburger that is 22% fat and 32 lb of the hamburger that is 12% fat.

You Try It 3**Strategy**

- Rate of the second plane: r
Rate of the first plane: $r + 30$

	Rate	Time	Distance
1st plane	$r + 30$	4	$4(r + 30)$
2nd plane	r	4	$4r$

- The total distance traveled by the two planes is 1160 mi.
 $4(r + 30) + 4r = 1160$

Solution

$$4(r + 30) + 4r = 1160$$

$$4r + 120 + 4r = 1160$$

$$8r + 120 = 1160$$

$$8r = 1040$$

$$r = 130$$

$$r + 30 = 130 + 30 = 160$$

The first plane is traveling 160 mph. The second plane is traveling 130 mph.

SECTION 2.4**You Try It 1**

$$2x - 1 < 6x + 7$$

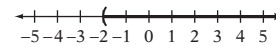
$$-4x - 1 < 7$$

$$-4x < 8$$

$$\frac{-4x}{-4} > \frac{8}{-4}$$

$$x > -2$$

$$\{x | x > -2\}$$



- Subtract $6x$ from each side.
- Add 1 to each side.
- Divide each side by -4 .

You Try It 2

$$5x - 2 \leq 4 - 3(x - 2)$$

$$5x - 2 \leq 4 - 3x + 6$$

$$5x - 2 \leq 10 - 3x$$

$$8x - 2 \leq 10$$

$$8x \leq 12$$

$$\frac{8x}{8} \leq \frac{12}{8}$$

$$x \leq \frac{3}{2}$$

$$\left(-\infty, \frac{3}{2}\right]$$

You Try It 3

$$-2 \leq 5x + 3 \leq 13$$

$$-2 - 3 \leq 5x + 3 - 3 \leq 13 - 3$$

$$-5 \leq 5x \leq 10$$

$$\frac{-5}{5} \leq \frac{5x}{5} \leq \frac{10}{5}$$

$$-1 \leq x \leq 2$$

$$[-1, 2]$$

- Subtract 3 from each of the three parts.

- Divide each of the three parts by 5.

You Try It 4

$$2 - 3x > 11$$

$$\text{or } 5 + 2x > 7$$

$$-3x > 9$$

$$2x > 2$$

$$x < -3$$

$$x > 1$$

$$\{x | x < -3\}$$

$$\{x | x > 1\}$$

$$\{x | x < -3\} \cup \{x | x > 1\}$$

$$= \{x | x < -3 \text{ or } x > 1\}$$

You Try It 5

Strategy To find the maximum height, substitute the given values in the inequality $\frac{1}{2}bh < A$ and solve.

Solution

$$\frac{1}{2}bh < A$$

$$\frac{1}{2}(12)(x + 2) < 50$$

$$6(x + 2) < 50$$

$$6x + 12 < 50$$

$$6x < 38$$

$$x < \frac{19}{3}$$

The largest integer less than $\frac{19}{3}$ is 6.

$$x + 2 = 6 + 2 = 8$$

The maximum height of the triangle is 8 in.

You Try It 6

Strategy To find the range of scores, write and solve an inequality using N to represent the score on the last test.

Solution

$$80 \leq \frac{72 + 94 + 83 + 70 + N}{5} \leq 89$$

$$80 \leq \frac{319 + N}{5} \leq 89$$

$$5 \cdot 80 \leq 5 \left(\frac{319 + N}{5} \right) \leq 5 \cdot 89$$

$$400 \leq 319 + N \leq 445$$

$$400 - 319 \leq 319 + N - 319 \leq 445 - 319$$

$$81 \leq N \leq 126$$

Because 100 is the maximum score, the range of scores to receive a B grade is $81 \leq N \leq 100$.

SECTION 2.5**You Try It 1**

$$|2x - 3| = 5$$

$$\begin{array}{ll} 2x - 3 = 5 & 2x - 3 = -5 \\ 2x = 8 & 2x = -2 \\ x = 4 & x = -1 \end{array} \quad \begin{array}{l} \bullet \text{ Add 3.} \\ \bullet \text{ Divide by 2.} \end{array}$$

The solutions are 4 and -1 .

$$\text{You Try It 2} \quad |x - 3| = -2$$

There is no solution to this equation because the absolute value of a number must be nonnegative.

You Try It 3

$$\begin{array}{ll} 5 - |3x + 5| = 3 & \\ -|3x + 5| = -2 & \bullet \text{ Subtract 5.} \\ |3x + 5| = 2 & \bullet \text{ Multiply by } -1. \\ 3x + 5 = 2 & 3x + 5 = -2 \\ 3x = -3 & 3x = -7 \\ x = -1 & x = -\frac{7}{3} \end{array}$$

The solutions are -1 and $-\frac{7}{3}$.

You Try It 4

$$\begin{array}{l} |3x + 2| < 8 \\ -8 < 3x + 2 < 8 \\ -8 - 2 < 3x + 2 - 2 < 8 - 2 \\ -10 < 3x < 6 \\ \frac{-10}{3} < \frac{3x}{3} < \frac{6}{3} \\ -\frac{10}{3} < x < 2 \\ \left\{ x \mid -\frac{10}{3} < x < 2 \right\} \end{array}$$

You Try It 5 $|3x - 7| < 0$

The absolute value of a number must be nonnegative.

The solution set is the empty set.

$\emptyset$

You Try It 6 $|2x + 7| \geq -1$

The absolute value of a number is nonnegative.

The solution set is the set of real numbers.

You Try It 7

$$|5x + 3| > 8$$

$$\begin{array}{rcl} 5x + 3 < -8 & \text{or} & 5x + 3 > 8 \\ 5x < -11 & & 5x > 5 \\ x < -\frac{11}{5} & & x > 1 \end{array}$$

$$\left\{x \mid x < -\frac{11}{5}\right\} \quad \{x \mid x > 1\}$$

$$\begin{aligned} &\left\{x \mid x < -\frac{11}{5}\right\} \cup \{x \mid x > 1\} \\ &= \left\{x \mid x < -\frac{11}{5} \text{ or } x > 1\right\} \end{aligned}$$

You Try It 8**Strategy**

Let b represent the diameter of the bushing, T the tolerance, and d the lower and upper limits of the diameter. Solve the absolute value inequality $|d - b| \leq T$ for d .

Solution

$$\begin{aligned} |d - b| &\leq T \\ |d - 2.55| &\leq 0.003 \\ -0.003 &\leq d - 2.55 \leq 0.003 \\ -0.003 + 2.55 &\leq d - 2.55 + 2.55 \leq 0.003 + 2.55 \\ 2.547 &\leq d \leq 2.553 \end{aligned}$$

The lower and upper limits of the diameter of the bushing are 2.547 in. and 2.553 in.

SOLUTIONS TO CHAPTER 3 “YOU TRY IT”**SECTION 3.1****You Try It 1**

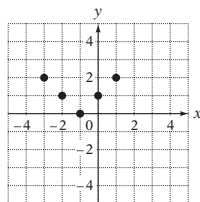
$$\begin{aligned} y &= -x^2 + 5 = -(-3)^2 + 5 \\ &= -9 + 5 = -4 \end{aligned}$$

- Replace x by -3 and solve for y .

The ordered-pair solution is $(-3, -4)$.

You Try It 2 $y = |x + 1|$

x	y
-3	2
-2	1
-1	0
0	1
1	2

**You Try It 3** Choose $P_1(5, -2)$ and $P_2(-4, 3)$.

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{[5 - (-4)]^2 + (-2 - 3)^2} \\ &= \sqrt{9^2 + (-5)^2} \\ &= \sqrt{81 + 25} \\ &= \sqrt{106} \approx 10.30 \end{aligned}$$

You Try It 4 Choose $P_1(-3, -5)$ and $P_2(-2, 3)$.

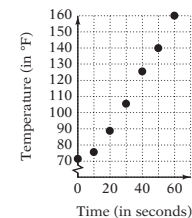
$$\begin{aligned} x_m &= \frac{x_1 + x_2}{2} & y_m &= \frac{y_1 + y_2}{2} \\ &= \frac{-3 + (-2)}{2} & &= \frac{-5 + 3}{2} \\ &= -\frac{5}{2} & &= -1 \end{aligned}$$

The midpoint is $\left(-\frac{5}{2}, -1\right)$.

You Try It 5**Strategy**

To draw a scatter diagram:

- Draw a coordinate grid with the horizontal axis representing the time in seconds and the vertical axis representing the temperature in $^{\circ}\text{F}$. Because the temperatures start at 72° , it is more convenient to start labeling the vertical axis at 70 (any number less than 72 could be used). The jagged line on the vertical axis indicates that the graph does not start at zero.
- Graph the ordered pairs $(0, 72)$, $(10, 76)$, $(20, 88)$, $(30, 106)$, $(40, 126)$, $(50, 140)$, and $(60, 160)$.

Solution**SECTION 3.2****You Try It 1** Domain: $\{-1, 3, 4, 6\}$
Range: $\{5\}$

- The domain is the set of first coordinates.

$$\begin{aligned} \text{You Try It 2} \quad G(x) &= \frac{3x}{x + 2} \\ G(-4) &= \frac{3(-4)}{-4 + 2} = \frac{-12}{-2} = 6 \end{aligned}$$

$$\begin{aligned} \text{You Try It 3} \quad f(x) &= x^2 - 11 \\ f(3h) &= (3h)^2 - 11 \\ &= 9h^2 - 11 \end{aligned}$$

You Try It 4 $h(z) = 3z + 1$

$$h(0) = 3(0) + 1 = 1$$

$$h\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) + 1 = 2$$

$$h\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 1 = 3$$

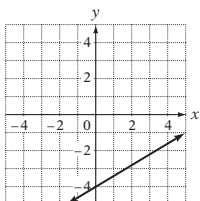
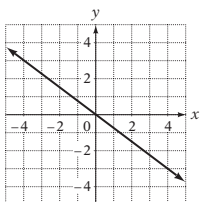
$$h(1) = 3(1) + 1 = 4$$

The range is $\{1, 2, 3, 4\}$.

You Try It 5 $f(x) = \frac{2}{x - 5}$

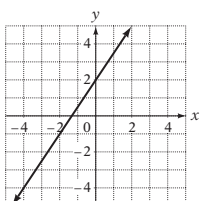
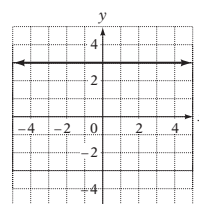
For $x = 5$, $f(5) = \frac{2}{5 - 5} = \frac{2}{0}$, which is not a real number.

5 is excluded from the domain of the function.

SECTION 3.3**You Try It 1****You Try It 2****You Try It 3** $-3x + 2y = 4$

$$2y = 3x + 4$$

$$y = \frac{3}{2}x + 2$$

**You Try It 4** $y - 3 = 0$
 $y = 3$ 

- The graph of $y = 3$ goes through the point $(0, 3)$.

You Try It 5

x-intercept:

$$3x - y = 2$$

$$3x - 0 = 2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

x-intercept: $\left(\frac{2}{3}, 0\right)$

y-intercept:

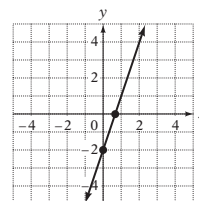
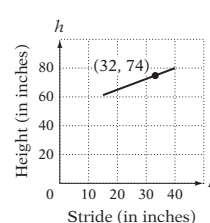
$$3x - y = 2$$

$$3(0) - y = 2$$

$$-y = 2$$

$$y = -2$$

y-intercept: $(0, -2)$

**You Try It 6**

- Graph $h = \frac{3}{4}L + 50$.
When $L = 20$,
 $h = 65$. When
 $L = 40$, $h = 80$.

The ordered pair $(32, 74)$ means that a person with a stride of 32 in. is 74 in. tall.

SECTION 3.4**You Try It 1** Choose $P_1(4, -3)$ and $P_2(2, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-3)}{2 - 4} = \frac{10}{-2} = -5$$

The slope is -5 .

You Try It 2 Choose $P_1(6, -1)$ and $P_2(6, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-1)}{6 - 6} = \frac{8}{0}$$

Division by zero is not defined.

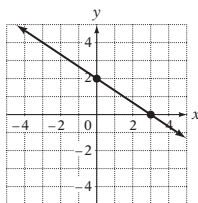
The slope of the line is undefined.

You Try It 3 Choose $P_1(5, 25,000)$ and $P_2(2, 55,000)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{55,000 - 25,000}{2 - 5} \bullet (x_1, y_1) = (5, 25,000), \\ &\quad (x_2, y_2) = (2, 55,000) \\ &= \frac{30,000}{-3} \\ &= -10,000 \end{aligned}$$

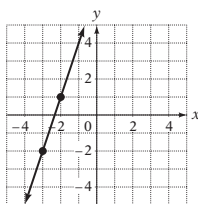
A slope of $-10,000$ means that the value of the recycling truck is decreasing by \$10,000 per year.

You Try It 4 $2x + 3y = 6$
 $3y = -2x + 6$
 $y = -\frac{2}{3}x + 2$
 $m = -\frac{2}{3} = \frac{-2}{3}$
 y-intercept = $(0, 2)$



You Try It 5 Locate $(-3, -2)$.

$$m = 3 = \frac{3}{1}$$



SECTION 3.5

You Try It 1 $m = -\frac{1}{3}$ $(x_1, y_1) = (-3, -2)$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-2) &= -\frac{1}{3}[x - (-3)] \\ y + 2 &= -\frac{1}{3}(x + 3) \\ y + 2 &= -\frac{1}{3}x - 1 \\ y &= -\frac{1}{3}x - 3 \end{aligned}$$

The equation of the line is

$$y = -\frac{1}{3}x - 3.$$

You Try It 2 $m = -3$ $(x_1, y_1) = (4, -3)$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= -3(x - 4) \\ y + 3 &= -3x + 12 \\ y &= -3x + 9 \end{aligned}$$

The equation of the line is $y = -3x + 9$.

You Try It 3 Choose $P_1(2, 0)$ and $P_2(5, 3)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{5 - 2} = \frac{3}{3} = 1 \\ y - y_1 &= m(x - x_1) \\ y - 0 &= 1(x - 2) \\ y &= 1(x - 2) \\ y &= x - 2 \end{aligned}$$

The equation of the line is $y = x - 2$.

You Try It 4 Choose $P_1(2, 3)$ and $P_2(-5, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{-5 - 2} = \frac{0}{-7} = 0$$

The line has zero slope, so the line is a horizontal line.

All points on the line have an ordinate of 3.
 The equation of the line is $y = 3$.

You Try It 5

Strategy

- Select the independent and dependent variables. The function is to predict the Celsius temperature, so that quantity is the dependent variable, y . The Fahrenheit temperature is the independent variable, x .
- From the given data, two ordered pairs are $(212, 100)$ and $(32, 0)$. Use these ordered pairs to determine the linear function.

Solution

Choose $P_1(32, 0)$ and $P_2(212, 100)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{100 - 0}{212 - 32} = \frac{100}{180} = \frac{5}{9} \\ y - y_1 &= m(x - x_1) \\ y - 0 &= \frac{5}{9}(x - 32) \\ y &= \frac{5}{9}(x - 32), \text{ or } C = \frac{5}{9}(F - 32) \end{aligned}$$

The linear function is

$$f(F) = \frac{5}{9}(F - 32).$$

SECTION 3.6

You Try It 1

$$m_1 = \frac{1 - (-3)}{7 - (-2)} = \frac{4}{9}$$

$$m_2 = \frac{-5 - 1}{6 - 4} = \frac{-6}{2} = -3$$

$$m_1 \cdot m_2 = \frac{4}{9}(-3) = -\frac{4}{3}$$

$$\bullet (x_1, y_1) = (-2, -3),$$

$$(x_2, y_2) = (7, 1)$$

$$\bullet (x_1, y_1) = (4, 1),$$

$$(x_2, y_2) = (6, -5)$$

No, the lines are not perpendicular.

You Try It 2 $5x + 2y = 2$

$$2y = -5x + 2$$

$$y = -\frac{5}{2}x + 1$$

$$m_1 = -\frac{5}{2}$$

$$5x + 2y = -6$$

$$2y = -5x - 6$$

$$y = -\frac{5}{2}x - 3$$

$$m_2 = -\frac{5}{2}$$

$$m_1 = m_2 = -\frac{5}{2}$$

Yes, the lines are parallel.

You Try It 3 $x - 4y = 3$

$$-4y = -x + 3$$

$$y = \frac{1}{4}x - \frac{3}{4}$$

$$m_1 = \frac{1}{4}$$

$$m_1 \cdot m_2 = -1$$

$$\frac{1}{4} \cdot m_2 = -1$$

$$m_2 = -4$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -4[x - (-2)] \quad \bullet (x_1, y_1) = (-2, 2)$$

$$y - 2 = -4(x + 2)$$

$$y - 2 = -4x - 8$$

$$y = -4x - 6$$

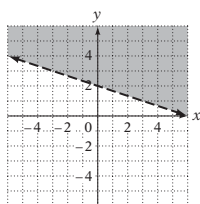
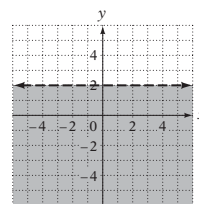
The equation of the line is $y = -4x - 6$.

SECTION 3.7

You Try It 1 $x + 3y > 6$

$$3y > -x + 6$$

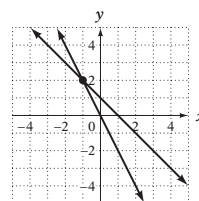
$$y > -\frac{1}{3}x + 2$$

You Try It 2 $y < 2$ 

SOLUTIONS TO CHAPTER 4 "YOU TRY IT"

SECTION 4.1

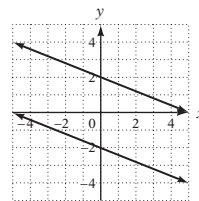
You Try It 1



- Find the point of intersection of the graphs of the equations.

The solution is $(-1, 2)$.

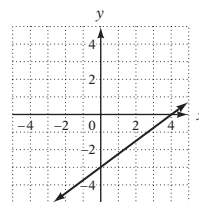
You Try It 2



- Graph the two equations.

The lines are parallel and therefore do not intersect. The system of equations has no solution. The system of equations is inconsistent.

You Try It 3



- Graph the two equations.

The two equations represent the same line. The system of equations is dependent. The solutions are the ordered pairs

$$\left(x, \frac{3}{4}x - 3\right).$$

You Try It 4 (1) $3x - y = 3$
 (2) $6x + 3y = -4$

Solve Equation (1) for y .

$$\begin{aligned} 3x - y &= 3 \\ -y &= -3x + 3 \\ y &= 3x - 3 \end{aligned}$$

Substitute into Equation (2).

$$\begin{aligned} 6x + 3y &= -4 \\ 6x + 3(3x - 3) &= -4 \\ 6x + 9x - 9 &= -4 \\ 15x - 9 &= -4 \\ 15x &= 5 \\ x &= \frac{5}{15} = \frac{1}{3} \end{aligned}$$

Substitute the value of x into Equation (1).

$$\begin{aligned} 3x - y &= 3 \\ 3\left(\frac{1}{3}\right) - y &= 3 \\ 1 - y &= 3 \\ -y &= 2 \\ y &= -2 \end{aligned}$$

The solution is $\left(\frac{1}{3}, -2\right)$.

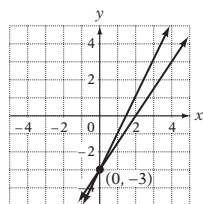
You Try It 5 (1) $y = 2x - 3$
 (2) $3x - 2y = 6$

$$\begin{aligned} 3x - 2y &= 6 \\ 3x - 2(2x - 3) &= 6 \\ 3x - 4x + 6 &= 6 \\ -x + 6 &= 6 \\ -x &= 0 \\ x &= 0 \end{aligned}$$

Substitute the value of x into Equation (1).

$$\begin{aligned} y &= 2x - 3 \\ y &= 2(0) - 3 \\ y &= 0 - 3 \\ y &= -3 \end{aligned}$$

The solution is $(0, -3)$.



• Graph the two equations.

You Try It 6 (1) $6x - 3y = 6$
 (2) $2x - y = 2$

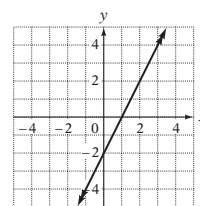
Solve Equation (2) for y .

$$\begin{aligned} 2x - y &= 2 \\ -y &= -2x + 2 \\ y &= 2x - 2 \end{aligned}$$

Substitute into Equation (1).

$$\begin{aligned} 6x - 3y &= 6 \\ 6x - 3(2x - 2) &= 6 \\ 6x - 6x + 6 &= 6 \\ 6 &= 6 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs $(x, 2x - 2)$.



• Graph the two equations.

You Try It 7

Strategy

- Amount invested at 4.2%: x
 Amount invested at 6%: y

	Principal	Rate	Interest
Amount at 4.2%	x	0.042	$0.042x$
Amount at 6%	y	0.06	$0.06y$

- The total investment is \$13,600.
 The two accounts earn the same interest.

Solution

$$\begin{aligned} x + y &= 13,600 \\ 0.042x &= 0.06y \\ x &= \frac{10}{7}y \end{aligned}$$

Substitute $\frac{10}{7}y$ for x in $x + y = 13,600$ and solve for y .

$$\begin{aligned} \frac{10}{7}y + y &= 13,600 \\ \frac{17}{7}y &= 13,600 \\ y &= 5600 \end{aligned}$$

$$\begin{aligned} x + 5600 &= 13,600 \\ x &= 8000 \end{aligned}$$

\$8000 must be invested at 4.2%, and \$5600 must be invested at 6%.

SECTION 4.2

You Try It 1 (1) $2x + 5y = 6$
 (2) $3x - 2y = 6x + 2$

Write Equation (2) in the form
 $Ax + By = C$.

$$\begin{aligned} 3x - 2y &= 6x + 2 \\ -3x - 2y &= 2 \end{aligned}$$

Solve the system:

$$\begin{aligned} 2x + 5y &= 6 \\ -3x - 2y &= 2 \end{aligned}$$

Eliminate y .

$$\begin{aligned} 2(2x + 5y) &= 2(6) \\ 5(-3x - 2y) &= 5(2) \\ \hline 4x + 10y &= 12 \\ -15x - 10y &= 10 \\ \hline -11x &= 22 && \bullet \text{ Add the equations.} \\ x &= -2 && \bullet \text{ Solve for } x. \end{aligned}$$

Replace x in Equation (1).

$$\begin{aligned} 2x + 5y &= 6 \\ 2(-2) + 5y &= 6 \\ -4 + 5y &= 6 \\ 5y &= 10 \\ y &= 2 \end{aligned}$$

The solution is $(-2, 2)$.

You Try It 2 $2x + y = 5$
 $4x + 2y = 6$

Eliminate y .

$$\begin{aligned} -2(2x + y) &= -2(5) \\ 4x + 2y &= 6 \\ \hline -4x - 2y &= -10 \\ 4x + 2y &= 6 \\ \hline 0x + 0y &= -4 && \bullet \text{ Add the equations.} \\ 0 &= -4 \end{aligned}$$

This is not a true equation.

The system is inconsistent and therefore has no solution.

You Try It 3 (1) $x - y + z = 6$
 (2) $2x + 3y - z = 1$
 (3) $x + 2y + 2z = 5$

Eliminate z . Add Equations (1) and (2).

$$\begin{aligned} x - y + z &= 6 \\ 2x + 3y - z &= 1 \\ \hline 3x + 2y &= 7 && \bullet \text{ Equation (4)} \end{aligned}$$

Multiply Equation (2) by 2 and add to Equation (3).

$$\begin{aligned} 4x + 6y - 2z &= 2 \\ x + 2y + 2z &= 5 \\ \hline 5x + 8y &= 7 && \bullet \text{ Equation (5)} \end{aligned}$$

Solve the system of two equations.

$$\begin{aligned} (4) \quad 3x + 2y &= 7 \\ (5) \quad 5x + 8y &= 7 \end{aligned}$$

Multiply Equation (4) by -4 and add to Equation (5).

$$\begin{aligned} -12x - 8y &= -28 \\ 5x + 8y &= 7 \\ \hline -7x &= -21 \\ x &= 3 \end{aligned}$$

Replace x by 3 in Equation (4).

$$\begin{aligned} 3x + 2y &= 7 \\ 3(3) + 2y &= 7 \\ 9 + 2y &= 7 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$

Replace x by 3 and y by -1 in Equation (1).

$$\begin{aligned} x - y + z &= 6 \\ 3 - (-1) + z &= 6 \\ 4 + z &= 6 \\ z &= 2 \end{aligned}$$

The solution is $(3, -1, 2)$.

SECTION 4.3

You Try It 1 $\begin{vmatrix} -1 & -4 \\ 3 & -5 \end{vmatrix} = -1(-5) - (-4)(3)$
 $= 5 + 12 = 17$

The value of the determinant is 17.

You Try It 2 Expand by cofactors of the first row.

$$\begin{aligned} &\begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} \\ &= 1 \begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 4 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} + (-2) \begin{vmatrix} 3 & 1 \\ 0 & -2 \end{vmatrix} \\ &= 1(2 + 2) - 4(6 - 0) - 2(-6 - 0) \\ &= 4 - 24 + 12 \\ &= -8 \end{aligned}$$

The value of the determinant is -8 .

You Try It 3 $\begin{vmatrix} 3 & -2 & 0 \\ 1 & 4 & 2 \\ -2 & 1 & 3 \end{vmatrix}$

$$\begin{aligned} &= 3 \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} - (-2) \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} + 0 \begin{vmatrix} 1 & 4 \\ -2 & 1 \end{vmatrix} \\ &= 3(12 - 2) + 2(3 + 4) + 0 \\ &= 3(10) + 2(7) \\ &= 30 + 14 \\ &= 44 \end{aligned}$$

The value of the determinant is 44.

You Try It 4

$$3x - y = 4$$

$$6x - 2y = 5$$

$$D = \begin{vmatrix} 3 & -1 \\ 6 & -2 \end{vmatrix} = 0$$

Because $D = 0$, $\frac{D_x}{D}$ is undefined.

Therefore, the system of equations is dependent or inconsistent. That is, the system is not independent.

You Try It 5

$$2x - y + z = -1$$

$$3x + 2y - z = 3$$

$$x + 3y + z = -2$$

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 3 & 1 \end{vmatrix} = 21$$

$$D_x = \begin{vmatrix} -1 & -1 & 1 \\ 3 & 2 & -1 \\ -2 & 3 & 1 \end{vmatrix} = 9$$

$$D_y = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 3 & -1 \\ 1 & -2 & 1 \end{vmatrix} = -3$$

$$D_z = \begin{vmatrix} 2 & -1 & -1 \\ 3 & 2 & 3 \\ 1 & 3 & -2 \end{vmatrix} = -42$$

$$x = \frac{D_x}{D} = \frac{9}{21} = \frac{3}{7}$$

$$y = \frac{D_y}{D} = \frac{-3}{21} = -\frac{1}{7}$$

$$z = \frac{D_z}{D} = \frac{-42}{21} = -2$$

The solution is $\left(\frac{3}{7}, -\frac{1}{7}, -2\right)$.

SECTION 4.4**You Try It 1****Strategy**

- Rate of the rowing team in calm water: t
Rate of the current: c

	Rate	Time	Distance
With current	$t + c$	2	$2(t + c)$
Against current	$t - c$	2	$2(t - c)$

- The distance traveled with the current is 18 mi.
The distance traveled against the current is 10 mi.

$$2(t + c) = 18$$

$$2(t - c) = 10$$

Solution

$$2(t + c) = 18$$

$$2(t - c) = 10$$

$$\frac{1}{2} \cdot 2(t + c) = \frac{1}{2} \cdot 18$$

$$\frac{1}{2} \cdot 2(t - c) = \frac{1}{2} \cdot 10$$

$$t + c = 9$$

$$t - c = 5$$

$$2t = 14$$

$$t = 7$$

$$t + c = 9$$

$$7 + c = 9$$

$$c = 2$$

• **Substitute 7 for t .**

The rate of the rowing team in calm water is 7 mph.

The rate of the current is 2 mph.

You Try It 2**Strategy**

- Number of dimes: d
Number of nickels: n
Number of quarters: q
- There are 19 coins in a bank that contains only nickels, dimes, and quarters. ($n + d + q = 19$)
The value of the coins is \$2.
($5n + 10d + 25q = 200$)
There are twice as many nickels as dimes. ($n = 2d$)

Solution

$$(1) \quad n + d + q = 19$$

$$(2) \quad n = 2d$$

$$(3) \quad 5n + 10d + 25q = 200$$

Solve the system of equations. Substitute $2d$ for n in Equation (1) and Equation (3).

$$2d + d + q = 19$$

$$5(2d) + 10d + 25q = 200$$

$$(4) \quad 3d + q = 19$$

$$(5) \quad 20d + 25q = 200$$

Solve the system of equations in two variables by multiplying Equation (4) by -25 and then adding it to Equation (5).

$$-75d - 25q = -475$$

$$20d + 25q = 200$$

$$-55d = -275$$

$$d = 5$$

Substituting the value of d into Equation (2), we have $n = 10$.

Substituting the values of d and n into Equation (1), we have $q = 4$.

There are 10 nickels, 5 dimes, and 4 quarters in the bank.

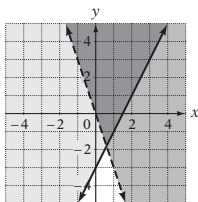
SECTION 4.5

You Try It 1 Shade above the solid line

$$y = 2x - 3.$$

Shade above the dashed line $y = -3x$.

The solution set of the system is the intersection of the solution sets of the individual inequalities.

**You Try It 2** $3x + 4y > 12$

$$4y > -3x + 12$$

$$y > -\frac{3}{4}x + 3$$

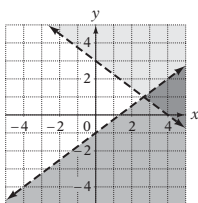
Shade above the dashed line

$$y = -\frac{3}{4}x + 3.$$

Shade below the dashed line

$$y = \frac{3}{4}x - 1.$$

The solution set of the system is the intersection of the solution sets of the individual inequalities.



SOLUTIONS TO CHAPTER 5 "YOU TRY IT"

SECTION 5.1

You Try It 1

$$\begin{aligned}
 (-3a^2b^4)(-2ab^3)^4 &= (-3a^2b^4)[(-2)^4a^4b^{12}] \\
 &= (-3a^2b^4)(16a^4b^{12}) \\
 &= -48a^6b^{16}
 \end{aligned}$$

$$\begin{aligned}
 \text{You Try It 2 } (-4ab^4)^2(2a^4b^2)^4 &= [(-4)^2a^2b^8][2^4a^{16}b^8] \\
 &= [16a^2b^8][16a^{16}b^8] \\
 &= 256a^{18}b^{16}
 \end{aligned}$$

$$\begin{aligned}
 \text{You Try It 3 } (y^{n-3})^2 &= y^{(n-3)2} \\
 &= y^{2n-6}
 \end{aligned}$$

• Multiply the exponents.

$$\begin{aligned}
 \text{You Try It 4 } \frac{20r^{-2}t^{-5}}{-16r^{-3}s^{-2}} &= -\frac{4 \cdot 5r^{-2-(-3)}s^2t^{-5}}{4 \cdot 4} \\
 &= -\frac{5rs^2}{4t^5}
 \end{aligned}$$

$$\begin{aligned}
 \text{You Try It 5 } \frac{(9u^{-6}v^4)^{-1}}{(6u^{-3}v^{-2})^{-2}} &= \frac{9^{-1}u^6v^{-4}}{6^{-2}u^6v^4} \\
 &= 9^{-1} \cdot 6^2u^0v^{-8} \\
 &= \frac{36}{9v^8} \\
 &= \frac{4}{v^8}
 \end{aligned}$$

$$\begin{aligned}
 \text{You Try It 6 } \frac{a^{2n+1}}{a^{n+3}} &= a^{2n+1-(n+3)} \\
 &= a^{2n+1-n-3} \\
 &= a^{n-2}
 \end{aligned}$$

• Subtract the exponents.

$$\text{You Try It 7 } 942,000,000 = 9.42 \times 10^8$$

$$\text{You Try It 8 } 2.7 \times 10^{-5} = 0.000027$$

$$\begin{aligned}
 \text{You Try It 9 } \frac{5,600,000 \times 0.000000081}{900 \times 0.000000028} &= \frac{5.6 \times 10^6 \times 8.1 \times 10^{-8}}{9 \times 10^2 \times 2.8 \times 10^{-8}} \\
 &= \frac{(5.6)(8.1) \times 10^{6+(-8)-2-(-8)}}{(9)(2.8)} \\
 &= 1.8 \times 10^4 = 18,000
 \end{aligned}$$

You Try It 10**Strategy** To find the number of arithmetic operations:

- Find the reciprocal of 9.74×10^{-16} , which is the number of operations performed in 1 s.
- Write the number of seconds in 1 min (60) in scientific notation.
- Multiply the number of arithmetic operations per second by the number of seconds in 1 min.

$$\text{Solution } \frac{1}{9.74 \times 10^{-16}} \approx 1.03 \times 10^{15}$$

$$60 = 6.0 \times 10$$

$$(1.03 \times 10^{15})(6.0 \times 10) = 6.18 \times 10^{16}$$

The computer can perform 6.18×10^{16} operations in 1 min.

SECTION 5.2

You Try It 1

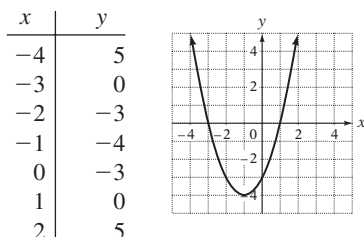
$$\begin{aligned}
 R(x) &= -2x^4 - 5x^3 + 2x - 8 \\
 R(2) &= -2(2)^4 - 5(2)^3 + 2(2) - 8 \\
 &= -2(16) - 5(8) + 4 - 8 \\
 &= -32 - 40 + 4 - 8 \\
 &= -76
 \end{aligned}$$

• Replace x by 2. Simplify.

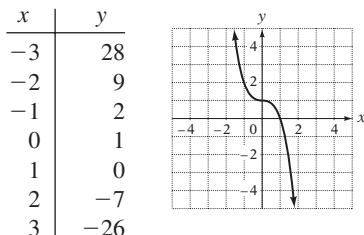
You Try It 2 The leading coefficient is -3 , the constant term is -12 , and the degree is 4.

- You Try It 3**
- Yes, this is a polynomial function.
 - No, this is not a polynomial function. A polynomial function does not have a variable expression raised to a negative power.
 - No, this is not a polynomial function. A polynomial function does not have a variable expression within a radical.

You Try It 4



You Try It 5



You Try It 6

$$\begin{array}{r} -3x^2 - 4x + 9 \\ -5x^2 - 7x + 1 \\ \hline -8x^2 - 11x + 10 \end{array}$$

You Try It 7 Add the additive inverse of $6x^2 + 3x - 7$ to $-5x^2 + 2x - 3$.

$$\begin{array}{r} -5x^2 + 2x - 3 \\ -6x^2 - 3x + 7 \\ \hline -11x^2 - x + 4 \end{array}$$

You Try It 8

$$\begin{aligned} S(x) &= (4x^3 - 3x^2 + 2) + (-2x^2 + 2x - 3) \\ &= 4x^3 - 5x^2 + 2x - 1 \end{aligned}$$

$$\begin{aligned} S(-1) &= 4(-1)^3 - 5(-1)^2 + 2(-1) - 1 \\ &= 4(-1) - 5(1) - 2 - 1 \\ &= -4 - 5 - 2 - 1 \\ &= -12 \end{aligned}$$

You Try It 9

$$\begin{aligned} D(x) &= P(x) - R(x) \\ D(x) &= (5x^{2n} - 3x^n - 7) - (-2x^{2n} - 5x^n + 8) \\ &= (5x^{2n} - 3x^n - 7) + (2x^{2n} + 5x^n - 8) \\ &= 7x^{2n} + 2x^n - 15 \end{aligned}$$

SECTION 5.3

You Try It 1

$$\begin{aligned} (2b^2 - 7b - 8)(-5b) &= 2b^2(-5b) - 7b(-5b) - 8(-5b) \\ &= -10b^3 + 35b^2 + 40b \end{aligned}$$

• Use the Distributive Property.

You Try It 2

$$\begin{aligned} x^2 - 2x[x - x(4x - 5) + x^2] &= x^2 - 2x[x - 4x^2 + 5x + x^2] \\ &= x^2 - 2x[6x - 3x^2] \\ &= x^2 - 12x^2 + 6x^3 \\ &= 6x^3 - 11x^2 \end{aligned}$$

You Try It 3

$$\begin{array}{r} -2b^2 + 5b - 4 \\ - 3b + 2 \\ \hline -4b^2 + 10b - 8 \\ 6b^3 - 15b^2 + 12b \\ \hline 6b^3 - 19b^2 + 22b - 8 \end{array}$$

• $2(-2b^2 + 5b - 4)$
• $-3b(-2b^2 + 5b - 4)$

You Try It 4

$$\begin{aligned} (3x - 4)(2x - 3) &= 6x^2 - 9x - 8x + 12 \\ &= 6x^2 - 17x + 12 \end{aligned}$$

• FOIL

You Try It 5

$$\begin{aligned} (3ab + 4)(5ab - 3) &= 15a^2b^2 - 9ab + 20ab - 12 \\ &= 15a^2b^2 + 11ab - 12 \end{aligned}$$

You Try It 6

$$\begin{aligned} (3x - 7)(3x + 7) &= 9x^2 - 49 \end{aligned}$$

• The sum and difference of two terms

You Try It 7

$$\begin{aligned} (2ab + 7)(2ab - 7) &= 4a^2b^2 - 49 \end{aligned}$$

• The sum and difference of two terms

You Try It 8

$$\begin{aligned} (3x - 4y)^2 &= 9x^2 - 24xy + 16y^2 \end{aligned}$$

• The square of a binomial

You Try It 9

$$\begin{aligned} (5xy + 4)^2 &= 25x^2y^2 + 40xy + 16 \end{aligned}$$

• The square of a binomial

You Try It 10

Strategy To find the area, replace the variables b and h in the equation $A = \frac{1}{2}bh$ by the given values, and solve for A .

Solution

$$\begin{aligned} A &= \frac{1}{2}bh \\ A &= \frac{1}{2}(2x + 6)(x - 4) \end{aligned}$$

$$\begin{aligned} A &= (x + 3)(x - 4) \\ A &= x^2 - 4x + 3x - 12 \\ A &= x^2 - x - 12 \end{aligned}$$

• FOIL

The area is $(x^2 - x - 12) \text{ ft}^2$.

You Try It 11

Strategy To find the volume, subtract the volume of the small rectangular solid from the volume of the large rectangular solid.

Large rectangular solid:

$$\text{Length} = L_1 = 12x$$

$$\text{Width} = W_1 = 7x + 2$$

$$\text{Height} = H_1 = 5x - 4$$

Small rectangular solid:

$$\text{Length} = L_2 = 12x$$

$$\text{Width} = W_2 = x$$

$$\text{Height} = H_2 = 2x$$

Solution

V = Volume of large rectangular solid – volume of small rectangular solid

$$V = (L_1 \cdot W_1 \cdot H_1) - (L_2 \cdot W_2 \cdot H_2)$$

$$V = (12x)(7x + 2)(5x - 4) - (12x)(x)(2x)$$

$$= (84x^2 + 24x)(5x - 4) - (12x^2)(2x)$$

$$= (420x^3 - 336x^2 + 120x^2 - 96x) - 24x^3$$

$$= 396x^3 - 216x^2 - 96x$$

The volume is $(396x^3 - 216x^2 - 96x) \text{ ft}^3$.

You Try It 12

Strategy To find the area, replace the variable r in the equation $A = \pi r^2$ by the given value, and solve for A .

Solution

$$A = \pi r^2$$

$$A \approx 3.14(2x + 3)^2$$

$$= 3.14(4x^2 + 12x + 9)$$

$$= 12.56x^2 + 37.68x + 28.26$$

The area is

$$(12.56x^2 + 37.68x + 28.26) \text{ cm}^2.$$

SECTION 5.4**You Try It 1**

$$\frac{4x^3y + 8x^2y^2 - 4xy^3}{2xy} = \frac{4x^3y}{2xy} + \frac{8x^2y^2}{2xy} - \frac{4xy^3}{2xy}$$

$$= 2x^2 + 4xy - 2y^2$$

$$\text{Check: } 2xy(2x^2 + 4xy - 2y^2) = 4x^3y + 8x^2y^2 - 4xy^3$$

You Try It 2

$$\begin{array}{r} 5x - 1 \\ 3x + 4 \overline{) 15x^2 + 17x - 20} \\ \underline{15x^2 + 20x} \\ -3x - 20 \\ \underline{-3x - 4} \\ -16 \end{array}$$

$$\frac{15x^2 + 17x - 20}{3x + 4} = 5x - 1 - \frac{16}{3x + 4}$$

You Try It 3

$$\begin{array}{r} x^2 + 3x - 1 \\ 3x - 1 \overline{) 3x^3 + 8x^2 - 6x + 2} \\ \underline{3x^3 - x^2} \\ 9x^2 - 6x \\ \underline{9x^2 - 3x} \\ -3x + 2 \\ \underline{-3x + 1} \\ 1 \end{array}$$

$$\frac{3x^3 + 8x^2 - 6x + 2}{3x - 1} = x^2 + 3x - 1 + \frac{1}{3x - 1}$$

You Try It 4

$$\begin{array}{r} 3x^2 - 2x + 4 \\ x^2 - 3x + 2 \overline{) 3x^4 - 11x^3 + 16x^2 - 16x + 8} \\ \underline{3x^4 - 9x^3 + 6x^2} \\ -2x^3 + 10x^2 - 16x \\ \underline{-2x^3 + 6x^2 - 4x} \\ 4x^2 - 12x + 8 \\ \underline{4x^2 - 12x + 8} \\ 0 \end{array}$$

$$\frac{3x^4 - 11x^3 + 16x^2 - 16x + 8}{x^2 - 3x + 2} = 3x^2 - 2x + 4$$

You Try It 5

$$\begin{array}{r} -2 \overline{) 6 8 -5} \\ \phantom{-2 \overline{) }} \underline{12 16} \\ \phantom{-2 \overline{) }} 6 -4 3 \end{array}$$

$$(6x^2 + 8x - 5) \div (x + 2)$$

$$= 6x - 4 + \frac{3}{x + 2}$$

You Try It 6

$$\begin{array}{r} 2 \overline{) 5 -12 -8 16} \\ \phantom{2 \overline{) }} \underline{10 -24} \\ \phantom{2 \overline{) }} 5 -2 -12 -8 \end{array}$$

$$(5x^3 - 12x^2 - 8x + 16) \div (x - 2)$$

$$= 5x^2 - 2x - 12 - \frac{8}{x - 2}$$

You Try It 7

$$\begin{array}{r} 3 \overline{) 2 -3 -8 0 -2} \\ \phantom{3 \overline{) }} \underline{6 9 3 9} \\ \phantom{3 \overline{) }} 2 3 1 3 7 \end{array}$$

$$(2x^4 - 3x^3 - 8x^2 - 2) \div (x - 3)$$

$$= 2x^3 + 3x^2 + x + 3 + \frac{7}{x - 3}$$

You Try It 8

2	2	-3	-5
		4	2
	2	1	-3

$$P(2) = -3$$

You Try It 9

-3	2	-5	0	7
		-6	33	-99
	2	-11	33	-92

$$P(-3) = -92$$

SECTION 5.5

You Try It 1 The GCF of $3x^3y$, $6x^2y^2$, and $3xy^3$ is $3xy$.

$$3x^3y - 6x^2y^2 - 3xy^3 = 3xy(x^2 - 2xy - y^2)$$

You Try It 2 The GCF of $6t^{2n}$ and $9t^n$ is $3t^n$.

$$6t^{2n} - 9t^n = 3t^n(2t^n - 3)$$

You Try It 3

$$\begin{aligned} 3(6x - 7y) - 2x^2(6x - 7y) \\ = (6x - 7y)(3 - 2x^2) \end{aligned}$$

• The common binomial factor is $(6x - 7y)$.

You Try It 4 $4a^2 - 6a - 6ax + 9x$

$$\begin{aligned} &= (4a^2 - 6a) - (6ax - 9x) \\ &= 2a(2a - 3) - 3x(2a - 3) \\ &= (2a - 3)(2a - 3x) \end{aligned}$$

You Try It 5 $x^2 - x - 20 = (x + 4)(x - 5)$

You Try It 6 $x^2 + 5xy + 6y^2 = (x + 2y)(x + 3y)$

You Try It 7 $4x^2 + 15x - 4 = (x + 4)(4x - 1)$

You Try It 8 $10x^2 + 39x + 14 = (2x + 7)(5x + 2)$

You Try It 9

The GCF of $3a^3b^3$, $3a^2b^2$, and $60ab$ is $3ab$.

$$\begin{aligned} 3a^3b^3 + 3a^2b^2 - 60ab \\ = 3ab(a^2b^2 + ab - 20) \\ = 3ab(ab + 5)(ab - 4) \end{aligned}$$

You Try It 10

We are looking for two linear functions that, when multiplied, equal $3x^2 - 17x - 6$.

We must factor $3x^2 - 17x - 6$.

$$3x^2 - 17x - 6 = (3x + 1)(x - 6)$$

$$f(x) = 3x + 1 \text{ and } g(x) = x - 6.$$

SECTION 5.6**You Try It 1**

$$\begin{aligned} x^2 - 36y^4 &= x^2 - (6y^2)^2 \\ &= (x + 6y^2)(x - 6y^2) \end{aligned}$$

• Difference of two squares

You Try It 2

$$9x^2 + 12x + 4 = (3x + 2)^2$$

• Perfect-square trinomial

You Try It 3

$$\begin{aligned} (a + b)^2 - (a - b)^2 &\quad \bullet \text{ Difference of two squares} \\ &= [(a + b) + (a - b)][(a + b) - (a - b)] \\ &= (a + b + a - b)(a + b - a + b) \\ &= (2a)(2b) = 4ab \end{aligned}$$

You Try It 4

$$\begin{aligned} a^3b^3 - 27 &= (ab)^3 - 3^3 \quad \bullet \text{ Difference of two cubes} \\ &= (ab - 3)(a^2b^2 + 3ab + 9) \end{aligned}$$

You Try It 5

$$\begin{aligned} 8x^3 + y^3z^3 &= (2x)^3 + (yz)^3 \quad \bullet \text{ Sum of two cubes} \\ &= (2x + yz)(4x^2 - 2xyz + y^2z^2) \end{aligned}$$

You Try It 6

$$\begin{aligned} (x - y)^3 + (x + y)^3 &\quad \bullet \text{ Sum of two cubes} \\ &= [(x - y) + (x + y)] \\ &\quad \times [(x - y)^2 - (x - y)(x + y) + (x + y)^2] \\ &= 2x[x^2 - 2xy + y^2 - (x^2 - y^2) + x^2 + 2xy + y^2] \\ &= 2x(x^2 - 2xy + y^2 - x^2 + y^2 + x^2 + 2xy + y^2) \\ &= 2x(x^2 + 3y^2) \end{aligned}$$

You Try It 7 Let $u = x^2$.

$$\begin{aligned} 3x^4 + 4x^2 - 4 &= 3u^2 + 4u - 4 \\ &= (u + 2)(3u - 2) \\ &= (x^2 + 2)(3x^2 - 2) \end{aligned}$$

You Try It 8

$$\begin{aligned} 18x^3 - 6x^2 - 60x &= 6x(3x^2 - x - 10) \quad \bullet \text{ GCF} \\ &= 6x(3x + 5)(x - 2) \end{aligned}$$

You Try It 9 $4x - 4y - x^3 + x^2y$ • Factor by grouping.

$$\begin{aligned} &= (4x - 4y) - (x^3 - x^2y) \\ &= 4(x - y) - x^2(x - y) \\ &= (x - y)(4 - x^2) \\ &= (x - y)(2 + x)(2 - x) \end{aligned}$$

You Try It 10

$$\begin{aligned} x^{4n} - x^{2n}y^{2n} &= x^{2n+2n} - x^{2n}y^{2n} \quad \bullet \text{ GCF} \\ &= x^{2n}(x^{2n} - y^{2n}) \\ &= x^{2n}[(x^n)^2 - (y^n)^2] \quad \bullet \text{ Difference of two squares} \\ &= x^{2n}(x^n + y^n)(x^n - y^n) \end{aligned}$$

SECTION 5.7**You Try It 1**

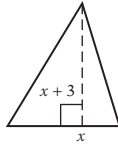
$$\begin{aligned} (x + 4)(x - 1) &= 14 \\ x^2 + 3x - 4 &= 14 \\ x^2 + 3x - 18 &= 0 \\ (x + 6)(x - 3) &= 0 \\ x + 6 = 0 &\quad x - 3 = 0 \\ x = -6 &\quad x = 3 \end{aligned}$$

• FOIL
• Write in standard form.
• Factor the left side.
• Principle of Zero Products

The solutions are -6 and 3 .

You Try It 2**Strategy**

Draw a diagram. Then use the formula for the area of a triangle.

**Solution**

$$A = \frac{1}{2}bh$$

$$54 = \frac{1}{2}x(x + 3)$$

$$108 = x(x + 3)$$

$$0 = x^2 + 3x - 108$$

$$0 = (x - 9)(x + 12)$$

$$x - 9 = 0$$

$$x + 12 = 0$$

$$x = 9$$

$$x = -12$$

$$x + 3 = 9 + 3 = 12$$

The base is 9 cm; the height is 12 cm.

SOLUTIONS TO CHAPTER 6 “YOU TRY IT”**SECTION 6.1****You Try It 1**

$$f(x) = \frac{3 - 5x}{x^2 + 5x + 6}$$

$$f(2) = \frac{3 - 5(2)}{2^2 + 5(2) + 6}$$

$$= \frac{3 - 10}{4 + 10 + 6}$$

$$= \frac{-7}{20}$$

$$= -\frac{7}{20}$$

• Replace x by 2.

You Try It 2

Set the denominator equal to zero. Then solve for x .

$$2x^2 - 7x + 3 = 0$$

$$(2x - 1)(x - 3) = 0$$

$$2x - 1 = 0 \quad x - 3 = 0$$

$$x = \frac{1}{2}$$

$$x = 3$$

• Solve the quadratic equation by factoring.

The domain is $\left\{x \mid x \neq \frac{1}{2}, 3\right\}$.

You Try It 3

$$\frac{21a^3b - 14a^3b^2}{7a^2b} = \frac{7a^3b(3 - 2b)}{7a^2b}$$

$$= a(3 - 2b)$$

• Factor out the GCF.

You Try It 4

$$\frac{6x^4 - 24x^3}{12x^3 - 48x^2} = \frac{6x^3(x - 4)}{12x^2(x - 4)}$$

$$= \frac{6x^3(\cancel{x - 4})}{12x^2(\cancel{x - 4})} = \frac{x}{2}$$

• Factor out the GCF.

You Try It 5

$$\begin{aligned} \frac{20x - 15x^2}{15x^3 - 5x^2 - 20x} &= \frac{5x(4 - 3x)}{5x(3x^2 - x - 4)} \\ &= \frac{5x(4 - 3x)}{5x(3x - 4)(x + 1)} \\ &= \frac{\cancel{5x}(4 - 3\cancel{x})}{\cancel{5x}(\cancel{3x} - 4)(x + 1)} \\ &= -\frac{1}{x + 1} \end{aligned}$$

You Try It 6

$$\begin{aligned} \frac{x^{2n} + x^n - 12}{x^{2n} - 3x^n} &= \frac{(x^n + 4)(x^n - 3)}{x^n(x^n - 3)} \\ &= \frac{(x^n + 4)(\cancel{x^n - 3})}{x^n(\cancel{x^n - 3})} \\ &= \frac{x^n + 4}{x^n} \end{aligned}$$

You Try It 7

$$\begin{aligned} \frac{12 + 5x - 3x^2}{x^2 + 2x - 15} \cdot \frac{2x^2 + x - 45}{3x^2 + 4x} &= \frac{(4 + 3x)(3 - x)}{(x + 5)(x - 3)} \cdot \frac{(2x - 9)(x + 5)}{x(3x + 4)} \\ &= \frac{(4 + 3x)(3 - x)(2x - 9)(x + 5)}{(x + 5)(x - 3) \cdot x(3x + 4)} \\ &= \frac{\cancel{(4 + 3x)}(\cancel{3 - x})(2x - 9)(\cancel{x + 5})}{(\cancel{x + 5})(\cancel{x - 3}) \cdot x(\cancel{3x + 4})} = -\frac{2x - 9}{x} \end{aligned}$$

You Try It 8

$$\begin{aligned} \frac{2x^2 - 13x + 20}{x^2 - 16} \cdot \frac{2x^2 + 9x + 4}{6x^2 - 7x - 5} &= \frac{(2x - 5)(x - 4)}{(x - 4)(x + 4)} \cdot \frac{(2x + 1)(x + 4)}{(3x - 5)(2x + 1)} \\ &= \frac{(2x - 5)(x - 4) \cdot (2x + 1)(x + 4)}{(x - 4)(x + 4) \cdot (3x - 5)(2x + 1)} \\ &= \frac{\cancel{(2x - 5)}(\cancel{x - 4})(\cancel{2x + 1})(\cancel{x + 4})}{(\cancel{x - 4})(\cancel{x + 4})(3x - 5)(\cancel{2x + 1})} = \frac{2x - 5}{3x - 5} \end{aligned}$$

You Try It 9

$$\begin{aligned} \frac{7a^3b^7}{15x^2y^3} \div \frac{21a^5b^2}{20x^4y} &= \frac{7a^3b^7}{15x^2y^3} \cdot \frac{20x^4y}{21a^5b^2} \\ &= \frac{7a^3b^7 \cdot 20x^4y}{15x^2y^3 \cdot 21a^5b^2} \\ &= \frac{4b^5x^2}{9a^2y^2} \end{aligned}$$

• Rewrite division as multiplication by the reciprocal.

You Try It 10

$$\begin{aligned}
 \frac{6x^2 - 3xy}{10ab^4} \div \frac{16x^2y^2 - 8xy^3}{15a^2b^2} &= \frac{6x^2 - 3xy}{10ab^4} \cdot \frac{15a^2b^2}{16x^2y^2 - 8xy^3} \\
 &= \frac{3x(2x - y)}{10ab^4} \cdot \frac{15a^2b^2}{8xy^2(2x - y)} \\
 &= \frac{3x(2x - y) \cdot 15a^2b^2}{10ab^4 \cdot 8xy^2(2x - y)} = \frac{9a}{16b^2y^2}
 \end{aligned}$$

• Rewrite division as multiplication by the reciprocal.

You Try It 11

$$\begin{aligned}
 \frac{6x^2 - 7x + 2}{3x^2 + x - 2} \div \frac{4x^2 - 8x + 3}{5x^2 + x - 4} &= \frac{6x^2 - 7x + 2}{3x^2 + x - 2} \cdot \frac{5x^2 + x - 4}{4x^2 - 8x + 3} \\
 &= \frac{(2x - 1)(3x - 2)}{(x + 1)(3x - 2)} \cdot \frac{(x + 1)(5x - 4)}{(2x - 1)(2x - 3)} \\
 &= \frac{(2x - 1)(3x - 2)(x + 1)(5x - 4)}{(x + 1)(3x - 2)(2x - 1)(2x - 3)} \\
 &= \frac{(2x - 1)(3x - 2)(x + 1)(5x - 4)}{(x + 1)(3x - 2)(2x - 1)(2x - 3)} = \frac{5x - 4}{2x - 3}
 \end{aligned}$$

SECTION 6.2**You Try It 1**

The LCM is $(2x - 5)(x + 4)$.

$$\begin{aligned}
 \frac{2x}{2x - 5} &= \frac{2x}{2x - 5} \cdot \frac{x + 4}{x + 4} = \frac{2x^2 + 8x}{(2x - 5)(x + 4)} \\
 \frac{3}{x + 4} &= \frac{3}{x + 4} \cdot \frac{2x - 5}{2x - 5} = \frac{6x - 15}{(2x - 5)(x + 4)}
 \end{aligned}$$

You Try It 2

$$2x^2 - 11x + 15 = (x - 3)(2x - 5); x^2 - 3x = x(x - 3)$$

The LCM is $x(x - 3)(2x - 5)$.

$$\begin{aligned}
 \frac{3x}{2x^2 - 11x + 15} &= \frac{3x}{(x - 3)(2x - 5)} \cdot \frac{x}{x} = \frac{3x^2}{x(x - 3)(2x - 5)} \\
 \frac{x - 2}{x^2 - 3x} &= \frac{x - 2}{x(x - 3)} \cdot \frac{2x - 5}{2x - 5} = \frac{2x^2 - 9x + 10}{x(x - 3)(2x - 5)}
 \end{aligned}$$

You Try It 3

$$2x - x^2 = x(2 - x) = -x(x - 2);$$

$$3x^2 - 5x - 2 = (x - 2)(3x + 1)$$

The LCM is $x(x - 2)(3x + 1)$.

$$\begin{aligned}
 \frac{2x - 7}{2x - x^2} &= \frac{2x - 7}{x(x - 2)} \cdot \frac{3x + 1}{3x + 1} = \frac{6x^2 - 19x - 7}{x(x - 2)(3x + 1)} \\
 \frac{3x - 2}{3x^2 - 5x - 2} &= \frac{3x - 2}{(x - 2)(3x + 1)} \cdot \frac{x}{x} = \frac{3x^2 - 2x}{x(x - 2)(3x + 1)}
 \end{aligned}$$

You Try It 4

The LCM is ab .

$$\begin{aligned}
 \frac{2}{b} - \frac{1}{a} + \frac{4}{ab} &= \frac{2}{b} \cdot \frac{a}{a} - \frac{1}{a} \cdot \frac{b}{b} + \frac{4}{ab} \\
 &= \frac{2a}{ab} - \frac{b}{ab} + \frac{4}{ab} = \frac{2a - b + 4}{ab}
 \end{aligned}$$

You Try It 5

The LCM is $a(a - 5)(a + 5)$.

$$\begin{aligned}
 \frac{a - 3}{a^2 - 5a} + \frac{a - 9}{a^2 - 25} &= \frac{a - 3}{a(a - 5)} + \frac{a - 9}{(a - 5)(a + 5)} \\
 &= \frac{a - 3}{a(a - 5)} \cdot \frac{a + 5}{a + 5} + \frac{a - 9}{(a - 5)(a + 5)} \cdot \frac{a}{a} \\
 &= \frac{(a - 3)(a + 5) + a(a - 9)}{a(a - 5)(a + 5)} \\
 &= \frac{(a^2 + 2a - 15) + (a^2 - 9a)}{a(a - 5)(a + 5)} \\
 &= \frac{a^2 + 2a - 15 + a^2 - 9a}{a(a - 5)(a + 5)} \\
 &= \frac{2a^2 - 7a - 15}{a(a - 5)(a + 5)} = \frac{(2a + 3)(a - 5)}{a(a - 5)(a + 5)} \\
 &= \frac{(2a + 3)(\cancel{a - 5})}{a(\cancel{a - 5})(a + 5)} = \frac{2a + 3}{a(a + 5)}
 \end{aligned}$$

You Try It 6

The LCM is $(x - 4)(x + 1)$.

$$\begin{aligned}
 \frac{2x}{x - 4} - \frac{x - 1}{x + 1} + \frac{2}{x^2 - 3x - 4} &= \frac{2x}{x - 4} \cdot \frac{x + 1}{x + 1} - \frac{x - 1}{x + 1} \cdot \frac{x - 4}{x - 4} + \frac{2}{(x - 4)(x + 1)} \\
 &= \frac{2x(x + 1) - (x - 1)(x - 4) + 2}{(x - 4)(x + 1)} \\
 &= \frac{(2x^2 + 2x) - (x^2 - 5x + 4) + 2}{(x - 4)(x + 1)} \\
 &= \frac{x^2 + 7x - 2}{(x - 4)(x + 1)}
 \end{aligned}$$

SECTION 6.3**You Try It 1**

The LCM of the denominators in the numerator and denominator is $x - 3$.

$$\begin{aligned}
 \frac{2x + 5 + \frac{14}{x - 3}}{4x + 16 + \frac{49}{x - 3}} &= \frac{2x + 5 + \frac{14}{x - 3}}{4x + 16 + \frac{49}{x - 3}} \cdot \frac{x - 3}{x - 3} \\
 &= \frac{(2x + 5)(x - 3) + \frac{14}{x - 3}(x - 3)}{(4x + 16)(x - 3) + \frac{49}{x - 3}(x - 3)} \\
 &= \frac{2x^2 - x - 15 + 14}{4x^2 + 4x - 48 + 49} = \frac{2x^2 - x - 1}{4x^2 + 4x + 1} \\
 &= \frac{(2x + 1)(x - 1)}{(2x + 1)(2x + 1)} = \frac{x - 1}{2x + 1}
 \end{aligned}$$

You Try It 2

The LCM of 2 and $\frac{1}{x}$ is x .

$$\begin{aligned} 2 - \frac{1}{2 - \frac{1}{x}} &= 2 - \frac{1}{2 - \frac{1}{x}} \cdot \frac{x}{x} \\ &= 2 - \frac{1 \cdot x}{2 \cdot x - \frac{1}{x} \cdot x} = 2 - \frac{x}{2x - 1} \end{aligned}$$

The LCM of the denominators is $2x - 1$.

$$\begin{aligned} 2 - \frac{x}{2x - 1} &= 2 \cdot \frac{2x - 1}{2x - 1} - \frac{x}{2x - 1} \\ &= \frac{4x - 2}{2x - 1} - \frac{x}{2x - 1} \\ &= \frac{4x - 2 - x}{2x - 1} = \frac{3x - 2}{2x - 1} \end{aligned}$$

SECTION 6.4**You Try It 1**

$$\begin{aligned} \frac{5}{x - 2} &= \frac{3}{4} \\ \frac{5}{x - 2} \cdot 4(x - 2) &= \frac{3}{4} \cdot 4(x - 2) \\ 5 \cdot 4 &= 3(x - 2) \\ 20 &= 3x - 6 \\ 26 &= 3x \\ \frac{26}{3} &= x \end{aligned}$$

The solution is $\frac{26}{3}$.

You Try It 2

$$\begin{aligned} \frac{5}{2x - 3} &= \frac{-2}{x + 1} \\ \frac{5}{2x - 3} (x + 1)(2x - 3) &= \frac{-2}{x + 1} (x + 1)(2x - 3) \\ 5(x + 1) &= -2(2x - 3) \\ 5x + 5 &= -4x + 6 \\ 9x + 5 &= 6 \\ 9x &= 1 \\ x &= \frac{1}{9} \end{aligned}$$

The solution is $\frac{1}{9}$.

You Try It 3**Strategy**

To find the cost, write and solve a proportion using x to represent the cost.

Solution

$$\begin{aligned} \frac{2}{12.40} &= \frac{15}{x} \\ \frac{2}{12.40} \cdot x(12.40) &= \frac{15}{x} \cdot x(12.40) \\ 2x &= 15(12.40) \\ 2x &= 186 \\ x &= 93 \end{aligned}$$

The cost of 15 lb of cashews is \$93.00.

SECTION 6.5**You Try It 1**

$$\begin{aligned} \frac{3x}{x - 3} &= 2 + \frac{9}{x - 3} \\ (x - 3) \left(\frac{3x}{x - 3} \right) &= (x - 3) \left(2 + \frac{9}{x - 3} \right) \\ 3x &= 2(x - 3) + 9 \\ 3x &= 2x - 6 + 9 \\ 3x &= 2x + 3 \\ x &= 3 \end{aligned}$$

3 does not check as a solution. The equation has no solution.

You Try It 2

$$\begin{aligned} \frac{5}{x - 2} &= 2x + \frac{3x - 1}{x - 2} \\ (x - 2) \left(\frac{5}{x - 2} \right) &= (x - 2) \left(2x + \frac{3x - 1}{x - 2} \right) \\ 5 &= (x - 2)(2x) + 3x - 1 \\ 5 &= 2x^2 - 4x + 3x - 1 \\ 0 &= 2x^2 - x - 6 \\ 0 &= (2x + 3)(x - 2) \\ 2x + 3 &= 0 & x - 2 &= 0 \\ x &= -\frac{3}{2} & x &= 2 \end{aligned}$$

2 does not check as a solution.

The solution is $-\frac{3}{2}$.

You Try It 3**Strategy**

- Time required for the small pipe to fill the tank: x

	Rate	Time	Part
Large pipe	$\frac{1}{9}$	6	$\frac{6}{9}$
Small pipe	$\frac{1}{x}$	6	$\frac{6}{x}$

- The sum of the part of the task completed by the large pipe and the part of the task completed by the small pipe is 1.

Solution

$$\frac{6}{9} + \frac{6}{x} = 1$$

$$\frac{2}{3} + \frac{6}{x} = 1 \quad \bullet \text{ Simplify } \frac{6}{9}.$$

$$3x\left(\frac{2}{3} + \frac{6}{x}\right) = 3x \cdot 1 \quad \bullet \text{ Multiply by the LCM of 3 and } x.$$

$$2x + 18 = 3x$$

$$18 = x$$

The small pipe, working alone, will fill the tank in 18 h.

You Try It 4

Strategy

- Rate of the wind: r

	Distance	Rate	Time
With wind	700	$150 + r$	$\frac{700}{150 + r}$
Against wind	500	$150 - r$	$\frac{500}{150 - r}$

- The time flying with the wind equals the time flying against the wind.

Solution

$$\frac{700}{150 + r} = \frac{500}{150 - r}$$

$$(150 + r)(150 - r)\left(\frac{700}{150 + r}\right)$$

$$= (150 + r)(150 - r)\left(\frac{500}{150 - r}\right)$$

$$(150 - r)700 = (150 + r)500$$

$$105,000 - 700r = 75,000 + 500r$$

$$30,000 = 1200r$$

$$25 = r$$

The rate of the wind is 25 mph.

SECTION 6.6

You Try It 1

Strategy

To find the distance:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k .
- Write the direct variation equation, replacing k by its value. Substitute 5 for t and solve for s .

Solution

$$s = kt^2 \quad \bullet \text{ Direct variation equation}$$

$$64 = k(2)^2 \quad \bullet \text{ Replace } s \text{ by } 64 \text{ and } t \text{ by } 2.$$

$$64 = k \cdot 4$$

$$16 = k$$

$$s = 16t^2 = 16(5)^2 = 400 \quad \bullet k = 16, t = 5$$

The object will fall 400 ft in 5 s.

You Try It 2

Strategy

To find the resistance:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k .
- Write the inverse variation equation, replacing k by its value. Substitute 0.02 for d and solve for R .

Solution

$$R = \frac{k}{d^2} \quad \bullet \text{ Inverse variation equation}$$

$$0.5 = \frac{k}{(0.01)^2} \quad \bullet \text{ Replace } R \text{ by } 0.5 \text{ and } d \text{ by } 0.01.$$

$$0.5 = \frac{k}{0.0001}$$

$$0.00005 = k$$

$$R = \frac{0.00005}{d^2} = \frac{0.00005}{(0.02)^2} = 0.125 \quad \bullet k = 0.00005, d = 0.02$$

The resistance is 0.125 ohm.

You Try It 3

Strategy

- Write the basic combined variation equation, replace the variables by the given values, and solve for k .
- Write the combined variation equation, replacing k by its value and substituting 4 for W , 8 for d , and 16 for L . Solve for s .

Solution

$$s = \frac{kWd^2}{L}$$

$$1200 = \frac{k(2)(12)^2}{12}$$

$$1200 = 24k$$

$$50 = k$$

$$s = \frac{50Wd^2}{L}$$

$$= \frac{50(4)8^2}{16}$$

$$= 800$$

The strength of the beam is 800 lb.

SOLUTIONS TO CHAPTER 7 "YOU TRY IT"

SECTION 7.1

You Try It 1

$$16^{-3/4} = (2^4)^{-3/4} \quad \bullet 16 = 2^4$$

$$= 2^{-3}$$

$$= \frac{1}{2^3} = \frac{1}{8}$$

You Try It 2

$(-81)^{3/4}$

The base of the exponential expression is negative, and the denominator of the exponent is a positive even number.

Therefore, $(-81)^{3/4}$ is not a real number.

You Try It 3

$$(x^{3/4}y^{1/2}z^{-2/3})^{-4/3} = x^{-1}y^{-2/3}z^{8/9}$$

$$= \frac{z^{8/9}}{xy^{2/3}}$$

You Try It 4

$$\left(\frac{16a^{-2}b^{4/3}}{9a^4b^{-2/3}}\right)^{-1/2} = \left(\frac{2^4a^{-6}b^2}{3^2}\right)^{-1/2}$$

- Use the Rule for Dividing Exponential Expressions.
- Use the Rule for Simplifying Powers of Products.

$$= \frac{2^{-2}a^3b^{-1}}{3^{-1}}$$

$$= \frac{3a^3}{2^2b} = \frac{3a^3}{4b}$$

You Try It 5

$$(2x^3)^{3/4} = \sqrt[4]{(2x^3)^3}$$

$$= \sqrt[4]{8x^9}$$

You Try It 6

$$-5a^{5/6} = -5(a^5)^{1/6}$$

$$= -5\sqrt[6]{a^5}$$

You Try It 7

$$\sqrt[3]{3ab} = (3ab)^{1/3}$$

You Try It 8

$$\sqrt[4]{x^4 + y^4} = (x^4 + y^4)^{1/4}$$

You Try It 9

$$\sqrt[3]{-8x^{12}y^3} = -2x^4y$$

You Try It 10

$$-\sqrt[4]{81x^{12}y^8} = -3x^3y^2$$

SECTION 7.2**You Try It 1**

$$\sqrt[5]{x^7} = \sqrt[5]{x^5 \cdot x^2}$$

$$= \sqrt[5]{x^5} \sqrt[5]{x^2}$$

- x^5 is a perfect fifth power.

$$= x \sqrt[5]{x^2}$$

You Try It 2

$$\sqrt[3]{-64x^8y^{18}} = \sqrt[3]{-64x^6y^{18}(x^2)}$$

$$= \sqrt[3]{-64x^6y^{18}} \sqrt[3]{x^2}$$

- $-64x^6y^{18}$ is a perfect third power.

$$= -4x^2y^6 \sqrt[3]{x^2}$$

You Try It 3

$$3xy\sqrt[3]{81x^5y} - \sqrt[3]{192x^8y^4}$$

$$= 3xy\sqrt[3]{27x^3 \cdot 3x^2y} - \sqrt[3]{64x^6y^3 \cdot 3x^2y}$$

$$= 3xy\sqrt[3]{27x^3} \sqrt[3]{3x^2y} - \sqrt[3]{64x^6y^3} \sqrt[3]{3x^2y}$$

$$= 3xy \cdot 3x \sqrt[3]{3x^2y} - 4x^2y \sqrt[3]{3x^2y}$$

$$= 9x^2y \sqrt[3]{3x^2y} - 4x^2y \sqrt[3]{3x^2y} = 5x^2y \sqrt[3]{3x^2y}$$

You Try It 4

$$5\sqrt{2}(\sqrt{6} + \sqrt{24})$$

- The Distributive Property
- Simplify each radical.

$$= 5\sqrt{12} + 5\sqrt{48}$$

$$= 5\sqrt{4 \cdot 3} + 5\sqrt{16 \cdot 3}$$

$$= 5 \cdot 2\sqrt{3} + 5 \cdot 4\sqrt{3}$$

$$= 10\sqrt{3} + 20\sqrt{3} = 30\sqrt{3}$$

You Try It 5

$$(4 - 2\sqrt{7})(1 + 3\sqrt{7})$$

- The FOIL method

$$= 4 + 12\sqrt{7} - 2\sqrt{7} - 6(\sqrt{7})^2$$

$$= 4 + 10\sqrt{7} - 6 \cdot 7$$

$$= 4 + 10\sqrt{7} - 42 = -38 + 10\sqrt{7}$$

You Try It 6

$$(4 - \sqrt{2x})^2$$

- The FOIL method

$$= (4 - \sqrt{2x})(4 - \sqrt{2x})$$

$$= 16 - 4\sqrt{2x} - 4\sqrt{2x} + (\sqrt{2x})^2$$

$$= 16 - 8\sqrt{2x} + 2x$$

$$= 2x - 8\sqrt{2x} + 16$$

You Try It 7

$$\sqrt{\frac{5}{6}} = \frac{\sqrt{5}}{\sqrt{6}}$$

- Rationalize the denominator.

$$= \frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$$

You Try It 8

$$\frac{3x}{\sqrt[3]{3x^2}} = \frac{3x}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{9x}}{\sqrt[3]{9x}} = \frac{3x\sqrt[3]{9x}}{\sqrt[3]{27x^3}}$$

- Rationalize the denominator.

$$= \frac{3x\sqrt[3]{9x}}{3x} = \sqrt[3]{9x}$$

You Try It 9

$$\frac{3}{2 - \sqrt{x}} = \frac{3}{2 - \sqrt{x}} \cdot \frac{2 + \sqrt{x}}{2 + \sqrt{x}}$$

- Rationalize the denominator.

$$= \frac{3(2 + \sqrt{x})}{2^2 - (\sqrt{x})^2} = \frac{6 + 3\sqrt{x}}{4 - x}$$

SECTION 7.3**You Try It 1**

$$\sqrt{x} - \sqrt{x+5} = 1$$

- Add $\sqrt{x+5}$ to each side.
- Square each side.

$$\sqrt{x} = 1 + \sqrt{x+5}$$

$$(\sqrt{x})^2 = (1 + \sqrt{x+5})^2$$

$$x = 1 + 2\sqrt{x+5} + x + 5$$

$$-6 = 2\sqrt{x+5}$$

$$-3 = \sqrt{x+5}$$

- Square each side.

$$(-3)^2 = (\sqrt{x+5})^2$$

$$9 = x + 5$$

$$4 = x$$

4 does not check as a solution. The equation has no solution.

You Try It 2

$$\sqrt[4]{x-8} = 3$$

- Raise each side to the fourth power.

$$(\sqrt[4]{x-8})^4 = 3^4$$

$$x - 8 = 81$$

$$x = 89$$

Check:

$$\sqrt[4]{x-8} = 3$$

$$\sqrt[4]{89-8} \quad 3$$

$$\sqrt[4]{81} \quad 3$$

$$3 = 3$$

The solution is 89.

You Try It 3**Strategy**

To find the diagonal, use the Pythagorean Theorem. One leg is the length of the rectangle. The second leg is the width of the rectangle. The hypotenuse is the diagonal of the rectangle.

Solution

$$\begin{aligned}
 c^2 &= a^2 + b^2 && \bullet \text{ Pythagorean Theorem} \\
 c^2 &= (6)^2 + (3)^2 && \bullet \text{ Replace } a \text{ by 6 and } b \text{ by 3.} \\
 c^2 &= 36 + 9 && \bullet \text{ Solve for } c. \\
 c^2 &= 45 \\
 (c^2)^{1/2} &= (45)^{1/2} && \bullet \text{ Raise each side to the } \frac{1}{2} \text{ power.} \\
 c &= \sqrt{45} && \bullet a^{1/2} = \sqrt{a} \\
 c &\approx 6.7
 \end{aligned}$$

The diagonal is 6.7 cm.

You Try It 4

Strategy To find the height, replace d in the equation with the given value and solve for h .

$$\begin{aligned}
 \text{Solution} \quad d &= \sqrt{1.5h} \\
 5.5 &= \sqrt{1.5h} && \bullet \text{ Replace } d \text{ by 5.5.} \\
 (5.5)^2 &= (\sqrt{1.5h})^2 && \bullet \text{ Square each side.} \\
 30.25 &= 1.5h \\
 20.17 &\approx h
 \end{aligned}$$

The periscope must be approximately 20.17 ft above the water.

You Try It 5

Strategy To find the distance, replace the variables v and a in the equation by their given values and solve for s .

$$\begin{aligned}
 \text{Solution} \quad v &= \sqrt{2as} \\
 88 &= \sqrt{2 \cdot 22s} && \bullet \text{ Replace } v \text{ by 88} \\
 88 &= \sqrt{44s} && \text{and } a \text{ by 22.} \\
 (88)^2 &= (\sqrt{44s})^2 && \bullet \text{ Square each side.} \\
 7744 &= 44s \\
 176 &= s
 \end{aligned}$$

The distance required is 176 ft.

SECTION 7.4

$$\text{You Try It 1} \quad \sqrt{-45} = i\sqrt{45} = i\sqrt{9 \cdot 5} = 3i\sqrt{5}$$

You Try It 2

$$\begin{aligned}
 -b + \sqrt{b^2 - 4ac} \\
 -(6) + \sqrt{(6)^2 - 4(1)(25)} &= -6 + \sqrt{36 - 100} \\
 &= -6 + \sqrt{-64} = -6 + i\sqrt{64} \\
 &= -6 + 8i
 \end{aligned}$$

$$\text{You Try It 3} \quad (-4 + 2i) - (6 - 8i) = -10 + 10i$$

$$\begin{aligned}
 \text{You Try It 4} \quad (16 - \sqrt{-45}) - (3 + \sqrt{-20}) \\
 &= (16 - i\sqrt{45}) - (3 + i\sqrt{20}) \\
 &= (16 - i\sqrt{9 \cdot 5}) - (3 + i\sqrt{4 \cdot 5}) \\
 &= (16 - 3i\sqrt{5}) - (3 + 2i\sqrt{5}) \\
 &= 13 - 5i\sqrt{5}
 \end{aligned}$$

$$\text{You Try It 5} \quad (-3i)(-10i) = 30i^2 = 30(-1) = -30$$

You Try It 6

$$\begin{aligned}
 -\sqrt{-8} \cdot \sqrt{-5} &= -i\sqrt{8} \cdot i\sqrt{5} = -i^2\sqrt{40} \\
 &= -(-1)\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}
 \end{aligned}$$

You Try It 7

$$\begin{aligned}
 -6i(3 + 4i) &= -18i - 24i^2 && \bullet \text{ The Distributive Property} \\
 &= -18i - 24(-1) \\
 &= 24 - 18i
 \end{aligned}$$

You Try It 8

$$\begin{aligned}
 (4 - 3i)(2 - i) &= 8 - 4i - 6i + 3i^2 && \bullet \text{ FOIL} \\
 &= 8 - 10i + 3i^2 \\
 &= 8 - 10i + 3(-1) \\
 &= 5 - 10i
 \end{aligned}$$

You Try It 9

$$\begin{aligned}
 (5 - 3i)^2 &= (5 - 3i)(5 - 3i) \\
 &= 25 - 15i - 15i + 9i^2 \\
 &= 25 - 30i + 9(-1) \\
 &= 25 - 30i - 9 = 16 - 30i
 \end{aligned}$$

You Try It 10

$$\begin{aligned}
 (3 + 6i)(3 - 6i) &= 3^2 + 6^2 && \bullet \text{ Conjugates} \\
 &= 9 + 36 \\
 &= 45
 \end{aligned}$$

You Try It 11

$$\begin{aligned}
 \frac{2 - 3i}{4i} &= \frac{2 - 3i}{4i} \cdot \frac{i}{i} \\
 &= \frac{2i - 3i^2}{4i^2} \\
 &= \frac{2i - 3(-1)}{4(-1)} \\
 &= \frac{3 + 2i}{-4} = -\frac{3}{4} - \frac{1}{2}i
 \end{aligned}$$

You Try It 12

$$\begin{aligned}
 \frac{2 + 5i}{3 - 2i} &= \frac{2 + 5i}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i} = \frac{6 + 4i + 15i + 10i^2}{3^2 + 2^2} \\
 &= \frac{6 + 19i + 10(-1)}{9 + 4} = \frac{-4 + 19i}{13} \\
 &= -\frac{4}{13} + \frac{19}{13}i
 \end{aligned}$$

SOLUTIONS TO CHAPTER 8 "YOU TRY IT"**SECTION 8.1****You Try It 1**

$$\begin{aligned}
 2x^2 &= 7x - 3 \\
 2x^2 - 7x + 3 &= 0 && \bullet \text{ Write in standard form.} \\
 (2x - 1)(x - 3) &= 0 && \bullet \text{ Solve by factoring.}
 \end{aligned}$$

$$\begin{aligned}
 2x - 1 &= 0 && x - 3 = 0 \\
 2x &= 1 && x = 3
 \end{aligned}$$

$$x = \frac{1}{2}$$

The solutions are $\frac{1}{2}$ and 3.

You Try It 2

$$\begin{aligned}
 x^2 - 3ax - 4a^2 &= 0 \\
 (x + a)(x - 4a) &= 0 \\
 x + a &= 0 && x - 4a = 0 \\
 x &= -a && x = 4a
 \end{aligned}$$

The solutions are $-a$ and $4a$.

You Try It 3

$$(x - r_1)(x - r_2) = 0$$

$$(x - 3)\left[x - \left(-\frac{1}{2}\right)\right] = 0 \quad \bullet \ r_1 = 3, r_2 = -\frac{1}{2}$$

$$(x - 3)\left(x + \frac{1}{2}\right) = 0$$

$$x^2 - \frac{5}{2}x - \frac{3}{2} = 0$$

$$2\left(x^2 - \frac{5}{2}x - \frac{3}{2}\right) = 2 \cdot 0$$

$$2x^2 - 5x - 3 = 0$$

You Try It 4

$$2(x + 1)^2 - 24 = 0$$

$$2(x + 1)^2 = 24$$

$$(x + 1)^2 = 12 \quad \bullet \text{ Solve for } (x + 1)^2.$$

$$\sqrt{(x + 1)^2} = \sqrt{12} \quad \bullet \text{ Take the square root of each side of the equation.}$$

$$x + 1 = \pm\sqrt{12}$$

$$x + 1 = \pm 2\sqrt{3}$$

$$x + 1 = 2\sqrt{3} \quad x + 1 = -2\sqrt{3} \quad \bullet \text{ Solve for } x.$$

$$x = -1 + 2\sqrt{3} \quad x = -1 - 2\sqrt{3}$$

The solutions are $-1 + 2\sqrt{3}$ and $-1 - 2\sqrt{3}$.

SECTION 8.2**You Try It 1**

$$4x^2 - 4x - 1 = 0 \quad \bullet \text{ Write in the form } ax^2 + bx = -c.$$

$$4x^2 - 4x = 1$$

$$\frac{1}{4}(4x^2 - 4x) = \frac{1}{4} \cdot 1 \quad \bullet \text{ Multiply both sides by } \frac{1}{a}.$$

$$x^2 - x = \frac{1}{4}$$

$$x^2 - x + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} \quad \bullet \text{ Complete the square.}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{2}{4} \quad \bullet \text{ Factor.}$$

$$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{\frac{2}{4}} \quad \bullet \text{ Take square roots.}$$

$$x - \frac{1}{2} = \pm \frac{\sqrt{2}}{2}$$

$$x - \frac{1}{2} = \frac{\sqrt{2}}{2} \quad x - \frac{1}{2} = -\frac{\sqrt{2}}{2} \quad \bullet \text{ Solve for } x.$$

$$x = \frac{1}{2} + \frac{\sqrt{2}}{2} \quad x = \frac{1}{2} - \frac{\sqrt{2}}{2}$$

The solutions are $\frac{1 + \sqrt{2}}{2}$ and $\frac{1 - \sqrt{2}}{2}$.

You Try It 2

$$x^2 + 4x + 8 = 0$$

$$x^2 + 4x = -8$$

$$x^2 + 4x + 4 = -8 + 4 \quad \bullet \text{ Complete the square.}$$

$$(x + 2)^2 = -4 \quad \bullet \text{ Factor.}$$

$$\sqrt{(x + 2)^2} = \sqrt{-4} \quad \bullet \text{ Take square roots.}$$

$$x + 2 = \pm 2i$$

$$x + 2 = 2i \quad x + 2 = -2i \quad \bullet \text{ Solve for } x.$$

$$x = -2 + 2i \quad x = -2 - 2i$$

The solutions are $-2 + 2i$ and $-2 - 2i$.

SECTION 8.3**You Try It 1**

$$x^2 - 2x + 10 = 0$$

$$a = 1, b = -2, c = 10$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{4 - 40}}{2} = \frac{2 \pm \sqrt{-36}}{2}$$

$$= \frac{2 \pm 6i}{2} = 1 \pm 3i$$

The solutions are $1 + 3i$ and $1 - 3i$.

You Try It 2

$$4x^2 = 4x - 1$$

$$4x^2 - 4x + 1 = 0 \quad \bullet \text{ Write in standard form.}$$

$$a = 4, b = -4, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2 \cdot 4}$$

$$= \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4 \pm \sqrt{0}}{8}$$

$$= \frac{4}{8} = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

You Try It 3

$$3x^2 - x - 1 = 0$$

$$a = 3, b = -1, c = -1$$

$$b^2 - 4ac =$$

$$(-1)^2 - 4(3)(-1) = 1 + 12 = 13$$

$$13 > 0$$

Because the discriminant is greater than zero, the equation has two unequal real number solutions.

SECTION 8.4**You Try It 1**

$$x - 5x^{1/2} + 6 = 0$$

$$(x^{1/2})^2 - 5(x^{1/2}) + 6 = 0$$

$$u^2 - 5u + 6 = 0$$

$$(u - 2)(u - 3) = 0$$

$$u - 2 = 0 \quad u - 3 = 0$$

$$u = 2 \quad u = 3$$

Replace u by $x^{1/2}$.

$$x^{1/2} = 2 \quad x^{1/2} = 3$$

$$\sqrt{x} = 2 \quad \sqrt{x} = 3$$

$$(\sqrt{x})^2 = 2^2 \quad (\sqrt{x})^2 = 3^2$$

$$x = 4 \quad x = 9$$

The solutions are 4 and 9.

You Try It 2

$$\sqrt{2x + 1} + x = 7$$

$$\sqrt{2x + 1} = 7 - x$$

$$(\sqrt{2x + 1})^2 = (7 - x)^2$$

$$2x + 1 = 49 - 14x + x^2$$

$$0 = x^2 - 16x + 48$$

$$0 = (x - 4)(x - 12)$$

$$x - 4 = 0 \quad x - 12 = 0$$

$$x = 4 \quad x = 12$$

4 checks as a solution.

12 does not check as a solution.

The solution is 4.

You Try It 3

$$\sqrt{2x - 1} + \sqrt{x} = 2$$

Solve for one of the radical expressions.

$$\sqrt{2x - 1} = 2 - \sqrt{x}$$

$$(\sqrt{2x - 1})^2 = (2 - \sqrt{x})^2$$

$$2x - 1 = 4 - 4\sqrt{x} + x$$

$$x - 5 = -4\sqrt{x}$$

$$(x - 5)^2 = (-4\sqrt{x})^2$$

$$x^2 - 10x + 25 = 16x$$

$$x^2 - 26x + 25 = 0$$

$$(x - 1)(x - 25) = 0$$

$$x - 1 = 0 \quad x - 25 = 0$$

$$x = 1 \quad x = 25$$

1 checks as a solution.

25 does not check as a solution.

The solution is 1.

You Try It 4

$$3y + \frac{25}{3y - 2} = -8$$

$$(3y - 2)\left(3y + \frac{25}{3y - 2}\right) = (3y - 2)(-8) \quad \bullet \text{ Clear denominators.}$$

$$(3y - 2)(3y) + (3y - 2)\left(\frac{25}{3y - 2}\right) = (3y - 2)(-8)$$

$$9y^2 - 6y + 25 = -24y + 16$$

$$9y^2 + 18y + 9 = 0 \quad \bullet \text{ Write in standard form.}$$

$$9(y^2 + 2y + 1) = 0 \quad \bullet \text{ Solve by factoring.}$$

$$9(y + 1)(y + 1) = 0$$

$$y + 1 = 0 \quad y + 1 = 0$$

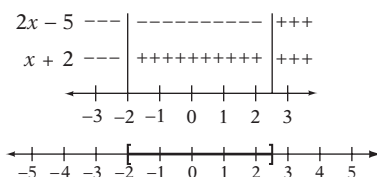
$$y = -1 \quad y = -1$$

The solution is -1 .

SECTION 8.5**You Try It 1**

$$2x^2 - x - 10 \leq 0$$

$$(2x - 5)(x + 2) \leq 0$$



$$\left\{x \mid -2 \leq x \leq \frac{5}{2}\right\}$$

SECTION 8.6**You Try It 1****Strategy**

- The unknown time for Olivia working alone: t
- The unknown time for William working alone: $t + 3$
- The time working together: 2 h

	Rate	•	Time	=	Part
Olivia	$\frac{1}{t}$	•	2	=	$\frac{2}{t}$
William	$\frac{1}{t + 3}$	•	2	=	$\frac{2}{t + 3}$

- The sum of the parts of the task completed by Olivia and William equals 1.

Solution

$$\begin{aligned}\frac{2}{t} + \frac{2}{t+3} &= 1 \\ t(t+3)\left(\frac{2}{t} + \frac{2}{t+3}\right) &= t(t+3) \cdot 1 \\ 2(t+3) + 2t &= t^2 + 3t \\ 2t + 6 + 2t &= t^2 + 3t \\ 0 &= t^2 - t - 6 \\ 0 &= (t+2)(t-3)\end{aligned}$$

$$\begin{aligned}t+2 &= 0 & t-3 &= 0 \\ t &= -2 & t &= 3\end{aligned}$$

The time cannot be negative. It takes Olivia 3 h, working alone, to detail the car. The time for William to detail the car is $t+3$.

$$\begin{aligned}t+3 &= 3+3=6\end{aligned}$$

It takes William 6 h, working alone, to detail the car.

You Try It 2**Strategy**

- Unknown rate of the wind: r

	Distance	÷	Rate	=	Time
With the wind	1200	÷	$250 + r$	=	$\frac{1200}{250 + r}$
Against the wind	1200	÷	$250 - r$	=	$\frac{1200}{250 - r}$

- The time with the wind was 2 h less than the time against the wind.

Solution

$$\begin{aligned}\frac{1200}{250+r} &= \frac{1200}{250-r} - 2 \\ (250+r)(250-r)\left(\frac{1200}{250+r}\right) &= (250+r)(250-r)\left(\frac{1200}{250-r} - 2\right) \\ (250-r)1200 &= (250+r)1200 - 2(250+r)(250-r) \\ 300,000 - 1200r &= 300,000 + 1200r - 2(62,500 - r^2) \\ 300,000 - 1200r &= 300,000 + 1200r - 125,000 + 2r^2 \\ 0 &= 2r^2 + 2400r - 125,000 \\ 0 &= 2(r^2 + 1200r - 62,500) \\ 0 &= 2(r+1250)(r-50)\end{aligned}$$

$$\begin{aligned}r+1250 &= 0 & r-50 &= 0 \\ r &= -1250 & r &= 50\end{aligned}$$

The rate cannot be negative. The rate of the wind was 50 mph.

You Try It 3**Strategy**

- Height of the triangle: h
- Base of the triangle: $2h+4$
- Use the formula $A = \frac{1}{2}bh$.

Solution

$$\begin{aligned}A &= \frac{1}{2}bh \\ 35 &= \frac{1}{2}(2h+4)h \\ 35 &= h^2 + 2h \\ 0 &= h^2 + 2h - 35 \\ 0 &= (h+7)(h-5) \\ h+7 &= 0 & h-5 &= 0 \\ h &= -7 & h &= 5\end{aligned}$$

The height cannot be negative.

The height of the triangle is 5 in.

SOLUTIONS TO CHAPTER 9 “YOU TRY IT”**SECTION 9.1****You Try It 1**

x -coordinate of vertex:

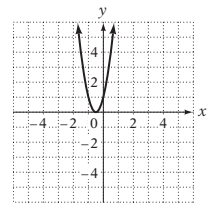
$$-\frac{b}{2a} = -\frac{4}{2(4)} = -\frac{1}{2}$$

y -coordinate of vertex:

$$\begin{aligned}y &= 4x^2 + 4x + 1 \\ &= 4\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 1 \\ &= 1 - 2 + 1 \\ &= 0\end{aligned}$$

$$\text{Vertex: } \left(-\frac{1}{2}, 0\right)$$

$$\text{Axis of symmetry: } x = -\frac{1}{2}$$

**You Try It 2**

x -coordinate of vertex:

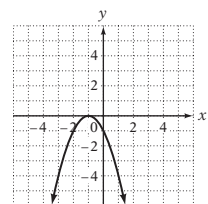
$$-\frac{b}{2a} = -\frac{-2}{2(-1)} = -1$$

y -coordinate of vertex:

$$\begin{aligned}f(x) &= -x^2 - 2x - 1 \\ f(-1) &= -(-1)^2 - 2(-1) - 1 \\ &= -1 + 2 - 1 \\ &= 0\end{aligned}$$

$$\text{Vertex: } (-1, 0)$$

The domain is $\{x | x \in \text{real numbers}\}$. The range is $\{y | y \leq 0\}$.



You Try It 3

$$y = x^2 + 3x + 4$$

$$0 = x^2 + 3x + 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(4)}}{2 \cdot 1}$$

$$= \frac{-3 \pm \sqrt{-7}}{2}$$

$$= \frac{-3 \pm i\sqrt{7}}{2}$$

• $a = 1, b = 3, c = 4$

The equation has no real number solutions. There are no x -intercepts.

You Try It 4

$$g(x) = x^2 - x + 6$$

$$0 = x^2 - x + 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 - 24}}{2}$$

$$= \frac{1 \pm \sqrt{-23}}{2} = \frac{1 \pm i\sqrt{23}}{2}$$

• $a = 1, b = -1, c = 6$

The zeros of the function are $\frac{1}{2} + \frac{\sqrt{23}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{23}}{2}i$.

You Try It 5

$$y = x^2 - x - 6$$

$$a = 1, b = -1, c = -6$$

$$b^2 - 4ac$$

$$(-1)^2 - 4(1)(-6) = 1 + 24 = 25$$

Because the discriminant is greater than zero, the parabola has two x -intercepts.

You Try It 6

$$f(x) = -3x^2 + 4x - 1$$

$$x = -\frac{b}{2a} = -\frac{4}{2(-3)} = \frac{2}{3}$$

• The x -coordinate of the vertex

$$f(x) = -3x^2 + 4x - 1$$

$$f\left(\frac{2}{3}\right) = -3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) - 1$$

• $x = \frac{2}{3}$

$$= -\frac{4}{3} + \frac{8}{3} - 1 = \frac{1}{3}$$

Because a is negative, the function has a maximum value.

The maximum value of the function is $\frac{1}{3}$.

You Try It 7**Strategy**

- To find the time it takes the ball to reach its maximum height, find the t -coordinate of the vertex.
- To find the maximum height, evaluate the function at the t -coordinate of the vertex.

Solution

$$t = -\frac{b}{2a} = -\frac{64}{2(-16)} = 2$$

• The t -coordinate of the vertex

The ball reaches its maximum height in 2 s.

$$s(t) = -16t^2 + 64t$$

$$s(2) = -16(2)^2 + 64(2) = -64 + 128 = 64$$

• $t = 2$

The maximum height is 64 ft.

You Try It 8**Strategy**

$$P = 2x + y$$

$$100 = 2x + y$$

$$100 - 2x = y$$

• $P = 100$
• Solve for y .

Express the area of the rectangle in terms of x .

$$A = xy$$

$$A = x(100 - 2x)$$

• $y = 100 - 2x$

$$A = -2x^2 + 100x$$

- To find the width, find the x -coordinate of the vertex of $f(x) = -2x^2 + 100x$.
- To find the length, replace x in $y = 100 - 2x$ by the x -coordinate of the vertex.

Solution

$$x = -\frac{b}{2a} = -\frac{100}{2(-2)} = 25$$

The width is 25 ft.

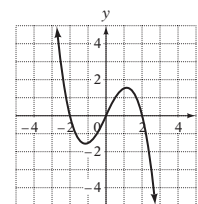
$$100 - 2x = 100 - 2(25)$$

$$= 100 - 50 = 50$$

The length is 50 ft.

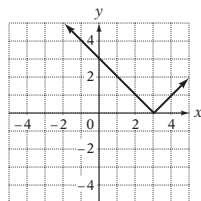
SECTION 9.2**You Try It 1**

Any vertical line intersects the graph at most once. The graph is the graph of a function.

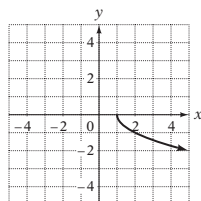
You Try It 2

Domain: $\{x \mid x \in \text{real numbers}\}$

Range: $\{y \mid y \in \text{real numbers}\}$

You Try It 3

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

You Try It 4

Domain: $\{x \mid x \geq 1\}$
Range: $\{y \mid y \leq 0\}$

SECTION 9.3**You Try It 1**

$$\begin{aligned}(f + g)(-2) &= f(-2) + g(-2) \\ &= [(-2)^2 + 2(-2)] + [5(-2) - 2] \\ &= (4 - 4) + (-10 - 2) \\ &= -12\end{aligned}$$

$$(f + g)(-2) = -12$$

You Try It 2

$$\begin{aligned}(f \cdot g)(3) &= f(3) \cdot g(3) \\ &= (4 - 3^2) \cdot [3(3) - 4] \\ &= (4 - 9) \cdot (9 - 4) \\ &= (-5)(5) \\ &= -25\end{aligned}$$

$$(f \cdot g)(3) = -25$$

You Try It 3

$$\begin{aligned}\left(\frac{f}{g}\right)(4) &= \frac{f(4)}{g(4)} \\ &= \frac{4^2 - 4}{4^2 + 2 \cdot 4 + 1} \\ &= \frac{16 - 4}{16 + 8 + 1} \\ &= \frac{12}{25}\end{aligned}$$

$$\left(\frac{f}{g}\right)(4) = \frac{12}{25}$$

You Try It 4

$$\begin{aligned}g(x) &= x^2 \\ g(-1) &= (-1)^2 = 1\end{aligned}$$

• Evaluate g at -1 .

$$\begin{aligned}f(x) &= 1 - 2x \\ f[g(-1)] &= f(1) = 1 - 2(1) = -1 \\ f[g(-1)] &= -1\end{aligned}$$

• Evaluate f at $g(-1) = 1$.

You Try It 5

$$M(s) = s^3 + 1$$

$$\begin{aligned}M[L(s)] &= (s + 1)^3 + 1 \\ &= s^3 + 3s^2 + 3s + 1 + 1 \\ &= s^3 + 3s^2 + 3s + 2\end{aligned}$$

$$M[L(s)] = s^3 + 3s^2 + 3s + 2$$

• Evaluate M at $L(s)$.

SECTION 9.4**You Try It 1**

Because any horizontal line intersects the graph at most once, the graph is the graph of a 1-1 function.

You Try It 2

$$f(x) = \frac{1}{2}x + 4$$

$$y = \frac{1}{2}x + 4$$

$$x = \frac{1}{2}y + 4$$

$$x - 4 = \frac{1}{2}y$$

$$2x - 8 = y$$

$$f^{-1}(x) = 2x - 8$$

The inverse of the function is given by $f^{-1}(x) = 2x - 8$.

You Try It 3

$$\begin{aligned}f[g(x)] &= 2\left(\frac{1}{2}x - 3\right) - 6 \\ &= x - 6 - 6 = x - 12\end{aligned}$$

No, $g(x)$ is not the inverse of $f(x)$.

You Try It 4

$$s(x) = \frac{3}{5}x + 1125$$

$$y = \frac{3}{5}x + 1125$$

• Replace $s(x)$ by y .

$$x = \frac{3}{5}y + 1125$$

• Interchange x and y .

$$x - 1125 = \frac{3}{5}y$$

• Solve for y .

$$\frac{5}{3}(x - 1125) = y$$

$$\frac{5}{3}x - 1875 = y$$

$$\frac{5}{3}x - 1875 = s^{-1}(x)$$

• Replace y by $s^{-1}(x)$.

$s^{-1}(x) = \frac{5}{3}x - 1875$. The equation gives the temperature of the air given the speed of sound.

SOLUTIONS TO CHAPTER 10 “YOU TRY IT”**SECTION 10.1****You Try It 1**

$$f(x) = \left(\frac{2}{3}\right)^x$$

$$f(3) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$\bullet x = 3$$

$$f(-2) = \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\bullet x = -2$$

You Try It 2

$$f(x) = 2^{2x+1}$$

$$f(0) = 2^{2(0)+1} = 2^1 = 2$$

$$\bullet x = 0$$

$$f(-2) = 2^{2(-2)+1} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\bullet x = -2$$

You Try It 3

$$f(x) = e^{2x-1}$$

$$f(2) = e^{2 \cdot 2 - 1} = e^3 \approx 20.0855$$

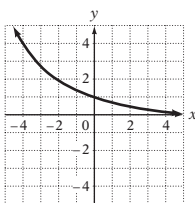
$$\bullet x = 2$$

$$f(-2) = e^{2(-2)-1} = e^{-5} \approx 0.0067$$

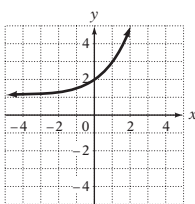
$$\bullet x = -2$$

You Try It 4

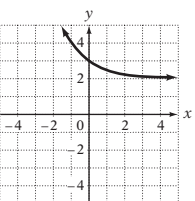
x	y
-4	4
-2	2
0	1
2	$\frac{1}{2}$
4	$\frac{1}{4}$

**You Try It 5**

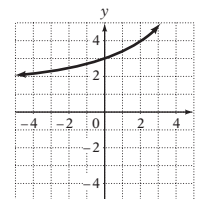
x	y
-2	$\frac{5}{4}$
-1	$\frac{3}{2}$
0	2
1	3
2	5

**You Try It 6**

x	y
-2	6
-1	4
0	3
1	$\frac{5}{2}$
2	$\frac{9}{4}$

**You Try It 7**

x	y
-4	$\frac{9}{4}$
-2	$\frac{5}{2}$
0	3
2	4
4	6

**SECTION 10.2****You Try It 1**

$$\log_4 64 = x$$

• Write the equivalent exponential form.

$$64 = 4^x$$

$$4^3 = 4^x$$

• $64 = 4^3$

$$3 = x$$

• The bases are the same.

$$\log_4 64 = 3$$

The exponents are equal.

You Try It 2

$$\log_2 x = -4$$

$$2^{-4} = x$$

• Write the equivalent exponential form.

$$\frac{1}{2^4} = x$$

$$\frac{1}{16} = x$$

The solution is $\frac{1}{16}$.

You Try It 3

$$\ln x = 3$$

• Write the equivalent exponential form.

$$e^3 = x$$

$$20.0855 \approx x$$

You Try It 4

$$\log_8 \sqrt[3]{xy^2} = \log_8 (xy^2)^{1/3} = \frac{1}{3} \log_8 (xy^2)$$

• Power Property

$$= \frac{1}{3} (\log_8 x + \log_8 y^2)$$

• Product Property

$$= \frac{1}{3} (\log_8 x + 2 \log_8 y)$$

• Power Property

$$= \frac{1}{3} \log_8 x + \frac{2}{3} \log_8 y$$

• Distributive Property

You Try It 5

$$\frac{1}{3} (\log_4 x - 2 \log_4 y + \log_4 z)$$

$$= \frac{1}{3} (\log_4 x - \log_4 y^2 + \log_4 z)$$

$$= \frac{1}{3} \left(\log_4 \frac{x}{y^2} + \log_4 z \right) = \frac{1}{3} \left(\log_4 \frac{xz}{y^2} \right)$$

$$= \log_4 \left(\frac{xz}{y^2} \right)^{1/3} = \log_4 \sqrt[3]{\frac{xz}{y^2}}$$

You Try It 6

$$\log_3 0.834 = \frac{\ln 0.834}{\ln 3} \approx -0.16523$$

You Try It 7

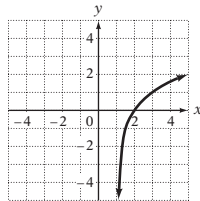
$$\log_7 6.45 = \frac{\log 6.45}{\log 7} \approx 0.95795$$

SECTION 10.3

You Try It 1

$$\begin{aligned}
 f(x) &= \log_2(x - 1) \\
 y &= \log_2(x - 1) & \bullet f(x) = y \\
 2^y &= x - 1 & \bullet \text{Write the equivalent} \\
 2^y + 1 &= x & \text{exponential equation.}
 \end{aligned}$$

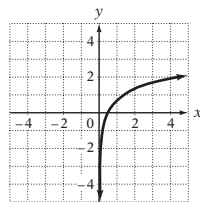
x	y
$\frac{5}{4}$	-2
$\frac{3}{2}$	-1
2	0
3	1
5	2



You Try It 2

$$\begin{aligned}
 f(x) &= \log_3(2x) \\
 y &= \log_3(2x) & \bullet f(x) = y \\
 3^y &= 2x & \bullet \text{Write the equivalent} \\
 \frac{3^y}{2} &= x & \text{exponential equation.}
 \end{aligned}$$

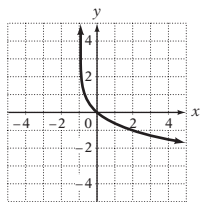
x	y
$\frac{1}{18}$	-2
$\frac{1}{6}$	-1
$\frac{1}{2}$	0
$\frac{3}{2}$	1
$\frac{9}{2}$	2



You Try It 3

$$\begin{aligned}
 f(x) &= -\log_3(x + 1) \\
 y &= -\log_3(x + 1) & \bullet f(x) = y \\
 -y &= \log_3(x + 1) & \bullet \text{Multiply both sides by } -1. \\
 3^{-y} &= x + 1 & \bullet \text{Write the equivalent} \\
 3^{-y} - 1 &= x & \text{exponential equation.}
 \end{aligned}$$

x	y
8	-2
2	-1
0	0
$-\frac{2}{3}$	1
$-\frac{8}{9}$	2



SECTION 10.4

You Try It 1

$$\begin{aligned}
 (1.06)^n &= 1.5 \\
 \log(1.06)^n &= \log 1.5 & \bullet \text{Take the log of each side.} \\
 n \log 1.06 &= \log 1.5 & \bullet \text{Power Property} \\
 n &= \frac{\log 1.5}{\log 1.06} & \bullet \text{Divide both sides by } \log 1.06. \\
 n &\approx 6.9585
 \end{aligned}$$

The solution is 6.9585.

You Try It 2

$$\begin{aligned}
 4^{3x} &= 25 \\
 \log 4^{3x} &= \log 25 & \bullet \text{Take the log of each side.} \\
 3x \log 4 &= \log 25 & \bullet \text{Power Property} \\
 3x &= \frac{\log 25}{\log 4} & \bullet \text{Divide both sides by } \log 4. \\
 3x &\approx 2.3219 \\
 x &\approx 0.7740 & \bullet \text{Divide both sides by } 3.
 \end{aligned}$$

The solution is 0.7740.

You Try It 3

$$\begin{aligned}
 \log_4(x^2 - 3x) &= 1 \\
 4^1 &= x^2 - 3x & \bullet \text{Write in exponential form.} \\
 4 &= x^2 - 3x \\
 0 &= x^2 - 3x - 4 \\
 0 &= (x + 1)(x - 4) \\
 x + 1 &= 0 & x - 4 &= 0 \\
 x &= -1 & x &= 4
 \end{aligned}$$

The solutions are -1 and 4.

You Try It 4

$$\begin{aligned}
 \log_3 x + \log_3(x + 3) &= \log_3 4 \\
 \log_3[x(x + 3)] &= \log_3 4 & \bullet \text{Product Property} \\
 x(x + 3) &= 4 & \bullet \text{1-1 Property of Logarithms} \\
 x^2 + 3x &= 4 \\
 x^2 + 3x - 4 &= 0 \\
 (x + 4)(x - 1) &= 0 & \bullet \text{Solve by factoring.} \\
 x + 4 &= 0 & x - 1 &= 0 \\
 x &= -4 & x &= 1
 \end{aligned}$$

-4 does not check as a solution. The solution is 1.

You Try It 5

$$\begin{aligned}
 \log_3 x + \log_3(x + 6) &= 3 \\
 \log_3[x(x + 6)] &= 3 & \bullet \text{Product Property} \\
 x(x + 6) &= 3^3 & \bullet \text{Write the equivalent} \\
 x^2 + 6x &= 27 & \text{exponential equation.} \\
 x^2 + 6x - 27 &= 0 \\
 (x + 9)(x - 3) &= 0 & \bullet \text{Solve by factoring.} \\
 x + 9 &= 0 & x - 3 &= 0 \\
 x &= -9 & x &= 3
 \end{aligned}$$

-9 does not check as a solution. The solution is 3.

SECTION 10.5**You Try It 1****Strategy**

To find the hydrogen ion concentration, replace pH by 2.9 in the equation $\text{pH} = -\log(\text{H}^+)$ and solve for H^+ .

Solution

$$\text{pH} = -\log(\text{H}^+)$$

$$2.9 = -\log(\text{H}^+)$$

$$-2.9 = \log(\text{H}^+)$$

$$10^{-2.9} = \text{H}^+$$

$$0.00126 \approx \text{H}^+$$

The hydrogen ion concentration is approximately 0.00126.

You Try It 2**Strategy**

Substitute 0.20 for p and 6 for k into the equation $\log p = -kd$ and solve for d .

Solution

$$\log p = -kd$$

$$\log 0.20 = -6d$$

$$\frac{\log 0.20}{-6} = d$$

$$-6$$

$$0.116 \approx d$$

The opaque glass must be 0.116 m thick.

SOLUTIONS TO CHAPTER 11 "YOU TRY IT"**SECTION 11.1****You Try It 1**

$$y = x^2 + 2x + 1$$

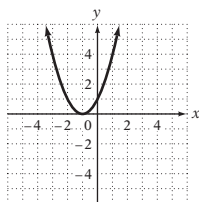
$$-\frac{b}{2a} = -\frac{2}{2(1)} = -1$$

Axis of symmetry:

$$x = -1$$

$$y = (-1)^2 + 2(-1) + 1 = 0$$

Vertex: $(-1, 0)$

**You Try It 2**

$$x = -y^2 - 2y + 2$$

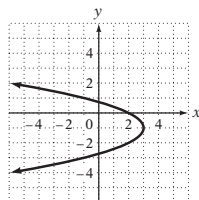
$$-\frac{b}{2a} = -\frac{-2}{2(-1)} = -1$$

Axis of symmetry:

$$y = -1$$

$$x = -(-1)^2 - 2(-1) + 2 = 3$$

Vertex: $(3, -1)$

**You Try It 3**

$$y = x^2 - 2x - 1$$

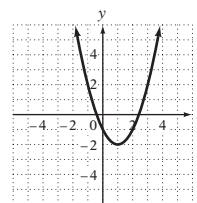
$$-\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

Axis of symmetry:

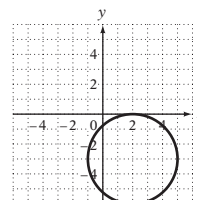
$$x = 1$$

$$y = 1^2 - 2(1) - 1 = -2$$

Vertex: $(1, -2)$

**SECTION 11.2****You Try It 1**

- Center: $(2, -3)$
- Radius: $\sqrt{9} = 3$

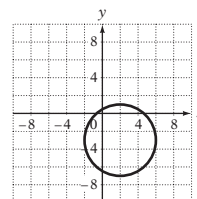
**You Try It 2**

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + [y - (-3)]^2 = 4^2$$

$$(x - 2)^2 + (y + 3)^2 = 16$$

- $h = 2$,
- $k = -3$,
- $r = 4$

**You Try It 3**

$$x^2 + y^2 - 2x - 15 = 0$$

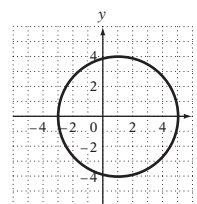
$$(x^2 - 2x) + y^2 = 15$$

$$(x^2 - 2x + 1) + y^2 = 15 + 1$$

$$(x - 1)^2 + y^2 = 16$$

Center: $(1, 0)$

Radius: 4

**You Try It 4**

$$x^2 + y^2 - 4x + 8y + 15 = 0$$

$$x^2 + y^2 - 4x + 8y = -15$$

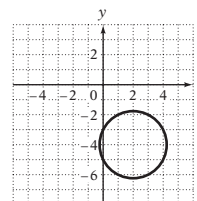
$$(x^2 - 4x) + (y^2 + 8y) = -15$$

$$(x^2 - 4x + 4) + (y^2 + 8y + 16) = -15 + 4 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 5$$

Center: $(2, -4)$

Radius: $\sqrt{5} \approx 2.2$



SECTION 11.3

You Try It 1

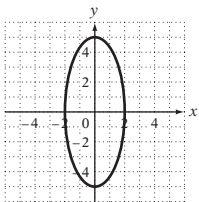
$$a^2 = 4, b^2 = 25$$

x-intercepts:

$$(2, 0) \text{ and } (-2, 0)$$

y-intercepts:

$$(0, 5) \text{ and } (0, -5)$$



You Try It 2

$$a^2 = 18, b^2 = 9$$

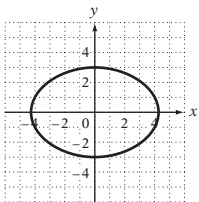
x-intercepts:

$$(3\sqrt{2}, 0) \text{ and } (-3\sqrt{2}, 0)$$

y-intercepts:

$$(0, 3) \text{ and } (0, -3)$$

$$\left(3\sqrt{2} \approx 4\frac{1}{4}\right)$$



You Try It 3

$$a^2 = 9, b^2 = 25$$

Axis of symmetry:

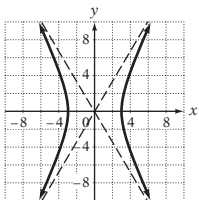
x-axis

Vertices:

$$(3, 0) \text{ and } (-3, 0)$$

Asymptotes:

$$y = \frac{5}{3}x \text{ and } y = -\frac{5}{3}x$$



You Try It 4

$$a^2 = 9, b^2 = 9$$

Axis of symmetry:

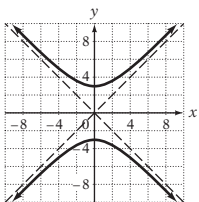
y-axis

Vertices:

$$(0, 3) \text{ and } (0, -3)$$

Asymptotes:

$$y = x \text{ and } y = -x$$



SECTION 11.4

You Try It 1

$$(1) y = 2x^2 + x - 3$$

$$(2) y = 2x^2 - 2x + 9$$

Use the substitution method.

$$\begin{aligned} y &= 2x^2 + x - 3 \\ 2x^2 - 2x + 9 &= 2x^2 + x - 3 \\ -3x + 9 &= -3 \\ -3x &= -12 \\ x &= 4 \end{aligned}$$

Substitute into Equation (1).

$$\begin{aligned} y &= 2x^2 + x - 3 \\ y &= 2(4)^2 + 4 - 3 \\ y &= 32 + 4 - 3 \\ y &= 33 \end{aligned}$$

The solution is (4, 33).

You Try It 2

$$(1) x^2 - y^2 = 10$$

$$(2) x^2 + y^2 = 8$$

Use the addition method. Add the two equations.

$$\begin{aligned} 2x^2 &= 18 \\ x^2 &= 9 \\ x &= \pm\sqrt{9} = \pm 3 \end{aligned}$$

Substitute into Equation (2).

$$\begin{aligned} x^2 + y^2 &= 8 & x^2 + y^2 &= 8 \\ 3^2 + y^2 &= 8 & (-3)^2 + y^2 &= 8 \\ 9 + y^2 &= 8 & 9 + y^2 &= 8 \\ y^2 &= -1 & y^2 &= -1 \\ y &= \pm\sqrt{-1} & y &= \pm\sqrt{-1} \end{aligned}$$

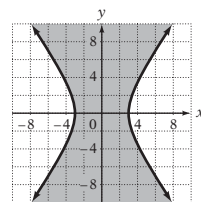
Because y is not a real number, the system of equations has no real number solution. The graphs do not intersect.

SECTION 11.5

$$\text{You Try It 1} \quad \frac{x^2}{9} - \frac{y^2}{16} \leq 1$$

Graph the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$

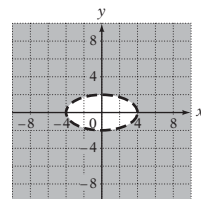
as a solid curve. Shade the region of the plane that includes (0, 0).



$$\text{You Try It 2} \quad \frac{x^2}{16} + \frac{y^2}{4} > 1$$

Graph the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$

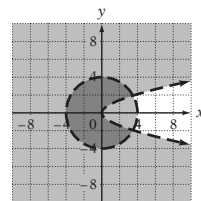
as a dashed curve. Shade the region of the plane that does not include (0, 0).



$$\text{You Try It 3} \quad \begin{aligned} x^2 + y^2 &< 16 \\ y^2 &> x \end{aligned}$$

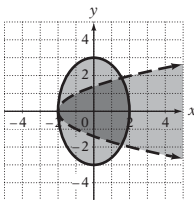
Draw the circle $x^2 + y^2 = 16$

as a dashed curve. Shade inside the circle. Draw the parabola $y^2 = x$ as a dashed curve. Shade outside the parabola. The solution set is the region of the plane that includes (-1, 0).



You Try It 4 $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$
 $x > y^2 - 2$

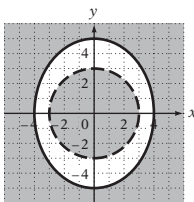
Draw the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ as a solid curve. Shade inside the ellipse. Draw the parabola $x = y^2 - 2$ as a dashed curve. Shade inside the parabola. The solution set is the region of the plane that includes $(0, 0)$.



You Try It 5 $\frac{x^2}{16} + \frac{y^2}{25} \geq 1$
 $x^2 + y^2 < 9$

Draw the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ as a solid curve. Shade outside the ellipse. Draw the circle $x^2 + y^2 = 9$ as a dashed curve. Shade inside the circle.

The solution sets of the two inequalities do not intersect. This system of inequalities has no real number solution.



SOLUTIONS TO CHAPTER 12 "YOU TRY IT"

SECTION 12.1

You Try It 1

$$a_n = n(n + 1)$$

$$a_1 = 1(1 + 1) = 2 \quad \text{The first term is 2.}$$

$$a_2 = 2(2 + 1) = 6 \quad \text{The second term is 6.}$$

$$a_3 = 3(3 + 1) = 12 \quad \text{The third term is 12.}$$

$$a_4 = 4(4 + 1) = 20 \quad \text{The fourth term is 20.}$$

You Try It 2

$$a_n = \frac{1}{n(n + 2)}$$

$$a_6 = \frac{1}{6(6 + 2)} = \frac{1}{48} \quad \text{The sixth term is } \frac{1}{48}.$$

$$a_9 = \frac{1}{9(9 + 2)} = \frac{1}{99} \quad \text{The ninth term is } \frac{1}{99}.$$

You Try It 3

$$\sum_{i=3}^6 (i^2 - 2) = (3^2 - 2) + (4^2 - 2) + (5^2 - 2) + (6^2 - 2)$$

$$= 7 + 14 + 23 + 34 = 78$$

You Try It 4 $\sum_{n=1}^4 \frac{x^n}{n} = \frac{x^1}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$

$$= x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$$

SECTION 12.2

You Try It 1

7, 9, 11, ..., 59

$$d = a_2 - a_1 = 9 - 7 = 2$$

• Find d .

$$a_n = a_1 + (n - 1)d$$

$$59 = 7 + (n - 1)2$$

• $a_n = 59, a_1 = 7, d = 2$

$$59 = 7 + 2n - 2$$

$$59 = 5 + 2n$$

$$54 = 2n$$

$$27 = n$$

There are 27 terms in the sequence.

You Try It 2

-4, -2, 0, 2, 4, ...

$$d = a_2 - a_1 = -2 - (-4) = 2$$

• Find d .

$$a_n = a_1 + (n - 1)d$$

$$a_{25} = -4 + (25 - 1)2 = -4 + (24)2$$

• Find a_{25} .

$$= -4 + 48$$

$$a_{25} = 44$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

• Use the formula.

$$S_{25} = \frac{25}{2}(-4 + 44) = \frac{25}{2}(40) = 25(20)$$

$$S_{25} = 500$$

You Try It 3 $\sum_{n=1}^{18} (3n - 2)$

$$a_n = 3n - 2$$

$$a_1 = 3(1) - 2 = 1$$

• Find a_1 .

$$a_{18} = 3(18) - 2 = 52$$

• Find a_{18} .

$$S_n = \frac{n}{2}(a_1 + a_n)$$

• Use the formula.

$$S_{18} = \frac{18}{2}(1 + 52) = 9(53) = 477$$

You Try It 4

Strategy

To find the value of the 20th-place prize:

- Write the equation for the n th-place prize.
- Find the 20th term of the sequence.

To find the total amount of prize money being awarded, use the Formula for the Sum of n Terms of an Arithmetic Sequence.

Solution

10,000, 9700, ...

$$d = a_2 - a_1 = 9700 - 10,000 = -300$$

• Find d .

$$a_n = a_1 + (n - 1)d$$

$$= 10,000 + (n - 1)(-300)$$

• Find the equation for the sequence.

$$= 10,000 - 300n + 300$$

$$= -300n + 10,300$$

$$a_{20} = -300(20) + 10,300$$

• Find the 20th term.

$$= -6000 + 10,300 = 4300$$

The value of the 20th-place prize is \$4300.

(continued)

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \bullet \text{ Use the formula.}$$

$$S_{20} = \frac{20}{2}(10,000 + 4300) = 10(14,300) \\ = 143,000$$

The total amount of prize money being awarded is \$143,000.

SECTION 12.3

You Try It 1

$$5, 2, \frac{4}{5}, \dots$$

$$r = \frac{a_2}{a_1} = \frac{2}{5} \quad \bullet \text{ Find } r. a_2 = 2, a_1 = 5$$

$$a_n = a_1 r^{n-1} \quad \bullet \text{ Use the formula.}$$

$$a_5 = 5\left(\frac{2}{5}\right)^{5-1} = 5\left(\frac{2}{5}\right)^4 \quad \bullet n = 5, a_1 = 5, r = \frac{2}{5}$$

$$a_5 = 5\left(\frac{16}{625}\right) = \frac{16}{125}$$

You Try It 2

$$3, a_2, a_3, -192, \dots$$

$$a_n = a_1 r^{n-1} \quad \bullet \text{ Use the formula.}$$

$$a_4 = 3r^{4-1} \quad \bullet n = 4, a_1 = 3$$

$$-192 = 3r^{4-1} \quad \bullet a_4 = -192$$

$$-192 = 3r^3$$

$$-64 = r^3$$

$$-4 = r \quad \bullet \text{ The common ratio is } -4.$$

$$a_n = a_1 r^{n-1}$$

$$a_3 = 3(-4)^{3-1} = 3(-4)^2 \quad \bullet n = 3, a_1 = 3, r = -4 \\ = 3(16) = 48$$

You Try It 3

$$1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$$

$$r = \frac{a_2}{a_1} = \frac{-\frac{1}{3}}{1} = -\frac{1}{3} \quad \bullet \text{ Find } r. a_2 = -\frac{1}{3}, a_1 = 1$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad \bullet \text{ Use the formula.}$$

$$S_4 = \frac{1\left[1 - \left(-\frac{1}{3}\right)^4\right]}{1 - \left(-\frac{1}{3}\right)} = \frac{1 - \frac{1}{81}}{\frac{4}{3}} = \frac{\frac{80}{81}}{\frac{4}{3}}$$

$$= \frac{80}{81} \cdot \frac{3}{4} = \frac{20}{27}$$

You Try It 4

$$\sum_{n=1}^5 \left(\frac{1}{2}\right)^n$$

$$a_n = \left(\frac{1}{2}\right)^n$$

$$a_1 = \left(\frac{1}{2}\right)^1 = \frac{1}{2} \quad \bullet \text{ Find } a_1.$$

$$a_2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad \bullet \text{ Find } a_2.$$

$$r = \frac{a_2}{a_1} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} \quad \bullet \text{ Find } r.$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad \bullet \text{ Use the formula.}$$

$$S_5 = \frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^5\right]}{1 - \frac{1}{2}} = \frac{\frac{1}{2}\left(1 - \frac{1}{32}\right)}{\frac{1}{2}} = \frac{\frac{1}{2}\left(\frac{31}{32}\right)}{\frac{1}{2}} \\ = \frac{31}{32}$$

You Try It 5

$$3, -2, \frac{4}{3}, -\frac{8}{9}, \dots$$

$$r = \frac{a_2}{a_1} = \frac{-2}{3} = -\frac{2}{3} \quad \bullet \text{ Find } r. a_2 = -2, a_1 = 3$$

$$S = \frac{a_1}{1 - r} = \frac{3}{1 - \left(-\frac{2}{3}\right)} \quad \bullet \text{ Use the formula.}$$

$$= \frac{3}{1 + \frac{2}{3}} = \frac{3}{\frac{5}{3}} = \frac{9}{5}$$

You Try It 6

$$0.\overline{66} = 0.6 + 0.06 + 0.006 + \dots$$

$$= \frac{6}{10} + \frac{6}{100} + \frac{6}{1000} + \dots$$

$$S = \frac{a_1}{1 - r} = \frac{\frac{6}{10}}{1 - \frac{1}{10}} = \frac{\frac{6}{10}}{\frac{9}{10}} = \frac{6}{9} = \frac{2}{3}$$

An equivalent fraction is $\frac{2}{3}$.

You Try It 7

$$0.\overline{3421} = 0.34 + 0.0021 + 0.000021 + \dots$$

$$= \frac{34}{100} + \frac{21}{10,000} + \frac{21}{1,000,000} + \dots$$

$$S = \frac{a_1}{1-r} = \frac{\frac{21}{10,000}}{1 - \frac{1}{100}} = \frac{\frac{21}{10,000}}{\frac{99}{100}} = \frac{21}{9900}$$

$$\begin{aligned} 0.342\overline{1} &= \frac{34}{100} + \frac{21}{9900} = \frac{3366}{9900} + \frac{21}{9900} \\ &= \frac{3387}{9900} = \frac{1129}{3300} \end{aligned}$$

An equivalent fraction is $\frac{1129}{3300}$.

You Try It 8

Strategy To find the total number of letters mailed, use the Formula for the Sum of n Terms of a Finite Geometric Series.

Solution $n = 6, a_1 = 3, r = 3$

$$\begin{aligned} S_n &= \frac{a_1(1-r^n)}{1-r} \\ S_6 &= \frac{3(1-3^6)}{1-3} = \frac{3(1-729)}{1-3} \\ &= \frac{3(-728)}{-2} = \frac{-2184}{-2} = 1092 \end{aligned}$$

From the first through the sixth mailings, 1092 letters will have been mailed.

You Try It 9

Strategy To find the value of the investment, use the Formula for the n th Term of a Geometric Sequence, $a_n = a_1 r^{n-1}$.

Solution $n = 11, a_1 = 20,000, r = 1.08$

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ a_{11} &= 20,000(1.08)^{11-1} = 20,000(1.08)^{10} \\ &\approx 43,178.50 \end{aligned}$$

The value of the investment after 10 years is \$43,178.50.

SECTION 12.4

You Try It 1

$$\begin{aligned} (3m-n)^4 &= \binom{4}{0}(3m)^4 + \binom{4}{1}(3m)^3(-n) \\ &\quad + \binom{4}{2}(3m)^2(-n)^2 + \binom{4}{3}(3m)(-n)^3 \\ &\quad + \binom{4}{4}(-n)^4 \\ &= 1(81m^4) + 4(27m^3)(-n) + 6(9m^2)(n^2) \\ &\quad + 4(3m)(-n^3) + 1(n^4) \\ &= 81m^4 - 108m^3n + 54m^2n^2 \\ &\quad - 12mn^3 + n^4 \end{aligned}$$

You Try It 2

$$\begin{aligned} (y-2)^{10} &= \binom{10}{0}y^{10} + \binom{10}{1}y^9(-2) \\ &\quad + \binom{10}{2}y^8(-2)^2 + \cdots \\ &= 1(y^{10}) + 10y^9(-2) + 45y^8(4) + \cdots \\ &= y^{10} - 20y^9 + 180y^8 - \cdots \end{aligned}$$

You Try It 3

$$\begin{aligned} (t-2s)^7 &= \binom{7}{r-1} t^{n-r+1} b^{r-1} \quad \bullet \quad n=7, a=t, b=-2s, r=3 \\ &= \binom{7}{3-1} (t)^{7-3+1} (-2s)^{3-1} = \binom{7}{2} (t)^5 (-2s)^2 \\ &= 21t^5(4s^2) = 84t^5s^2 \end{aligned}$$

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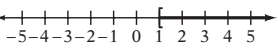
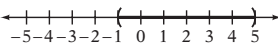
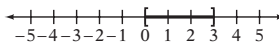
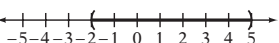
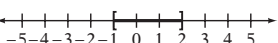
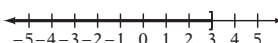
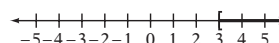
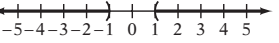
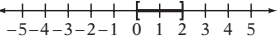
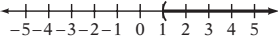
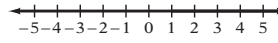
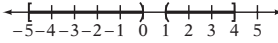
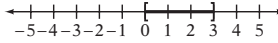
Answers to Selected Exercises

ANSWERS TO CHAPTER 1 SELECTED EXERCISES

PREP TEST

1. $\frac{13}{20}$ 2. $\frac{11}{60}$ 3. $\frac{2}{9}$ 4. $\frac{2}{3}$ 5. 44.405 6. 73.63 7. 7.446 8. 54.06 9. a. Yes b. No c. Yes
d. Yes 10. a. C b. D c. A d. B

SECTION 1.1

1. a. Integers: 0, -3 b. Rational numbers: $-\frac{15}{2}$, 0, -3, $2.\overline{33}$ c. Irrational numbers: π , 4.232232223..., $\frac{\sqrt{5}}{4}$, $\sqrt{7}$
d. Real numbers: all 3. -27 5. $-\frac{3}{4}$ 7. 0 9. $\sqrt{33}$ 11. 91 13. 7 15. -2, -1 17. 9, 0, -9
19. 4, 0, 4 21. Yes; negative real numbers 23. $\{-2, -1, 0, 1, 2, 3, 4\}$ 25. $\{2, 4, 6, 8, 10, 12\}$
27. $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$ 29. $\{x | x > 4, x \in \text{integers}\}$ 31. $\{x | x \geq -2\}$ 33. $\{x | 0 < x < 1\}$
35. $\{x | 1 \leq x \leq 4\}$ 37. Yes 39. No 41. No 43. 
45.  47.  49. 
51. (-2, 4) 53. [-1, 5] 55. $(-\infty, 1)$ 57. $[-2, \infty)$ 59. $\{x | 0 < x < 8\}$ 61. $\{x | -5 \leq x \leq 7\}$
63. $\{x | -3 \leq x < 6\}$ 65. $\{x | x \leq 4\}$ 67. $\{x | x > 5\}$ 69. 
71.  73.  75. 
77. $A \cup B = \{1, 2, 4, 6, 9\}$ 79. $A \cup B = \{2, 3, 5, 8, 9, 10\}$ 81. $A \cup B = \{-4, -2, 0, 2, 4, 8\}$
83. $A \cup B = \{1, 2, 3, 4, 5\}$ 85. $A \cap B = \{6\}$ 87. $A \cap B = \{5, 10, 20\}$ 89. $A \cap B = \emptyset$ 91. $A \cap B = \{4, 6\}$
93. (iii) 95.  97.  99. 
101.  103.  105. 
107. A 109. B 111. A 113. R 115. R 117. 0

SECTION 1.2

3. -30 5. -17 7. -96 9. -6 11. 5 13. 11,200 15. 20 17. 96 19. 23 21. 7 23. 17
25. -4 27. -62 29. -25 31. 7 33. 368 35. 26 37. -83 39. Negative 43. $\frac{43}{48}$
45. $-\frac{67}{45}$ 47. $-\frac{13}{36}$ 49. $\frac{11}{24}$ 51. $\frac{13}{24}$ 53. $-\frac{3}{56}$ 55. $-\frac{2}{3}$ 57. $-\frac{11}{14}$ 59. $-\frac{1}{24}$ 61. -12.974 63. -6.008
65. 1.9215 67. -6.02 69. -6.7 71. 1, 3, or 5 73. -1.11 75. -2030.44 77. 125
79. -8 81. -125 83. 324 85. -36 87. -72 89. 216 91. -16 93. 12 95. 2,654,208
97. Negative 99. Positive 103. -7 105. $\frac{177}{11}$ 107. $\frac{16}{3}$ 109. -40 111. 32 113. 24 115. $\frac{2}{15}$
117. $-\frac{1}{2}$ 119. $-\frac{97}{72}$ 121. 6.284 123. 5.4375 125. 2 127. (ii) 129. 0 131. No, the number zero has a
multiplicative inverse that is undefined. 133. 9 135. 625 137. First find b^c ; then find $a^{(b^c)}$.

SECTION 1.3

1. 3 3. 3 5. 0 7. 6 9. 0 11. 1 13. $(2 \cdot 3)$ 15. The Division Property of Zero
17. The Inverse Property of Multiplication 19. The Addition Property of Zero 21. The Division Property of Zero
23. The Distributive Property 25. The Associative Property of Multiplication 27. Zero 29. 10 31. 4
33. 0 35. $-\frac{1}{7}$ 37. $\frac{9}{2}$ 39. $\frac{1}{2}$ 41. 2 43. 3 45. -12 47. 2 49. 6 51. -2 53. -14
55. 56 57. 256 59. Negative 61. $12x$ 63. $7x$ 65. $-8a - 7b$ 67. y 69. z 71. $3a - 15$
73. $-5x + 45$ 75. $-x - y$ 77. $4x - 6y + 15$ 79. $15x + 90$ 81. $-3x - 12y$ 83. $48a - 75$
85. $-12x + 12y$ 87. $33a - 25b$ 89. $31x + 34y$ 91. $4 - 8x - 5y$ 93. $2x + 9$
95. a. Negative b. Positive c. Zero 97. $0.124x + 72$ 99. $\frac{t}{12}$

SECTION 1.4

1. $n - 8$ 3. $\frac{4}{5}n$ 5. $\frac{n}{14}$ 7. No 9. $n - (n + 2) = -2$ 11. $5(8n) = 40n$ 13. $17n - 2n = 15n$
15. $n^2 - (12 + n^2) = -12$ 17. $15n + (5n + 12) = 20n + 12$ 19. $(15 - x + 2) + 2x = x + 17$ 21. $\frac{5(34 - x)}{x - 3}$
23. a. Historical average temperature: t ; $t + 3.5$ b. Historical maximum temperature: t ; $t + 1.5$
25. Distance from Earth to the moon: d ; $390d$ 27. $10,000 - x$ 29. Measure of angle B : x ; measure of angle A : $2x$; measure of angle C : $4x$ 31. The selling price of the house in 2005 33. The sum of twice x and 3 35. Twice the sum of x and 3
37. a. ma b. Av^2

CHAPTER 1 CONCEPT REVIEW*

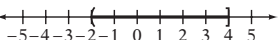
- The symbol $<$ indicates that the number to the left of the symbol is less than the number to the right of the symbol. For example, $5 < 8$. [1.1A]
- The absolute value of a positive number is the number itself. The absolute value of a negative number is the opposite of the number. The absolute value of zero is zero. [1.1A]
- The roster method encloses a list of the elements of a set in braces; for example, $\{1, 2, 3, 4, 5\}$. Set-builder notation uses a rule to describe the elements of the set; for example, $\{x \mid x > 4, x \in \text{real numbers}\}$. [1.1B]
- To indicate an infinite set when using the roster method, use three dots to indicate that the pattern of numbers continues without end. [1.1B]
- A half-open interval is used to represent a set when one endpoint is included and the other is not. [1.1B]
- To divide real numbers, change the division symbol to a multiplication symbol. Then multiply the dividend by the multiplicative inverse of the divisor. [1.2B]
- The steps of the Order of Operations Agreement are:
 - Perform operations inside grouping symbols. Grouping symbols include parentheses $()$, brackets $[\]$, braces $\{ \}$, the absolute value symbol, and the fraction bar.
 - Simplify exponential expressions.
 - Do multiplication and division as they occur from left to right.
 - Do addition and subtraction as they occur from left to right. [1.2D]
- The Commutative Property of Multiplication states that two numbers can be multiplied in either order; the product is the same. The Associative Property of Multiplication states that changing the grouping of three or more factors does not change their product. In the Commutative Property of Multiplication, the order in which the numbers appear changes, whereas in the Associative Property of Multiplication, the order in which the numbers appear does not change. [1.3A]
- The union of two sets, $A \cup B$, is the set of all elements that belong to either set A or set B . The intersection of two sets, $A \cap B$, is the set of all elements that are common to both set A and set B . [1.1C]
- To simplify a variable expression, first use the Distributive Property to remove parentheses if the expression contains parentheses. Then combine like terms. [1.3C]

**Note:* The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

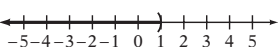
11. **a.** The phrase *less than* indicates subtraction. **b.** The phrase *the total of* indicates addition. **c.** The phrase *the quotient of* indicates division. **d.** The phrase *the product of* indicates multiplication. [1.4A]

12. The Inverse Property of Multiplication is used to evaluate $7 \cdot \frac{1}{7}$. [1.3A]

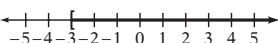
CHAPTER 1 REVIEW EXERCISES

1. $\{-2, -1, 0, 1, 2, 3\}$ [1.1B] 2. $A \cap B = \{2, 3\}$ [1.1C] 3.  [1.1B]

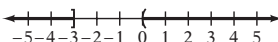
4. The Associative Property of Multiplication [1.3A] 5. -2.84 [1.2B] 6. $\frac{7}{6}$ [1.3B] 7. -288 [1.2C]

8. $16y - 15x + 18$ [1.3C] 9. $\frac{3}{4}$ [1.1A] 10. $\{x | x < -3\}$ [1.1B] 11.  [1.1B]

12. -15 [1.2A] 13. $-\frac{7}{30}$ [1.2B] 14. 4 [1.3A] 15. $-\frac{5}{8}$ [1.2B] 16. $0, 2$ [1.1A]

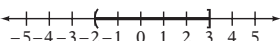
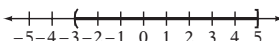
17. 20 [1.3B] 18. 14 [1.2A] 19. 52 [1.2D] 20.  [1.1B]

21. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ [1.1C] 22. 12 [1.2A] 23. $\{x | -2 \leq x \leq 3\}$ [1.1B] 24. $\frac{2}{15}$ [1.2B]

25. 3 [1.3A] 26.  [1.1C] 27. $-6x + 14$ [1.3C] 28. $-4, 0, -7$ [1.1A]

29. The Inverse Property of Addition [1.3A] 30. 3.1 [1.2B] 31. 87 [1.1A] 32. $-1, 4$ [1.1A]

33. $\{-3, -2, -1, 0, 1\}$ [1.1B] 34. $\{x | x < 7\}$ [1.1B] 35. $A \cup B = \{-4, -2, 0, 2, 4, 5, 10\}$ [1.1C]

36. $A \cap B = \emptyset$ [1.1C] 37.  [1.1C] 38.  [1.1C]

39. 5 [1.2A] 40. $-\frac{1}{2}$ [1.2B] 41. 24 [1.2C] 42. -107 [1.2D] 43. -6 [1.3B] 44. $5a + 9b$ [1.3C]

45. $4(x + 4); 4x + 16$ [1.4A] 46. Flying time between San Diego and New York: t ; $13 - t$ [1.4B]

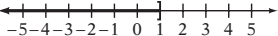
47. Number of calories burned by walking at 4 mph for 1 h: C ; $C + 396$ [1.4B] 48. $2(x - 2) + 8; 2x + 4$ [1.4A]

49. First integer: x ; $4x + 5$ [1.4B] 50. $12 - \frac{x+3}{4} = \frac{-x+45}{4}$ [1.4A] 51. $2x + (40 - x) + 5 = x + 45$ [1.4A]

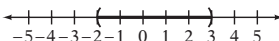
52. The width of the rectangle: w ; $3w - 3$ [1.4B]

CHAPTER 1 TEST

1. -30 [1.2A, How To 4] 2. $A \cap B = \{5, 7\}$ [1.1C, How To 8] 3. -72 [1.2C, Example 6]

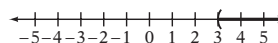
4.  [1.1B, You Try It 8] 5. $A \cap B = \{-1, 0, 1\}$ [1.1C, How To 8]

6. -5 [1.3B, Example 3] 7. 2 [1.2A, Example 2] 8. $14x + 48y$ [1.3C, Example 6] 9. 12 [1.1A, Example 2]

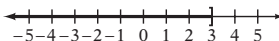
10. -100 [1.2C, Example 6] 11.  [1.1C, Example 12] 12. 12 [1.2A, Example 1]

13. $\frac{25}{36}$ [1.2B, Example 4] 14. 4 [1.3A, Example 1] 15. $-\frac{4}{27}$ [1.2B, Example 5] 16. -5 [1.1A, Example 1]

17. 2 [1.3B, Example 3] 18. -15 [1.2A, How To 5] 19. 10 [1.2D, How To 13]

20.  [1.1B, You Try It 8] 21. $A \cup B = \{1, 2, 3, 4, 5, 7\}$ [1.1C, How To 8]

22. $13x - y$ [1.3C, Example 6] 23. 6 [1.2D, How To 14] 24. $\frac{2}{15}$ [1.2B, Example 5]

25. The Distributive Property [1.3A, Example 2] 26.  [1.1C, Example 11]

27. -1.41 [1.2B, How To 6] 28. $A \cup B = \{-2, -1, 0, 1, 2, 3\}$ [1.2B, How To 8]

29. $(x + 1) - 2(9 - x) = 3x - 17$ [1.4A, Example 3] 30. Amount of cocoa produced in Ghana: x ; $3x$ [1.4B, Example 5]

ANSWERS TO CHAPTER 2 SELECTED EXERCISES**PREP TEST**

1. -4 [1.2A] 2. -6 [1.2A] 3. 3 [1.2A] 4. 1 [1.2B] 5. $-\frac{1}{2}$ [1.2B] 6. $10x - 5$ [1.3C]
 7. $6x - 9$ [1.3C] 8. $3n + 6$ [1.3C] 9. $0.03x + 20$ [1.3C] 10. $20 - n$ [1.4B]

SECTION 2.1

5. Yes 7. Yes 9. 9 11. -15 13. 4 15. $-\frac{2}{3}$ 17. $\frac{17}{6}$ 19. $\frac{1}{6}$ 21. $\frac{15}{2}$ 23. $-\frac{32}{25}$ 25. -21
 27. $-\frac{2}{3}$ 29. 1 31. $\frac{15}{16}$ 33. 18 35. 800 37. $\frac{3}{2}$ 39. 0 41. Greater than 43. 8 45. 6
 47. 6 49. No solution 51. -3 53. 1 55. $\frac{4}{3}$ 57. 1 59. $-\frac{4}{3}$ 61. -1.25 63. 2 65. $\frac{3}{2}$
 67. -55 69. Positive 71. 6 73. -6 75. $\frac{5}{6}$ 77. $-\frac{31}{7}$ 79. $\frac{2}{3}$ 81. $\frac{35}{12}$ 83. -22 85. 6
 87. $\frac{11}{2}$ 89. $-\frac{4}{29}$ 91. 9.4 93. 1500 95. $\frac{17}{8}$ 97. 0 99. (i) 101. $r = \frac{C}{2\pi}$ 103. $h = \frac{2A}{b}$ 105. $M = \frac{IC}{100}$
 107. $r = \frac{A - P}{Pt}$ 109. $c = 2s - a - b$ 111. $h = \frac{S - 2\pi r^2}{2\pi r}$ 113. $R = Pn + C$ 115. $P = \frac{A}{1 + i}$ 117. Subtract

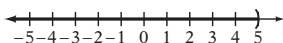
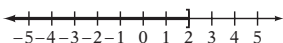
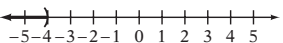
SECTION 2.2

1. The number is 5. 3. The integers are 3 and 7. 5. The integers are 21 and 29. 7. The numbers are 21, 44, and 58.
 9. The integers are -20 , -19 , and -18 . 11. No solution 13. The integers are 5, 7, and 9.
 15. a. $39n + 41m$ b. $0.39n + 0.41m$ 17. There are 12 dimes in the bank. 19. There are eight 20¢ stamps and sixteen 15¢ stamps.
 21. There are five 3¢ stamps in the collection. 23. There are seven 18¢ stamps in the collection. 25. No solution

SECTION 2.3

1. (iii), (iv) 3. The cost of the mixture is \$5.00 per pound. 5. 320 adult tickets were sold. 7. The mixture must contain 22.5 L of imitation maple syrup.
 9. 15 oz of pure gold and 35 oz of the alloy were used. 11. The cost of the mixture is \$4.11 per pound.
 13. The mixture must contain 10 gal of cranberry juice. 15. (ii), (iii), (iv), (v)
 17. 200 lb of the 15% aluminum alloy must be used. 19. The solution should contain 2.25 oz of 70% rubbing alcohol and 1.25 oz of water.
 21. 45 oz of pure water must be added. 23. 300 ml of alcohol must be added. 25. The remaining fruit drink is 6% fruit juice.
 27. 4 qt will have to be replaced with pure antifreeze. 29. a. Equal to b. Greater than
 31. It takes the police officer $\frac{2}{3}$ min to catch up. 33. The speed of the first car is 58 mph. The speed of the second car is 66 mph.
 35. The distance between the two airports is 375 mi. 37. The rate of the freight train is 30 mph. The rate of the passenger train is 48 mph.
 39. The cyclist traveled 44 mi. 41. 15 oz of the 12-karat gold should be used. 43. The cars are $3.3\bar{3}$ (or $3\frac{1}{3}$) mi apart 2 min before impact.

SECTION 2.4

3. (i), (iii) 5. $\{x | x < 5\}$  7. $\{x | x \leq 2\}$ 
 9. $\{x | x < -4\}$  11. $\{x | x > 3\}$ 13. $\{x | x > 4\}$ 15. $\{x | x \leq 2\}$ 17. $\{x | x > -2\}$
 19. $\{x | x \geq 2\}$ 21. $\{x | x > -2\}$ 23. $\{x | x \leq 3\}$ 25. $\{x | x < 2\}$ 27. $\{x | x < -3\}$ 29. $\{x | x \leq 5\}$
 31. $\{x | x \geq -\frac{1}{2}\}$ 33. Both positive and negative numbers 35. Both positive and negative numbers
 37. $(-\infty, \frac{23}{16})$ 39. $(-\infty, \frac{8}{3})$ 41. $(1, \infty)$ 43. $(\frac{14}{11}, \infty)$ 45. $(-\infty, 1]$ 47. $(-\infty, \frac{7}{4}]$
 49. $[-\frac{5}{4}, \infty)$ 51. $(-\infty, 2]$ 55. $[-2, 4]$ 57. $(-\infty, 3) \cup (5, \infty)$ 59. $(-4, 2)$ 61. $(-\infty, -4) \cup (6, \infty)$
 63. $(-\infty, -3)$ 65. $\emptyset$ 67. $(-2, 1)$ 69. All real numbers 71. Two intervals of real numbers

73. $\{x|x < -2 \text{ or } x > 2\}$ 75. $\{x|2 < x < 6\}$ 77. $\{x|-3 < x < -2\}$ 79. $\{x|x > 5 \text{ or } x < -\frac{5}{3}\}$ 81. $\emptyset$

83. The set of real numbers 85. $\{x|\frac{17}{7} \leq x \leq \frac{45}{7}\}$ 87. $\{x|-5 < x < \frac{17}{3}\}$ 89. The set of real numbers

91. $\emptyset$ 93. $t \leq 42$ 95. $t \leq 42$ 97. The smallest number is -12 . 99. The maximum width of the rectangle is 11 cm. 101. The advertisement can run for a maximum of 104 days. 103. The homeowner can pay a maximum of \$19 per gallon of paint. 105. $32^\circ < F < 86^\circ$ 107. George's amount of sales must be \$44,000 or more. 109. The first account is less expensive if more than 200 checks are written. 111. $58 \leq n \leq 100$ 113. The three even integers are 10, 12, and 14; or 12, 14, and 16; or 14, 16, and 18. 115. a. Always true b. Sometimes true c. Sometimes true d. Sometimes true e. Always true

SECTION 2.5

1. Yes 3. Yes 5. 7 and -7 7. 4 and -4 9. 6 and -6 11. 7 and -7 13. No solution
 15. No solution 17. 1 and -5 19. 8 and 2 21. 2 23. No solution 25. $-\frac{3}{2}$ and 3 27. $\frac{3}{2}$
 29. No solution 31. 7 and -3 33. 2 and $-\frac{10}{3}$ 35. 1 and 3 37. $\frac{3}{2}$ 39. No solution 41. $\frac{11}{6}$ and $-\frac{1}{6}$
 43. $-\frac{1}{3}$ and -1 45. No solution 47. 3 and 0 49. No solution 51. 1 and $\frac{13}{3}$ 53. No solution
 55. $\frac{7}{3}$ and $\frac{1}{3}$ 57. $-\frac{1}{2}$ 59. $-\frac{1}{2}$ and $-\frac{7}{2}$ 61. $-\frac{8}{3}$ and $\frac{10}{3}$ 63. No solution 65. Two positive solutions
 67. Two negative solutions 69. $\{x|x > 3 \text{ or } x < -3\}$ 71. $\{x|x > 1 \text{ or } x < -3\}$ 73. $\{x|4 \leq x \leq 6\}$
 75. $\{x|x \geq 5 \text{ or } x \leq -1\}$ 77. $\{x|-3 < x < 2\}$ 79. $\{x|x > 2 \text{ or } x < -\frac{14}{5}\}$ 81. $\emptyset$
 83. The set of real numbers 85. $\{x|x \leq -\frac{1}{3} \text{ or } x \geq 3\}$ 87. $\{x|-2 \leq x \leq \frac{9}{2}\}$ 89. $\{x|x = 2\}$
 91. $\{x|x < -2 \text{ or } x > \frac{22}{9}\}$ 93. $\{x|-\frac{3}{2} < x < \frac{9}{2}\}$ 95. $\{x|x < 0 \text{ or } x > \frac{4}{5}\}$ 97. $\{x|x > 5 \text{ or } x < 0\}$
 99. All negative solutions 101. The desired dosage is 3 ml. The tolerance is 0.2 ml. 103. The lower and upper limits of the diameter of the bushing are 1.742 in. and 1.758 in. 105. The lower and upper limits of the percent of voters who felt the economy was the most important election issue are 38% and 44%. 107. The lower and upper limits of the length of the piston rod are $9\frac{19}{32}$ in. and $9\frac{21}{32}$ in. 109. The lower and upper limits of the resistor are 28,420 ohms and 29,580 ohms.
 111. a. $\{x|x \geq -3\}$ b. $\{a|a \leq 4\}$ 113. a. $\leq$ b. $\geq$ c. $\geq$ d. $=$

CHAPTER 2 CONCEPT REVIEW*

- The Addition Property of Equations is used to remove a term from one side of an equation by adding the opposite of that term to each side of the equation. [2.1A]
- The root of an equation is just another name for the solution of an equation. They are the same thing. [2.1A]
- To check the solution of an equation, substitute the proposed solution for the variable in the original equation. Simplify the resulting numerical expressions. If the left and right sides are equal, the proposed answer is a solution of the equation. If the left and right sides are not equal, the proposed solution is not a solution of the equation. [2.1A]
- To solve an equation containing parentheses, first use the Distributive Property to remove the parentheses. Combine like terms on each side of the equation. Use the Addition Property of Equations to rewrite the equation with only one variable term. Then use the Addition Property of Equations to rewrite the equation with only one constant term. Use the Multiplication Property of Equations to rewrite the equation in the form *variable* = *constant*. The constant is the solution. [2.1C]
- Consecutive integers differ by 1; for example, 8 and 9 are consecutive integers. Consecutive even integers are even numbers that differ by 2; for example, 8 and 10 are consecutive even integers. [2.2A]
- In writing an expression for the total value of a number of stamps, the value of the stamp is multiplied by the expression for the number of stamps. [2.2B]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

7. In solving a percent mixture problem, use the formula $Ar = Q$, where A is the amount of solution or alloy, r is the percent of concentration, and Q is the quantity of a substance in the solution or alloy. [2.3B]
8. In solving a uniform motion problem, use the formula $rt = d$, where r is the rate of travel, t is the time spent traveling, and d is the distance traveled. [2.3C]
9. The Multiplication Property of Equations states that both sides of an equation can be multiplied by the same nonzero number without changing the solution of the equation. The Multiplication Property of Inequalities consists of two rules:
(1) Each side of an inequality can be multiplied by the same positive number without changing the solution set; (2) If each side of an inequality is multiplied by the same negative number, the inequality symbol must be reversed in order to keep the solution set of the inequality unchanged. [2.4A]
10. The compound inequality $-c < ax + b < c$ is equivalent to an absolute value inequality of the form $|ax + b| < c$, $c > 0$. [2.5B]
11. The compound inequality $ax + b < -c$ or $ax + b > c$ is equivalent to an absolute value inequality of the form $|ax + b| > c$. [2.5B]
12. To check the solution to an absolute value equation, substitute the proposed solution for the variable in the original equation. Simplify the resulting numerical expressions. If the left and right sides are equal, the proposed solution is a solution of the equation. If the left and right sides are not equal, the proposed solution is not a solution of the equation. [2.5A]

CHAPTER 2 REVIEW EXERCISES

1. 6 [2.1B] 2. $(\frac{5}{3}, \infty)$ [2.4A] 3. $L = \frac{P - 2W}{2}$ [2.1D] 4. -9 [2.1A] 5. $\{x | -3 < x < \frac{4}{3}\}$ [2.4B]
6. $-\frac{40}{7}$ [2.1B] 7. $-\frac{2}{3}$ [2.1A] 8. -1 and 9 [2.5A] 9. $\{x | 1 < x < 4\}$ [2.5B] 10. $-\frac{9}{19}$ [2.1C]
11. $\frac{26}{17}$ [2.1C] 12. $\{x | x > 2 \text{ or } x < -2\}$ [2.4B] 13. $\{x | x \geq 2 \text{ or } x \leq \frac{1}{2}\}$ [2.5B] 14. $C = R - Pn$ [2.1D]
15. $-\frac{17}{2}$ [2.1C] 16. No solution [2.5A] 17. $(-\infty, \infty)$ [2.4B] 18. $\frac{5}{2}$ [2.1C] 19. -15 [2.1A]
20. $-\frac{1}{12}$ [2.1A] 21. 7 [2.1A] 22. $\frac{2}{3}$ [2.1A] 23. $\frac{8}{5}$ [2.1B] 24. 3 [2.1B] 25. 9 [2.1C]
26. 8 [2.1C] 27. $(-\infty, 1)$ [2.4A] 28. $[-4, \infty)$ [2.4A] 29. $\{x | x \leq -\frac{39}{2}\}$ [2.4A] 30. $\{x | x \geq 1\}$ [2.4A]
31. $(-1, 2)$ [2.4B] 32. $-\frac{5}{2}, \frac{11}{2}$ [2.5A] 33. $-\frac{8}{5}$ [2.5A] 34. $\emptyset$ [2.5B] 35. The island is 16 mi from the dock. [2.3C]
36. The mixture must contain 13 gal of apple juice. [2.3A] 37. The executive's amount of sales must be \$47,500 or more. [2.4C]
38. There are 7 quarters in the collection. [2.2B] 39. The lower and upper limits of the diameter of the bushing are 2.747 in. and 2.753 in. [2.5C]
40. The integers are 6 and 14. [2.2A]
41. $82 \leq x \leq 100$ [2.4C] 42. The speed of the first plane is 520 mph. The speed of the second plane is 440 mph. [2.3C]
43. 375 lb of the 30% tin alloy and 125 lb of the 70% tin alloy were used. [2.3B] 44. The lower and upper limits of the length of the piston rod are $10\frac{11}{32}$ in. and $10\frac{13}{32}$ in. [2.5C]

CHAPTER 2 TEST

1. -2 [2.1A, How To 1] 2. $-\frac{1}{8}$ [2.1A, How To 2] 3. $\frac{5}{6}$ [2.1A, How To 3] 4. 4 [2.1B, Example 3]
5. $\frac{32}{3}$ [2.1B, Example 3] 6. $-\frac{1}{5}$ [2.1B, Example 3] 7. 1 [2.1C, Example 4] 8. $R = \frac{E - Ir}{I}$ [2.1D, How To 8]
9. $\frac{12}{7}$ [2.1C, How To 7] 10. $\{x | x \leq -3\}$ [2.4A, How To 5] 11. $(-1, \infty)$ [2.4A, How To 6]
12. $\{x | x > -2\}$ [2.4B, How To 10] 13. $\emptyset$ [2.4B, Example 4] 14. $-\frac{9}{5}$ and 3 [2.5A, How To 1]
15. 7 and -2 [2.5A, Example 3] 16. $\{x | \frac{1}{3} \leq x \leq 3\}$ [2.5B, Example 4] 17. $\{x | x < -\frac{1}{2} \text{ or } x > 2\}$ [2.5B, Example 7]
18. It costs less to rent from Gambelli Agency if the car is driven less than 72 mi. [2.4C, Example 5] 19. The lower and upper limits of the diameter of the bushing are 2.648 in. and 2.652 in. [2.5C, You Try It 8]
20. The integers are 4 and 11. [2.2A, Example 1] 21. There are six 24¢ stamps. [2.2B, How To 2] 22. The cost of the hamburger mixture is \$3.58 per pound. [2.3A, Example 1]
23. The jogger ran a total distance of 12 mi. [2.3C, How To 3]

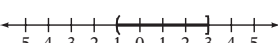
24. The rate of the slower train is 60 mph. The rate of the faster train is 65 mph. [2.3C, You Try It 3]

25. It is necessary to add 100 oz of pure water. [2.3B, Example 2]

CUMULATIVE REVIEW EXERCISES

1. -11 [1.2A] 2. -108 [1.2C] 3. 3 [1.2D] 4. -64 [1.2D] 5. -8 [1.3B] 6. 1 [1.3B]

7. The Commutative Property of Addition [1.3A] 8. $3n + (3n + 6) = 6n + 6$ [1.4A] 9. $B = \frac{Fc}{ev}$ [2.1D]

10. $25y$ [1.3C] 11. $\{-4, 0\}$ [1.1C] 12.  [1.1C] 13. $y = \frac{-Ax - C}{B}$ [2.1D]

14. $\frac{1}{2}$ [2.1A] 15. 1 [2.1B] 16. 24 [2.1B] 17. 2 [2.1C] 18. 2 [2.1C] 19. 1 [2.1C]

20. $-\frac{13}{5}$ [2.1C] 21. $(-\infty, 1]$ [2.4A] 22. $\{x | -4 < x \leq 1\}$ [2.4B] 23. -1 and 4 [2.5A]

24. 7 and -4 [2.5A] 25. $\{x | x < -\frac{4}{3} \text{ or } x > 2\}$ [2.5B] 26. $\{x | -2 < x < 6\}$ [2.5B] 27. The second account costs less if the customer writes fewer than 50 checks. [2.4C]

28. The first integer is -3 . [2.2A] 29. There are 15 dimes. [2.2B]

30. 88 oz of pure silver were used in the mixture. [2.3A] 31. The speed of the slower plane is 220 mph. [2.3C]

32. The lower and upper limits of the diameter of the bushing are 2.449 in. and 2.451 in. [2.5C]

33. 3 L of 12% acid solution must be used in the mixture. [2.3B]

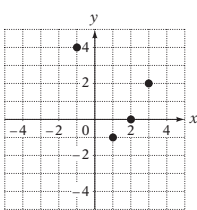
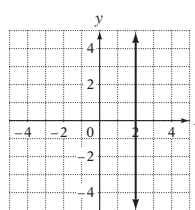
ANSWERS TO CHAPTER 3 SELECTED EXERCISES

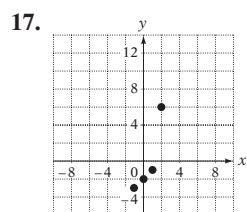
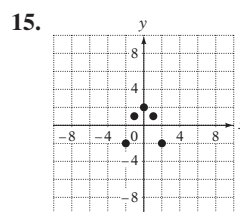
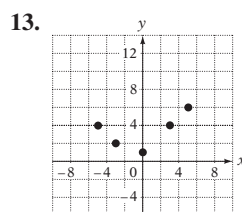
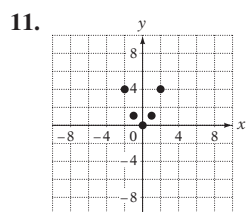
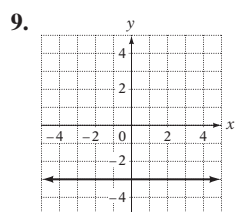
PREP TEST

1. $-4x + 12$ [1.3C] 2. 10 [1.2D] 3. -2 [1.2D] 4. 11 [1.3B] 5. 2.5 [1.3B] 6. 5 [1.3B]

7. 1 [1.3B] 8. 4 [2.1B] 9. $y = 2x - 7$ [2.1D]

SECTION 3.1

1. a. I or IV b. III or IV 3.  5. $A(0, 3), B(1, 1), C(3, -4), D(-4, 4)$ 7. 

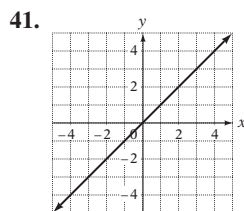
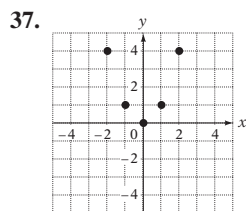
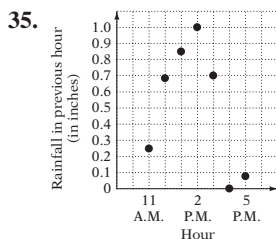


19. Length: 4.47; midpoint: $(4, 3)$ 21. Length: 2.24; midpoint: $(-1, \frac{7}{2})$

23. Length: 5.10; midpoint: $(-\frac{1}{2}, -\frac{9}{2})$ 25. Length: 9.90; midpoint: $(\frac{3}{2}, \frac{3}{2})$ 27. Length: 3; midpoint: $(\frac{7}{2}, -5)$

29. Length: 4.18; midpoint: $(\frac{1}{2}, \frac{1}{2})$ 31. The x -coordinates are equal. 33. a. 25 g of cerium selenate will dissolve at 50°C .

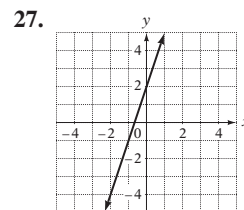
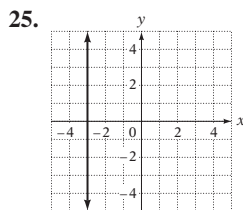
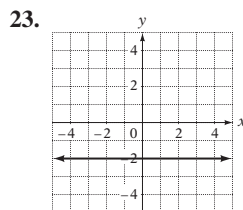
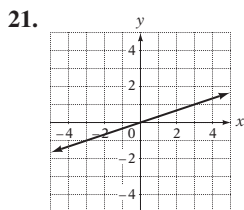
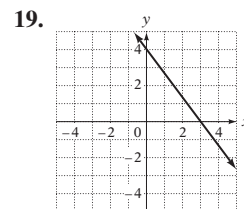
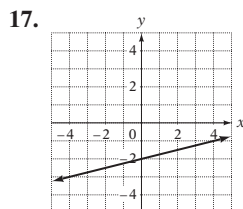
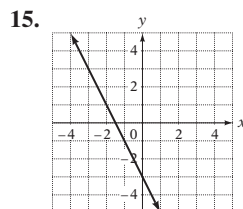
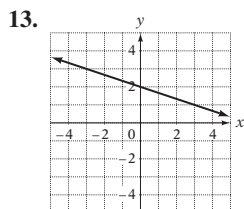
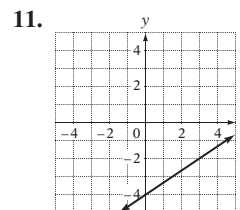
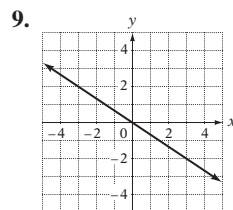
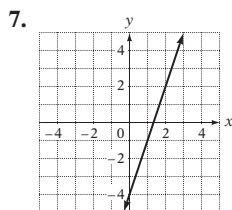
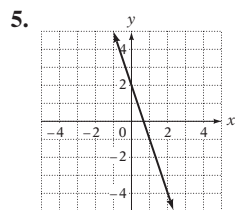
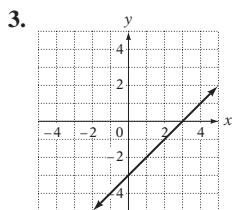
b. 5 g of cerium selenate will dissolve at 80°C .



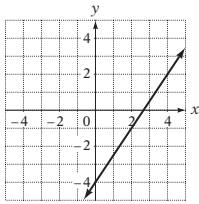
SECTION 3.2

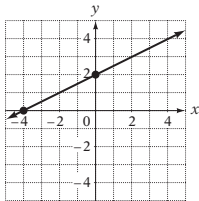
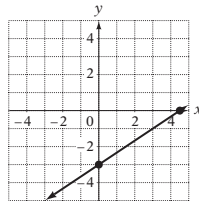
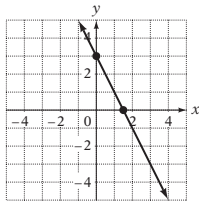
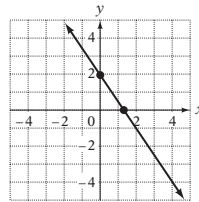
3. Yes 5. Yes 7. No 9. Function 11. Function 13. Function 15. Not a function 17. a. Yes
b. $y = \$125$ 19. True 21. $f(3) = 11$ 23. $f(0) = -4$ 25. $G(0) = 4$ 27. $G(-2) = 10$ 29. $q(3) = 5$
31. $q(-2) = 0$ 33. $F(4) = 24$ 35. $F(-3) = -4$ 37. $H(1) = 1$ 39. $H(t) = \frac{3t}{t+2}$ 41. $s(-1) = 6$
43. $s(a) = a^3 - 3a + 4$ 45. $4h$ 47. The annual cost savings is \$114.29. 49. a. \$4.75 per game b. \$4.00 per game
51. Domain = $\{1, 2, 3, 4, 5\}$ 53. Domain = $\{0, 2, 4, 6\}$ 55. Domain = $\{1, 3, 5, 7, 9\}$ 57. Domain = $\{-2, -1, 0, 1, 2\}$
Range = $\{1, 4, 7, 10, 13\}$ Range = $\{1, 2, 3, 4\}$ Range = $\{0\}$ Range = $\{0, 1, 2\}$
59. Domain = $\{-2, -1, 0, 1, 2\}$ 61. 1 63. -8 65. None 67. None 69. 0 71. None 73. None
Range = $\{-3, 3, 6, 7, 9\}$
75. -2 77. None 79. Range = $\{-3, 1, 5, 9\}$ 81. Range = $\{-23, -13, -8, -3, 7\}$ 83. Range = $\{0, 1, 4\}$
85. Range = $\{2, 14, 26, 42\}$ 87. Range = $\{-5, \frac{5}{3}, 5\}$ 89. Range = $\{-1, -\frac{1}{2}, -\frac{1}{3}, 1\}$ 91. Range = $\{-38, -8, 2\}$
95. a. $\{(-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8)\}$ b. Yes, the set of ordered pairs defines a function because each member of the domain is assigned to exactly one member of the range. 97. The power produced will be 50.625 watts.
99. a. The speed of the paratrooper 11.5 s after beginning the jump is 36.3 ft/s. b. The paratrooper is falling at a speed of approximately 30 ft/s. 101. a. 275,000 malware attacks b. 700,000 malware attacks

SECTION 3.3



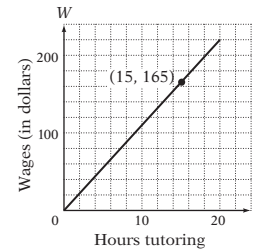
29.


 31. No. If $B = 0$, it is not possible to solve $Ax + By = C$ for y .

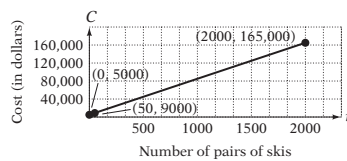
 33. x -intercept: $(-4, 0)$; y -intercept: $(0, 2)$

 35. x -intercept: $(\frac{9}{2}, 0)$; y -intercept: $(0, -3)$

 37. x -intercept: $(\frac{3}{2}, 0)$; y -intercept: $(0, 3)$

 39. x -intercept: $(\frac{4}{3}, 0)$; y -intercept: $(0, 2)$

 41. No. The graph of the equation $x = a$, $a \neq 0$, is a vertical line and has no y -intercept.

45. Marlys receives \$165 for tutoring 15 h.

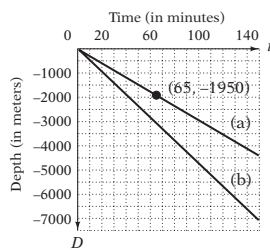
43. The heart of the hummingbird will



47. The cost of manufacturing 50 pairs of skis is \$9000.



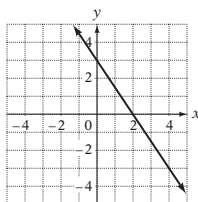
49. a.



After 65 min, Alvin is 1950 m below sea level. b. Below

51. Yes. The points with y -coordinate less than -6500 .

57.



SECTION 3.4

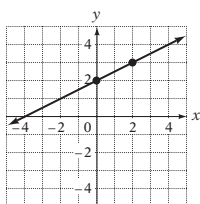
1. -1 3. $\frac{1}{3}$ 5. $-\frac{2}{3}$ 7. $-\frac{3}{4}$ 9. Undefined 11. $\frac{7}{5}$ 13. 0 15. $-\frac{1}{2}$ 17. Undefined 19. a and c

21. $m = 40$. The average speed of the motorist is 40 mph. 23. $m = -5$. The temperature of the oven decreases $5^\circ/\text{min}$.

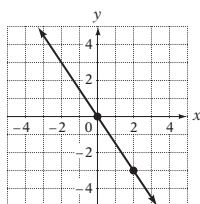
25. $m = -0.05$. For each mile the car is driven, approximately 0.05 gal of fuel is used. 27. $m \approx 352.4$. The average

speed of the runner was 352.4 m/min. 29. a. No b. Yes 31. $-3, 5, -3, (0, 5)$ 33. $4, 0, 4, (0, 0)$

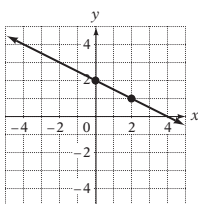
35.



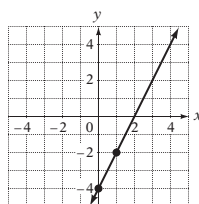
37.



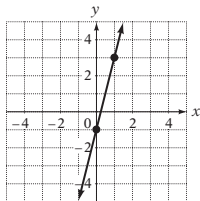
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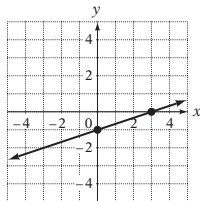
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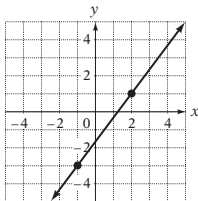
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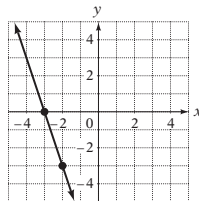
45.



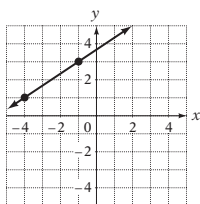
47.



49.



51.



53. a. Below

b. Negative

55. Increases by 2

57. Increases by 2

59. Decreases by $\frac{2}{3}$

61. i. D; ii. C; iii. B; iv. F; v. E; vi. A

63. $k = 10$

65. $k = -4$

SECTION 3.5

3. Check that the coefficient of x is the given slope. Check that the coordinates of the given point are a solution of your equation.

5. $y = 2x + 5$ 7. $y = \frac{1}{2}x + 2$ 9. $y = \frac{5}{4}x + \frac{21}{4}$ 11. $y = -\frac{5}{3}x + 5$ 13. $y = -3x + 9$ 15. $y = -3x + 4$

17. $y = \frac{2}{3}x - \frac{7}{3}$ 19. $y = \frac{1}{2}x$ 21. $y = 3x - 9$ 23. $y = -\frac{2}{3}x + 7$ 25. $y = -x - 3$ 27. $y = \frac{7}{5}x - \frac{27}{5}$

29. $y = -\frac{2}{5}x + \frac{3}{5}$ 31. $x = 3$ 33. $y = -\frac{5}{4}x - \frac{15}{2}$ 35. $y = -3$ 37. $y = -2x + 3$ 39. $x = -5$

41. Check that the coordinates of each given point are a solution of your equation. 43. $y = x + 2$ 45. $y = -2x - 3$

47. $y = \frac{2}{3}x + \frac{5}{3}$ 49. $y = \frac{1}{3}x + \frac{10}{3}$ 51. $y = \frac{3}{2}x - \frac{1}{2}$ 53. $y = -\frac{3}{2}x + 3$ 55. $y = -1$ 57. $y = x - 1$

59. $y = -x + 1$ 61. $y = -\frac{8}{3}x + \frac{25}{3}$ 63. $y = \frac{1}{2}x - 1$ 65. $y = -4$ 67. $y = \frac{3}{4}x$ 69. $y = -\frac{4}{3}x + \frac{5}{3}$

71. $x = -2$ 73. $y = x - 1$ 75. $y = \frac{4}{3}x + \frac{7}{3}$ 77. $y = -x + 3$ 79. a. $f(x) = 1200x, 0 \leq x \leq 26\frac{2}{3}$ b. The height

of the plane 11 min after takeoff is 13,200 ft. 81. a. $f(x) = 0.625x - 1170.5$ b. In 2012, 87% of trees at 2600 ft will be

hardwoods. 83. a. $f(x) = -0.032x + 16, 0 \leq x \leq 500$ b. After 150 mi are driven, 11.2 gal are left in the tank.

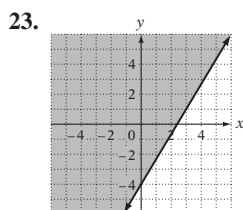
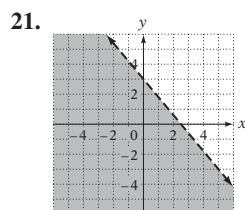
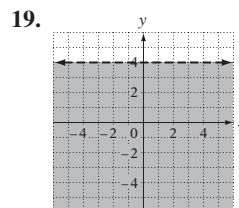
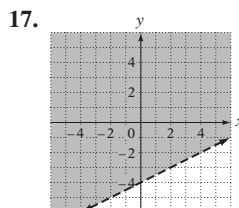
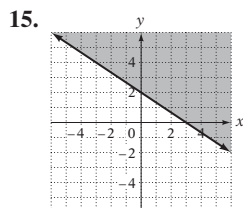
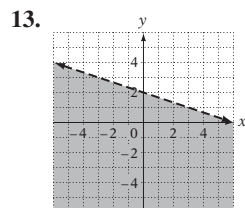
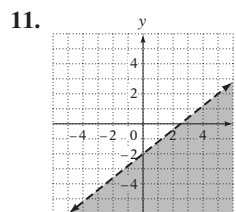
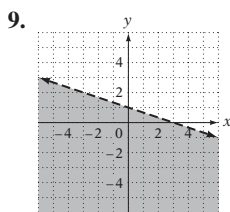
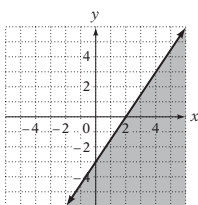
85. a. $f(x) = -20x + 230,000$ b. 60,000 motorcycles would be sold at \$8500 each. 87. a. $f(x) = 63x$ b. There are 315 Calories in a 5-ounce serving. 89. Substitute 15,000 for $f(x)$ and solve for x . 91. $f(x) = x + 3$ 93. 0 95. a. -10
 b. 6 99. Answers will vary. Possible answers include (0, 4), (3, 2), and (9, -2). 101. $y = -2x + 1$

SECTION 3.6

3. No. If two nonvertical lines are perpendicular, the product of their slopes is -1 . Therefore, one line must have a positive slope and one line must have a negative slope. 5. -5 7. $-\frac{1}{4}$ 9. Yes 11. No 13. No 15. Yes 17. Yes 19. Yes
 21. No 23. Yes 25. $y = 2x - 8$ 27. $y = \frac{3}{2}x + 2$ 29. $y = \frac{2}{3}x - \frac{8}{3}$ 31. $y = \frac{1}{3}x - \frac{1}{3}$ 33. $y = -\frac{5}{3}x - \frac{14}{3}$
 35. $y = -2x + 15$ 37. $\frac{A_1}{B_1} = \frac{A_2}{B_2}$

SECTION 3.7

3. Yes 5. No 7.



25. Quadrant I

CHAPTER 3 CONCEPT REVIEW*

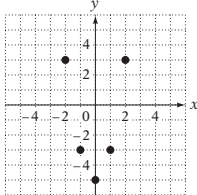
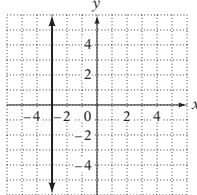
- To find the midpoint (x_m, y_m) of a line segment with endpoints (x_1, y_1) and (x_2, y_2) , use the formulas

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}. \quad [3.1B]$$
- The value of a dependent variable y depends on the value of the independent variable x . The value of the independent variable x is not dependent on the value of any other variable. [3.2A]
- For a rational function, any values of the variable that result in a denominator of zero are excluded from the domain of the function. For a radical function, any values of the variable that result in a negative radicand are excluded from the domain of the function. [3.2A]
- A constant function is of the form $y = b$. The y -intercept is the point $(0, b)$. [3.3B]
- To graph the equation of a line using the x - and y -intercepts, first find the x -intercept by letting $y = 0$ and solving for x . Then find the y -intercept by letting $x = 0$ and solving for y . Plot the x - and y -intercepts on a coordinate grid. Draw a straight line through the two points. [3.3C]
- The slope of a line describes the slant of the line. Only one number is needed to describe whether the line rises or falls from left to right and how steep the line is. [3.4A]
- The slope of a vertical line is undefined. [3.4A]
- When graphing a line using the slope and y -intercept, start at the y -intercept. Then use the slope to find a second point on the line. [3.4B]
- When the slope of a line is undefined, the equation of the line is of the form $x = a$. [3.5A]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

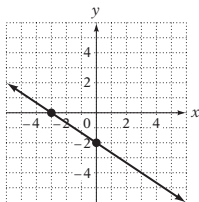
10. A vertical line is perpendicular to a horizontal line. The slope of a vertical line is undefined. [3.6A]
11. After graphing the line of a linear inequality, choose a point on one side of the line. For example, choose the point $(0, 0)$. Substitute 0 for x and 0 for y in the original inequality. If the result is a true inequality, shade the half-plane that contains the point $(0, 0)$. If the result is not a true inequality, shade the half-plane that does not contain the point $(0, 0)$. [3.7A]
12. Given two points, the first step in finding the equation of the line is to find the slope of the line through the two points. Then you can use the point-slope formula to find the equation of the line. [3.5B]

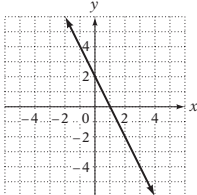
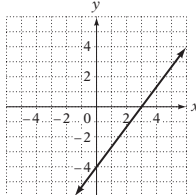
CHAPTER 3 REVIEW EXERCISES

1. $(4, 2)$ [3.1A] 2. $P(-2) = -2$ [3.2A] 3.  [3.1A] 4.  [3.1A]

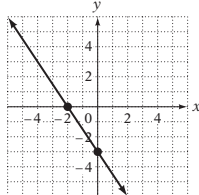
5. Range = $\{-1, 1, 5\}$ [3.2A] 6. Domain = $\{-1, 0, 1, 5\}$ [3.2A] 7. Midpoint: $\left(\frac{1}{2}, \frac{9}{2}\right)$; length: 5.10 [3.1B]

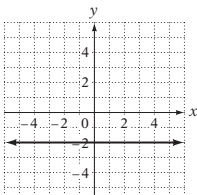
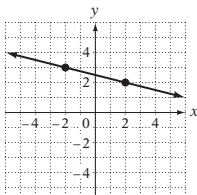
8. -4 [3.2A] 9. x -intercept: $(-3, 0)$ [3.3C]
 y -intercept: $(0, -2)$

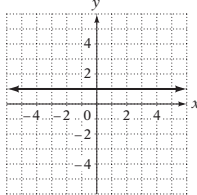


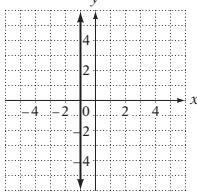
10. x -intercept: $(-2, 0)$ [3.3C] 11.  [3.3A] 12.  [3.3B]

y -intercept: $(0, -3)$

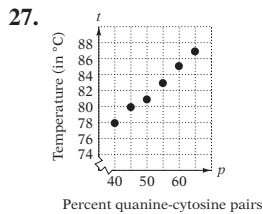


13. -1 [3.4A] 14. $y = \frac{5}{2}x + \frac{23}{2}$ [3.5A] 15.  [3.1A] 16.  [3.4B]

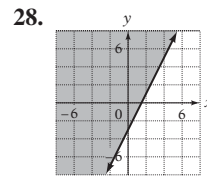
17. $\{-2, -1, 2\}$ [3.2A] 18. a. $f(x) = -x + 295$; $0 \leq x \leq 295$ b. When the rate is \$120, 175 rooms will be occupied. [3.5C] 19. $y = -4x - 5$ [3.6A] 20. $y = \frac{5}{2}x + 8$ [3.6A] 21.  [3.3B]

22.  [3.3B] 23. $y = -\frac{2}{3}x + 1$ [3.5A] 24. $y = \frac{1}{4}x + 4$ [3.5B] 25. 10 [3.1B]

26. (1, 3) [3.1B]



[3.1C]



[3.7A]

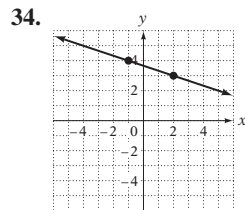
29. [3.7A]

30. $y = -\frac{7}{6}x + \frac{5}{3}$ [3.5B]

31. $y = 2x$ [3.6A]

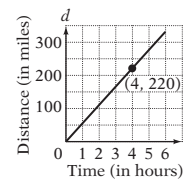
32. $y = -3x + 7$ [3.6A]

33. $y = \frac{3}{2}x + 2$ [3.6A]



[3.4B]

35. After 4 h, the car has traveled 220 mi.



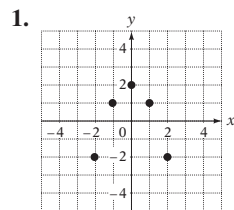
[3.3D]

36. The slope is 20. The manufacturing cost is \$20 per calculator. [3.4A]

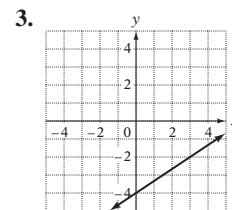
37. a. $f(x) = 80x + 25,000$

b. The house will cost \$185,000 to build. [3.5C]

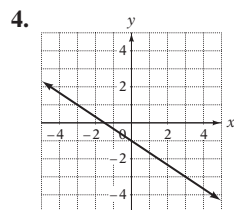
CHAPTER 3 TEST



[3.1A, Example 2]

 2. $(-3, 0)$ [3.1A, Example 1]


[3.3A, You Try It 1]



[3.3B, Example 3]

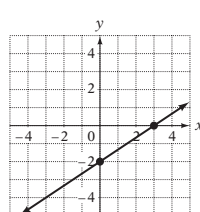
 5. $x = -2$ [3.5A, How To 3]

 6. Length: 10.82; midpoint: $(-\frac{1}{2}, 5)$ [3.1B, Example 3, Example 4]

 7. $-\frac{1}{6}$ [3.4A, Example 1]

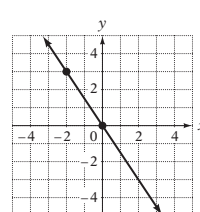
 8. $P(2) = 9$ [3.2A, How To 2]

9.



[3.3C, Example 5]

10.



[3.4B, Example 5]

11. $y = \frac{2}{5}x + 4$ [3.5A, How To 2]

12. 0 [3.2A, You Try It 5]

13. $y = -\frac{7}{5}x + \frac{1}{5}$ [3.5B, How To 4]

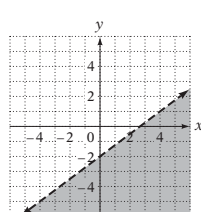
14. $y = -3$ [3.5B, You Try It 4]

15. Domain = $\{-4, -2, 0, 3\}$
Range = $\{0, 2, 5\}$ [3.2A, How To 1]

16. $y = -\frac{3}{2}x + \frac{7}{2}$ [3.6A, How To 2]

17. $y = 2x + 1$ [3.6A, How To 6]

18.



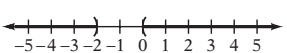
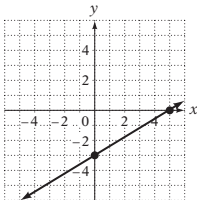
[3.7A, Example 1]

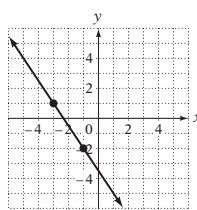
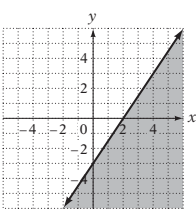
 19. The slope is $-10,000$. The value of the house decreases by \$10,000 each year. [3.4A, You Try It 3]

20. a. $f(x) = -\frac{3}{10}x + 175$

b. When the tuition is \$300, 85 students will enroll. [3.5C, Example 5]

CUMULATIVE REVIEW EXERCISES

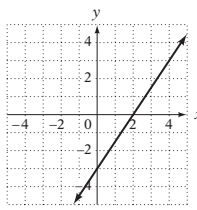
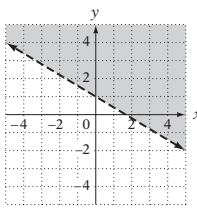
1. The Commutative Property of Multiplication [1.3A]
2. $\frac{9}{2}$ [2.1B]
3. $\frac{8}{9}$ [2.1C]
4. $-\frac{1}{14}$ [2.1C]
5. $\{x|x < -1 \text{ or } x > \frac{1}{2}\}$ [2.4B]
6. $\frac{5}{2}$ and $-\frac{3}{2}$ [2.5A]
7. $\{x|0 < x < \frac{10}{3}\}$ [2.5B]
8. 8 [1.2D]
9. -4.5 [1.3B]
10.  [1.1C]
11. $C = R - Pn$ [2.1D]
12. $x = 3 - \frac{3}{2}y$ [2.1D]
13. $\emptyset$ [2.4B]
14. 14 [3.2A]
15. $(-8, 13)$ [3.1A]
16. $-\frac{7}{4}$ [3.4A]
17. $y = \frac{3}{2}x + \frac{13}{2}$ [3.5A]
18. $y = -\frac{5}{4}x + 3$ [3.5B]
19. $y = -\frac{3}{2}x + 7$ [3.6A]
20. $y = -\frac{2}{3}x + \frac{8}{3}$ [3.6A]
21. There are 7 dimes in the coin purse. [2.2B]
22. The first plane is traveling at 200 mph and the second plane is traveling at 400 mph. [2.3C]
23. The mixture consists of 40 lb of \$9.00 coffee and 20 lb of \$6.00 coffee. [2.3A]
24.  [3.3C]

25.  [3.4B]
26.  [3.7A]

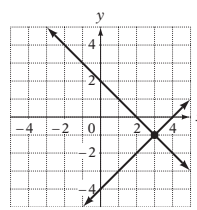
27. a. $y = -5000x + 30,000$ b. The value of the truck decreases by \$5000 per year. [3.5C]

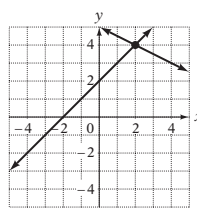
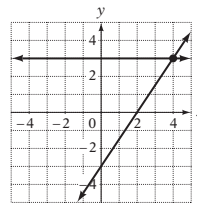
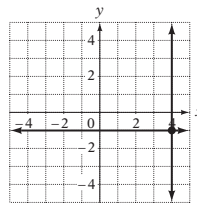
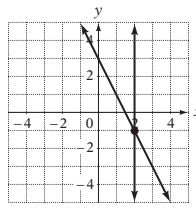
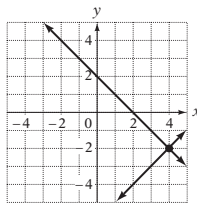
ANSWERS TO CHAPTER 4 SELECTED EXERCISES

PREP TEST

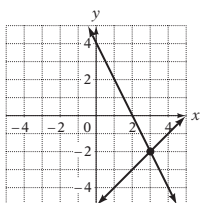
1. $6x + 5y$ [1.3C]
2. 7 [1.3B]
3. 0 [2.1B]
4. -3 [2.1C]
5. 1000 [2.1C]
6.  [3.3B]
7.  [3.7A]

SECTION 4.1

1. No
3. Yes
5. Independent
7. Inconsistent
9. Independent
11.  (3, -1)

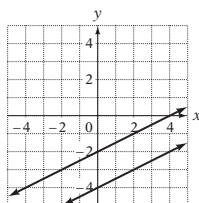
13.  (2, 4)
15.  (4, 3)
17.  (4, -1)
19.  (2, -1)
21.  (4, -2)

23.



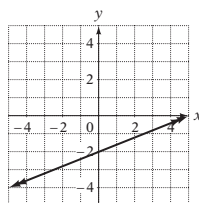
(3, -2)

25.



No solution

27.


 $\left(x, \frac{2}{5}x - 2\right)$

 29. (2, -1) 31. (1, 2) 33. (3, -2) 35. $\left(-\frac{5}{2}, -4\right)$ 37. (-1, 4) 39. (-2, 5) 41. (1, 5) 43. (2, 0)

 45. $\left(\frac{2}{3}, -1\right)$ 47. No solution 49. (-2, 5) 51. (0, 0) 53. $\left(x, \frac{1}{2}x - 4\right)$ 55. (1, 5) 57. $\left(\frac{2}{3}, 3\right)$ 59. $\frac{2}{3}$

61. 5.5% and 7.2% 63. The amount invested in the insured bond fund is \$5500. 65. The amount invested at 6.5% must be \$4000. 67. The amount invested at 3.5% is \$23,625. The amount invested at 4.5% is \$18,375. 69. There is \$6000 invested in the mutual bond fund. 71. (1.20, 1.40) 73. (0.54, 1.03)

SECTION 4.2

 1. Answers may vary. Possible answers are 6 and -5. 3. (6, 1) 5. (1, 1) 7. (2, 1) 9. (-2, 1) 11. $(x, 3x - 4)$

 13. $\left(-\frac{1}{2}, 2\right)$ 15. No solution 17. (-1, -2) 19. (-5, 4) 21. (2, 5) 23. $\left(\frac{1}{2}, \frac{3}{4}\right)$ 25. (0, 0) 27. (-1, 3)

 29. $\left(\frac{2}{3}, -\frac{2}{3}\right)$ 31. (-2, 3) 33. (2, -1) 35. (10, -5) 37. $\left(-\frac{1}{2}, \frac{2}{3}\right)$ 39. $\left(\frac{5}{3}, \frac{1}{3}\right)$ 41. No solution 43. (1, -1)

45. (2, 1, 3) 47. (1, -1, 2) 49. (1, 2, 4) 51. (2, -1, -2) 53. (-2, -1, 3) 55. No solution 57. (1, 4, 1)

59. (1, 3, 2) 61. (1, -1, 3) 63. (0, 2, 0) 65. (1, 5, 2) 67. (-2, 1, 1) 69. a. (iii) b. (ii) c. (i) 71. (1, -1)

73. (1, -1)

SECTION 4.3

 3. 11 5. 18 7. 0 9. 15 11. -30 13. 0 15. 0 17. (3, -4) 19. (4, -1) 21. $\left(\frac{11}{14}, \frac{17}{21}\right)$ 23. $\left(\frac{1}{2}, 1\right)$

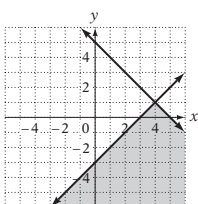
 25. Not independent 27. (-1, 0) 29. (1, -1, 2) 31. (2, -2, 3) 33. Not independent 35. No 37. a. Sometimes true b. Always true c. Sometimes true 39. The area is 239 ft².

SECTION 4.4

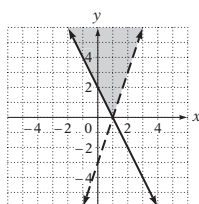
1. Less than 3. The rate of the motorboat in calm water is 15 mph. The rate of the current is 3 mph. 5. The rate of the plane in calm air is 502.5 mph. The rate of the wind is 47.5 mph. 7. The rate of the team in calm water is 8 km/h. The rate of the current is 2 km/h. 9. The rate of the plane in calm air is 180 mph. The rate of the wind is 20 mph. 11. The rate of the plane in calm air is 110 mph. The rate of the wind is 10 mph. 13. Greater than 15. There are 30 nickels in the bank. 17. The cost of the pine is \$1.10/ft. The cost of the redwood is \$3.30/ft. 19. The cost of the wool carpet is \$52/yd. 21. The company plans to manufacture 25 mountain bikes during the week. 23. The owner drove 322 mi in the city and 72 mi on the highway. 25. The chemist should use 480 g of the first alloy and 40 g of the second alloy. 27. The Model VI computer costs \$4000. 29. The investor placed \$10,400 in the 8% account, \$5200 in the 6% account, and \$9400 in the 4% account. 31. The measures of the two angles are 35° and 145°. 33. The age of the oil painting is 85 years, and the age of the watercolor is 50 years.

SECTION 4.5

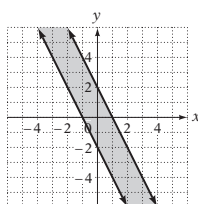
1.



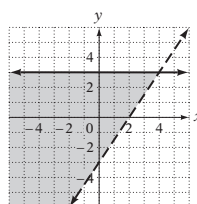
3.

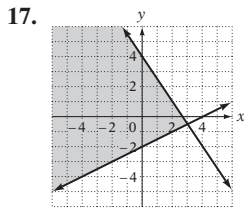
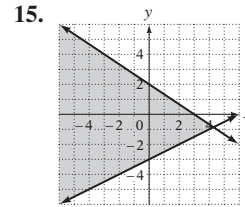
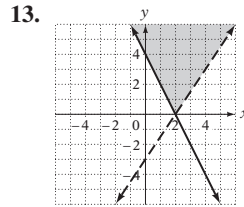
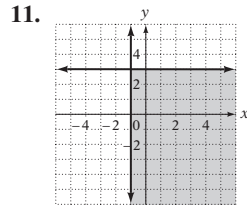
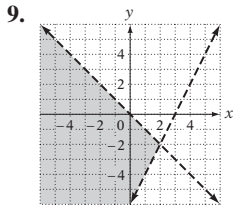


5.

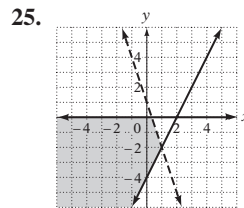
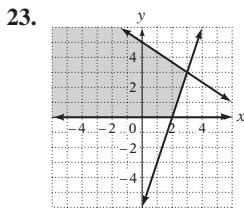


7.





19. The points below the line $x + y = b$ 21. The region between the parallel lines $x + y = a$ and $x + y = b$



CHAPTER 4 CONCEPT REVIEW*

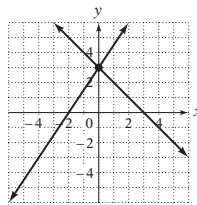
- To determine the solution after graphing a system of linear equations, find the point of intersection of the lines. The point of intersection is the ordered-pair solution. [4.1A]
- To solve a simple interest problem, use the formula $Pr = I$, where P is the principal, r is the simple interest rate, and I is the simple interest. [4.1C]
- If a system of equations is dependent, the graphs are the same line, and they intersect at infinitely many points. There are infinitely many solutions of the system of equations. We can solve one of the equations for y , writing it in the form $y = ax + b$. Then we can state the solution of the system of equations as the ordered pairs $(x, ax + b)$. [4.1A]
- To solve a system of linear equations by the addition method, rewrite one or both of the equations in the system so that the coefficients of one of the variables are opposites. Suppose that the coefficients of the y terms are opposites. Add the two equations. The result is an equation with no y term; solve it for x . The solution of this equation is the first coordinate of the ordered-pair solution of the system of equations. Substitute the value of x into either of the original equations in the system, and solve the resulting equation for y . This is the second coordinate of the ordered-pair solution of the system of equations. [4.2A]
- If, when solving a system of linear equations by the addition method, the result is $0 = 0$, it means that the system is a dependent system of equations. [4.2A]
- To check the solution of a system of three equations in three variables, substitute the values of the three variables into each of the equations of the system. In each case, simplify the resulting numerical expression. If the left and right sides are equal in each case, the ordered triple is the solution of the system. If the left and right sides are not equal in all three cases, the proposed solution is not the solution of the system. [4.2B]
- To evaluate the determinant of a 3×3 matrix, expand by cofactors of any row or any column. The cofactor of an element of a matrix is $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is the column number of the element. [4.3A]
- When solving a system of equations using Cramer's Rule, if the value of the coefficient matrix D is zero, the system is dependent or inconsistent. [4.3B]
- Let b be the rate of the boat in calm water, and let c be the rate of the current. We can represent the rate of the boat going against the current by the expression $b - c$. [4.4A]
- To solve a system of linear inequalities, first graph the solution set of each linear inequality. The solution set of the system of linear inequalities is the region of the plane represented by the intersection of the shaded areas. [4.5A]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

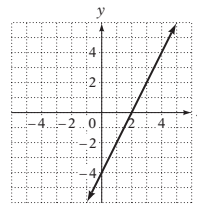
11. The minor of an element of a 3×3 determinant is the 2×2 determinant that is obtained by eliminating the row and column that contain that element. [4.3A]
12. The value of the cofactor of an element of a matrix is $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is the column number of the element. [4.3A]

CHAPTER 4 REVIEW EXERCISES

1. $(6, -\frac{1}{2})$ [4.1B] 2. $(-4, 7)$ [4.2A] 3. [4.1A] 4. [4.1A]



$(0, 3)$

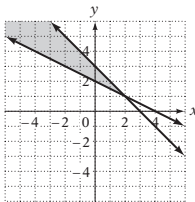
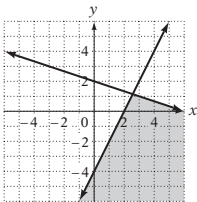


$(x, 2x - 4)$

5. $(x, -\frac{1}{4}x + \frac{3}{2})$ [4.1B] 6. $(x, \frac{1}{3}x - 2)$ [4.2A] 7. $(3, -1, -2)$ [4.2B] 8. $(5, -2, 3)$ [4.2B] 9. 28 [4.3A]

10. 0 [4.3A] 11. $(3, -1)$ [4.3B] 12. $(\frac{110}{23}, \frac{25}{23})$ [4.3B] 13. $(-1, -3, 4)$ [4.3B] 14. $(2, 3, -5)$ [4.3B]

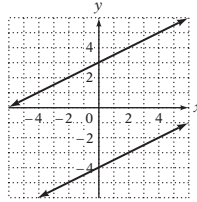
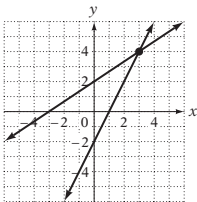
15. [4.5A] 16. [4.5A]



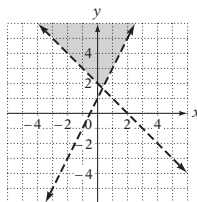
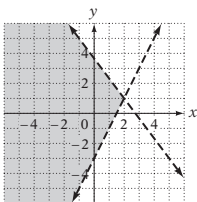
17. The rate of the cabin cruiser in calm water is 16 mph. The rate of the current is 4 mph. [4.4A] 18. The rate of the plane in calm air is 175 mph. The rate of the wind is 25 mph. [4.4A] 19. On Friday, 100 children attended. [4.4B] 20. The amount invested at 3% is \$5000. The amount invested at 7% is \$15,000. [4.1C]

CHAPTER 4 TEST

1. $(3, 4)$ [4.1A, Example 1] 2. No solution [4.1A, Example 2]



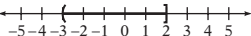
3. [4.5A, Example 3] 4. [4.5A, Example 3] 5. $(\frac{3}{4}, \frac{7}{8})$ [4.1B, Example 4]

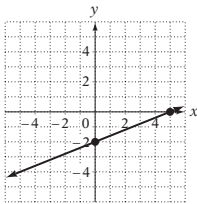
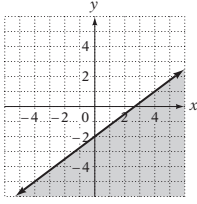
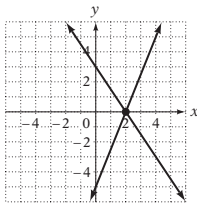


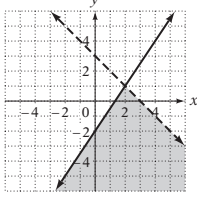
6. $(-3, -4)$ [4.1B, How To 3] 7. $(2, -1)$ [4.1B, Example 5] 8. $(-2, 1)$ [4.2A, How To 1] 9. No solution [4.2A, You Try It 2] 10. $(1, 1)$ [4.2A, Example 1] 11. No solution [4.2B, How To 4] 12. $(2, -1, -2)$ [4.2B, How To 6] 13. 10 [4.3A, Example 1] 14. -32 [4.3A, How To 4] 15. $(-\frac{1}{3}, -\frac{10}{3})$ [4.3B, How To 5] 16. $(\frac{59}{19}, -\frac{62}{19})$ [4.3B, How To 5] 17. $(\frac{1}{5}, -\frac{6}{5}, \frac{3}{5})$ [4.3B, How To 6] 18. The rate of the plane in calm air is 150 mph. The rate of the wind is 25 mph. [4.4A, Example 1] 19. The cost of cotton is \$9/yd. The cost of wool is \$14/yd. [4.4B, How To 2]

20. The amount invested at 2.7% is \$9000. The amount invested at 5.1% is \$6000. [4.1C, Example 2]

CUMULATIVE REVIEW EXERCISES

1. $-\frac{11}{28}$ [2.1C] 2. $y = 5x - 11$ [3.5B] 3. $3x - 24$ [1.3C] 4. -4 [1.3B] 5. $(-\infty, 6)$ [2.4B]
 6. $\{x | -4 < x < 8\}$ [2.5B] 7. $\{x | x < -1 \text{ or } x > 4\}$ [2.5B] 8. $f(-3) = -98$ [3.2A]
 9. The range is $\{0, 1, 5, 8, 16\}$. [3.2A] 10. 1 [3.2A] 11. $3h$ [3.2A] 12.  [1.1C]
 13. $y = -\frac{2}{3}x + \frac{5}{3}$ [3.5A] 14. $y = -\frac{3}{2}x + \frac{1}{2}$ [3.6A] 15. 6.32 [3.1B] 16. $(-\frac{1}{2}, 4)$ [3.1B]

17.  [3.4B] 18.  [3.7A] 19.  (2, 0) [4.1A]

20.  [4.5A] 21. (1, 0, -1) [4.2B] 22. 3 [4.3A] 23. (2, -3) [4.3B]

24. $(-5, -11)$ [4.1B] 25. There are 16 nickels in the purse. [2.2B] 26. The amount of water that should be added is 80 ml. [2.3B] 27. The rate of the wind is 12.5 mph. [4.4A] 28. The cost of 1 lb of steak is \$6. [4.4B] 29. The lower and upper limits of the resistor are 10,200 ohms and 13,800 ohms. [2.5C] 30. The slope of the line is 40. The commission rate of the executive is \$40 for every \$1000 worth of sales. [3.4A]

ANSWERS TO CHAPTER 5 SELECTED EXERCISES

PREP TEST

1. $-12y$ [1.3C] 2. -8 [1.2C] 3. $3a - 8b$ [1.3C] 4. $11x - 2y - 2$ [1.3C] 5. $-x + y$ [1.3C]
 6. $2 \cdot 2 \cdot 2 \cdot 5$ [1.2B] 7. 4 [1.2B] 8. -13 [1.3B] 9. $-\frac{1}{3}$ [2.1B]

SECTION 5.1

1. a^4b^4 3. $-18x^3y^4$ 5. x^8y^{16} 7. $81x^8y^{12}$ 9. $729a^{10}b^6$ 11. $64a^{24}b^{18}$ 13. x^5y^{11} 15. $-45a^7b^5$ 17. $-192x^{14}y$

19. $42a^6b^2c^5$ 21. $54a^{13}b^{17}$ 23. $-432a^7b^{11}$ 25. $-6x^4y^4z^5$ 27. $-6x^5y^5z^4$ 29. $-12a^2b^9c^2$ 31. x^{2n+1} 33. y^{6n-2}

35. a^{2n^2-6n} 37. x^{15n+10} 39. 33 41. 243 43. y^3 45. $\frac{a^3b^2}{4}$ 47. $\frac{x}{y^4}$ 49. $\frac{1}{2}$ 51. $-\frac{1}{9}$ 53. $\frac{1}{x^6y^{10}}$ 55. $\frac{15}{a^6b}$

57. $\frac{4z^2}{9y^9}$ 59. $\frac{x^{13}}{4y^{18}}$ 61. $\frac{3}{4x^3}$ 63. $-\frac{1}{2x^2}$ 65. $\frac{1}{y^8}$ 67. x^5y^5 69. $\frac{a^8}{b^9}$ 71. $-\frac{1}{243a^5b^{10}}$ 73. $\frac{3a^4}{4b^3}$ 75. $\frac{16x^2}{y^6}$

77. $\frac{ab}{72}$ 79. $-\frac{32x^{19}}{y^{14}}$ 81. $\frac{1}{b^{4n}}$ 83. $-\frac{1}{y^{6n}}$ 85. y^{n-2} 87. $\frac{x^{n-5}}{y^6}$ 89. $\frac{8b^{15}}{3a^{18}}$ 91. 0 93. 4.67×10^{-6}

95. 1.7×10^{-10} 97. 2×10^{11} 99. 0.000000123 101. 8,200,000,000,000,000 103. 0.039 105. 150,000

107. 20,800,000 109. 0.000000015 111. 0.0000000000178 113. 11,000,000 115. 0.000008 117. Greater than

119. It would take a spaceship 2.6×10^{11} years to cross the galaxy. 121. The mass of a proton is 1.83664508×10^3 times larger

- than the mass of an electron. 123. The mass of the sun is 3.3898305×10^5 times larger than the mass of Earth. 125. The

- signals from Earth to Mars traveled 1.081×10^7 mi/min. 127. The lodgepole pine trees released 1.6×10^2 seeds for each

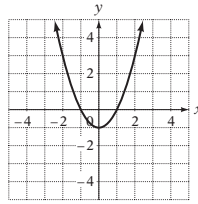
surviving seedling.

SECTION 5.2

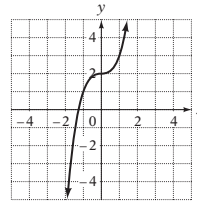
1. $P(3) = 13$ 3. $R(2) = 10$ 5. $f(-1) = -11$ 7. Polynomial: a. -1 b. 8 c. 2 9. Not a polynomial

11. Not a polynomial 13. Polynomial: a. 3 b. π c. 5 15. Polynomial: a. -5 b. 2 c. 3

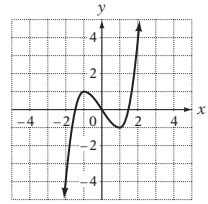
17. Polynomial: a. 14 b. 14 c. 0 19.



21.



23.


 25. $f(c) - g(c) > 0; f(c) - g(c) < 0$ 27. $6x^2 - 6x + 5$ 29. $-x^2 + 1$ 31. $5y^2 - 15y + 2$ 33. $7a^2 - a + 2$
 35. $3x^2 - 3xy - 2y^2$ 37. $8x^2 - 2xy + 5y^2$ 39. $S(x) = 3x^4 - 8x^2 + 2x; S(2) = 20$ 41. a. $k = 8$ b. $k = -4$

SECTION 5.3

 1. Distributive Property 3. $2x^2 - 6x$ 5. $6x^4 - 3x^3$ 7. $6x^2y - 9xy^2$ 9. $x^{n+1} + x^n$ 11. $x^{2n} + x^n y^n$ 13. $-4b^2 + 10b$
 15. $-6a^4 + 4a^3 - 6a^2$ 17. $-3y^4 - 4y^3 + 2y^2$ 19. $-20x^2 + 15x^3 - 15x^4 - 20x^5$ 21. $-2x^4y + 6x^3y^2 - 4x^2y^3$
 23. $-3x^3 - 12x^2 + 28x$ 25. $5y^2 - 11y$ 27. $6y^2 - 31y$ 29. $9b^5 - 9b^3 + 24b$ 31. $x^2 + 5x - 14$ 33. $8y^2 + 2y - 21$
 35. $4a^2 - 7ac - 15c^2$ 37. $25x^2 - 70x + 49$ 39. $8x^2 + 8xy - 30y^2$ 41. $x^2y^2 + xy - 12$ 43. $2x^4 - 15x^2 + 25$
 45. $10x^4 - 15x^2y + 5y^2$ 47. $x^3 + 2x^2 - 11x + 20$ 49. $10a^3 - 27a^2b + 26ab^2 - 12b^3$ 51. $4x^4 - 13x^2 + 16x - 15$
 53. $4x^5 - 16x^4 + 15x^3 - 4x^2 + 28x - 45$ 55. $x^4 - 3x^3 - 6x^2 + 29x - 21$ 57. $2a^3 + 7a^2 - 43a + 42$
 59. $6x^3 + 5x^2 - 61x - 60$ 61. $2y^5 - 10y^4 - y^3 - y^2 + 3$ 63. 5 65. $9x^2 - 4$ 67. $36 - x^2$ 69. $4a^2 - 9b^2$
 71. $9a^2b^2 - 16$ 73. $x^4 - 1$ 75. $x^2 - 10x + 25$ 77. $9a^2 + 30ab + 25b^2$ 79. $x^4 - 6x^2 + 9$ 81. $4x^4 - 12x^2y^2 + 9y^4$
 83. $9m^2n^2 - 30mn + 25$ 85. $-x^2 + 2xy$ 87. $-4xy$ 89. False 91. ft^2 93. $(3x^2 + 10x - 8) \text{ft}^2$ 95. $(x^2 + 3x) \text{m}^2$
 97. $(x^3 + 9x^2 + 27x + 27) \text{cm}^3$ 99. $(2x^3) \text{in}^3$ 101. $(78.5x^2 + 125.6x + 50.24) \text{in}^2$ 103. a. $a^3 - b^3$ b. $x^3 + y^3$
 105. a. $k = 5$ b. $k = 1$ 107. $a^2 + b^2$

SECTION 5.4

 1. $x - 2$ 3. $-x + 2$ 5. $xy + 2$ 7. $x^2 + 3x - 5$ 9. $3b^3 + 4b^2 + 2b$ 11. $a^4 - 6a^2 + 1$ 13. $6x^3 + 21x^2 - 15x$
 15. $x + 8$ 17. $x^2 + 3x + 6 + \frac{20}{x-3}$ 19. $3x + 5 + \frac{3}{2x+1}$ 21. $5x + 7 + \frac{2}{2x-1}$ 23. $4x^2 + 6x + 9 + \frac{18}{2x-3}$
 25. $3x^2 + 1 + \frac{1}{2x^2-5}$ 27. $x^2 - 3x - 10$ 29. $x^2 - 2x + 1 - \frac{1}{x-3}$ 31. $2x^3 - 3x^2 + x - 4$ 33. $2x + \frac{x+2}{x^2+2x-1}$
 35. $x^2 + 4x + 6 + \frac{10x+8}{x^2-2x-1}$ 37. $x^2 + 4 + \frac{3}{2x+1}$ 39. False 41. 3 43. $2x - 8$ 45. $3x - 8$
 47. $3x + 3 - \frac{1}{x-1}$ 49. $2x^2 - 3x + 9$ 51. $4x^2 + 8x + 15 + \frac{12}{x-2}$ 53. $2x^2 - 3x + 7 - \frac{8}{x+4}$
 55. $3x^3 + 2x^2 + 12x + 19 + \frac{33}{x-2}$ 57. $3x^3 - x + 4 - \frac{2}{x+1}$ 59. $3x + 1 + \frac{8}{x-2}$ 61. $x - 3$ 63. $P(3) = 8$
 65. $R(4) = 43$ 67. $P(-2) = -39$ 69. $Z(-3) = -60$ 71. $Q(2) = 31$ 73. $F(-3) = 178$ 75. $P(5) = 122$
 77. $R(-3) = 302$ 79. $Q(2) = 0$ 81. a. $a^2 - ab + b^2$ b. $x^4 - x^3y + x^2y^2 - xy^3 + y^4$ c. $x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5$

SECTION 5.5

 1. $3a(2a - 5)$ 3. $x^2(4x - 3)$ 5. Nonfactorable 7. $x(x^4 - x^2 - 1)$ 9. $4(4x^2 - 3x + 6)$ 11. $5b^2(1 - 2b + 5b^2)$
 13. $x^n(x^n - 1)$ 15. $x^{2n}(x^n - 1)$ 17. $a^2(a^{2n} + 1)$ 19. $6x^2y(2y - 3x + 4)$ 21. $4a^2b^2(6a - 1 - 4b^2)$ 23. $3x, 2x + 1$
 25. $(a + 2)(x - 2)$ 27. $(x - 2)(a + b)$ 29. $(a - 2b)(x - y)$ 31. $(x + 4)(y - 2)$ 33. $(a + b)(x - y)$
 35. $(y - 3)(x^2 - 2)$ 37. $(3 + y)(2 + x^2)$ 39. $(2a + b)(x^2 - 2y)$ 41. $(2b + a)(3x - 2y)$ 43. $(y - 5)(x^n + 1)$
 45. $(2x - 1)(x^2 + 2)$ 47. a. (i), (ii), (iii) b. (ii) 49. $-11, -4, -1, 1, 4, 11$ 51. $(x + 10)(x + 2)$ 53. $(a + 9)(a - 8)$
 55. $(a + 6)(a + 1)$ 57. $(y - 6)(y - 12)$ 59. $(x + 12)(x - 11)$ 61. $(x + 10)(x + 5)$ 63. $(b + 2)(b - 8)$
 65. $(a - b)(a - 2b)$ 67. $(a + 11b)(a - 3b)$ 69. $(x + 3y)(x + 2y)$ 71. $-(x - 2)(x + 1)$ 73. $-(x - 5)(x + 1)$
 75. $(x - 3)(x - 2)$ 77. $(2x + 1)(x + 3)$ 79. $(2y + 3)(3y - 2)$ 81. $(3b + 7)(2b - 5)$ 83. $(3y - 13)(y - 3)$
 85. Nonfactorable over the integers 87. $(a + 1)(4a - 5)$ 89. $(5x - 2)(2x - 5)$ 91. Nonfactorable over the integers
 93. $(6x - y)(x + 7y)$ 95. $(7a - 3b)(a + 7b)$ 97. $(6x + 5y)(3x + 2y)$ 99. $-(5x - 3)(x + 2)$
 101. Nonfactorable over the integers 103. $-(4b - 7)(2b + 5)$ 105. $y^2(5y - 4)(y - 5)$ 107. $-2x^2(5x - 2)(3x + 5)$
 109. $y(4y - 3)(4y - 1)$ 111. Answers will vary. One possibility is $2x^2 + 3x + 5$. 113. $f(x) = 2x - 1; g(x) = x - 2$
 115. $f(a) = 3a - 1; g(a) = a + 4$ 117. $f(x) = 2x - 3; g(x) = x + 8$ 119. $g(x) = 2x - 1; h(x) = 3x + 5$

121. $f(t) = 6t + 1$; $g(t) = t - 3$ 123. a. $-9, -6, 6$, and 9 b. $-5, -1, 1$, and 5 c. $-7, -5, 5$, and 7 d. $-9, -3, 3$, and 9
 e. $-16, -8, 8$, and 16 f. $-5, -1, 1$, and 5

SECTION 5.6

1. $4; 25x^6; 100x^4y^4$ 3. $4z^4$ 5. $9a^2b^3$ 7. (iv) 9. $(x + 4)(x - 4)$ 11. $(2x + 1)(2x - 1)$ 13. $(4x + 11)(4x - 11)$
 15. $(1 + 3a)(1 - 3a)$ 17. $(xy + 10)(xy - 10)$ 19. Nonfactorable over the integers 21. $(5 + ab)(5 - ab)$
 23. $(a^n + 1)(a^n - 1)$ 25. $(x - 6)^2$ 27. $(b - 1)^2$ 29. $(4x - 5)^2$ 31. Nonfactorable over the integers
 33. Nonfactorable over the integers 35. $(x + 3y)^2$ 37. $(5a - 4b)^2$ 39. $(x^n + 3)^2$ 41. $(x - 7)(x - 1)$
 43. $(x - y - a - b)(x - y + a + b)$ 45. $8; x^9; 27c^{15}d^{18}$ 47. $2x^3$ 49. $4a^2b^6$ 51. Yes 53. Yes
 55. $(x - 3)(x^2 + 3x + 9)$ 57. $(2x - 1)(4x^2 + 2x + 1)$ 59. $(x - y)(x^2 + xy + y^2)$ 61. $(m + n)(m^2 - mn + n^2)$
 63. $(4x + 1)(16x^2 - 4x + 1)$ 65. $(3x - 2y)(9x^2 + 6xy + 4y^2)$ 67. $(xy + 4)(x^2y^2 - 4xy + 16)$ 69. Nonfactorable over the integers
 71. Nonfactorable over the integers 73. $(a - 2b)(a^2 - ab + b^2)$ 75. $(x^{2n} + y^n)(x^{4n} - x^{2n}y^n + y^{2n})$
 77. $(x^n + 2)(x^{2n} - 2x^n + 4)$ 79. No, polynomials cannot have square roots as variable terms. 81. $(xy - 5)(xy - 3)$
 83. $(xy - 12)(xy - 5)$ 85. $(x^2 - 3)(x^2 - 6)$ 87. $(b^2 + 5)(b^2 - 18)$ 89. $(x^2y^2 - 2)(x^2y^2 - 6)$ 91. $(x^n + 1)(x^n + 2)$
 93. $(3xy - 5)(xy - 3)$ 95. $(2ab - 3)(3ab - 7)$ 97. $(x^2 + 1)(2x^2 - 15)$ 99. $(2x^n - 1)(x^n - 3)$ 101. $(2a^n + 5)(3a^n + 2)$
 103. $3(2x - 3)^2$ 105. $a(3a - 1)(9a^2 + 3a + 1)$ 107. $5(2x + 1)(2x - 1)$ 109. $y^3(y + 11)(y - 5)$
 111. $(4x^2 + 9)(2x + 3)(2x - 3)$ 113. $2a(2 - a)(4 + 2a + a^2)$ 115. $b^3(ab - 1)(a^2b^2 + ab + 1)$ 117. $2x^2(2x - 5)^2$
 119. $(x^2 + y^2)(x + y)(x - y)$ 121. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$ 123. Nonfactorable over the integers
 125. $2a(2a - 1)(4a^2 + 2a + 1)$ 127. $a^2b^2(a + 4b)(a - 12b)$ 129. $(x - 2)^2(x + 2)$ 131. $(x - 4)(x + 4)(2x + 1)$
 133. $(2x + 1)(2x - 1)(x + y)(x - y)$ 135. $b^n(b + 2)(3b - 2)$ 137. 8

SECTION 5.7

1. No; if $a = 0$, then b could be any number. 3. -3 and 5 5. -7 and 8 7. $-4, 0$, and $\frac{2}{3}$ 9. -5 and 3 11. 1 and 3
 13. -2 and 5 15. $-\frac{1}{2}$ and $\frac{3}{2}$ 17. $\frac{1}{2}$ 19. -3 and 3 21. $-\frac{1}{2}$ and $\frac{1}{2}$ 23. -3 and 5 25. -4 and 7 27. -1 and $\frac{13}{2}$
 29. -2 and $\frac{4}{3}$ 31. $-5, 0$, and 2 33. $-3, -1$, and 3 35. $-2, -\frac{2}{3}$, and 2 37. $-2, -\frac{2}{5}$, and 2 39. The number is -15 or 14 .
 41. The base of the triangle is 8 cm, and the height is 16 cm. 43. The length of the electronic paper rectangle was 12 cm, and the width was 5 cm. 45. Larry incorrectly assumed that if $ab = 15$, then $a = 3$ and $b = 5$. Yet he found the correct result.
 47. $x^2 - 10x + 21 = 0$ 49. $x^2 + 4x + 3 = 0$ 51. The length of the cardboard is 18 in., and the width is 8 in.

CHAPTER 5 CONCEPT REVIEW*

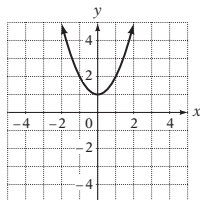
- The degree of a monomial is the sum of the exponents on the variables. [5.1A]
- A number written in scientific notation is of the form $a \times 10^n$, where a is a number between 1 and 10 . To write a very small number in scientific notation, move the decimal point to the right of the first nonzero digit. This is the value of a . The exponent on 10 is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved. [5.1C]
- To multiply two binomials, use the FOIL method: add the products of the first terms, the outer terms, the inner terms, and the last terms. [5.3B]
- To square a binomial, multiply it times itself. [5.3C]
- After you divide a polynomial by a binomial, check the answer by using the following equation:
 $\text{Dividend} = (\text{quotient} \times \text{divisor}) + \text{remainder}$. [5.4B]
- A binomial is a factor of a polynomial if, when the polynomial is divided by the binomial, there is no remainder. [5.4B]
- To evaluate a polynomial function at a given value of the variable, use synthetic division to divide the polynomial by the value at which the polynomial is being evaluated. The remainder is the value. [5.4D]
- To use synthetic division for division of a polynomial, the divisor must be of the form $x - a$. [5.4C]
- When using synthetic division with a polynomial that has a missing term, insert a zero for the missing term in the polynomial. [5.4C]
- When factoring a polynomial by grouping, after grouping the first two terms and the last two terms, factor the GCF from each group. [5.5B]
- The binomial factors of the difference of two perfect squares, $a^2 - b^2$, are $a + b$ and $a - b$. [5.6A]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

12. To solve a quadratic equation by factoring, the equation must be set equal to zero in order to use the Principle of Zero Products, which states that if the product of two factors is zero, then at least one of the factors must be zero. [5.7A]
13. To factor a polynomial completely means to write the polynomial as a product of factors, each of which is nonfactorable over the integers. [5.6D]

CHAPTER 5 REVIEW EXERCISES

1. $6a^2b(3a^3b - 2ab^2 + 5)$ [5.5A] 2. $5x + 4 + \frac{6}{3x-2}$ [5.4B] 3. $144x^2y^{10}z^{14}$ [5.1B]
4. $(a + 2b)(2x - 3y)$ [5.5B] 5. $-(x - 4)(x + 3)$ [5.5C] 6. $P(2) = 1$ [5.4D] 7. $4x^2 - 8xy + 5y^2$ [5.2B]
8. $(6x + 5)(4x + 3)$ [5.5D] 9. $(2x + 3y)^2$ [5.6A] 10. $-6a^3b^6$ [5.1A] 11. $(4a - 3b)(16a^2 + 12ab + 9b^2)$ [5.6B]
12. $4x^2 + 3x - 8 + \frac{50}{x+6}$ [5.4C] 13. $P(-2) = -7$ [5.2A] 14. $(x - 8)(x + 5)$ [5.5C]
15. $(xy + 3)(xy - 3)$ [5.6A] 16. $12x^5y^3 + 8x^3y^2 - 28x^2y^4$ [5.3A] 17. $(x^n - 6)^2$ [5.6A] 18. $\frac{5}{2}$ and 4 [5.7A]
19. $33x^2 + 24x$ [5.3A] 20. $3a^2(a^2 - 6)(a^2 + 1)$ [5.6D] 21. $16x^2 - 24xy + 9y^2$ [5.3C]
22. $x^3 + 4x^2 + 16x + 64 + \frac{252}{x-4}$ [5.4C] 23. $2x^2 - 5x - 2$ [5.2B] 24. $70xy^2z^6$ [5.1B] 25. $-\frac{x^3}{4y^2z^3}$ [5.1B]
26. 9.48×10^8 [5.1C] 27. 2×10^{-6} [5.1C] 28. 68 [5.4D] 29. $4x^4 - 2x^2 + 5$ [5.4A]
30. $2x - 3 - \frac{4}{6x+1}$ [5.4B] 31. $a^7 - 5a^4 + 2a^3$ [5.3A] 32. $x^4 + 3x^3 - 23x^2 - 29x + 6$ [5.3B]
33. $5ab^2(2a^2b - 4ab^2 + 7)$ [5.5A] 34. $x^2(5x^3 + x + 4)$ [5.5A] 35. $(y - 3)(x - 4)$ [5.5B]
36. $(x - 7)(x - 9)$ [5.5C] 37. $(8x - 1)(3x + 8)$ [5.5D] 38. $f(x) = 5x - 2; g(x) = x + 1$ [5.5D]
39. $(6 + a^n)(6 - a^n)$ [5.6A] 40. $(2 - y^n)(4 + 2y^n + y^{2n})$ [5.6B] 41. $(6x^4 - 5)(6x^4 - 1)$ [5.6C]
42. $3ab(a - b)(a^2 + ab + b^2)$ [5.6D] 43. $(x + 2)^2(x - 2)^2$ [5.6D] 44. -2, 0, 3 [5.7A] 45. -4, 0, 4 [5.7A]
46. -1, -6, 6 [5.7A] 47. $(3x^2 + 2)(5x^2 - 3)$ [5.6C] 48. $\frac{c^{10}}{2b^{17}}$ [5.1B] 49. $6x^3 - 29x^2 + 14x + 24$ [5.3B]
50. $(7x^2y^2 + 3)(3x^2y^2 + 2)$ [5.6C] 51. -4, 1, and 4 [5.7A] 52. $25a^2 - 4b^2$ [5.3C] 53. 0.00254 [5.1C]
54. $(3x - 2)(2x - 9)$ [5.5D] 55. [5.2A] 56. a. 3 b. 8 c. 5 [5.2A]



57. The mass of the moon is 8.103×10^{19} tons. [5.1D] 58. The number is -8 or 7. [5.7B] 59. The Great Galaxy of Andromeda is 1.291224×10^{19} mi from Earth. [5.1D] 60. The area is $(10x^2 - 29x - 21)$ cm². [5.3D]

CHAPTER 5 TEST

1. $(4t + 3)^2$ [5.6A, How To 2] 2. $-18r^2s^2 + 12rs^3 + 18rs^2$ [5.3A, How To 1] 3. $P(2) = -3$ [5.2A, How To 1]
4. $(3x - 2)(9x^2 + 6x + 4)$ [5.6B, How To 5] 5. $(4x + 5)(4x - 5)$ [5.6A, How To 1] 6. $6t^5 - 8t^4 - 15t^3 + 22t^2 - 5$ [5.3B, Example 3] 7. $35x^2 - 55x$ [5.3A, Example 2] 8. $3x(2x - 3)(2x + 5)$ [5.6D, Example 8] 9. -1, $-\frac{1}{6}$, and 1 [5.7A, How To 3] 10. $2x^3 - 4x^2 + 6x - 14$ [5.2B, How To 6] 11. 5.01×10^{-7} [5.1C, Example 7] 12. $2x + 1 + \frac{4}{7x-3}$ [5.4B, Example 2] 13. $49 - 25x^2$ [5.3C, How To 8] 14. $(2a^2 - 5)(3a^2 + 1)$ [5.6C, How To 7]
15. $6a^2 - 13ab - 28b^2$ [5.3B, Example 4] 16. $(3x^2 + 4)(x - 3)(x + 3)$ [5.6D, Example 8] 17. $64a^7b^7$ [5.1A, Example 1]
18. $-\frac{1}{3}$ and $\frac{1}{2}$ [5.7A, How To 2] 19. $P(-2) = -8$ [5.4D, How To 6] 20. $\frac{2b^7}{a^{10}}$ [5.1B, Example 5]
21. $x^2 - 5x + 10 - \frac{23}{x+3}$ [5.4C, How To 4] 22. $(2x - 3)(3x - 4)$ [5.5D, Example 7] 23. $(3x - 2)(2x - a)$ [5.5B, How To 5] 24. There are 6.048×10^5 s in 1 week. [5.1D, Example 10] 25. The arrow will be 64 ft above the ground at 1 s and at 2 s after the arrow has been released. [5.7B, Example 2] 26. The area is $(10x^2 - 3x - 1)$ ft². [5.3D, Example 10]

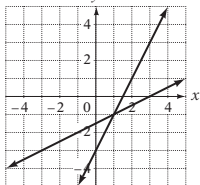
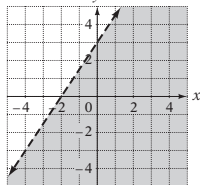
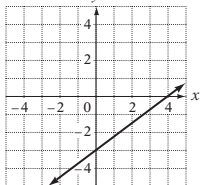
CUMULATIVE REVIEW EXERCISES

1. 4 [1.2D] 2. $-\frac{5}{4}$ [1.3B] 3. The Inverse Property of Addition [1.3A] 4. $-18x + 8$ [1.3C]
5. $-\frac{1}{6}$ [2.1A] 6. $-\frac{11}{4}$ [2.1B] 7. $x^2 + 3x + 9 + \frac{24}{x-3}$ [5.4C] 8. -1 and $\frac{7}{3}$ [2.5A]

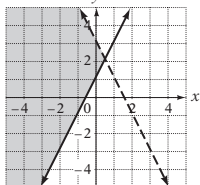
9. $P(-2) = 18$ [3.2A] 10. $x = -2$ [3.2A] 11. $\{-4, -1, 8\}$ [3.2A] 12. The slope is $-\frac{1}{6}$. [3.4A]

13. $y = -\frac{3}{2}x + \frac{1}{2}$ [3.5A] 14. $y = \frac{2}{3}x + \frac{16}{3}$ [3.6A] 15. $\left(-\frac{7}{5}, -\frac{8}{5}\right)$ [4.3B] 16. $\left(-\frac{9}{7}, \frac{2}{7}, \frac{11}{7}\right)$ [4.2B]

17. [3.3C] 18. [3.7A] 19. $(1, -1)$ [4.1A]



20. [4.5A] 21. $\frac{b^5}{a^8}$ [5.1B] 22. $\frac{y^2}{25x^6}$ [5.1B] 23. $\frac{21}{8}$ [5.1B] 24. $4x^3 - 7x + 3$ [5.3B]



25. $-2x(2x - 3)(x - 2)$ [5.5D] 26. $(x - y)(a + b)$ [5.5B] 27. $(x^2 + 4)(x + 2)(x - 2)$ [5.6D] 28. $2(x - 2)(x^2 + 2x + 4)$ [5.6D] 29. The integers are 9 and 15. [2.2A] 30. 40 oz of pure gold must be mixed with the alloy. [2.3A] 31. The slower cyclist travels at 5 mph and the faster cyclist at 7.5 mph. [2.3C] 32. The vehicle will reach the moon in 12 h. [5.1D] 33. $m = 50$. The average speed is 50 mph. [3.4A]

ANSWERS TO CHAPTER 6 SELECTED EXERCISES

PREP TEST

1. 50 [1.2B] 2. $-\frac{1}{6}$ [1.2B] 3. $-\frac{3}{2}$ [1.2B] 4. $\frac{1}{24}$ [1.2B] 5. $\frac{5}{24}$ [1.2B] 6. $-\frac{2}{9}$ [1.2D] 7. $\frac{1}{3}$ [1.3B] 8. -2 [2.1C] 9. $\frac{10}{7}$ [2.1C] 10. The rates of the planes are 110 mph and 130 mph. [2.3C]

SECTION 6.1

3. 2 5. -2 7. $\frac{1}{9}$ 9. $\frac{1}{35}$ 11. $\frac{3}{4}$ 13. The domain is $\{x | x \neq 3\}$. 15. The domain is $\{x | x \neq -4\}$. 17. The domain is $\{x | x \neq -3\}$. 19. The domain is $\{x | x \neq \frac{2}{3}, 4\}$. 21. The domain is $\{x | x \neq -3, 2\}$. 23. The domain is $\{x | x \in \text{real numbers}\}$. 25. (i), (ii), (iii) 29. $1 - 2x$ 31. $3x - 1$ 33. $x^2y^2 - 4xy + 5$ 35. $2x$ 37. $-\frac{2}{a}$ 39. The expression is in simplest form. 41. $\frac{x-3}{x-5}$ 43. $\frac{2x+1}{2x+3}$ 45. $\frac{x+4}{x-4}$ 47. $\frac{a-b}{a^2-ab+b^2}$ 49. $\frac{4x^2+2xy+y^2}{2x+y}$ 51. $\frac{(a-2)(x+1)}{ax}$ 53. $\frac{x^2-3}{x^2+1}$ 55. $\frac{2xy+1}{3xy-1}$ 57. $\frac{a^4+4}{a^4+1}$ 59. $\frac{a^n+b^n}{a^n-b^n}$ 61. $\frac{a+b}{(x+1)(x-1)}$ 63. 4 65. $\frac{abx}{2}$ 67. $\frac{x}{2}$ 69. $\frac{y(x-1)}{x^2(x+1)}$ 71. $\frac{-x+5}{x-2}$ 73. $\frac{(x+1)(x-1)(x-4)}{x-2}$ 75. $\frac{x+y}{3}$ 77. $\frac{x^4+4}{x^4-1}$ 79. $p(x) = x^2 + 2x - 15$ 81. $\frac{4by}{3ax}$ 83. $\frac{4(x-y)^2}{9x^2y}$ 85. $\frac{2x-3y}{4y^2}$ 87. $-\frac{2x+5}{2x-5}$ 89. -1 91. $\frac{(x+y)(x-y)}{x^3}$ 93. $\frac{x+3}{x-3}$ 95. $(x^n+1)^2$ 97. $p(x) = x^2 - 4$ 99. $\frac{5(3y^2+2)}{y^2}$

SECTION 6.2

1. a. Six b. Four 3. $\frac{9y^3}{12x^2y^4}, \frac{17x}{12x^2y^4}$ 5. $\frac{2x^2-4x}{6x^2(x-2)}, \frac{3x-6}{6x^2(x-2)}$ 7. $\frac{3x-1}{2x(x-5)}, -\frac{6x^3-30x^2}{2x(x-5)}$ 9. $\frac{6x^2+9x}{(2x+3)(2x-3)}, \frac{10x^2-15x}{(2x+3)(2x-3)}$ 11. $\frac{2x}{(x+3)(x-3)}, \frac{x^2+4x+3}{(x+3)(x-3)}$ 13. $\frac{6}{6(x+2y)(x-2y)}, \frac{5x+10y}{6(x+2y)(x-2y)}$ 15. $\frac{3x^2-3x}{(x+1)(x-1)^2}, \frac{5x^2+5x}{(x+1)(x-1)^2}$ 17. $-\frac{x-3}{(x-2)(x^2+2x+4)}, \frac{2x-4}{(x-2)(x^2+2x+4)}$ 19. $\frac{2x^2+6x}{(x-1)(x+3)^2}, -\frac{x^2-x}{(x-1)(x+3)^2}$ 21. $-\frac{12x^2-8x}{(2x-3)(2x-5)(3x-2)}, \frac{6x^2-9x}{(2x-3)(2x-5)(3x-2)}$ 23. $\frac{5}{(3x-4)(2x-3)}, -\frac{4x^2-6x}{(3x-4)(2x-3)}, \frac{3x^2-x-4}{(3x-4)(2x-3)}$ 25. $\frac{2x^2+10x}{(x+5)(x-3)}, -\frac{2x-6}{(x+5)(x-3)}, -\frac{x-1}{(x+5)(x-3)}$ 27. True 29. $\frac{1}{2x^2}$ 31. $\frac{1}{x+2}$ 33. $\frac{12ab-9b+8a}{30a^2b^2}$ 35. $\frac{5-16b+12a}{40ab}$ 37. $\frac{7}{12x}$ 39. $\frac{2xy-8x+3y}{10x^2y^2}$ 41. $-\frac{a(2a-13)}{(a+1)(a-2)}$ 43. $\frac{5x^2-6x+10}{(5x-2)(2x-5)}$ 45. $\frac{a}{b(a-b)}$ 47. $\frac{a^2+18a-9}{a(a-3)}$

$$\begin{array}{llll}
 49. \frac{17x^2 + 20x - 25}{x(6x - 5)} & 51. \frac{6}{(x - 3)^2(x + 3)} & 53. \frac{-2(x - 1)}{(x + 2)^2} & 55. \frac{-5x^2 - 17x + 8}{(x + 4)(x - 2)} \\
 61. \frac{x^2 - 52x + 160}{4(x + 3)(x - 3)} & 63. \frac{3x - 1}{4x + 1} & 65. \frac{2(5x - 3)}{(x + 4)(x - 3)(x + 3)} & 67. \frac{x - 2}{x + 3} \\
 77. \frac{3 - a}{3a} & 79. \frac{-2x^2 + 5x - 2}{(x + 2)(x + 1)} & 81. \frac{b - a}{b + 2a} & 83. \frac{2}{x + 2}
 \end{array}$$

57. $\frac{3x^n + 2}{(x^n + 1)(x^n - 1)}$ 59. 1
 69. 1 71. $\frac{x + 1}{2x - 1}$ 73. $\frac{1}{2x - 1}$ 75. $\frac{1}{x^2 + 4}$
 85. a. $f(4) = \frac{2}{3}$; $g(4) = 4$; $S(4) = 4\frac{2}{3}$ b. Yes

SECTION 6.3

$$\begin{array}{llll}
 3. \frac{5}{23} & 5. \frac{2}{5} & 7. \frac{x}{x - 1} & 9. \frac{-a}{a + 2} \\
 11. \frac{-1}{x(x + h)} & 13. \frac{(x - 1)(x + 1)}{x^2 + 1} & 15. \frac{-a - 1}{a + 1} & 17. \frac{2}{5} \\
 19. \frac{-1}{2} & 21. \frac{x + 1}{x - 4} & 23. \frac{x - 2}{x - 3} & 25. \frac{x - 3}{x + 4} \\
 27. \frac{x + 2}{x + 5} & 29. \frac{x + 2}{x - 1} & 31. \frac{-x + 4}{2x + 3} & 33. \frac{x + y}{x - y} \\
 35. \frac{x^2 - x - 1}{x^2 + x + 1} & 37. \frac{-2(a + 1)}{7a - 4} & 39. \frac{-2x}{x^2 + 1} & 41. \frac{a(a^2 + a + 1)}{a^2 + 1} \\
 43. \frac{-a^2}{1 - 2a} & 45. \frac{-3x + 2}{x - 2} & 47. \frac{a - 1}{a} & 49. \frac{3}{10}
 \end{array}$$

51. $\frac{2}{x + 2}$

SECTION 6.4

$$3. 9 \quad 5. \frac{15}{2} \quad 7. 3 \quad 9. \frac{10}{3} \quad 11. -2 \quad 13. \frac{2}{5} \quad 15. -4 \quad 17. \frac{3}{4} \quad 19. -\frac{6}{5} \quad 21. -2 \quad 23. \text{True}$$

25. There are 56.75 g of protein in a 454-gram box of the pasta. 27. There would be 3500 computers with defective USB ports.
 29. The gallon of milk would cost \$3.81. 31. The dimensions of the room are 18 ft by 24 ft. 33. A person must walk 21.54 mi to lose 1 lb. 35. The knee braces would provide 12 min of talk time after walking for 1 min. 37. The value is \$.71.

SECTION 6.5

$$\begin{array}{llll}
 1. -3 \text{ and } 4 & 3. -5 & 5. \frac{5}{2} & 7. -1 \\
 9. 8 & 11. -3 & 13. -4 & 15. \text{No solution} \\
 17. 7 & 19. 0 \text{ and } 4 & 21. -1 & 23. -1 \text{ and } -\frac{5}{3} \\
 25. -\frac{1}{3} \text{ and } 5 & 27. -\frac{11}{3} & 29. \text{No solution} & 31. \text{Working alone, the experienced bricklayer can do the job in 14 h.} \\
 33. \text{Working alone, the second machine would take 60 min to complete the order.} & 35. \text{With all three machines working, it would take 4 h to fill the bottles.} & 37. \text{It would take 90 min to empty the tank.} & 39. \text{It will take 120 min to have 76 balloons.} \\
 41. \text{The two clerks, working together, can complete the task in 40 min (or 2400 s).} & 43. \text{The time } t \text{ is less than } n. & 45. \text{The rate of the runner is 8 mph.} & 47. \text{The rate of the tortoise is 0.4 ft/s. The rate of the hare is 72 ft/s.} \\
 49. \text{The rate of the cyclist is 10 mph.} & 51. \text{The rate of the jet was 525 mph.} & 53. \text{The rate of the current is 3 mph.} & 55. \text{The rate of the wind is 60.80 mph.} \\
 57. \text{The rate of the current is 2 mph.} & 59. \text{a. 3} & \text{b. 0 and } y & \text{c. } 3y \text{ and } \frac{3}{2}y \\
 61. \frac{a - b}{b}
 \end{array}$$

SECTION 6.6

1. The profit is \$80,000. 3. The pressure is 6.75 lb/in². 5. In 10 s, the object will fall 1600 ft. 7. It takes five computers 100 s to solve the problem. 9. The load on the sail is 360 lb. 11. The agency would receive a commission of \$13,800.
 13. The current is 7.5 amps. 15. The intensity is 48 foot-candles when the distance is 5 ft. 17. y is doubled.
 19. inversely 21. inversely

CHAPTER 6 CONCEPT REVIEW*

- To determine the excluded values of the domain in a rational function, set the denominator equal to zero and solve the equation for the unknown. The solutions are the excluded values of the domain. [6.1A]
- A rational expression is in simplest form when the numerator and denominator have no common factors. [6.1B]
- No, you do not need a common denominator when multiplying or dividing rational expressions. [6.1C, 6.1D]
- After adding two rational expressions with the same denominator, simplify the sum by factoring the numerator and denominator and then dividing the numerator and denominator by the common factors. [6.2B]
- The first step in simplifying a complex fraction is to find the LCM of the denominators of the fractions in the numerator and denominator. [6.3A]
- To write a proportion, put quantities having the same units in the numerators and quantities having the same units in the denominators. [6.4B]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

7. If a job is completed in a given number of hours x , then the rate of work is $\frac{1}{x}$ of the work each hour. [6.5B]
8. It is important to check the solutions to a rational equation because when both sides of an equation are multiplied by a variable expression, the resulting equation may have a solution that is not a solution of the original equation. [6.5A]
9. To find the value of the constant of proportionality, substitute the given values of the other variables into the variation equation and solve for the constant of proportionality. [6.6A]
10. The formula for uniform motion is $d = rt$. To solve the formula for t , divide each side of the equation by r . [6.5C]
11. We cannot cross out common *terms* in the numerator and denominator of a rational expression; we can only divide by common *factors*. Note that terms indicate addition and subtraction. Factors indicate multiplication. [6.1B]
12. In multiplying rational expressions, we begin by factoring the numerator and denominator of each fraction. We do this so that we can then divide by the common factors in the numerators and denominators. This can simplify the expression considerably. [6.1C]

CHAPTER 6 REVIEW EXERCISES

1. $\frac{ab}{a-1}$ [6.1C] 2. $-\frac{5x^2-17x-9}{(x-3)(x+2)}$ [6.2B] 3. $P(4) = 4$ [6.1A] 4. 4 [6.4A] 5. $-\frac{9x^2+16}{24x}$ [6.3A]
6. $\frac{16x^2+4x}{(4x+1)(4x-1)}$, $\frac{12x^2-7x+1}{(4x+1)(4x-1)}$ [6.2A] 7. 1 and 2 [6.5A] 8. $P(-2) = \frac{2}{21}$ [6.1A] 9. $\frac{2}{5}$ [6.4A]
10. The domain is $\{x|x \neq -3, 2\}$. [6.1A] 11. $\frac{3x^2-1}{3x^2+1}$ [6.1B] 12. The domain is $\{x|x \neq 3\}$. [6.1A]
13. x [6.1C] 14. $\frac{4x-1}{x+2}$ [6.2B] 15. 15 [6.5A] 16. No solution [6.5A] 17. $\frac{x^2-2x-5}{x^2-7x+10}$ [6.3A]
18. $\frac{x^2-7x+12}{(x-5)(x-4)}$, $\frac{x}{(x-5)(x-4)}$, $-\frac{x-5}{(x-5)(x-4)}$ [6.2A] 19. 10 [6.5A] 20. $\frac{3x+4}{x^2-3x-4}$ [6.2B] 21. $\frac{3x+2}{x}$ [6.1D]
22. $\frac{9x^2+x+4}{(3x-1)(x-2)}$ [6.2B] 23. $\frac{x^2+3x+9}{x+3}$ [6.1B] 24. $\frac{2x^2-x+15}{(x-3)(x+2)}$ [6.2B] 25. The domain is $\{x|x \in \text{real numbers}\}$. [6.1A]
26. $\frac{2x^2-8x-5}{(x+1)(x-2)}$ [6.2B] 27. $-\frac{x+4}{x(x+2)}$ [6.1B] 28. $\frac{4x^2+6x+9}{2x+3}$ [6.1B]
29. $-\frac{(x+4)(x+3)}{6(x-4)}$ [6.1C] 30. $\frac{x+6}{x-3}$ [6.1D] 31. $\frac{8(x-2)(2x-1)}{x(2x+1)}$ [6.1D] 32. $-\frac{x-3}{x+3}$ [6.1D]
33. $\frac{9+5ab}{6a^3b^2}$ [6.2B] 34. $\frac{3x+26}{(3x-2)(3x+2)}$ [6.2B] 35. $\frac{x-4}{x+5}$ [6.3A] 36. $\frac{x-1}{4}$ [6.3A] 37. 1, 13 [6.4A]
38. $\frac{15}{13}$ [6.5A] 39. $R = \frac{V}{I}$ [6.5A] 40. 6 [6.5A] 41. It would take 40 min to empty the tub with both pipes open. [6.5B]
42. The rate of the bus is 45 mph. [6.5C] 43. The rate of the helicopter is 180 mph. [6.5C]
44. To read 150 pages, it will take 375 min. [6.4B] 45. The current is 2 amps. [6.6A] 46. 48 mi would be represented by 12 in. [6.4B]
47. The stopping distance for a car traveling at 65 mph is 287.3 ft. [6.6A]
48. It would take the apprentice 104 min to complete the job working alone. [6.5B]

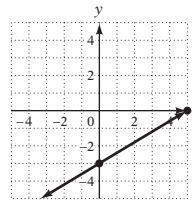
CHAPTER 6 TEST

1. 2 [6.4A, Example 1] 2. $\frac{x+2}{x-1}$ [6.1D, Example 11] 3. $\frac{x^2-9x+3}{(x-3)(x+2)}$ [6.2B, How To 4]
4. $\frac{x^2-2x-3}{(x+3)(x-3)(x-2)}$, $\frac{2x^2-4x}{(x+3)(x-3)(x-2)}$ [6.2A, Example 2] 5. No solution [6.5A, How To 2]
6. $\frac{5x-6}{(x+5)(x-2)}$ [6.2B, How To 5] 7. $\frac{6(x-2)}{5}$ [6.1C, Example 7] 8. The domain is $\{x|x \neq 3, -3\}$. [6.1A, Example 2]
9. $\frac{x-4}{x+3}$ [6.3A, How To 1] 10. $\frac{x+4}{x+2}$ [6.3A, How To 2] 11. $\frac{x+1}{3x-4}$ [6.1D, Example 11]
12. 1 and 2 [6.5A, How To 1] 13. $-\frac{2(a-2)}{3a+2}$ [6.1B, How To 4] 14. -2 and 6 [6.5A, Example 2]
15. $f(-1) = 2$ [6.1A, Example 1] 16. $\frac{-x^2-5x+2}{(x+1)(x+4)(x-1)}$ [6.2B, Example 5] 17. -3 and 5 [6.4A, Example 2]
18. The rate of the cyclist is 10 mph. [6.5C, How To 5] 19. The resistance is 0.4 ohm. [6.6A, How To 5]
20. The office requires 14 rolls of wallpaper. [6.4B, Example 3] 21. Working together, the landscapers could complete the task in 10 min. [6.5B, How To 4] 22. The intensity is 128 decibels. [6.6A, How To 3]

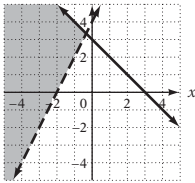
CUMULATIVE REVIEW EXERCISES

1. $\frac{36}{5}$ [1.2D] 2. $\frac{15}{2}$ [2.1C] 3. 7 and 1 [2.5A] 4. The domain is $\{x|x \neq 3\}$. [6.1A] 5. $P(-2) = \frac{3}{7}$ [6.1A]
6. 3.5×10^{-8} [5.1C] 7. $\frac{a^5}{b^6}$ [5.1B] 8. $(-\infty, 4]$ [2.4A] 9. $-4a^4 + 6a^3 - 2a^2$ [5.3A]

10. $(2x - 1)(x + 2)$ [5.5D] 11. $(xy - 3)(x^2y^2 + 3xy + 9)$ [5.6B] 12. $x + 3y$ [6.1B] 13. $y = \frac{3}{2}x + 2$ [3.6A]
 14. $-\frac{3}{4}$ and $\frac{3}{2}$ [5.7A] 15. $4x^2 + 10x + 10 + \frac{21}{x-2}$ [5.4C] 16. $\frac{3xy}{x-y}$ [6.1D]
 17. $\frac{x^3y - 2x^2y}{2x^2(x+1)(x-2)^2}$ [6.2A] 18. $-\frac{x}{(3x+2)(x+1)}$ [6.2B] 19. [3.3C]



20. [4.5A] 21. -28 [4.3A] 22. $\frac{x-3}{x+3}$ [6.3A] 23. (1, 1, 1) [4.2B]



24. $\{x | x < -\frac{2}{3} \text{ or } x > 2\}$ [2.5B] 25. 9 [6.4A] 26. 13 [6.5A] 27. $r = \frac{E - IR}{I}$ [2.1D] 28. $\frac{x}{x-1}$ [5.1B]

29. The smaller integer is 5 and the larger integer is 10. [2.2A] 30. 50 lb of almonds must be used. [2.3A]
 31. 75,000 people are expected to vote. [6.4B] 32. It would take the new computer 14 min to do the job working alone. [6.5B]
 33. The rate of the wind is 60 mph. [6.5C] 34. The first person has made 12 laps and is at the starting point. The second person has made 20 laps and is at the starting point. [2.3C]

ANSWERS TO CHAPTER 7 SELECTED EXERCISES

PREP TEST

1. 16 [1.2B] 2. 32 [1.2C] 3. 9 [1.2B] 4. $\frac{1}{12}$ [1.2B] 5. $-5x - 1$ [1.3C] 6. $\frac{xy^5}{4}$ [5.1B]
 7. $9x^2 - 12x + 4$ [5.3C] 8. $-12x^2 + 14x + 10$ [5.3B] 9. $36x^2 - 1$ [5.3C] 10. -1 and 15 [5.7A]

SECTION 7.1

1. (i), (iii) 3. 2 5. 27 7. $\frac{1}{9}$ 9. 4 11. Not a real number 13. $\frac{343}{125}$ 15. x 17. $y^{1/2}$ 19. $x^{1/12}$ 21. $a^{7/12}$
 23. $\frac{1}{a}$ 25. $\frac{1}{y}$ 27. $y^{3/2}$ 29. $\frac{1}{x}$ 31. $\frac{1}{x^4}$ 33. a 35. $x^{3/10}$ 37. a^3 39. $\frac{1}{x^{1/2}}$ 41. $y^{1/9}$ 43. $\frac{x}{y^2}$ 45. $\frac{x^{3/2}}{y^{1/4}}$
 47. $\frac{x^2}{y^8}$ 49. $\frac{1}{x^{11/12}}$ 51. $\frac{1}{y^{5/2}}$ 53. $\frac{1}{b^{7/8}}$ 55. a^5b^{13} 57. $\frac{x^2}{4y^{3/2}}$ 59. $\frac{1}{3xy^{11}}$ 61. $\frac{b^4}{2a}$ 63. $\frac{y^{17/2}}{x^3}$ 65. $\frac{16b^2}{a^{1/3}}$ 67. $y^2 - y$
 69. $a - a^2$ 71. $x^{3n/2}$ 73. $y^{3m/2}$ 75. False 77. $\sqrt[4]{3}$ 79. $\sqrt{a^3}$ 81. $\sqrt{32t^5}$ 83. $-2\sqrt[3]{x^2}$ 85. $\sqrt[3]{a^4b^2}$
 87. $\sqrt[5]{a^6b^{12}}$ 89. $\sqrt[4]{(4x-3)^3}$ 91. $\frac{1}{\sqrt[3]{x^2}}$ 93. $14^{1/2}$ 95. $x^{1/3}$ 97. $x^{4/3}$ 99. $b^{3/5}$ 101. $(2x^2)^{1/3}$ 103. $-(3x^5)^{1/2}$
 105. $3xy^{2/3}$ 107. $(a^2 - 2)^{1/2}$ 109. Positive 111. Not a real number 113. x^8 115. $-x^4$ 117. xy^3 119. $-x^5y$
 121. $4a^2b^6$ 123. Not a real number 125. $3x^3$ 127. $-4x^3y^4$ 129. $-x^2y^3$ 131. x^4y^2 133. $3xy^5$ 135. $2ab^2$
 137. a. False; 2 b. True c. True d. False; $(a^n + b^n)^{1/n}$ e. False; $a + 2a^{1/2}b^{1/2} + b$ f. False; $a^{n/m}$

SECTION 7.2

1. No 3. Yes 5. $x^2yz^2\sqrt{yz}$ 7. $2ab^4\sqrt{2a}$ 9. $3xyz^2\sqrt{5yz}$ 11. $2xyz\sqrt[4]{3yz^2}$ 13. $a^5b^2\sqrt[3]{ab^2}$ 15. $-5y\sqrt[3]{x^2y}$
 17. $abc^2\sqrt[3]{ab^2}$ 19. $2x^2y\sqrt[4]{xy}$ 21. True 23. $-6\sqrt{x}$ 25. $-2\sqrt{2x}$ 27. $3\sqrt{2b} + 5\sqrt{3b}$ 29. $-2xy\sqrt{2y}$
 31. $6ab^2\sqrt{3ab} + 3ab\sqrt{3ab}$ 33. $-\sqrt[3]{2}$ 35. $8b\sqrt[3]{2b^2}$ 37. $3a\sqrt[4]{2a}$ 39. $17\sqrt{2} - 15\sqrt{5}$ 41. $5b\sqrt{b}$
 43. $-8xy\sqrt{2x} + 2xy\sqrt{xy}$ 45. $2y\sqrt[3]{2x}$ 47. $-4ab\sqrt[4]{2b}$ 49. 16 51. $2\sqrt[3]{4}$ 53. $xy^3\sqrt{x}$ 55. $8xy\sqrt{x}$
 57. $2x^2y\sqrt[3]{2}$ 59. $2ab\sqrt[4]{3a^2b}$ 61. 6 63. $x - \sqrt{2x}$ 65. $4x - 8\sqrt{x}$ 67. $-4 - \sqrt{5}$ 69. $29 - 7\sqrt{7}$ 71. 12
 73. -3 75. $-10x + 14\sqrt{2x} + 3$ 77. $10x + 12\sqrt{x} + 2$ 79. $x + 4\sqrt{x} + 4$ 81. $2x - 8\sqrt{2x+1} + 17$
 83. $-42 - 33\sqrt{2}$ 85. True 89. $\sqrt[3]{4x}$ 91. $\sqrt{3} + x$ 93. $y\sqrt{5y}$ 95. $b\sqrt{13b}$ 97. $\frac{\sqrt{2}}{2}$ 99. $\frac{2\sqrt{3y}}{3y}$ 101. $\frac{3\sqrt{3a}}{a}$
 103. $\frac{\sqrt{2y}}{2}$ 105. $\frac{5\sqrt[3]{3}}{3}$ 107. $\frac{5\sqrt[3]{9y^2}}{3y}$ 109. $\frac{b\sqrt{2a}}{2a^2}$ 111. $\frac{\sqrt{15x}}{5x}$ 113. $2 + 2\sqrt{2}$ 115. $-\frac{12 + 4\sqrt{2}}{7}$ 117. $-\frac{10 + 5\sqrt{7}}{3}$
 119. $-\frac{7\sqrt{x} + 21}{x - 9}$ 121. $-\sqrt{6} + 3 - 2\sqrt{2} + 2\sqrt{3}$ 123. $-\frac{17 + 5\sqrt{5}}{4}$ 125. $\frac{8a - 10\sqrt{ab} + 3b}{16a - 9b}$ 127. $\frac{3 - 7\sqrt{y} + 2y}{1 - 4y}$

129. a. False; $\sqrt[6]{432}$ b. True c. False; $x^{2/3}$ d. False; $\sqrt{x} + \sqrt{y}$ is in simplest form. e. False; $\sqrt[2]{2} + \sqrt[3]{3}$ is in simplest form. f. True 131. $\sqrt[4]{a+b}$

SECTION 7.3

1. -2 3. 9 5. No solution 7. 35 9. 27 11. -12 13. 9 15. 9 17. 2 19. 2 21. 1
 23. $\sqrt{x} < \sqrt{x+5}$. Therefore, $\sqrt{x} - \sqrt{x+5} < 0$ and cannot be equal to a positive number. 25. On the moon, an object will fall 24.75 ft in 3 s. 27. a. The height of the water is 5.0 ft after the valve has been opened for 10 s. b. It takes 27.7 s to empty the tank. 29. The length of the pendulum is 7.30 ft. 31. The HDTV screen is approximately 7.15 in. wider. 33. The longest pole that can be placed in the box is 5.4 ft. 35. 2

SECTION 7.4

3. $i\sqrt{a}$ 5. $5i$ 7. $7i\sqrt{2}$ 9. $3 + 3i\sqrt{5}$ 11. $-6 - 10i$ 13. $-4 + 2i$ 15. $4 + 8i$ 17. $8 + 2i\sqrt{2}$
 19. $-2 + 6i\sqrt{3}$ 21. $-5 + i\sqrt{23}$ 23. $-4 + 2i\sqrt{14}$ 25. $8 - i$ 27. $-8 + 4i$ 29. $6 - 6i$ 31. $11 - 4i$
 33. $-6 - 2i$ 35. The real parts of the complex numbers are additive inverses. 37. 63 39. -4 41. 29
 43. $-4 + 12i$ 45. $-3 - 4i$ 47. $17 - i$ 49. $8 + 27i$ 51. $-21 + 20i$ 53. 1 55. True 57. $-3i$
 59. $\frac{3}{4} + \frac{1}{2}i$ 61. $\frac{10}{13} - \frac{2}{13}i$ 63. $\frac{4}{5} + \frac{2}{5}i$ 65. $-i$ 67. $\frac{12}{17} + \frac{3}{17}i$ 69. $\frac{3}{10} - \frac{11}{10}i$ 71. $\frac{6}{5} + \frac{7}{5}i$ 73. True
 75. a. Yes b. Yes

CHAPTER 7 CONCEPT REVIEW*

- When writing an expression containing a rational exponent as a radical expression, the denominator of the rational exponent is the index of the radical. The numerator of the rational exponent becomes the power of the radicand. [7.1B]
- Expressions that contain rational exponents do not always represent real numbers when the base of the exponential expression is a negative number. For this reason, all variables in this chapter represent positive numbers. [7.1C]
- A radical expression is in simplest form when the radicand contains no factor that is a perfect power. [7.2A]
- To rationalize the denominator of a radical expression that has two terms in the denominator, multiply the numerator and denominator by the conjugate of the denominator. [7.2D]
- You can add two radical expressions that have the same radicand and the same index. [7.2B]
- When solving an equation containing a radical expression, you isolate the radical so that when you raise each side of the equation to the appropriate power, the resulting equation does not contain a radical expression. [7.3A]
- A solution is an extraneous solution if, when that value is substituted into the original equation, the result is an equation that is not true. [7.3A]
- To add two complex numbers, add the real parts and add the imaginary parts. [7.4B]
- To rewrite $\sqrt{36} + \sqrt{-36}$ as a complex number, first rewrite the expression in the form $a + bi$: $\sqrt{36} + i\sqrt{36}$. Then simplify each radical: $6 + 6i$. [7.4A]
- To find the product of two variable expressions with the same base and fractional exponents, add the exponents. The base remains unchanged. [7.1A]

CHAPTER 7 REVIEW EXERCISES

1. $20x^2y^2$ [7.1A] 2. 7 [7.3A] 3. $39 - 2i$ [7.4C] 4. $7x^{2/3}y$ [7.1B] 5. $6\sqrt{3} - 13$ [7.2C] 6. -2 [7.3A]
 7. $\frac{1}{x^2}$ [7.1A] 8. $\frac{8\sqrt{3}y}{3y}$ [7.2D] 9. $-2a^2b^4$ [7.1C] 10. $2a^2b\sqrt{2b}$ [7.2B] 11. $4 + \sqrt{2}$ [7.2D]
 12. $\frac{2}{3} - \frac{5}{3}i$ [7.4D] 13. $3ab^3\sqrt{2a}$ [7.2A] 14. $2 + 12i$ [7.4B] 15. $5x^3y^3\sqrt[3]{2x^2y}$ [7.2B] 16. $4xy^2\sqrt[3]{x^2}$ [7.2C]
 17. $7 + 3i$ [7.4C] 18. $\frac{a^{2/3}}{2b^5}$ [7.1A] 19. $-2ab^2\sqrt[5]{2a^3b^2}$ [7.2A] 20. $-2 + 7i$ [7.4D] 21. $-6\sqrt{2}$ [7.4C]
 22. 30 [7.3A] 23. $3a^2b^3$ [7.1C] 24. $\frac{3\sqrt[3]{9x^2}}{x}$ [7.2D] 25. $-12 + 10i$ [7.4B] 26. $2x + 4\sqrt{2x-1} + 3$ [7.2C]
 27. $6x^2\sqrt{3y}$ [7.2B] 28. $\frac{1}{3}$ [7.1A] 29. $\frac{1}{a^{10}}$ [7.1A] 30. $-7x^3y^8$ [7.1C] 31. $4\sqrt[3]{a^2}$ [7.1B] 32. $\frac{x}{3}$ [7.1A]
 33. $xy^2z^2\sqrt[4]{x^2z^2}$ [7.2A] 34. $5\sqrt{6}$ [7.2B] 35. $4x^2\sqrt{3xy} - 4x^2\sqrt{5y}$ [7.2B] 36. 40 [7.2C] 37. $3x + 3\sqrt{3x}$ [7.2C]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

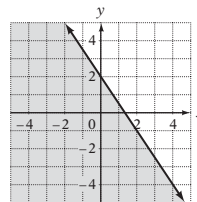
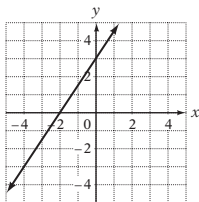
38. $5x\sqrt{x}$ [7.2D] 39. $\frac{-9-16\sqrt{7}}{29}$ [7.2D] 40. $6i$ [7.4A] 41. $8+6i$ [7.4A] 42. $-2+4i\sqrt{2}$ [7.4A]
 43. $9-i$ [7.4B] 44. -11 [7.2C] 45. $-4-9i$ [7.4B] 46. $15-8i$ [7.4C] 47. $6i$ [7.4D]
 48. $\frac{14}{5}+\frac{7}{5}i$ [7.4D] 49. 8 [7.3A] 50. -24 [7.3A] 51. The width is 5 in. [7.3B] 52. The amount of power is 120 watts. [7.3B]
 53. The distance required is 242 ft. [7.3B] 54. The distance is 6.63 ft. [7.3B]

CHAPTER 7 TEST

1. $\frac{1}{2}x^{3/4}$ [7.1B, How To 8] 2. $-2x^2y\sqrt[3]{2x}$ [7.2B, Example 3] 3. $3\sqrt[5]{y^2}$ [7.1B, Example 6]
 4. $18+16i$ [7.4C, How To 7] 5. $4x+12\sqrt{x}+9$ [7.2C, Example 6] 6. $r^{1/6}$ [7.1A, Example 4]
 7. 4 [7.3A, How To 3] 8. $2a^2b^2\sqrt[4]{2b^2}$ [7.2C, How To 6] 9. $-4x\sqrt{3}$ [7.2C, How To 7]
 10. $-3+2i$ [7.4B, How To 3] 11. $4x^2y^3\sqrt{2y}$ [7.2A, How To 2] 12. $14+10\sqrt{3}$ [7.2C, How To 8]
 13. -10 [7.4C, How To 5] 14. 2 [7.2D, Example 9] 15. $8a\sqrt{2a}$ [7.2B, Example 3]
 16. $2a-\sqrt{ab}-15b$ [7.2C, How To 8] 17. $\frac{64x^3}{y^6}$ [7.1A, How To 6] 18. $2\sqrt[3]{25x}$ [7.2D, You Try It 8]
 19. $-\frac{4}{5}+\frac{7}{5}i$ [7.4D, How To 10] 20. -3 [7.3A, How To 2] 21. $\frac{b^3}{8a^6}$ [7.1A, How To 6]
 22. $3abc^2\sqrt[3]{ac}$ [7.2A, Example 2] 23. $\frac{4x^2}{y}$ [7.2D, How To 10] 24. $\sqrt{5x}$ [7.2D, How To 11]
 25. The distance is 576 ft. [7.3B, Example 4]

CUMULATIVE REVIEW EXERCISES

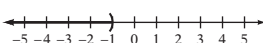
1. The Distributive Property [1.3A] 2. $f(-3) = 34$ [3.2A] 3. $\frac{3}{2}$ [2.1B] 4. $\frac{2}{3}$ [2.1C]
 5. $\frac{1}{3}$ and $\frac{7}{3}$ [2.5A] 6. $\{x|x>1\}$ [2.4A] 7. $\{x|-6\leq x\leq 3\}$ [2.5B] 8. $(9x+y)(9x-y)$ [5.6A]
 9. $x(x^2+3)(x+1)(x-1)$ [5.6D] 10. $y=\frac{1}{3}x+\frac{7}{3}$ [3.5B] 11. 3 [4.3A] 12. $C=R-nP$ [2.1D]
 13. $2x^2y^2$ [5.1B] 14. $\frac{x(2x-3)}{y(x+4)}$ [6.1C] 15. $-x\sqrt{10x}$ [7.2B] 16. $-\frac{1}{2}$ and 3 [6.5A]
 17. The slope is $\frac{3}{2}$, and the y-intercept is $(0, 3)$. [3.3B] 18. [3.7A]



19. $-\frac{1}{5}+\frac{3}{5}i$ [7.4D] 20. -20 [7.3A] 21. $\frac{x^2+12x-12}{(2x-3)(x+4)}$ [6.2B] 22. $(\frac{17}{4}, \frac{9}{2})$ [4.3B] 23. There are nineteen 18¢ stamps. [2.2B]
 24. The rate of the plane is 250 mph. [2.3C] 25. The time is 1.25 s. [5.1D]
 26. The height of the periscope is 32.7 ft. [7.3B] 27. $m=0.08$. The annual income is 8% of the investment. [3.4A]

ANSWERS TO CHAPTER 8 SELECTED EXERCISES

PREP TEST

1. $3\sqrt{2}$ [7.2A] 2. $3i$ [7.4A] 3. $\frac{2x-1}{x-1}$ [6.2B] 4. 8 [1.3A] 5. Yes [5.6A] 6. $(2x-1)^2$ [5.6A]
 7. $(3x+2)(3x-2)$ [5.6A] 8.  [1.1C] 9. -3 and 5 [5.7A] 10. 4 [6.4A]

SECTION 8.1

3. $2x^2-4x-5=0$; $a=2$, $b=-4$, $c=-5$ 5. $4x^2-5x+6=0$; $a=4$, $b=-5$, $c=6$ 7. 3 and 5
 9. -7 and -1 11. 0 and 4 13. -5 and 5 15. -2 and 3 17. 3 19. 0 and 2 21. -2 and 5
 23. 2 and 5 25. $-\frac{3}{2}$ and 6 27. $\frac{1}{4}$ and 2 29. -4 and $\frac{1}{3}$ 31. $-\frac{2}{3}$ and $\frac{9}{2}$ 33. -4 and $\frac{1}{4}$ 35. -2 and 9

37. -2 and $-\frac{3}{4}$ 39. -5 and 2 41. -4 and $-\frac{3}{2}$ 43. $2b$ and $7b$ 45. $-c$ and $7c$ 47. $-b$ and $-\frac{b}{2}$
 49. $\frac{2a}{3}$ and $4a$ 51. $x^2 - 7x + 10 = 0$ 53. $x^2 + 6x + 8 = 0$ 55. $x^2 - 5x - 6 = 0$ 57. $x^2 - 9 = 0$
 59. $x^2 - 8x + 16 = 0$ 61. $x^2 - 5x = 0$ 63. $x^2 - 3x = 0$ 65. $2x^2 - 7x + 3 = 0$ 67. $4x^2 - 5x - 6 = 0$
 69. $3x^2 + 11x + 10 = 0$ 71. $9x^2 - 4 = 0$ 73. $6x^2 - 5x + 1 = 0$ 75. $10x^2 - 7x - 6 = 0$
 77. $8x^2 + 6x + 1 = 0$ 79. $50x^2 - 25x - 3 = 0$ 81. 0 83. -7 and 7 85. $-2i$ and $2i$ 87. -2 and 2
 89. $-\frac{9}{2}$ and $\frac{9}{2}$ 91. $-7i$ and $7i$ 93. $-4\sqrt{3}$ and $4\sqrt{3}$ 95. $-5\sqrt{3}$ and $5\sqrt{3}$ 97. $-3i\sqrt{2}$ and $3i\sqrt{2}$
 99. -5 and 7 101. -6 and 0 103. -7 and 3 105. 0 and 1 107. $-5 - \sqrt{6}$ and $-5 + \sqrt{6}$
 109. $3 - 3i\sqrt{5}$ and $3 + 3i\sqrt{5}$ 111. $-\frac{2-9\sqrt{2}}{3}$ and $-\frac{2+9\sqrt{2}}{3}$ 113. Two complex solutions
 115. Two equal real solutions 117. $x^2 - 2 = 0$ 119. $x^2 + 1 = 0$ 121. $x^2 - 8 = 0$ 123. $x^2 + 2 = 0$
 125. $-\frac{1}{2}$ 127. -8 and 2

SECTION 8.2

1. -1 and 5 3. -9 and 1 5. 3 7. $-2 - \sqrt{11}$ and $-2 + \sqrt{11}$ 9. $3 - \sqrt{2}$ and $3 + \sqrt{2}$ 11. $1 - i$ and $1 + i$
 13. -3 and 8 15. -9 and 4 17. $\frac{3-\sqrt{5}}{2}$ and $\frac{3+\sqrt{5}}{2}$ 19. $\frac{1-\sqrt{5}}{2}$ and $\frac{1+\sqrt{5}}{2}$ 21. $3 - \sqrt{13}$ and $3 + \sqrt{13}$
 23. 3 and 5 25. $2 - 3i$ and $2 + 3i$ 27. $-3 - 2i$ and $-3 + 2i$ 29. $1 - 3\sqrt{2}$ and $1 + 3\sqrt{2}$
 31. $\frac{1-\sqrt{17}}{2}$ and $\frac{1+\sqrt{17}}{2}$ 33. $1 - 2i\sqrt{3}$ and $1 + 2i\sqrt{3}$ 35. $\frac{1}{2} - i$ and $\frac{1}{2} + i$ 37. $\frac{1}{3} - \frac{1}{3}i$ and $\frac{1}{3} + \frac{1}{3}i$
 39. $\frac{2-\sqrt{14}}{2}$ and $\frac{2+\sqrt{14}}{2}$ 41. $-\frac{3}{2}$ and 1 43. $1 - \sqrt{5}$ and $1 + \sqrt{5}$ 45. $\frac{1}{2}$ and 5
 47. $2 - \sqrt{5}$ and $2 + \sqrt{5}$ 49. -3.236 and 1.236 51. 0.293 and 1.707 53. -0.809 and 0.309 55. $c \leq 4$
 57. $-a$ and $2a$ 59. $-5a$ and $2a$ 61. No, the ball will have gone only 197.2 ft when it hits the ground.

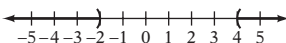
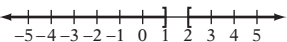
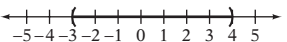
SECTION 8.3

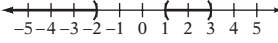
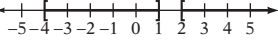
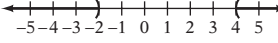
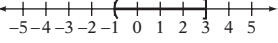
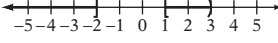
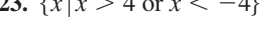
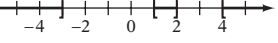
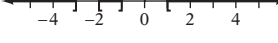
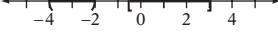
3. -2 and 5 5. -9 and 4 7. $4 - 2\sqrt{22}$ and $4 + 2\sqrt{22}$ 9. -8 and 3 11. $\frac{-5-\sqrt{33}}{4}$ and $\frac{-5+\sqrt{33}}{4}$
 13. $-\frac{1}{4}$ and $\frac{3}{2}$ 15. $7 - 3\sqrt{5}$ and $7 + 3\sqrt{5}$ 17. $\frac{1-\sqrt{3}}{2}$ and $\frac{1+\sqrt{3}}{2}$ 19. $-1 - i$ and $-1 + i$
 21. $1 - 2i$ and $1 + 2i$ 23. $2 - 3i$ and $2 + 3i$ 25. $\frac{1-\sqrt{11}}{2}$ and $\frac{1+\sqrt{11}}{2}$ 27. $-\frac{3}{2} - \frac{1}{2}i$ and $-\frac{3}{2} + \frac{1}{2}i$
 29. $\frac{3}{4} - \frac{3\sqrt{3}}{4}i$ and $\frac{3}{4} + \frac{3\sqrt{3}}{4}i$ 31. 0.394 and 7.606 33. -4.236 and 0.236 35. -1.351 and 1.851
 37. Two complex number solutions 39. Two equal real number solutions 41. Two unequal real number solutions
 43. $\sqrt{4ac}$ 45. No. The arrow does not reach a height of 275 ft. (The discriminant is less than zero.) 47. $\{p | p < 9\}$
 49. $(1, \infty)$ 51. $-2i$ and i

SECTION 8.4

1. Yes 3. No 5. $-2, 2, -3$, and 3 7. $-2, 2, -\sqrt{2}$, and $\sqrt{2}$ 9. 1 and 4 11. 16 13. $-2i, 2i, -1$, and 1
 15. $-4i, 4i, -2$, and 2 17. 16 19. 1 and 512 21. $-\frac{2}{3}, \frac{2}{3}, -1$, and 1 23. Exercises 28, 29, 30, 34, 35, 36, 38, 39, 42
 25. 3 27. 9 29. -1 and 2 31. 0 and 2 33. $-\frac{1}{2}$ and 2 35. -2 37. 1 39. 1 41. -3
 43. $y + 2$ 45. -1 and 10 47. -3 and 1 49. -1 and 0 51. $-\frac{1}{3}$ and $\frac{1}{2}$ 53. $-\frac{2}{3}$ and 6 55. $\frac{4}{3}$ and 3
 57. -2 and 3 59. 4

SECTION 8.5

3. a. True b. False c. False 5. a. False b. True 7.  $\{x | x < -2 \text{ or } x > 4\}$
 9.  $\{x | x \leq 1 \text{ or } x \geq 2\}$ 11.  $\{x | -3 < x < 4\}$

13.  $\{x | x < -2 \text{ or } 1 < x < 3\}$
 15.  $\{x | -4 \leq x \leq 1 \text{ or } x \geq 2\}$
17.  $\{x | x < -2 \text{ or } x > 4\}$
 19.  $\{x | -1 < x \leq 3\}$
21.  $\{x | x \leq -2 \text{ or } 1 \leq x < 3\}$
 23.  $\{x | x > 4 \text{ or } x < -4\}$
 25. $\{x | -3 \leq x \leq 12\}$
27. $\{x | \frac{1}{2} < x < \frac{3}{2}\}$
 29. $\{x | x < 1 \text{ or } x > \frac{5}{2}\}$
 31. $\{x | x < -1 \text{ or } 1 < x \leq 2\}$
 33. $\{x | \frac{1}{2} < x \leq 1\}$
35. $\{x | 2 < x \leq 3\}$
 37. $\{x | x > 5 \text{ or } -4 < x < -1\}$
 39. 
41. 
 43. 

SECTION 8.6

1. The maximum speed is 33 mph.
 3. The rocket takes 12.5 s to return to Earth.
 5. a. The maximum height is 27 ft.
- b. The height of the arch is 24 ft.
 c. The arch is 8 ft tall 20.13 ft from the center.
 7. The length of a side of the square base is 177 m.
9. It would take the larger pipe 6 min to fill the tank. It would take the smaller pipe 12 min to fill the tank.
11. It would take the faster computer 2 h to run the program.
 13. It would take the apprentice carpenter 6.8 h to install the floor.
15. The rate of the wind was approximately 108 mph.
 17. The rate of the jet stream is 93 mph.
 19. The dimensions of Colorado are 272 mi by 383 mi.
21. The height is 3 cm. The base is 14 cm.
 23. The piece of cardboard needed is 90 cm by 90 cm.
25. The dimensions of the work area are 10 ft by 30 ft.
 27. The radius of the cone is 1.5 in.

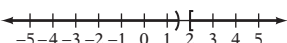
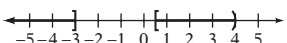
CHAPTER 8 CONCEPT REVIEW*

- Given the solutions of a quadratic equation, write the quadratic equation by using the equation $(x - r_1)(x - r_2) = 0$, where r_1 and r_2 are the given solutions. [8.1B]
- The symbol $\pm$ means plus or minus. For example, ± 6 means plus 6 or minus 6. [8.1C]
- To complete the square on $x^2 - 18x$, square the product of $\frac{1}{2}$ and the coefficient of x : $\left[\frac{1}{2}(-18)\right]^2 = 81$. Add 81 to the binomial: $x^2 - 18x + 81$. [8.2A]
- To approximate $4 + 3\sqrt{2}$, find the approximation of $\sqrt{2}$: 1.4142. Substitute the approximate value of $\sqrt{2}$ in $4 + 3\sqrt{2}$ and simplify: $4 + 3(1.4142) = 8.2426$. To approximate $4 - 3\sqrt{2}$, substitute the approximate value of $\sqrt{2}$ in $4 - 3\sqrt{2}$ and simplify: $4 - 3(1.4142) = -0.2426$. [8.2A]
- To solve a quadratic equation by completing the square:
 - Write the equation in the form $ax^2 + bx = -c$.
 - Multiply both sides of the equation by $\frac{1}{a}$.
 - Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
 - Factor the perfect-square trinomial.
 - Take the square root of each side of the equation.
 - Solve the resulting equation for x .
 - Check the solutions. [8.2A]
- The quadratic formula states that the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. [8.3A]
- In the quadratic formula, the discriminant is the quantity $b^2 - 4ac$. If $b^2 - 4ac = 0$, the equation has two equal real number solutions, a double root. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions. If $b^2 - 4ac < 0$, the equation has two complex number solutions. [8.3A]
- It is important to check the solutions to a radical equation because when both sides of an equation are squared, the resulting equation may have a solution that is not a solution of the original equation. [8.4A]
- A quadratic inequality is one that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where $a \neq 0$. The symbols $\leq$ and $\geq$ can also be used. [8.5A]
- In a quadratic inequality, the symbol $<$ or $>$ indicates that the endpoints are not included in the solution set. The symbol $\leq$ or $\geq$ indicates that the endpoints are included in the solution set. [8.5A]

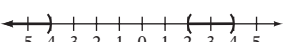
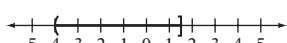
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11. In a rational inequality, we must check that no element of the solution results in a denominator of zero when substituted for the variable in the original inequality. [8.5A]

CHAPTER 8 REVIEW EXERCISES

1. 0 and $\frac{3}{2}$ [8.1A] 2. $-2c$ and $\frac{c}{2}$ [8.1A] 3. $-4\sqrt{3}$ and $4\sqrt{3}$ [8.1C] 4. $-\frac{1}{2} - 2i$ and $-\frac{1}{2} + 2i$ [8.1C]
 5. -3 and -1 [8.2A] 6. $\frac{7-2\sqrt{7}}{7}$ and $\frac{7+2\sqrt{7}}{7}$ [8.2A] 7. $\frac{3}{4}$ and $\frac{4}{3}$ [8.3A] 8. $\frac{1}{2} - \frac{\sqrt{31}}{2}i$ and $\frac{1}{2} + \frac{\sqrt{31}}{2}i$ [8.3A]
 9. $x^2 + 3x = 0$ [8.1B] 10. $12x^2 - x - 6 = 0$ [8.1B] 11. $1 - i\sqrt{7}$ and $1 + i\sqrt{7}$ [8.2A] 12. $-2i$ and $2i$ [8.2A]
 13. $\frac{11-\sqrt{73}}{6}$ and $\frac{11+\sqrt{73}}{6}$ [8.3A] 14. Two complex number solutions [8.3A] 15. $\{x \mid -3 < x < \frac{5}{2}\}$ [8.5A]
 16. $\{x \mid x \leq -4 \text{ or } -\frac{3}{2} \leq x \leq 2\}$ [8.5A] 17. -64 and 27 [8.4A] 18. $\frac{5}{4}$ [8.4B] 19. -1 and 3 [8.4C]
 20. -1 [8.4C] 21.  $\{x \mid x < \frac{3}{2} \text{ or } x \geq 2\}$ [8.5A] 22. 
 $\{x \mid x \leq -3 \text{ or } \frac{1}{2} \leq x < 4\}$ [8.5A] 23. 4 [8.4B] 24. 5 [8.4B] 25. $\frac{-3-\sqrt{249}}{10}$ and $\frac{-3+\sqrt{249}}{10}$ [8.4C]
 26. $\frac{-11-\sqrt{129}}{2}$ and $\frac{-11+\sqrt{129}}{2}$ [8.4C] 27. $3x^2 + 8x - 3 = 0$ [8.1B] 28. -5 and $\frac{1}{2}$ [8.1A]
 29. $-1 + 3\sqrt{2}$ and $-1 - 3\sqrt{2}$ [8.1C] 30. $-3 + i$ and $-3 - i$ [8.3A] 31. 2 and 3 [8.4C]
 32. $-5, 5, -\sqrt{3}$, and $\sqrt{3}$ [8.4A] 33. $\frac{25}{18}$ [8.4B] 34. -8 and $\frac{1}{8}$ [8.4A] 35. 2 [8.4B]
 36. $5 + 3\sqrt{2}$ and $5 - 3\sqrt{2}$ [8.2A] 37. 5 and $-\frac{4}{9}$ [8.4C] 38. $\frac{1+\sqrt{5}}{6}$ and $\frac{1-\sqrt{5}}{6}$ [8.3A] 39. $\frac{5}{4}$ [8.4B]
 40. $\frac{-13+\sqrt{217}}{2}$ and $\frac{-13-\sqrt{217}}{2}$ [8.4C] 41. Two unequal real number solutions [8.3A] 42. $\{x \mid -2 \leq x \leq 5\}$ [8.5A]
 43. The sculling crew's rate of rowing in calm water is 6 mph. [8.6A] 44. The width of the rectangle is 5 cm. The length of the rectangle is 12 cm. [8.6A] 45. The integers are 2, 4, and 6 or $-6, -4$, and -2 . [8.6A] 46. Working alone, the new computer can print the payroll in 12 min. [8.6A] 47. The rate of the first car is 40 mph. The rate of the second car is 50 mph. [8.6A]

CHAPTER 8 TEST

1. -4 and $\frac{2}{3}$ [8.1A, How To 1] 2. $-\frac{2}{3}$ and $\frac{3}{2}$ [8.1A, How To 1] 3. $x^2 - 9 = 0$ [8.1B, How To 3]
 4. $2x^2 + 7x - 4 = 0$ [8.1B, Example 3] 5. $2 - 2\sqrt{2}$ and $2 + 2\sqrt{2}$ [8.1C, Example 4]
 6. $3 - \sqrt{11}$ and $3 + \sqrt{11}$ [8.2A, How To 3] 7. $\frac{3-\sqrt{15}}{3}$ and $\frac{3+\sqrt{15}}{3}$ [8.2A, How To 4]
 8. $\frac{1-\sqrt{3}}{2}$ and $\frac{1+\sqrt{3}}{2}$ [8.3A, How To 2] 9. $-2 - 2i\sqrt{2}$ and $-2 + 2i\sqrt{2}$ [8.3A, How To 3]
 10. $\frac{1}{4}$ [8.4A, How To 1] 11. $\sqrt{3}, -\sqrt{3}, 1$, and -1 [8.4A, Example 1] 12. 4 [8.4B, How To 2]
 13. No solution [8.4B, Example 3] 14. -9 and 2 [8.4C, How To 3] 15. 
 $\{x \mid x < -4 \text{ or } 2 < x < 4\}$ [8.5A, How To 2] 16.  $\{x \mid -4 < x \leq \frac{3}{2}\}$ [8.5A, How To 3]
 17. Two equal real number solutions [8.3A, How To 4] 18. The ball hits the basket 1.88 s after it is released. [8.6A, How To 1] 19. It would take Cora 6 h to stain the bookcase. [8.6A, Example 1]
 20. The rate of the canoe in calm water is 4 mph. [8.6A, Example 2]

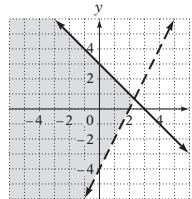
CUMULATIVE REVIEW EXERCISES

1. 14 [1.3B] 2. -28 [2.1C] 3. $-\frac{3}{2}$ [3.4A] 4. $y = x + 1$ [3.6A] 5. $-3xy(x^2 - 2xy + 3y^2)$ [5.5A]
 6. $(2x - 5)(3x + 4)$ [5.5D] 7. $(x + y)(a^n - 2)$ [5.5B] 8. $x^2 - 3x - 4 - \frac{6}{3x-4}$ [5.4B] 9. $\frac{x}{2}$ [6.1C]

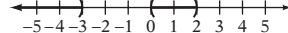
10. $2\sqrt{5}$ [3.1B] 11. $b = \frac{2S - an}{n}$ [2.1D] 12. $-8 - 14i$ [7.4C] 13. $1 - a$ [7.1A] 14. $\frac{x\sqrt[3]{4y^2}}{2y}$ [7.2D]

15. $-\frac{3}{2}$ and -1 [8.4C] 16. $-\frac{1}{2}$ [6.5A] 17. $2, -2, \sqrt{2}$, and $-\sqrt{2}$ [8.4A] 18. 0 and 1 [8.4B]

19. $\{x \mid -2 < x < \frac{10}{3}\}$ [2.5B] 20. $(\frac{5}{2}, 0)$ and $(0, -3)$ [3.3C] 21. [4.5A]



22. $(1, -1, 2)$ [4.3B] 23. $-\frac{7}{3}$ [6.1A] 24. $\{x \mid x \neq -3, 5\}$ [6.1A] 25. $\{x \mid x < -3 \text{ or } 0 < x < 2\}$ [8.5A]



26. $\{x \mid -3 < x \leq 1 \text{ or } x \geq 5\}$ [8.5A] 27. The lower limit is $9\frac{23}{64}$ in. The upper limit is

$9\frac{25}{64}$ in. [2.5C] 28. The area is $(x^2 + 6x - 16)$ ft². [5.3D] 29. Two complex number solutions [8.3A]

30. $m = -\frac{25,000}{3}$. The building depreciates $\frac{\$25,000}{3}$, or about \$8333, each year. [3.4A]

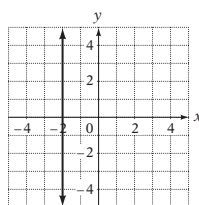
ANSWERS TO CHAPTER 9 SELECTED EXERCISES

PREP TEST

1. 1 [1.3B] 2. -7 [1.3B] 3. 30 [3.2A] 4. $h^2 + 4h - 1$ [3.2A] 5. $-\frac{2}{3}$ and 3 [8.1A]

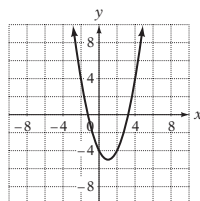
6. $2 - \sqrt{3}$ and $2 + \sqrt{3}$ [8.3A] 7. $y = \frac{1}{2}x - 2$ [2.1D] 8. Domain: $\{-2, 3, 4, 6\}$; Range: $\{4, 5, 6\}$

The relation is a function. [3.2A] 9. 8 [3.2A] 10. [3.3B]



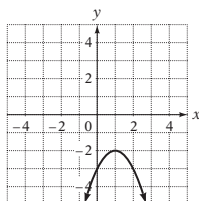
SECTION 9.1

5. -5 7. $x = 7$ 9.



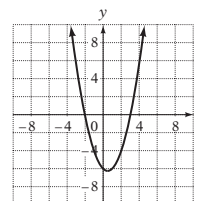
Vertex: $(1, -5)$
 Axis of symmetry: $x = 1$

11.



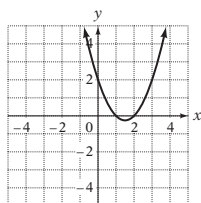
Vertex: $(1, -2)$
 Axis of symmetry: $x = 1$

13.



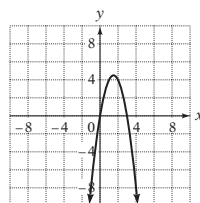
Vertex: $(\frac{1}{2}, -\frac{25}{4})$
 Axis of symmetry: $x = \frac{1}{2}$

15.



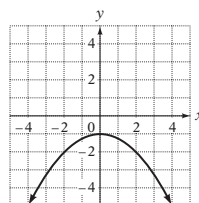
Vertex: $(\frac{3}{2}, -\frac{1}{4})$
 Axis of symmetry: $x = \frac{3}{2}$

17.



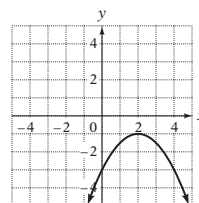
Vertex: $(\frac{3}{2}, \frac{9}{2})$
 Axis of symmetry: $x = \frac{3}{2}$

19.



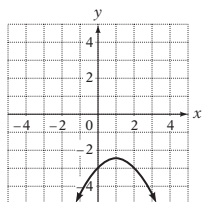
Vertex: $(0, -1)$
 Axis of symmetry: $x = 0$

21.



Vertex: $(2, -1)$
 Axis of symmetry: $x = 2$

23.


Vertex: $\left(1, -\frac{5}{2}\right)$

Axis of symmetry: $x = 1$

25. Domain: $\{x|x \in \text{real numbers}\}$

Range: $\{y|y \geq 2\}$

27. Domain: $\{x|x \in \text{real numbers}\}$

Range: $\{y|y \geq -5\}$

29. Domain: $\{x|x \in \text{real numbers}\}$

Range: $\{y|y \leq 0\}$

31. Domain: $\{x|x \in \text{real numbers}\}$

Range: $\{y|y \geq -7\}$

35. a. -4 and 1 b. -4 and 1

37. (3, 0) and (-3, 0)

39. (0, 0) and (-2, 0)

41. (4, 0) and (-2, 0)

43. $\left(-\frac{1}{2}, 0\right)$ and (3, 0)

45. $(-2 + \sqrt{7}, 0)$ and $(-2 - \sqrt{7}, 0)$

47. No x -intercepts

49. -1 and 5

51. $-\frac{4}{3}$ and 2

53. $\frac{1}{2}$

55. 0 and $\frac{4}{3}$

57. -2 and 2

59. $-3\sqrt{3}$ and $3\sqrt{3}$

61. $1 - 3\sqrt{2}$ and $1 + 3\sqrt{2}$

63. $-2 - i$ and $-2 + i$

65. $-2 - 3i$ and $-2 + 3i$

67. Two

69. One

71. No x -intercepts

73. Two

75. No x -intercepts

77. Two

79. a. $a > 0$ b. $a = 0$ c. $a < 0$

81. a. Minimum

83. a. Minimum

85. Minimum value: -2

87. Maximum value: -3

89. Maximum value: $\frac{9}{8}$

91. Minimum value: $-\frac{11}{4}$

93. Maximum value: $\frac{9}{4}$

95. Minimum value: $-\frac{1}{12}$

97. A price of \$250 will give the maximum revenue.

99. The vertex of the parabola is

approximately (42, 8500), so the maximum height of the plane is about 8500 m.

101. The minimum height is 24.36 ft.

103. The numbers are 10 and 10.

105. The numbers are -7 and 7.

107. The length is 100 ft and the width is 50 ft.

109. The dimensions are 350 ft by 525 ft.

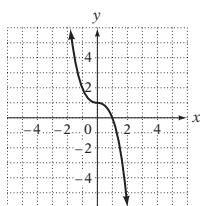
111. $-\frac{3}{2}$

113. -2 and 3

SECTION 9.2

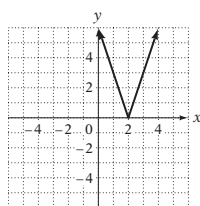
1. Yes 3. No 5. Yes 7. No

9.


Domain: $\{x|x \in \text{real numbers}\}$

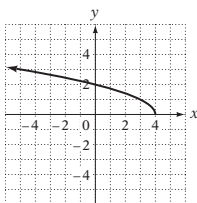
Range: $\{y|y \in \text{real numbers}\}$

11.


Domain: $\{x|x \in \text{real numbers}\}$

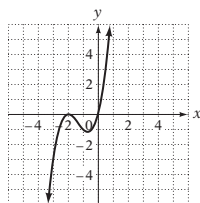
Range: $\{y|y \geq 0\}$

13.


Domain: $\{x|x \leq 4\}$

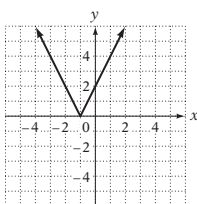
Range: $\{y|y \geq 0\}$

15.


Domain: $\{x|x \in \text{real numbers}\}$

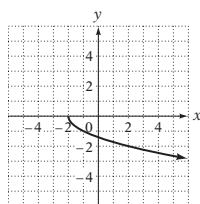
Range: $\{y|y \in \text{real numbers}\}$

17.


Domain: $\{x|x \in \text{real numbers}\}$

Range: $\{y|y \geq 0\}$

19.


Domain: $\{x|x \geq -2\}$

Range: $\{y|y \leq 0\}$

21. $a = 18$

23. 17

25. $f(14) = 8$

27. $\{x|-2 < x < 2\}$

29. $x = 1$

SECTION 9.3

 1. a. Yes b. Yes c. Yes d. No 3. 5 5. 1 7. 0 9. $-\frac{29}{4}$ 11. 2 13. 39 15. -8 17. Undefined

 19. -2 21. 7 25. 5 27. -13 29. -29 31. $8x - 13$ 33. 4 35. 3 37. $x + 4$ 39. 1 41. 5

 43. $x^2 + 1$ 45. 5 47. 11 49. $3x^2 + 3x + 5$ 51. -3 53. -27 55. $x^3 - 6x^2 + 12x - 8$

 57. a. $S[M(x)] = 80 + \frac{16,000}{x}$ b. \$83.20 c. When 5000 digital cameras are manufactured, the camera store sells each camera

 for \$83.20. 59. a. $I[n(m)] = 50,000m$ b. \$150,000 c. The garage's income from conversions during 3 months is \$150,000.

 61. a. $d[r(p)] = 0.90p - 1350$ b. $r[d(p)] = 0.90p - 1500$ c. $r[d(p)]$ (The cost is less.) 63. 0 65. -2 67. 7

 69. $h^2 + 6h$ 71. $2 + h$ 73. $2a + h$ 75. -1 77. -6 79. $6x - 13$
SECTION 9.4

3. a. Yes b. No 5. Yes 7. No 9. Yes 11. No 13. No 15. No 19. No 21. a. 4 b. 3 c. 6

 23. $\{(0, 1), (3, 2), (8, 3), (15, 4)\}$ 25. No inverse 27. $\{(-2, 0), (5, -1), (3, 3), (6, -4)\}$ 29. No inverse

 31. $f^{-1}(x) = \frac{1}{4}x + 2$ 33. $f^{-1}(x) = \frac{1}{2}x - 2$ 35. $f^{-1}(x) = 2x + 2$ 37. $f^{-1}(x) = -\frac{1}{2}x + 1$ 39. $f^{-1}(x) = \frac{3}{2}x - 6$

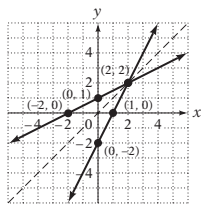
 41. $f^{-1}(x) = -3x + 3$ 43. $f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}$ 45. $f^{-1}(x) = \frac{1}{5}x + \frac{2}{5}$ 47. $f^{-1}(x) = \frac{1}{6}x + \frac{1}{2}$ 49. $\frac{5}{3}$ 51. 3

 53. Yes; Yes 55. Yes 57. No 59. Yes 61. Yes 63. $f^{-1}(x) = 16x$; The inverse function converts pounds to ounces.

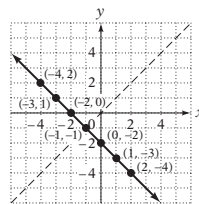
 65. $f^{-1}(x) = x - 30$; The inverse function converts a dress size in France to a dress size in the United States.

 67. $f^{-1}(x) = \frac{1}{90}x - \frac{13}{18}$; The inverse function gives the intensity of training percent for a given target heart rate for the athlete.

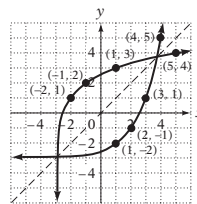
69.



71.



73.


CHAPTER 9 CONCEPT REVIEW*

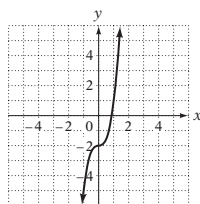
- Given a quadratic equation $y = ax^2 + bx + c$, the axis of symmetry is $x = -\frac{b}{2a}$. The coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. [9.1A]
- Given its vertex and axis of symmetry, you cannot accurately graph a parabola because the vertex and axis of symmetry are not enough to determine how wide or narrow the parabola is. [9.1A]
- If you are given the vertex of a parabola, you can determine the range of the function. If the parabola opens up, the range is all y values greater than or equal to the y -coordinate of the vertex. If the parabola opens down, the range is all y values less than or equal to the y -coordinate of the vertex. [9.1A]
- The following describes the effect of the discriminant on the x -intercepts of a parabola:
 - If $b^2 - 4ac = 0$, the parabola has one x -intercept.
 - If $b^2 - 4ac > 0$, the parabola has two x -intercepts.
 - If $b^2 - 4ac < 0$, the parabola has no x -intercepts. [9.1B]
- For the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, the coefficient of the x^2 term determines how wide or narrow the parabola is. The larger the absolute value of the coefficient of the x^2 term, the narrower the parabola; the smaller the absolute value of the coefficient of the x^2 term, the wider the parabola. [9.1A]
- The vertical-line test states that a graph defines a function if any vertical line intersects the graph at no more than one point. [9.2A]
- The graph of the absolute value of a linear polynomial is V-shaped. [9.2A]
- The four basic operations on functions are:
 - $(f + g)(x) = f(x) + g(x)$
 - $(f - g)(x) = f(x) - g(x)$
 - $(f \cdot g)(x) = f(x) \cdot g(x)$
 - $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$ [9.3A]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

9. The notation $(f \circ g)(x)$ refers to the composite function $f[g(x)]$. [9.3B]
 10. The horizontal-line test states that the graph of a function represents a 1–1 function if any horizontal line intersects the graph at no more than one point. [9.4A]
 11. For a function to have an inverse, it must be a 1–1 function. [9.4B]

CHAPTER 9 REVIEW EXERCISES

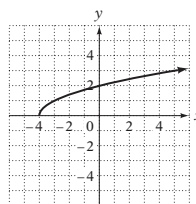
1. Yes [9.2A] 2. Yes [9.4A] 3.



Domain: $\{x | x \in \text{real numbers}\}$

Range: $\{y | y \in \text{real numbers}\}$ [9.2A]

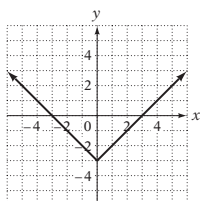
4.



Domain: $\{x | x \geq -4\}$

Range: $\{y | y \geq 0\}$ [9.2A]

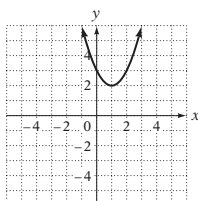
5.



Domain: $\{x | x \in \text{real numbers}\}$

Range: $\{y | y \geq -4\}$ [9.2A]

6.



Vertex: (1, 2)

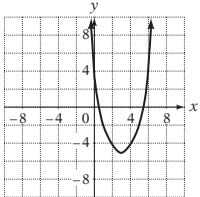
Axis of symmetry: $x = 1$ [9.1A]

7. Two x -intercepts [9.1B] 8. No x -intercepts [9.1B]

9. (0, 0) and $(-3, 0)$ [9.1B] 10. $(3 - \sqrt{2}, 0)$ and $(3 + \sqrt{2}, 0)$ [9.1B] 11. $-\frac{3}{2}$ and 5 [9.1B]
 12. $1 - 3i$ and $1 + 3i$ [9.1B] 13. 3 [9.1C] 14. $-\frac{17}{4}$ [9.1C] 15. 5 [9.3B] 16. 10 [9.3B]
 17. $12x^2 + 12x - 1$ [9.3B] 18. $6x^2 + 3x - 16$ [9.3B] 19. 7 [9.3A] 20. -9 [9.3A] 21. 70 [9.3A]
 22. $\frac{12}{7}$ [9.3A] 23. $f^{-1}(x) = -\frac{1}{6}x + \frac{2}{3}$ [9.4B] 24. $f^{-1}(x) = \frac{3}{2}x + 18$ [9.4B] 25. Yes [9.4B] 26. No [9.4B]
 27. $p^{-1}(x) = 2.5x - 37.5$; The inverse function gives a diver's depth below the surface of the water for a given pressure on the diver. [9.4B] 28. Fifty gloves should be made each month. The maximum profit for a month is \$5000. [9.1D]
 29. The dimensions are 7 ft by 7 ft. [9.1D]

CHAPTER 9 TEST

1.

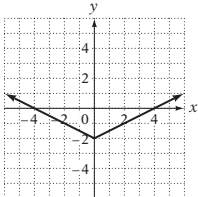


Vertex: (3, -5)

Axis of symmetry: $x = 3$

[9.1A, How To 2]

2.

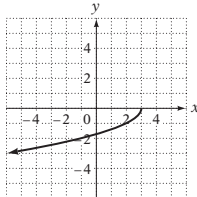


Domain: $\{x | x \in \text{real numbers}\}$

Range: $\{y | y \geq -2\}$

[9.2A, How To 3]

3.

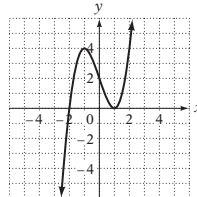


Domain: $\{x | x \leq 3\}$

Range: $\{y | y \leq 0\}$

[9.2A, How To 4]

4.



Domain: $\{x | x \in \text{real numbers}\}$

Range: $\{y | y \in \text{real numbers}\}$

[9.2A, How To 2]

5. No x -intercepts [9.1B, How To 7] 6. $(-\frac{2}{3}, 0)$ and (3, 0) [9.1B, How To 4] 7. 9 [9.1C, How To 8]

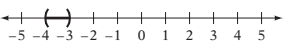
8. Domain: $\{x | x \in \text{real numbers}\}$; Range: $\{y | y \geq -7\}$ [9.1A, You Try It 2] 9. -2 [9.3A, Example 1]

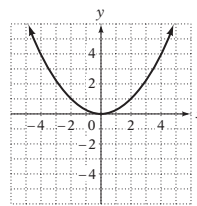
10. 234 [9.3A, Example 2] 11. $-\frac{13}{2}$ [9.3A, Example 3] 12. -5 [9.3A, Example 1] 13. -17 [9.3B, Example 4]

14. 31 [9.3B, Example 4] 15. $3x^2 - 1$ [9.3B, How To 4] 16. $f[g(x)] = 2x^2 - 4x - 5$ [9.3B, How To 4]

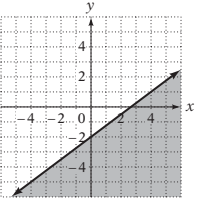
17. No [9.4B, How To 1] 18. $\{(6, 2), (5, 3), (4, 4), (3, 5)\}$ [9.4B, How To 1] 19. $f^{-1}(x) = \frac{1}{4}x + \frac{1}{2}$ [9.4B, How To 2]
 20. $f^{-1}(x) = 4x + 16$ [9.4B, You Try It 2] 21. Yes [9.4B, Example 3] 22. Yes [9.4B, Example 3]
 23. No [9.4A, Example 1] 24. $C^{-1}(x) = 0.8x - 4$; The inverse function gives the number of miles to the location for the given cost. [9.4B, Example 4] 25. Twenty-five speakers should be made. The minimum daily production cost is \$50. [9.1D, Example 7] 26. 14 and 14 [9.1D, Example 8] 27. Dimensions of 50 cm by 50 cm would give a maximum area of 2500 cm². [9.1D, How To 9]

CUMULATIVE REVIEW EXERCISES

1. $-\frac{23}{4}$ [1.3B] 2.  [1.1C] 3. 3 [2.1C] 4. $\{x | x < -2 \text{ or } x > 3\}$ [2.4B]
 5. The set of all real numbers [2.5B] 6. $-\frac{a^{10}}{12b^4}$ [5.1B] 7. $2x^3 - 4x^2 - 17x + 4$ [5.3B] 8. $\left(\frac{1}{2}, 3\right)$ [4.2A]
 9. $xy(x + 3y)(x - 2y)$ [5.6D] 10. -3 and 8 [5.7A] 11. $\{x | x < -3 \text{ or } x > 5\}$ [8.5A]
 12. $\frac{5}{2x - 1}$ [6.2B] 13. -2 [6.5A] 14. $-3 - 2i$ [7.4D] 15.



Vertex: (0, 0) [9.1A]
 Axis of symmetry: $x = 0$

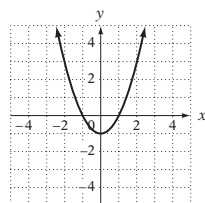
16.  [3.7A] 17. $y = -2x - 2$ [3.5B] 18. $y = -\frac{3}{2}x - \frac{7}{2}$ [3.6A]

19. $\frac{1}{2} + \frac{\sqrt{3}}{6}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{6}i$ [8.3A] 20. 3 [8.4B] 21. -3 [9.1C] 22. $\{1, 2, 4, 5\}$ [3.2A] 23. Yes [3.2A]
 24. 2 [7.3A] 25. 10 [9.3B] 26. $f^{-1}(x) = -\frac{1}{3}x + 3$ [9.4B] 27. The cost per pound of the mixture is \$3.96. [2.3A] 28. 25 lb of the 80% copper alloy must be used. [2.3B] 29. An additional 4.5 oz of insecticide are required. [6.4B] 30. It would take the larger pipe 4 min to fill the tank. [8.6A] 31. A force of 40 lb will stretch the spring 24 in. [6.6A] 32. The frequency is 80 vibrations/min. [6.6A]

ANSWERS TO CHAPTER 10 SELECTED EXERCISES

PREP TEST

1. $\frac{1}{9}$ [5.1B] 2. 16 [5.1B] 3. -3 [5.1B] 4. 0; 108 [3.2A] 5. -6 [2.1B] 6. -2 and 8 [8.1A]
 7. 6326.60 [1.3B] 8. [9.1A]

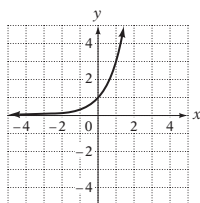


SECTION 10.1

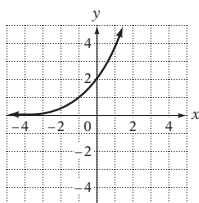
3. (iii) 5. a. 9 b. 1 c. $\frac{1}{9}$ 7. a. 16 b. 4 c. $\frac{1}{4}$ 9. a. 1 b. $\frac{1}{8}$ c. 16 11. a. 7.3891 b. 0.3679
 c. 1.2840 13. a. 54.5982 b. 1 c. 0.1353 15. a. 16 b. 16 c. 1.4768 17. a. 0.1353 b. 0.1353 c. 0.0111

19. $f(a) > f(b)$

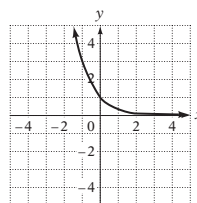
21.



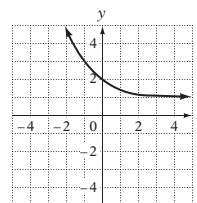
23.



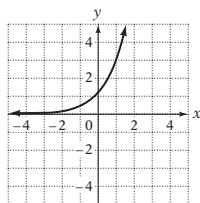
25.



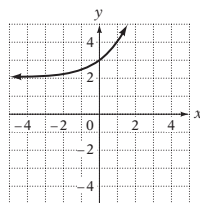
27.



29.



31.

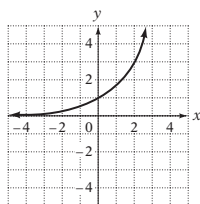


33. (0, 1)

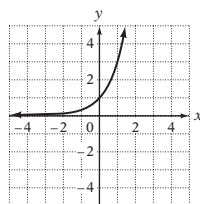
35. No x-intercept; (0, 1)

37. (i) and (iii), (ii) and (iv)

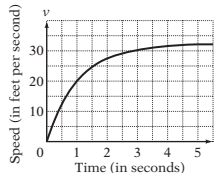
39.



41.



43. a.



b. The point (2, 27.7) means that after 2 s, the object is falling at a speed of 27.7 ft/s.

SECTION 10.2

3. $\log_5 25 = 2$ 5. $\log_4 \frac{1}{16} = -2$ 7. $\log_{10} x = y$ 9. $\log_a w = x$ 11. $3^2 = 9$ 13. $10^{-2} = 0.01$ 15. $e^y = x$

17. $b^v = u$ 19. 4 21. 7 23. 2 25. 3 27. 0 29. 4 31. 9 33. 64 35. $\frac{1}{7}$ 37. 1 39. 316.23

41. 0.02 43. 7.39 45. 0.61 47. 1 51. False 53. 0 55. 1 57. x 59. v 61. $x^2 + 1$ 63. $x^2 - x - 1$

65. $\log_8 x + \log_8 z$ 67. $5 \log_3 x$ 69. $\log_b r - \log_b s$ 71. $2 \log_3 x + 6 \log_3 y$ 73. $3 \log_7 u - 4 \log_7 v$

75. $2 \log_2 r + 2 \log_2 s$ 77. $2 \ln x + \ln y + \ln z$ 79. $\log_5 x + 2 \log_5 y - 4 \log_5 z$ 81. $2 \log_8 x - \log_8 y - 2 \log_8 z$

83. $\frac{3}{2} \log_4 x + \frac{1}{2} \log_4 y$ 85. $\frac{3}{2} \log_7 x - \frac{1}{2} \log_7 y$ 87. $\log_3 t - \frac{1}{2} \log_3 x$ 89. $\log_3 (x^3 y^2)$ 91. $\ln \frac{x^4}{y^2}$ 93. $\log_7 x^3$

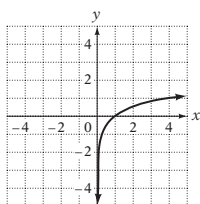
95. $\ln(x^3 y^4)$ 97. $\log_4 (x^2 y^2)$ 99. $\log_3 \frac{x^2 z^2}{y}$ 101. $\ln \frac{x}{y^2 z}$ 103. $\log_6 \sqrt{\frac{x}{y}}$ 105. $\log_4 \frac{s^2 r^2}{t^4}$ 107. $\ln \frac{x}{y^2 z^2}$ 109. $\log_4 \sqrt{\frac{x^3 z}{y^2}}$

111. $\log_2 \frac{\sqrt{xz}}{\sqrt[3]{y^2}}$ 113. 0.8617 115. 2.1133 117. -0.6309 119. 0.2727 121. 1.6826 123. 1.9266 125. 0.6752

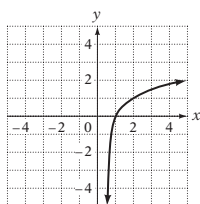
127. 2.6125 129. $\log_5 x = \frac{\log x}{\log 5}$ 131. a. False b. True c. False d. True e. False f. True

SECTION 10.3

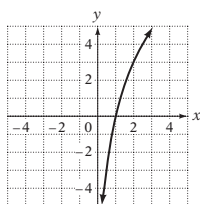
3.



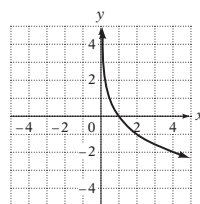
5.



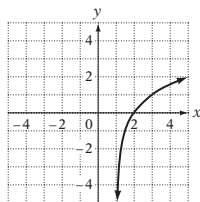
7.



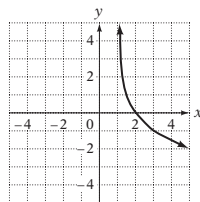
9.



11.

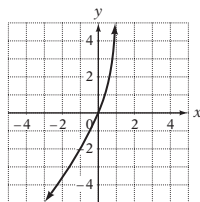


13.

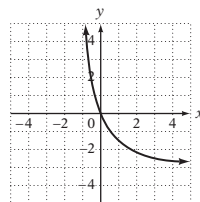


15. (1, 0)

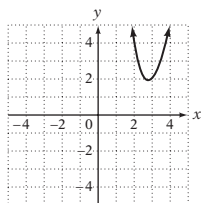
17.



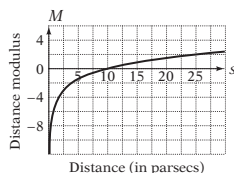
19.



21.



23. a.



b. The point (25.1, 2) means that a star that is 25.1 parsecs from Earth has a distance modulus of 2.

SECTION 10.4

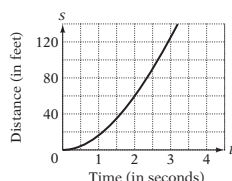
3. $x < 0$ 5. $-\frac{1}{3}$ 7. -3 9. 1 11. 6 13. $\frac{4}{7}$ 15. $\frac{9}{8}$ 17. 1.1133 19. -1.7626 21. 0.7740 23. -2.8074

25. 3.5850 27. 1.1309 29. -2.5850 31. 1.2224 33. 4.1166 35. 4.4629 37. $\frac{1}{2}$ 39. No solution 41. 3

43. 2 45. $\frac{11}{2}$ 47. 17.5327 49. 1 51. $\frac{5}{3}$ 53. No solution 55. $x - 2 < x$, and therefore $\log(x - 2) < \log x$.

This means that $\log(x - 2) - \log x < 0$ and cannot equal the positive number 3. 57. -6.1285 59. 3.0932

61. a.



b. It will take 2.64 s for the object to fall 100 ft.

SECTION 10.5

1. The value of the investment after 2 years is \$1172.
3. The investment will be worth \$15,000 in approximately 18 years.
5. a. After 3 h, the technetium level will be 21.2 mg. b. The technetium level will reach 20 mg after 3.5 h. 7. The half-life is 2.5 years.
9. The intensity of the earthquake was $794,328,235I_0$.
11. The Honshu earthquake was approximately 3.2 times stronger than the Quetta earthquake.
13. The magnitude of the earthquake for the seismogram given is 5.3.
15. The magnitude of the earthquake for the seismogram given is 5.6.
17. The pH of the baking soda solution is 8.4.
19. The range of hydrogen ion concentration is 2.5×10^{-7} to 5.0×10^{-6} .
21. A blue whale emits 188 decibels of sound.
23. The sound intensity of a cat's purr is 3.16×10^{-14} watts/cm².
25. 79.4% of the light will pass through the glass.
27. The thickness of the copper is 0.4 cm.
29. a. When $v = 160$, $f(375) = 0.43$ ft. b. When $v = 166.4$, $f(375) = 24.4$ ft.
31. The ball would travel 324 ft farther if air resistance were ignored.
33. The ratio of the current amount of uranium-235 to the original amount of uranium-235 is 0.016.
35. a. The value of the investment after 3 years is \$5986.09.
- b. The investor needs an annual interest rate of 11.2%.

CHAPTER 10 CONCEPT REVIEW*

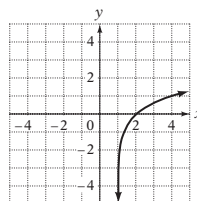
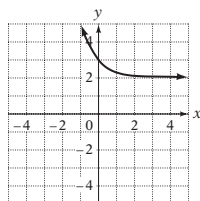
1. You know that a function is an exponential function if the base is a constant and the exponent contains a variable. [10.1A]
2. There are limitations on the value of the base of an exponential function; it is required to be positive. If the base were a negative number, the value of the function would be a complex number for some values of the variable. To avoid complex number values of a function, the base of the exponential function is always a positive number. Also, $b \neq 1$ because if $b = 1$ in the expression b^x , then the expression is always equal to 1. [10.1A]
3. The number e is an irrational number that is approximately equal to 2.71828183. [10.1A]
4. Because a logarithm is the inverse function of the exponential function, it has the same restrictions on the base b : $b > 0$, $b \neq 1$. See the answer to question 2. [10.2A]
5. The relationship between logarithmic functions and exponential functions is that $y = \log_b x$ is equivalent to $x = b^y$. [10.2A]
6. Remember that a logarithm is an exponent. When we multiply monomials, we add the exponents. Therefore, $\log_b(xy) = \log_b x + \log_b y$. [10.2B]
7. Remember that a logarithm is an exponent. When we divide monomials, we subtract the exponents. Therefore, $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$. [10.2B]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

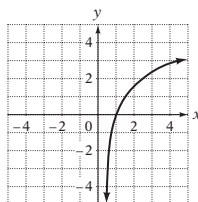
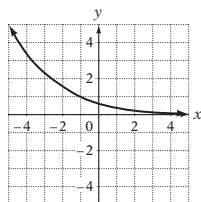
8. Using the Change-of-Base Formula, we can rewrite a logarithm in any base as a common logarithm or a natural logarithm. [10.2C]
9. The domain of a logarithmic function is $\{x | x > 0\}$. [10.3A]
10. The compound interest formula is $A = P(1 + i)^n$, where P is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods. [10.5A]
11. The Richter scale magnitude of an earthquake uses a logarithmic function to convert the intensity of shock waves I into a number M , which for most earthquakes is in the range of 0 to 10. The intensity I of an earthquake is often given in terms of the constant I_0 , where I_0 is the intensity of the smallest earthquake, called a zero-level earthquake, that can be measured on a seismograph near the earthquake's epicenter. An earthquake with an intensity I has a Richter scale magnitude of $M = \log \frac{I}{I_0}$, where I_0 is the measure of the intensity of a zero-level earthquake. [10.5A]

CHAPTER 10 REVIEW EXERCISES

1. 1 [10.1A] 2. $5^2 = 25$ [10.2A] 3. [10.1B] 4. [10.3A]



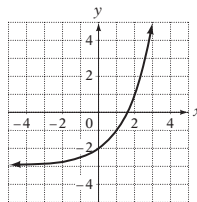
5. $\frac{2}{5} \log_3 x + \frac{4}{5} \log_3 y$ [10.2B] 6. $\log_3 \frac{x^2}{y^5}$ [10.2B] 7. -12 [10.4A] 8. $\frac{1}{4}$ [10.4B] 9. 1.7251 [10.2C]
10. 32 [10.2A] 11. 79 [10.4B] 12. 1000 [10.2A] 13. $\log_7 \sqrt[3]{xy^4}$ [10.2B] 14. $\frac{5}{2} \log_8 x - 3 \log_8 y$ [10.2B]
15. $\log_2 32 = 5$ [10.2A] 16. 0.4278 [10.2C] 17. -0.535 [10.4A] 18. $\frac{3}{2}$ [10.1A] 19. $\frac{11}{7}$ [10.4B]
20. 3 [10.4B] 21. [10.1B] 22. [10.3A] 23. 2 [10.2A]



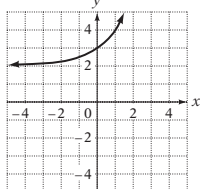
24. $\log_2 \sqrt[3]{\frac{x}{y}}$ [10.2B] 25. 1 [10.4A] 26. 1.5938 [10.4A] 27. $\frac{1}{5}$ [10.2A] 28. $\log_3 81 = 4$ [10.2A]
29. 5 [10.4B] 30. $\frac{2}{3} \log_5 x + \frac{1}{3} \log_5 y$ [10.2B] 31. -0.5207 [10.4A] 32. $\frac{1}{7}$ [10.1A] 33. 4 [10.2A]
34. 2 [10.4B] 35. 2.3219 [10.2C] 36. 3 [10.4A] 37. 625 [10.2A] 38. $\log_b \frac{x^3}{y^7}$ [10.2B]
39. 5 [10.1A] 40. 3.2091 [10.4A] 41. The value of the investment in 2 years is \$4692. [10.5A]
42. The Richter scale magnitude of the earthquake is 8.3. [10.5A] 43. The half-life is 27 days. [10.5A]
44. The sound emitted from the busy street corner is 107 decibels. [10.5A]

CHAPTER 10 TEST

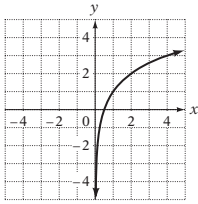
1. $f(0) = 1$ [10.1A, Example 1] 2. $f(-2) = \frac{1}{3}$ [10.1A, Example 2] 3. [10.1B, Example 5]



4. [10.1B, You Try It 5] 5. 2 [10.2A, You Try It 1] 6. $\frac{1}{9}$ [10.2A, How To 4]

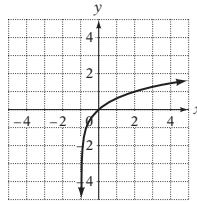


7.



[10.3A, You Try It 2]

8.



[10.3A, Example 3]

9. $\frac{1}{2}\log_6 x + \frac{3}{2}\log_6 y$ [10.2B, You Try It 5] 10. $\log_3 \sqrt{\frac{x}{y}}$ [10.2B, Example 6] 11. $\ln x - \frac{1}{2}\ln z$ [10.2B, How To 13]

12. $\ln \frac{x^3}{y\sqrt{z}}$ [10.2B, How To 14] 13. -2 [10.4A, How To 1] 14. -3 [10.4A, How To 2]

15. 2.5789 [10.4A, Example 2] 16. 6 [10.4B, How To 7] 17. 3 [10.4B, How To 7]

18. 1.3652 [10.2C, How To 16] 19. 2.6801 [10.2C, How To 16] 20. 3.1826 [10.4A, How To 4]

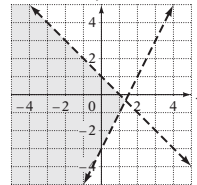
21. 6.0163 [10.4A, How To 4] 22. 5 [10.4B, How To 6] 23. The shard is 3099 years old. [10.5A, How To 2]

24. The intensity of the sound is 3.16×10^{-9} watts/cm². [10.5A, Example 1] 25. The half-life is 33 h. [10.5A, How To 2]

CUMULATIVE REVIEW EXERCISES

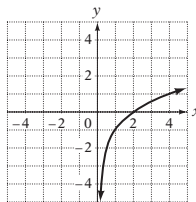
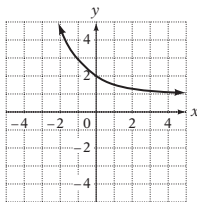
1. $\frac{8}{7}$ [2.1C] 2. $y = 2x - 6$ [3.6A] 3. $(4x^n + 3)(x^n + 1)$ [5.6C] 4. $\frac{x-3}{x+3}$ [6.3A] 5. $\frac{x\sqrt{y} + y\sqrt{x}}{x-y}$ [7.2D]

6. $2 + \sqrt{10}$ and $2 - \sqrt{10}$ [8.2A] 7. $3x^2 + 8x - 3 = 0$ [8.1B] 8. [4.5A] 9. $(0, -1, 2)$ [4.2B]



10. $-\frac{2x^2 - 17x + 13}{(x-2)(2x-3)}$ [6.2B] 11. $\{x | -5 \leq x \leq 1\}$ [8.5A] 12. $\{x | 1 \leq x \leq 4\}$ [2.5B]

13. [10.1B] 14. [10.3A] 15. 4 [10.1A]



16. 125 [10.2A] 17. $\log_b \frac{x^3}{y^5}$ [10.2B] 18. 1.7712 [10.2C] 19. 2 [10.4A] 20. $\frac{1}{2}$ [10.4B]

21. The customer can write at most 49 checks. [2.4C] 22. The cost per pound of the mixture is \$3.10. [2.3A]

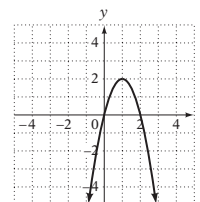
23. The rate of the wind is 25 mph. [6.5C] 24. The spring will stretch 10.2 in. [6.6A] 25. The cost of redwood is \$.40 per foot. The cost of fir is \$.25 per foot. [4.4B] 26. In approximately 10 years, the investment will be worth \$10,000. [10.5A]

ANSWERS TO CHAPTER 11 SELECTED EXERCISES

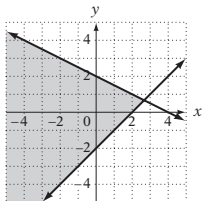
PREP TEST

1. 7.21 [3.1B] 2. $x^2 - 8x + 16; (x-4)^2$ [8.2A] 3. 0; -4 and 4 [8.1C] 4. $(1, -1)$ [4.1B]

5. $(1, -5)$ [4.2A] 6. Axis of symmetry: $x = 2$; Vertex: $(2, -2)$ [9.1A] 7. [9.1A]



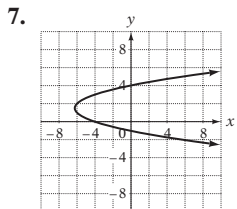
8. [4.5A]

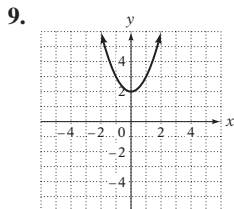


SECTION 11.1

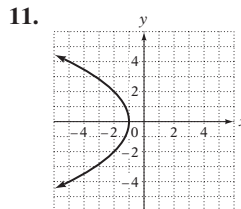
1. a. Vertical line b. The parabola opens up. 3. a. Horizontal line b. The parabola opens right.

5. a. Horizontal line b. The parabola opens left.

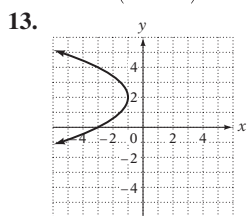

Axis of symmetry: $y = \frac{3}{2}$

Vertex: $\left(-\frac{25}{4}, \frac{3}{2}\right)$

Axis of symmetry: $x = 0$

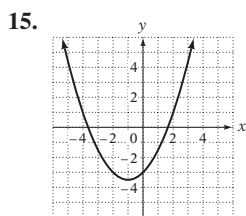
Vertex: (0, 2)

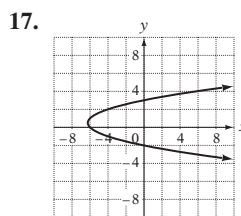

Axis of symmetry: $y = 0$

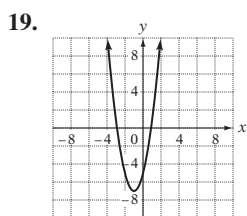
Vertex: (-1, 0)


Axis of symmetry: $y = 2$

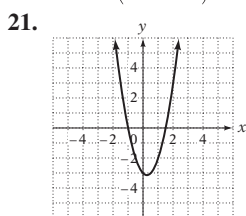
Vertex: (-1, 2)

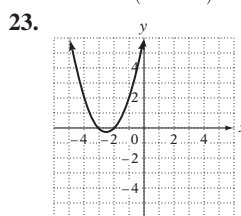

Axis of symmetry: $x = -1$

Vertex: $\left(-1, -\frac{7}{2}\right)$

Axis of symmetry: $y = \frac{1}{2}$

Vertex: $\left(-\frac{25}{4}, \frac{1}{2}\right)$

Axis of symmetry: $x = -1$

Vertex: (-1, -7)

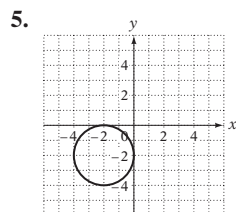
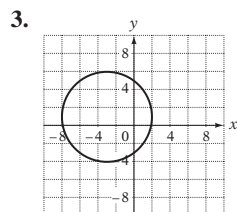
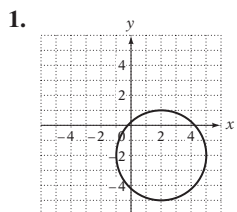

Axis of symmetry: $x = \frac{1}{4}$

Vertex: $\left(\frac{1}{4}, -\frac{25}{8}\right)$

Axis of symmetry: $x = -\frac{5}{2}$

Vertex: $\left(-\frac{5}{2}, -\frac{1}{4}\right)$

25. $\left(0, \frac{1}{8}\right)$

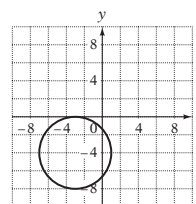
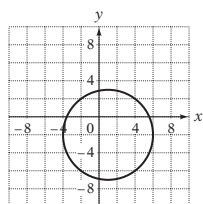
SECTION 11.2



7. $(x - 2)^2 + (y + 1)^2 = 4$

9. $x^2 + (y + 4)^2 = 7$ 11. $(x + 3)^2 + y^2 = 81$ 13. $(x + 1)^2 + (y - 1)^2 = 5$ 15. $(x + 1)^2 + y^2 = 20$

17. $x^2 + (y - 3)^2 = 65$ 19. $x^2 + y^2 = r^2$ 21. $(x - 1)^2 + (y + 2)^2 = 25$ 23. $(x + 3)^2 + (y + 4)^2 = 16$

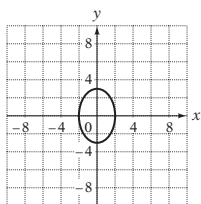


25. $(x - 5)^2 + (y - 4)^2 = 50$ 27. $(x - 6)^2 + y^2 = 24$ 29. $x^2 + (y + 7)^2 = 36$ 31. $(x - 2)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{85}{4}$

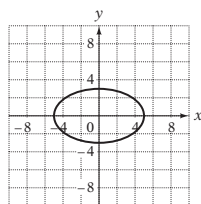
33. $\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{41}{2}$ 35. $x^2 + (y - 1)^2 = 13$ 37. $(x - 3)^2 + (y - 3)^2 = 9$

SECTION 11.3

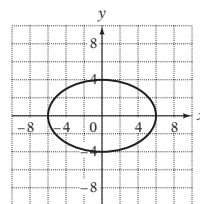
- 1.
- x
- intercepts:
- $(-2, 0)$
- and
- $(2, 0)$
-
- y
- intercepts:
- $(0, -3)$
- and
- $(0, 3)$



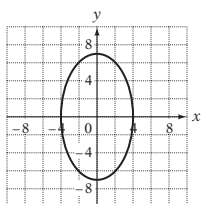
- 3.
- x
- intercepts:
- $(-5, 0)$
- and
- $(5, 0)$
-
- y
- intercepts:
- $(0, -3)$
- and
- $(0, 3)$



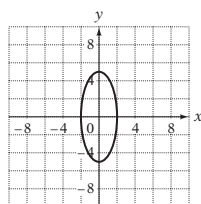
- 5.
- x
- intercepts:
- $(-6, 0)$
- and
- $(6, 0)$
-
- y
- intercepts:
- $(0, -4)$
- and
- $(0, 4)$



- 7.
- x
- intercepts:
- $(-4, 0)$
- and
- $(4, 0)$
-
- y
- intercepts:
- $(0, -7)$
- and
- $(0, 7)$

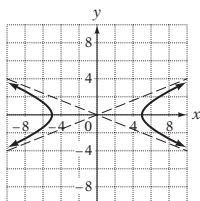


- 9.
- x
- intercepts:
- $(-2, 0)$
- and
- $(2, 0)$
-
- y
- intercepts:
- $(0, -5)$
- and
- $(0, 5)$

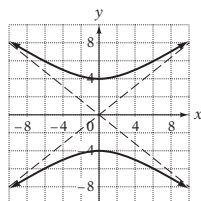


11. Rounder

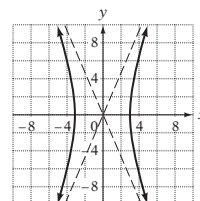
13. Vertices:
- $(-5, 0)$
- and
- $(5, 0)$
-
- Asymptotes:
- $y = -\frac{2}{5}x$
- and
- $y = \frac{2}{5}x$



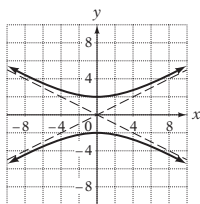
15. Vertices:
- $(0, -4)$
- and
- $(0, 4)$
-
- Asymptotes:
- $y = -\frac{4}{5}x$
- and
- $y = \frac{4}{5}x$



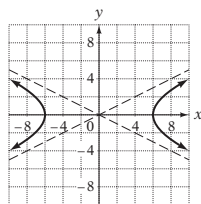
17. Vertices:
- $(-3, 0)$
- and
- $(3, 0)$
-
- Asymptotes:
- $y = -\frac{7}{3}x$
- and
- $y = \frac{7}{3}x$



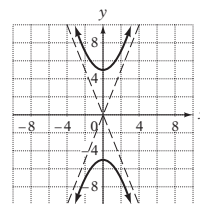
19. Vertices:
- $(0, -2)$
- and
- $(0, 2)$
-
- Asymptotes:
- $y = -\frac{1}{2}x$
- and
- $y = \frac{1}{2}x$



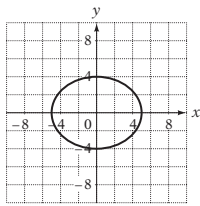
21. Vertices:
- $(-6, 0)$
- and
- $(6, 0)$
-
- Asymptotes:
- $y = -\frac{1}{2}x$
- and
- $y = \frac{1}{2}x$



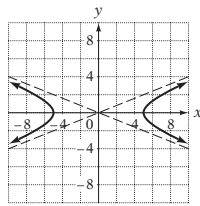
23. Vertices:
- $(0, -5)$
- and
- $(0, 5)$
-
- Asymptotes:
- $y = -\frac{5}{2}x$
- and
- $y = \frac{5}{2}x$



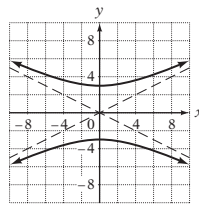
25. $\frac{x^2}{25} + \frac{y^2}{16} = 1$, ellipse



27. $\frac{x^2}{25} - \frac{y^2}{4} = 1$, hyperbola



29. $\frac{y^2}{9} - \frac{x^2}{36} = 1$, hyperbola



SECTION 11.4

1. (5, 19) and (-2, 5) 3. (-1, -2) and (2, 1) 5. (2, -2) 7. (2, 2) and $(-\frac{2}{9}, -\frac{22}{9})$ 9. (3, 2) and (2, 3)

11. $(\frac{\sqrt{3}}{2}, 3)$ and $(-\frac{\sqrt{3}}{2}, 3)$ 13. No real number solution 15. (1, 2), (1, -2), (-1, 2), and (-1, -2)

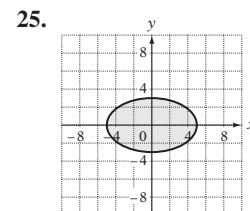
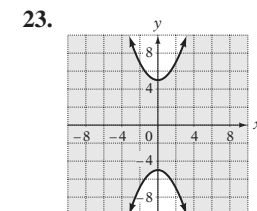
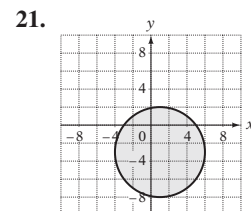
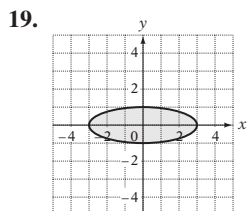
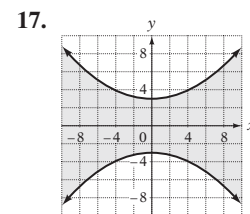
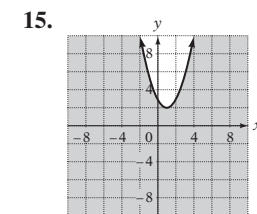
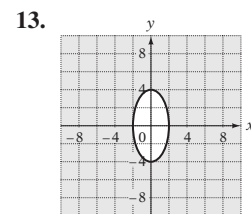
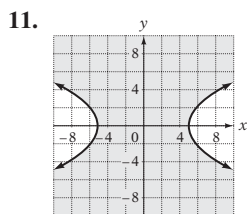
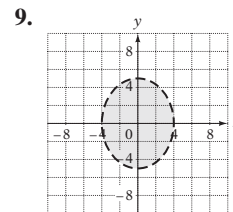
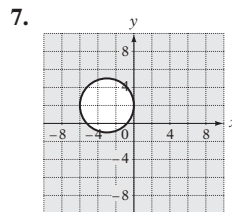
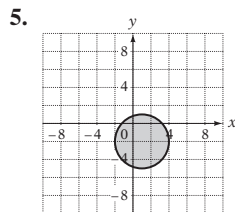
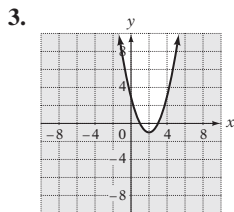
17. (3, 2), (3, -2), (-3, 2), and (-3, -2) 19. (2, 7) and (-2, 11) 21. (3, $\sqrt{2}$), (3, $-\sqrt{2}$), (-3, $\sqrt{2}$), and (-3, $-\sqrt{2}$)

23. No real number solution 25. ($\sqrt{2}$, 3), ($\sqrt{2}$, -3), ($-\sqrt{2}$, 3), and ($-\sqrt{2}$, -3) 27. (2, -1) and (8, 11) 29. (1, 0)

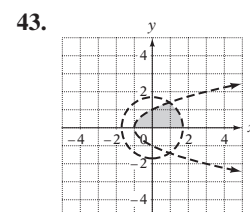
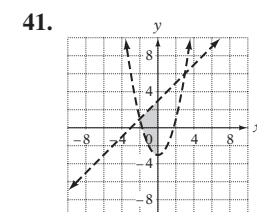
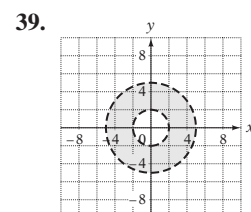
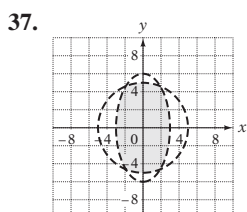
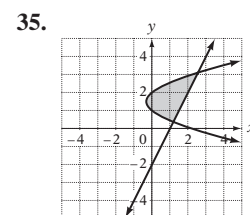
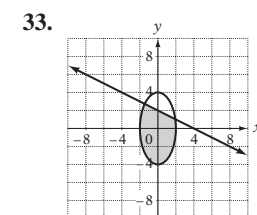
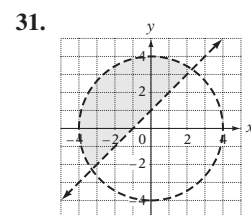
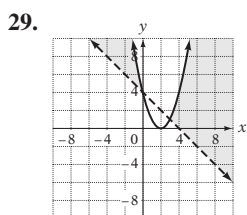
31. (2, -1) and (1, -2) 33. $(-\frac{1}{2}, \frac{9}{4})$ and (-4, 4) 35. (1, 2) 37. (1.755, 0.811) and (0.505, -0.986)

39. (5.562, -1.562) and (0.013, 3.987)

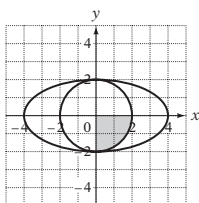
SECTION 11.5



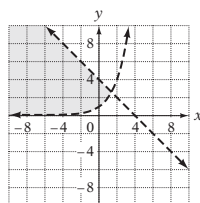
27. No



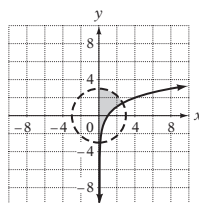
45.



47.



49.



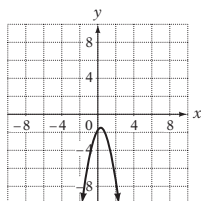
CHAPTER 11 CONCEPT REVIEW*

- The four conic sections described in this section are the parabola, the circle, the ellipse, and the hyperbola. They are constructed from the intersection of a plane and a right circular cone. [11.1A]
- Parabolas open not only up or down, but also to the left or right. Graphs of equations of the form $y = ax^2 + bx + c$, $a \neq 0$, open up or down. Graphs of equations of the form $x = ay^2 + by + c$, $a \neq 0$, open to the left or right. [11.1A]
- The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where r is the radius and (h, k) are the coordinates of the center of the circle. [11.2A]
- The general form of the equation of a circle is $x^2 + y^2 + ax + by + c = 0$. [11.2B]
- The distance formula is used to find the radius of a circle. In finding the radius, we are finding the distance from a point on the circle to the center of the circle. [11.2A]
- A circle is a special case of an ellipse. A circle occurs when $a^2 = b^2$ in the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [11.3A]
- The standard form of the equation of an ellipse with center at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [11.3A]
- The standard form of the equation of a hyperbola with center at the origin is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$. [11.3B]
- The asymptotes of a hyperbola are two lines that are “approached” by the hyperbola. As a point on the hyperbola gets farther from the origin, the hyperbola “gets closer to” the asymptotes. The equations of the asymptotes for a hyperbola with center at the origin are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$. [11.3B]
- False. A nonlinear system of equations is a system in which one or more of the equations is not a linear equation. In other words, a nonlinear system of equations can contain a linear equation. [11.4A]
- False. A nonlinear system of equations does not always have a solution set. The graphs of the equations in the system may not intersect, in which case the system does not have a solution. [11.4A]
- For the graph of the solution set of a quadratic inequality, dashed lines are used when the inequality is $<$ or $>$. Solid lines are used when the inequality is $\leq$ or $\geq$. [11.5A]
- When graphing the solution set of a nonlinear system of inequalities, it is helpful to use different kinds of shading for each region. This makes it easier to see the region of the plane represented by the intersection of the two shaded regions; it is where both kinds of shading overlap. [11.5B]

CHAPTER 11 REVIEW EXERCISES

1. Axis of symmetry: $x = \frac{1}{4}$

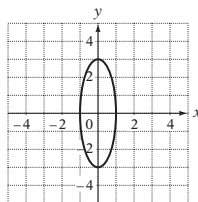
Vertex: $\left(\frac{1}{4}, -\frac{15}{8}\right)$



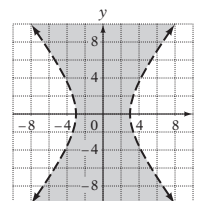
[11.1A]

2. Vertices: $(-1, 0)$, $(1, 0)$, $(0, -3)$, $(0, 3)$

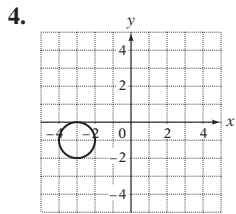
[11.3A]



3. [11.5A]

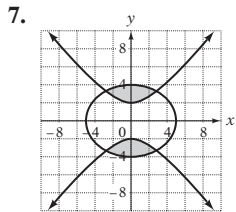


*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

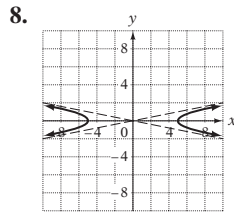


[11.2A]

5. Axis of symmetry: $x = 2$; vertex: $(2, 4)$ [11.1A]

6. $(1, \sqrt{5})$ and $(1, -\sqrt{5})$ [11.4A]


[11.5B]



[11.3B]

9. $(x + 1)^2 + (y - 2)^2 = 18$ [11.2A]

10. Axis of symmetry: $x = \frac{7}{2}$; vertex: $(\frac{7}{2}, \frac{17}{4})$ [11.1A]

11. $(1, \frac{3}{2})$ and $(\frac{2}{9}, \frac{1}{3})$ [11.4A]

12. $x^2 + (y + 3)^2 = 97$ [11.2A]

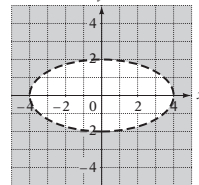
13. $(x + 1)^2 + (y - 5)^2 = 36$ [11.2A]

14. $(\sqrt{5}, 3)$, $(\sqrt{5}, -3)$, $(-\sqrt{5}, 3)$, and $(-\sqrt{5}, -3)$ [11.4A]

15. $(x + 2)^2 + (y - 1)^2 = 9$ [11.2B]

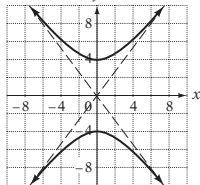
16. $(-2, -12)$ [11.4A]

17. [11.5A]


18. Vertices: $(0, -4)$ and $(0, 4)$

Asymptotes: $y = -\frac{4}{3}x$ and $y = \frac{4}{3}x$

[11.3B]

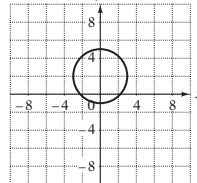
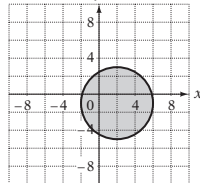


19.

[11.5A]

20.

[11.2A]


21. Vertices: $(-5, 0)$, $(5, 0)$, $(0, -3)$, and $(0, 3)$

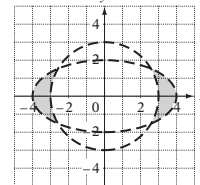
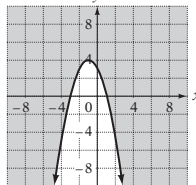
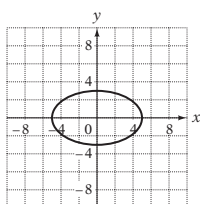
[11.3A]

22.

[11.5A]

23.

[11.5B]



24. [11.5B]

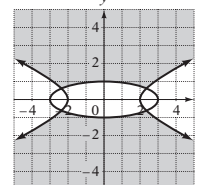
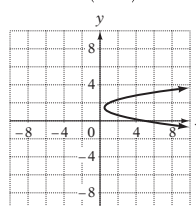
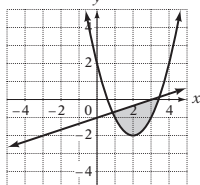
25. Axis of symmetry: $y = \frac{3}{2}$

Vertex: $(\frac{1}{2}, \frac{3}{2})$

[11.1A]

26.

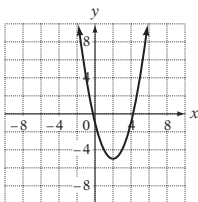
[11.5B]



27. Axis of symmetry: $x = 2$

Vertex: $(2, -5)$

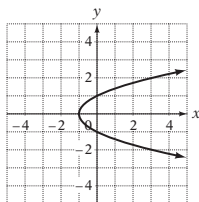
[11.1A]



28. Axis of symmetry: $y = 0$

Vertex: $(-1, 0)$

[11.1A]

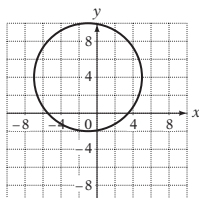


30. $(x - 3)^2 + (y + 2)^2 = 36$ [11.2B]

31. Center: $(-1, 4)$

Radius: 6

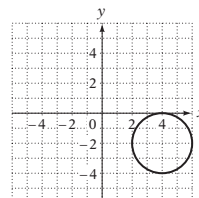
[11.2A]



32. Center: $(4, -2)$

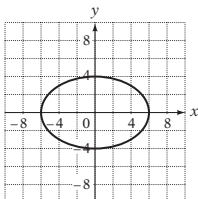
Radius: 2

[11.2B]



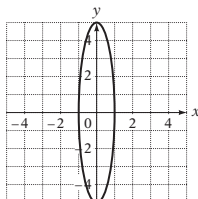
33. Vertices: $(-6, 0)$, $(6, 0)$, $(0, -4)$, and $(0, 4)$

[11.3A]



34. Vertices: $(-1, 0)$, $(1, 0)$, $(0, -5)$, and $(0, 5)$

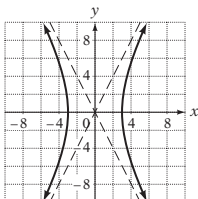
[11.3A]



35. Vertices: $(-3, 0)$ and $(3, 0)$

Asymptotes: $y = -2x$ and $y = 2x$

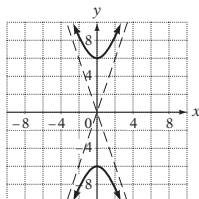
[11.3B]



36. Vertices: $(0, -6)$ and $(0, 6)$

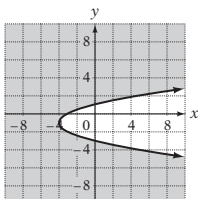
Asymptotes: $y = -3x$ and $y = 3x$

[11.3B]



- 38.

[11.5A]



CHAPTER 11 TEST

1. $(x + 3)^2 + (y + 3)^2 = 16$ [11.2A, Example 2]

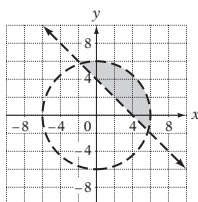
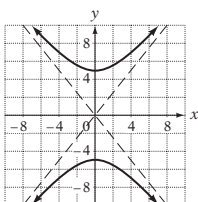
2. $(x + 2)^2 + (y - 1)^2 = 32$ [11.2A, How To 3]

3. [11.3B, Example 4]

- 4.

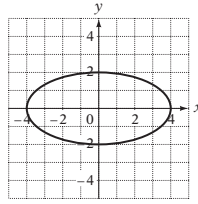
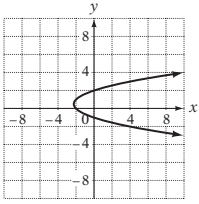
- [11.5B, Example 4]

5. $x = 3$ [11.1A, Example 1]

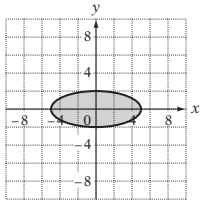


6. $(5, 1)$, $(-5, 1)$, $(5, -1)$, and $(-5, -1)$ [11.4A, Example 2] 7. $\left(\frac{3}{2}, \frac{1}{4}\right)$ [11.1A, Example 1]

8. [11.1A, Example 2] 9. [11.3A, Example 1]

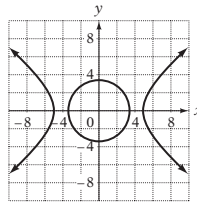


10. [11.5A, Example 1] 11. $(x + 2)^2 + (y - 4)^2 = 9$ [11.2A, Example 2]

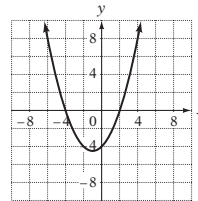
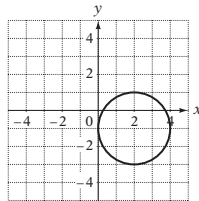


12. $\left(-\frac{9}{4}, \frac{1}{2}\right)$ and $(36, -4)$ [11.4A, Example 1] 13. $\left(\frac{2}{3}, \frac{4}{3}\right)$ and $(2, 0)$ [11.4A, How To 2]

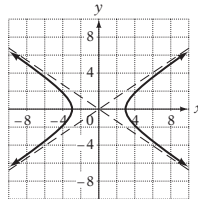
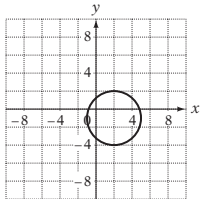
14. $(x + 1)^2 + (y + 3)^2 = 58$ [11.2A, How To 3] 15. $\emptyset$ [11.5B, Example 5]



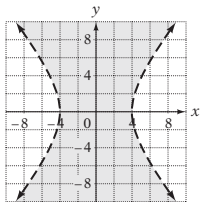
16. $(x - 2)^2 + (y + 1)^2 = 4$ [11.2B, Example 3] 17. [11.1A, Example 3]



18. [11.2A, Example 1] 19. [11.3B, Example 3]



20. [11.5A, You Try It 1]



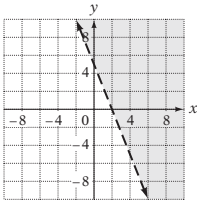
CUMULATIVE REVIEW EXERCISES

1. $\left\{x \mid -5 < x < 3\right\}$ [1.1C] 2. $\frac{38}{53}$ [2.1C] 3. $\left\{x \mid -\frac{4}{3} < x < 0\right\}$ [2.5B] 4. $y = -\frac{3}{2}x$ [3.5A]

5. $y = x - 6$ [3.6A] 6. $12a^3b - 9a^2b^3$ [5.3A] 7. $(x - y - 1)(x^2 - 2x + 1 + xy - y + y^2)$ [5.6B]

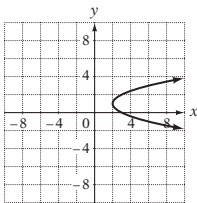
8. $\{x \mid -4 < x \leq 3\}$ [8.5A] 9. $\frac{x}{x + y}$ [6.1B] 10. $\frac{x - 5}{3x - 2}$ [6.2B] 11. $\frac{5}{2}$ [6.5A]

12. [3.7A] 13. $\frac{4a}{3b^{12}}$ [5.1B] 14. $2x^{3/4}$ [7.1B] 15. $42 + 2i$ [7.4C]



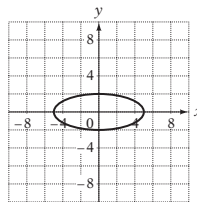
16. $\frac{-1 + \sqrt{7}}{2}$ and $\frac{-1 - \sqrt{7}}{2}$ [8.3A] 17. $(4, 2)$, $(4, -2)$, $(-4, 2)$, and $(-4, -2)$ [11.4A] 18. 6 [8.4B] 19. -20 [3.2A]
 20. $f^{-1}(x) = \frac{1}{4}x - 2$ [9.4B] 21. 0 [9.1C] 22. 400 [9.1D] 23. 5 [3.1B] 24. $(x + 1)^2 + (y - 2)^2 = 17$ [11.2A]

25. Axis of symmetry: $y = 1$
 Vertex: $(2, 1)$

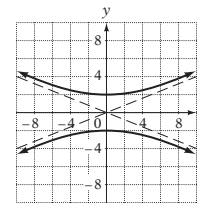


[11.1A]

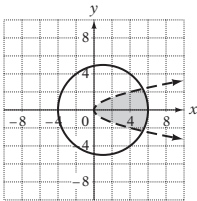
26. Vertices: $(-5, 0)$, $(5, 0)$, $(0, 2)$, and $(0, -2)$ [11.3A]



27. Vertices: $(0, -2)$ and $(0, 2)$
 Asymptotes: $y = -\frac{2}{5}x$ and $y = \frac{2}{5}x$ [11.3B]



28. [11.5B] 29. There were 82 adult tickets sold. [2.3A]



30. The rate of the motorcycle is 60 mph. [6.5C] 31. The rowing rate of the crew is 4.5 mph. [6.5C]
 32. The gear will make 18 revolutions/min. [6.6A]

ANSWERS TO CHAPTER 12 SELECTED EXERCISES

PREP TEST

1. 12 [1.2D] 2. $\frac{3}{4}$ [3.2A] 3. 18 [1.3B] 4. 96 [1.3B] 5. -410 [1.3B] 6. $\frac{4}{9}$ [1.2D]
 7. $x^2 + 2xy + y^2$ [5.3C] 8. $x^3 + 3x^2y + 3xy^2 + y^3$ [5.3C]

SECTION 12.1

3. 8 5. 2, 3, 4, 5 7. 3, 5, 7, 9 9. 0, -2, -4, -6 11. 2, 4, 8, 16 13. 2, 5, 10, 17 15. $0, \frac{7}{2}, \frac{26}{3}, \frac{63}{4}$
 17. 40 19. 110 21. 225 23. $\frac{1}{256}$ 25. 380 27. $-\frac{1}{36}$ 29. $a_n = n^2$ 31. 45 33. 20 35. 91
 37. 0 39. 432 41. $\frac{5}{6}$ 43. $2x + 2x^2 + 2x^3 + 2x^4 + 2x^5$ 45. $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$ 47. $x^2 + x^4 + x^6 + x^8 + x^{10}$
 49. $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16}$ 51. $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}$ 53. $\left(\sum_{i=1}^n i\right)^2$

SECTION 12.2

1. 141 3. 50 5. 17 7. $\frac{25}{2}$ 9. -109 11. $a_n = 3n - 2$ 13. $a_n = -3n + 6$ 15. $a_n = -2.5n + 9.5$
 17. 20 19. 20 21. 13 23. 20 25. No 27. Yes 29. 2500 31. -760 33. 189 35. 345
 37. $\frac{187}{2}$ 39. -180 41. $S_n = n^2$ 43. In 11 weeks, the person will walk 60 min per day. 45. There are 4077 seats in the loge seating section. 47. It will take 40 h of running to reach a weight of 186 lb. 49. $\log 8$

SECTION 12.3

3. 131,072 5. $\frac{128}{243}$ 7. 32 9. 6 and 4 11. -2 and $\frac{4}{3}$ 13. 12 and -48 15. No 17. Yes
19. 2186 21. $\frac{55}{27}$ 23. $\frac{2343}{64}$ 25. 62 27. $\frac{121}{243}$ 29. Positive 31. 9 33. $\frac{7}{9}$ 35. $\frac{8}{9}$ 37. $\frac{2}{9}$
39. $\frac{5}{11}$ 41. $\frac{1}{6}$ 43. Yes 45. No 47. The ball bounces to a height of 1.8 ft on the sixth bounce.
49. The average annual percent increase was 5.4%. 51. 19.7 million tons of waste were generated.
53. a. False b. False c. False d. False

SECTION 12.4

1. 6 3. 40,320 5. 1 7. 10 9. 1 11. 84 13. 21 15. 45 17. 1 19. 20 21. 11 23. 6
25. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ 27. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$ 29. $16m^4 + 32m^3 + 24m^2 + 8m + 1$
31. $32r^5 - 240r^4 + 720r^3 - 1080r^2 + 810r - 243$ 33. $a^{10} + 10a^9b + 45a^8b^2 + \cdots$ 35. $a^{11} - 11a^{10}b + 55a^9b^2 - \cdots$
37. $256x^8 + 1024x^7y + 1792x^6y^2 + \cdots$ 39. $65,536x^8 - 393,216x^7y + 1,032,192x^6y^2 - \cdots$ 41. $x^7 + 7x^5 + 21x^3 + \cdots$
43. $-560x^4$ 45. $-6x^{10}y^2$ 47. $126y^5$ 49. $5n^3$ 51. False 53. a. False b. False c. False d. False
- e. False f. True

CHAPTER 12 CONCEPT REVIEW*

- The general term of a sequence is the n th term of the sequence, written a_n . [12.1A]
- A sequence is an ordered list of numbers. A series is the indicated sum of the terms of a sequence. [12.1B]
- The index of a summation is a letter, often n or i , used in the summation notation. For example, for the summation notation $\sum_{n=1}^s 3n$, n is the index of the summation. [12.1B]
- To find the common difference of an arithmetic sequence, subtract the first term from the second term. [12.2A]
- The Formula for the n th Term of an Arithmetic Sequence is $a_n = a_1 + (n - 1)d$, where a_1 is the first term of the sequence and d is the common difference of the sequence. [12.2A]
- The Formula for the Sum of n Terms of an Arithmetic Series is $S_n = \frac{n}{2}(a_1 + a_n)$, where a_1 is the first term of the sequence. [12.2B]
- To find the common ratio of a geometric sequence, divide the second term of the sequence by the first term of the sequence. [12.3A]
- The Formula for the n th Term of a Geometric Sequence is $a_n = a_1r^{n-1}$, where a_1 is the first term of the sequence and r is the common ratio of the sequence. [12.3A]
- The Formula for the Sum of n Terms of a Finite Geometric Series is $S_n = \frac{a_1(1 - r^n)}{1 - r}$, where a_1 is the first term of the sequence and r is the common ratio of the sequence. [12.3B]
- We can find the sum of an infinite geometric series by using the formula $S_n = \frac{a_1}{1 - r}$, where a_1 is the first term of the sequence and r is the common ratio of the sequence, where $|r| < 1$. Note that this is the Formula for the Sum of n Terms of a Finite Geometric Series without the factor $(1 - r^n)$ in the numerator. In an infinite geometric series, r^n is approximately 0 for large values of n , so $(1 - r^n)$ approaches $(1 - 0) = 1$. [12.3C]
- In Pascal's Triangle, the first row consists of two 1's. The first and last number of each subsequent row is a 1. Each of the other numbers in a row can be obtained by finding the sum of the two closest numbers above it. The first six rows of Pascal's Triangle are:

		1		1										
		1		2		1								
		1		3		3		1						
		1		4		6		4		1				
		1		5		10		10		5		1		
		1		6		15		20		15		6		1

 [12.4A]
- $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ [12.4A]

*Note: The numbers in brackets following the answers in the Concept Review are a reference to the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Prep Tests, Concept Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

$$13. \binom{6}{4} = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = 15 \quad [12.4A]$$

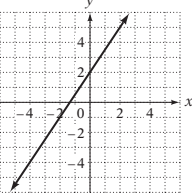
CHAPTER 12 REVIEW EXERCISES

1. $2 + 2x + 2x^2 + 2x^3$ [12.1B] 2. 28 [12.1A] 3. $\frac{19}{30}$ [12.3C] 4. $\frac{1}{3}$ [12.3C] 5. 1830 [12.2B]
 6. 35 [12.2B] 7. $\frac{23}{99}$ [12.3C] 8. 84 [12.4A] 9. 726 [12.3B] 10. 22 terms [12.2A] 11. 0.996 [12.3B]
 12. $5670x^4y^4$ [12.4A] 13. $-\frac{5}{27}$ [12.1A] 14. 1127 [12.3B] 15. $a_n = 5n - 12$ [12.2A]
 16. $-42,240x^4y^7$ [12.4A] 17. $243\sqrt{3}$ [12.3A] 18. 2000 [12.2B] 19. 220 [12.4A] 20. $-\frac{13}{60}$ [12.1B]
 21. $2x + 2x^2 + \frac{8}{3}x^3 + 4x^4 + \frac{32}{5}x^5$ [12.1B] 22. $\frac{1}{729}$ [12.3A]
 23. $x^5 - 15x^4y^2 + 90x^3y^4 - 270x^2y^6 + 405xy^8 - 243y^{10}$ [12.4A] 24. $\frac{16}{5}$ [12.3C] 25. 6 [12.3C] 26. 99 [12.4A]
 27. 77 [12.2A] 28. $4x + 4x^2 + 4x^3 + 4x^4 + 4x^5 + 4x^6$ [12.1B] 29. 15 [12.2A] 30. 135 [12.3A]
 31. 9 [12.3C] 32. $\frac{1}{2}$ [12.1A] 33. 44 [12.2A] 34. 729 [12.2B] 35. -168 [12.3B] 36. 21 [12.4A]
 37. $2268x^3y^6$ [12.4A] 38. 45 [12.2B] 39. $\frac{5}{8}$ [12.1A] 40. $a_n = 4n - 8$ [12.2A] 41. $\frac{2}{125}$ [12.3A]
 42. $\frac{11}{30}$ [12.3C] 43. 143 [12.2A] 44. 341 [12.3B] 45. A person will be walking 60 min each day in
 16 weeks. [12.2C] 46. The temperature of the spa after 8 h is 67.7°F . [12.3D] 47. The total salary for the
 nine-month period is \$24,480. [12.2C] 48. At the beginning of the seventh hour, the amount of radioactive material in the
 sample is 3.125 mg. [12.3D]

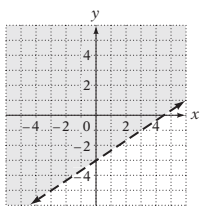
CHAPTER 12 TEST

1. 34 [12.2B, How To 5] 2. 84 [12.4A, How To 5] 3. 32 [12.3A, Example 1] 4. $\frac{1}{2}$ [12.1A, Example 2]
 5. $3x + 3x^2 + 3x^3 + 3x^4$ [12.1B, How To 6] 6. 468 [12.2B, Example 2] 7. 16 [12.3C, Example 5]
 8. 16 [12.2A, How To 3] 9. $\frac{7}{30}$ [12.3C, Example 6] 10. 70 [12.4A, How To 4]
 11. $\frac{2}{27}$ [12.3A, How To 1] 12. $-280x^4y^3$ [12.4A, Example 3] 13. -115 [12.2A, Example 1]
 14. $a_n = -3n + 15$ [12.2A, How To 2] 15. -66 [12.3B, How To 3] 16. $\frac{7}{6}, \frac{8}{7}$ [12.1A, How To 2]
 17. $\frac{665}{32}$ [12.3B, How To 3] 18. 1575 [12.2B, Example 2] 19. The inventory after the October 1 shipment was 2550 skeins.
 [12.2C, Example 4] 20. There will be 20 mg of radioactive material in the sample at the beginning of the fifth day.
 [12.3D, Example 8]

CUMULATIVE REVIEW EXERCISES

1.  [3.3B] 2. $2(x^2 + 2)(x^4 - 2x^2 + 4)$ [5.6D] 3. $\frac{x^2 + 5x - 2}{(x + 2)(x - 1)}$ [6.2B]
 4. $f(-2) = 14$ [3.2A] 5. $4y\sqrt{x} - y\sqrt{2}$ [7.2C] 6. $\frac{1}{4} + \frac{\sqrt{55}}{4}i$ and $\frac{1}{4} - \frac{\sqrt{55}}{4}i$ [8.3A] 7. 4 [7.3A]
 8. $(x + 1)^2 + (y + 1)^2 = 34$ [11.2A] 9. $(\frac{7}{6}, \frac{1}{2})$ [4.2A] 10. -10 [4.3A] 11. $\{x | x < -2 \text{ or } x > 2\}$ [2.4B]

12. [3.7A] 13. $\frac{1}{2} \log_5 x - \frac{1}{2} \log_5 y$ [10.2B] 14. 3 [10.4A]



15. $a_5 = 20; a_6 = 30$ [12.1A] 16. 6 [12.1B] 17. -103 [12.2A] 18. $\frac{9}{5}$ [12.3C]

19. $\frac{7}{15}$ [12.3C] 20. $12xy^5$ [12.4A] 21. The amount of water that must be added is 120 oz. [2.3B]

22. The new computer takes 24 min to complete the payroll. The old computer takes 40 min to complete the payroll. [6.5B]

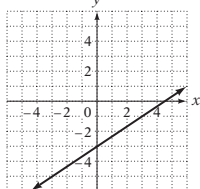
23. The rate of the boat was 6.25 mph. The rate of the current was 1.25 mph. [6.5C] 24. The half-life is 55 days. [10.5A]

25. There are 1536 seats in the theater. [12.2C] 26. The height of the fifth bounce is 2.6 ft. [12.3D]

FINAL EXAM

1. -31 [1.2D] 2. -1 [1.3B] 3. -16 [9.3B] 4. 8 [2.1B] 5. $\frac{2}{3}$ [2.1C] 6. $-\frac{2}{3}$ and 4 [2.5A]

7. [3.3C] 8. $y = -3x + 7$ [3.5B] 9. $y = -\frac{2}{3}x - \frac{1}{3}$ [3.6A] 10. $6a^3 - 5a^2 + 10a$ [5.3A]



x-intercept: $(\frac{9}{2}, 0)$

y-intercept: $(0, -3)$

11. $(2 - xy)(4 + 2xy + x^2y^2)$ [5.6B] 12. $(x - y)(1 - x)(1 + x)$ [5.6D] 13. $x^2 - 2x - 3 - \frac{5}{2x - 3}$ [5.4B]

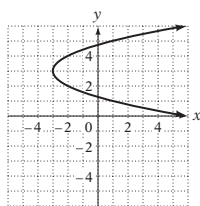
14. $\frac{x(x - 1)}{2x - 5}$ [6.1D] 15. $\frac{-10x}{(x + 2)(x - 3)}$ [6.2B] 16. $\frac{x + 3}{x + 1}$ [6.3A] 17. $-\frac{7}{4}$ [6.5A] 18. $d = \frac{a_n - a_1}{n - 1}$ [2.1D]

19. $\frac{y^4}{162x^3}$ [5.1B] 20. $\frac{1}{64x^8y^5}$ [7.1A] 21. $-2x^2y\sqrt{2y}$ [7.2B] 22. $\frac{x^2\sqrt{2y}}{2y^2}$ [7.2D] 23. $\frac{7}{25} - \frac{26}{25}i$ [7.4D]

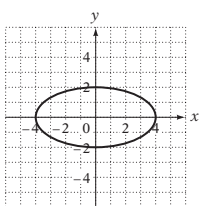
24. $2x^2 - 3x - 2 = 0$ [8.1B] 25. $\frac{3 + \sqrt{17}}{4}$ and $\frac{3 - \sqrt{17}}{4}$ [8.3A] 26. 27 and -8 [8.4A] 27. $\frac{3}{2}$ and -2 [8.4C]

28. Axis of symmetry: $y = 3$ 29. Vertices: $(-4, 0)$, $(4, 0)$, $(0, -2)$, and $(0, 2)$ 30. $f^{-1}(x) = \frac{3}{2}x + 6$ [9.4B]

Vertex: $(-3, 3)$



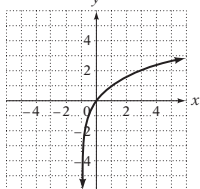
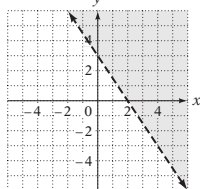
[11.1A]



[11.3A]

31. $(3, 4)$ [4.2A] 32. 10 [4.3A] 33. $(\frac{5}{2}, -\frac{3}{2})$ [11.4A] 34. $(\frac{3}{2}, \infty)$ [2.4B]

35. $\{x | -4 < x < -1\}$ [2.5B] 36. [3.7A] 37. [10.3A]



38. $\log_2 \frac{a^2}{b^2}$ [10.2B] 39. $\frac{27}{8}$ [10.4B] 40. 625 [12.2B] 41. $\frac{5}{9}$ [12.3C] 42. $144x^7y^2$ [12.4A]
43. The range of scores is $69 \leq x \leq 100$. [2.4C] 44. The cyclist traveled 40 mi. [2.3C] 45. The amount invested at 8.5% is \$8000. The amount invested at 6.4% is \$4000. [4.1C] 46. The width is 7 ft. The length is 20 ft. [8.6A]
47. 200 additional shares would need to be purchased. [6.4B] 48. The rate of the plane is 420 mph. [4.4B]
49. The distance traveled is 88 ft. [7.3B] 50. The rate of the plane for the first 360 mi is 120 mph. [8.6A]
51. The intensity is 200 foot-candles. [6.6A] 52. The rate of the boat in calm water is 12.5 mph. The rate of the current is 2.5 mph. [4.4A] 53. The value of the investment is \$4785.65. [10.5A] 54. The golf ball will bounce to a height of 1.64 ft. [12.3D]

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Glossary

abscissa The first number of an ordered pair; it measures a horizontal distance and is also called the first coordinate of an ordered pair. [3.1]

absolute value of a number The distance of the number from zero on the number line. [1.1, 2.5]

absolute value equation An equation containing the absolute-value symbol. [2.5]

absolute value function A function containing an absolute value symbol. [9.2]

addition method An algebraic method of finding an exact solution of a system of linear equations. [4.2]

additive inverse of a polynomial The polynomial with the sign of every term changed. [5.2]

additive inverses Numbers that are the same distance from zero on the number line but lie on different sides of zero; also called opposites. [1.1]

analytic geometry Geometry in which a coordinate system is used to study relationships between variables. [3.1]

arithmetic progression A sequence in which the difference between any two consecutive terms is constant; also called an arithmetic sequence. [12.2]

arithmetic sequence A sequence in which the difference between any two consecutive terms is constant; also called an arithmetic progression. [12.2]

arithmetic series The indicated sum of the terms of an arithmetic sequence. [12.2]

asymptotes The two straight lines that a hyperbola “approaches.” [11.3]

axes The two number lines that form a rectangular coordinate system; also called coordinate axes. [3.1]

axes of symmetry of a hyperbola The line that passes through the vertices of the hyperbola. [11.3]

axis of symmetry of a parabola A line of symmetry that passes through the vertex of the parabola and is parallel to the y-axis for an equation of the form $y = ax^2 + bx + c$ or parallel to the x-axis for an equation of the form $x = ay^2 + by + c$. [9.1, 11.1]

base In an exponential expression, the number that is taken as a factor as many times as indicated by the exponent. [1.2]

binomial A polynomial of two terms. [5.2]

center of a circle The central point that is equidistant from all the points that make up a circle. [11.2]

center of an ellipse The intersection of the two axes of symmetry of the ellipse. [11.3]

circle The set of all points (x, y) in the plane that are a fixed distance from a given point (h, k) called the center. [11.2]

clearing denominators Removing denominators from an equation that contains fractions by multiplying each side of the equation by the LCM of the denominators. [6.5]

closed interval In interval notation, an interval that contains its endpoints. [1.1]

coefficient The number part of a variable term. [1.3]

cofactor of an element of a matrix $(-1)^{i+j}$ times the minor of that element, where i is the row number of the element and j is its column number. [4.3]

combined variation A variation in which two or more types of variation occur at the same time. [6.6]

combining like terms Using the Distributive Property to add the coefficients of like variable terms; adding like terms of a variable expression. [1.3]

common difference of an arithmetic sequence The difference between any two consecutive terms in an arithmetic sequence. [12.2]

common logarithms Logarithms to the base 10. [10.2]

common monomial factor A monomial factor that is a factor of the terms in a polynomial. [5.5]

common ratio of a sequence In a geometric sequence, each successive term of the sequence is the same nonzero constant multiple of the preceding term. This common multiple is called the common ratio of the sequence. [12.3]

completing the square Adding to a binomial the constant term that makes it a perfect-square trinomial. [8.2]

complex fraction A fraction whose numerator or denominator contains one or more fractions. [1.2, 6.3]

complex number A number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. [7.4]

composite number A number that has whole-number factors besides 1 and itself. For example, 12 is a composite number. [1.1]

composition of two functions The operation on two functions f and g denoted by $f \circ g$. The value of the composition of f and g is given by $(f \circ g)(x) = f[g(x)]$. [9.3]

compound inequality Two inequalities joined with a connective word such as *and* or *or*. [2.4]

compound interest Interest that is computed not only on the original principal but also on the interest already earned. [10.5]

compound interest formula

$A = P(1 + i)^n$, where P is the original value of an investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods. [10.5]

conditional equation An equation that is true if the variable it contains is replaced by the proper value. $x + 2 = 5$ is a conditional equation. [2.1]

conic section A curve that can be constructed from the intersection of a plane and a right circular cone. The four conic sections are the parabola, hyperbola, ellipse, and circle. [11.1]

conjugates Binomial expressions that differ only in the sign of a term. The expressions $a + b$ and $a - b$ are conjugates. [7.2]

consecutive even integers Even integers that follow one another in order. [2.2]

consecutive integers Integers that follow one another in order. [2.2]

consecutive odd integers Odd integers that follow one another in order. [2.2]

constant function A function given by $f(x) = b$, where b is a constant. Its graph is a horizontal line passing through $(0, b)$. [3.3]

constant of proportionality k in a variation equation; also called the constant of variation. [6.6]

constant of variation k in a variation equation; also called the constant of proportionality. [6.6]

constant term A term that contains no variable part. [1.3, 5.2]

contradiction An equation in which any replacement for the variable will result in a false equation. $x = x + 1$ is a contradiction. [2.1]

coordinate axes The two number lines that form a rectangular coordinate system; also called axes. [3.1]

coordinates of a point The numbers in the ordered pair that is associated with the point. [3.1]

cube root of a perfect cube One of the three equal factors of the perfect cube. [5.6]

cubic function A third-degree polynomial function. [5.2]

degree of a monomial The sum of the exponents on the variables. [5.1]

degree of a polynomial The greatest of the degrees of any of its terms. [5.2]

dependent system of equations A system of equations whose graphs coincide. [4.1]

dependent variable A variable whose value depends on that of another variable known as the independent variable. [3.2]

descending order The terms of a polynomial in one variable are arranged in descending order when the exponents on the variable decrease from left to right. [5.2]

determinant A number associated with a square matrix. [4.3]

difference of two perfect squares A polynomial in the form $a^2 - b^2$. [5.6]

difference of two perfect cubes A polynomial in the form $a^3 - b^3$. [5.6]

direct variation A special function that can be expressed as the equation $y = kx$, where k is a constant called the constant of variation or the constant of proportionality. [6.6]

discriminant For an equation of the form $ax^2 + bx + c = 0$, the quantity $b^2 - 4ac$ is called the discriminant. [8.3]

domain The set of the first coordinates of all the ordered pairs of a function. [3.2]

double function A function that pairs a number with twice that number. [3.2]

double root When a quadratic equation has two solutions that are the same number, the solution is called a double root of the equation. [8.1]

element of a matrix A number in a matrix. [4.3]

elements of a set The objects in the set. [1.1]

ellipse An oval shape that is one of the conic sections. [11.3]

empty set The set that contains no elements. [1.1]

endpoints of an interval In interval notation, the values that mark the interval's beginning and end, whether or not either or both of those values are included in the interval. [1.1]

equation A statement of the equality of two mathematical expressions. [2.1]

equation in two variables An equation in which two different variables appear. [3.1]

equivalent equations Equations that have the same solution. [2.1]

evaluating a function Determining $f(x)$ for a given value of x . [3.2]

evaluating a variable expression Replacing the variables in a variable expression by numerical values and then simplifying the resulting expression. [1.3]

even integer An integer that is divisible by 2. [2.2]

expanding by cofactors A technique for finding the value of a 3×3 or larger determinant. [4.3]

expansion of the binomial $(a + b)^n$

To write the expression as the sum of its terms. [12.4]

exponent In an exponential expression, the raised number that indicates how many times the factor, or base, occurs in the multiplication. [1.2]

exponential decay equation Any equation that can be written in the form $A = A_0 b^{kt}$, where A is the size at time t , A_0 is the initial size, b is between 0 and 1, and k is a positive real number. [10.5]

exponential equation An equation in which a variable occurs in the exponent. [10.4]

exponential form The expression 2^6 is in exponential form. Compare *factored form*. [1.2]

exponential function The exponential function with base b is defined by $f(x) = b^x$, where b is a positive real number not equal to 1. [10.1]

exponential growth equation Any equation that can be written in the form $A = A_0 b^{kt}$, where A is the size at time t , A_0 is the initial size, b is greater than 1, and k is a positive real number. [10.5]

extraneous solution When each side of an equation is raised to an even power, the resulting equation may have a solution that is not a solution of the original equation. Such a solution is called an extraneous solution. [7.3]

factor One number is a factor of another when it can be divided into that other number with a remainder of zero. [5.5]

factored form The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in factored form. Compare *exponential form*. [1.2]

factoring a polynomial Writing the polynomial as a product of other polynomials. [5.5]

factoring a quadratic trinomial Expressing the trinomial as the product of two binomials. [5.5]

factoring by grouping Grouping the terms of the polynomial in such a way that a common binomial factor is found. [5.5]

finite sequence A sequence that contains a finite number of terms. [12.1]

finite set A set for which all the elements can be listed. [1.1]

first coordinate of an ordered pair

The first number of the ordered pair; it measures a horizontal distance and is also called the abscissa. [3.1]

first-degree equation in one variable

An equation in which all variables have an exponent of 1 and all variable terms include one and the same variable. [2.1]

FOIL A method of finding the product of two binomials. The letters stand for First, Outer, Inner, and Last. [5.3]

formula A literal equation that states a rule about measurement. [2.1]

function A relation in which no two ordered pairs that have the same first coordinate have different second coordinates. [3.2]

function notation Notation used for those equations that define functions. The letter f is commonly used to name a function. [3.2]

general term of a sequence In the sequence $a_1, a_2, a_3, \dots, a_n, \dots$, the general term of the sequence is a_n . [12.1]

geometric progression A sequence in which each successive term of the sequence is the same nonzero constant multiple of the preceding term; also called a geometric sequence. [12.3]

geometric sequence A sequence in which each successive term of the sequence is the same nonzero constant multiple of the preceding term; also called a geometric progression. [12.3]

geometric series The indicated sum of the terms of a geometric sequence. [12.3]

graph of a function A graph of the ordered pairs that belong to the function. [3.3]

graph of a quadratic inequality in two variables A region of the plane that is bounded by one of the conic sections. [11.5]

graph of a real number A heavy dot placed directly above the number on the number line. [1.1]

graph of an ordered pair The dot drawn at the coordinates of the point in the plane. [3.1]

graph of $x = a$ A vertical line passing through the point $(a, 0)$. [3.3]

graph of $y = b$ A horizontal line passing through the point $(0, b)$. [3.3]

graphing a point in the plane Placing a dot at the location given by the ordered pair; also called plotting a point in the plane. [3.1]

graphing, or plotting, an ordered pair

Placing a dot, on a rectangular coordinate system, at the location given by the ordered pair. [3.1]

greater than A number that lies to the right of another number on the number line is said to be greater than that number. [1.1]

greatest common factor (GCF) The GCF of two or more numbers is the largest integer that divides evenly into all of them. [1.2]

grouping symbols Parentheses (), brackets [], braces { }, the absolute value symbol, and the fraction bar. [1.2]

half-open interval In set-builder notation, an interval that contains one of its endpoints. [1.1]

half-plane The solution set of a linear inequality in two variables. [3.7]

horizontal-line test A graph of a function represents the graph of a one-to-one function if any horizontal line intersects the graph at no more than one point. [9.4]

hyperbola A conic section formed by the intersection of a cone and a plane perpendicular to the base of the cone. [11.3]

hypotenuse In a right triangle, the side opposite the 90° angle. [3.1, 7.3]

identity An equation in which any replacement for the variable will result in a true equation. $x + 2 = x + 2$ is an identity. [2.1]

imaginary number A number of the form ai , where a is a real number and $i = \sqrt{-1}$. [7.4]

imaginary part of a complex number For the complex number $a + bi$, b is the imaginary part. [7.4]

inconsistent system of equations A system of equations that has no solution. [4.1]

independent system of equations

A system of equations whose graphs intersect at only one point. [4.1]

independent variable A variable whose value determines that of another variable known as the dependent variable. [3.2]

index In the expression $\sqrt[n]{a}$, n is the index of the radical. [7.1]

index of a summation The variable used in summation notation. [12.1]

infinite geometric series The indicated sum of the terms of an infinite geometric sequence. [12.3]

infinite sequence A sequence that contains an infinite number of terms. [12.1]

infinite set A set in which the list of elements continues without end. [1.1]

input In a function, the value of the independent variable. [9.1]

input/output table A table that shows the results of evaluating a function for various values of the independent variable. [9.1]

integers The numbers $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$ [1.1]

intersection of two sets The set that contains all elements that are common to both of the sets. [1.1]

interval notation A type of set notation in which the property that distinguishes the elements of the set is their location within a specified interval. [1.1]

inverse of a function The set of ordered pairs formed by reversing the coordinates of each ordered pair of the function. [9.4]

inverse variation A function that can be expressed as the equation $y = \frac{k}{x}$, where k is a constant. [6.6]

irrational number The decimal representation of an irrational number never terminates or repeats and can only be approximated. [1.1]

joint variation A variation in which a variable varies directly as the product of two or more variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. [6.6]

leading coefficient In a polynomial, the coefficient of the variable with the largest exponent. [5.2]

least common multiple (LCM) The LCM of two or more numbers is the smallest number that is a multiple of each of those numbers. [1.2]

least common multiple (LCM) of two or more polynomials The simplest polynomial of least degree that contains the factors of each polynomial. [6.2]

leg In a right triangle, one of the two sides that are not opposite the 90° angle. [3.1, 7.3]

less than A number that lies to the left of another number on the number line is said to be less than that number. [1.1]

like terms Terms of a variable expression that have the same variable part. Having no variable part, constant terms are like terms. [1.3]

linear equation in three variables An equation of the form $Ax + By + Cz = D$, where A , B , and C are coefficients of the variables and D is a constant. [4.2]

linear equation in two variables An equation of the form $y = mx + b$ or $Ax + By = C$. [3.3]

linear function A function that can be expressed in the form $f(x) = mx + b$. Its graph is a straight line. [3.3, 5.2]

linear inequality in two variables An inequality of the form $y > mx + b$ or $Ax + By > C$. (The symbol $>$ could be replaced by $\geq$, $<$, or $\leq$.) [3.7]

literal equation An equation that contains more than one variable. [2.1]

logarithm For b greater than zero and not equal to 1, the statement $y = \log_b x$ (the logarithm of x to the base b) is equivalent to $x = b^y$. [10.2]

lower limit In a tolerance, the lowest acceptable value. [2.5]

main fraction bar The fraction bar that is placed between the numerator and denominator of a complex fraction. [1.2]

matrix A rectangular array of numbers. [4.3]

maximum value of a function The greatest value that the function can take on. [9.1]

midpoint of a line segment The point on a line segment that is equidistant from its endpoints. [3.1]

minimum value of a function The least value that the function can take on. [9.1]

minor of an element The minor of an element in a 3×3 determinant is the 2×2 determinant obtained by eliminating the row and column that contain that element. [4.3]

monomial A number, a variable, or a product of a number and variables; a polynomial of one term. [5.1, 5.2]

multiplicative inverse The multiplicative inverse of a nonzero real number a is $\frac{1}{a}$; also called the reciprocal. [1.2]

n factorial The product of the first n natural numbers; n factorial is written $n!$ [12.4]

natural exponential function The function defined by $f(x) = e^x$, where $e \approx 2.71828$. [10.1]

natural logarithm When e (the base of the natural exponential function) is used as the base of a logarithm, the logarithm is referred to as the natural logarithm and is abbreviated $\ln x$. [10.2]

natural numbers The numbers 1, 2, 3, ...; also called the positive integers. [1.1]

negative integers The numbers ..., -3, -2, -1. [1.1]

negative reciprocal The negative reciprocal of a nonzero real number a is $-1/a$. [3.6]

negative slope The slope of a line that slants downward to the right. [3.4]

nonfactorable over the integers A polynomial is nonfactorable over the integers if it does not factor using only integers. [5.5]

nonlinear system of equations A system of equations in which one or more of the equations are not linear equations. [11.4]

n th root of a A number b such that $b^n = a$. The n th root of a can be written $a^{1/n}$ or $\sqrt[n]{a}$. [7.1]

null set The set that contains no elements. [1.1]

numerical coefficient The number part of a variable term. [1.3]

odd integer An integer that is not divisible by 2. [2.2]

one-to-one function In a one-to-one function, given any y , there is only one x that can be paired with the given y . [9.4]

open interval In set-builder notation, an interval that does not contain its endpoints. [1.1]

opposites Numbers that are the same distance from zero on the number line but lie on different sides of zero; also called additive inverses. [1.1]

order $m \times n$ A matrix of m rows and n columns is of order $m \times n$. [4.3]

Order of Operations Agreement Rules that specify the order in which we perform operations in simplifying numerical expressions. [1.2]

ordered pair A pair of numbers expressed in the form (a, b) and used to locate a point in the plane determined by a rectangular coordinate system. [3.1]

ordered triple Three numbers expressed in the form (x, y, z) and used to locate a point in the xyz -coordinate system. [4.2]

ordinate The second number of an ordered pair; it measures a vertical distance and is also called the second coordinate of an ordered pair. [3.1]

origin The point of intersection of the two number lines that form a rectangular coordinate system. [3.1]

output The result of evaluating a function. [9.1]

parabola The graph of a quadratic function is called a parabola. [9.1]

parallel lines Lines that have the same slope and thus do not intersect. [3.6]

Pascal's Triangle A pattern for the coefficients of the terms of the expansion of the binomial $(a + b)^n$ that can be formed by writing the coefficients in a triangular array known as Pascal's Triangle. [12.4]

percent mixture problem A problem that involves combining two solutions or alloys that have different concentrations of the same substance. [2.3]

perfect cube The product of the same three factors. [5.6]

perfect square The product of a term and itself. [5.6]

perfect-square trinomial The square of a binomial. [5.6]

perpendicular lines Lines that intersect at right angles. The slopes of perpendicular lines are negative reciprocals of each other. [3.6]

plane The infinitely extending, two-dimensional space in which a rectangular coordinate system lies; may be pictured as a large, flat piece of paper. [3.1]

plotting a point in the plane Placing a dot at the location given by the ordered pair; also called graphing a point in the plane. [3.1]

point-slope formula The equation $y - y_1 = m(x - x_1)$, where m is the slope of a line and (x_1, y_1) is a point on the line. [3.5]

polynomial A variable expression in which the terms are monomials. [5.2]

polynomial function An expression whose terms are monomials. [5.2]

positive integers The numbers 1, 2, 3, ...; also called the natural numbers. [1.1]

positive slope The slope of a line that slants upward to the right. [3.4]

power In an exponential expression, the number of times (indicated by the exponent) that the factor, or base, occurs in the multiplication. [1.2]

prime number A number whose only whole-number factors are 1 and itself. For example, 17 is a prime number. [1.1]

prime polynomial A polynomial that is nonfactorable over the integers. [5.5]

principal square root The positive square root of a number. [7.1]

Principle of Zero Products If the product of two factors is zero, then at least one of the factors must be zero. [5.7]

product of the sum and difference of two terms A polynomial that can be expressed in the form $(a + b)(a - b)$. [5.3, 5.6]

proportion An equation that states the equality of two ratios or rates. [6.4]

Pythagorean Theorem The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. [3.1]

quadrant One of the four regions into which a rectangular coordinate system divides the plane. [3.1]

quadratic equation An equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$; also called a second-degree equation. [5.7, 8.1]

quadratic equation in standard form A quadratic equation written in descending order and set equal to zero. [5.7]

quadratic formula A general formula, derived by applying the method of completing the square to the standard form of a quadratic equation, used to solve quadratic equations. [8.3]

quadratic function A function that can be expressed by the equation $f(x) = ax^2 + bx + c$, where a is not equal to zero. [5.2, 9.1]

quadratic inequality An inequality that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where a is not equal to zero. The symbols $\leq$ and $\geq$ can also be used. [8.5]

quadratic in form A trinomial is quadratic in form if it can be written as $au^2 + bu + c$. [5.6, 8.4]

quadratic trinomial A trinomial of the form $ax^2 + bx + c$, where a and b are nonzero coefficients and c is a nonzero constant. [5.5]

radical In a radical expression, the symbol $\sqrt{\quad}$. [7.1]

radical equation An equation that contains a variable expression in a radicand. [7.3]

radical function A function containing a radical. [9.2]

radicand The expression under a radical sign. [7.1]

radius of a circle The fixed distance, from the center of a circle, of all points that make up the circle. [11.2]

range The set of the second coordinates of all the ordered pairs of a function. [3.2]

rate The quotient of two quantities that have different units. [6.4]

rate of work That part of a task that is completed in one unit of time. [6.5]

ratio The quotient of two quantities that have the same unit. [6.4]

rational expression A fraction in which the numerator or denominator is a polynomial. [6.1]

rational function A function that is written in terms of a rational expression. [6.1]

rational number A number of the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero. [1.1]

rationalizing the denominator The procedure used to remove a radical from the denominator of a fraction. [7.2]

real numbers The rational numbers and the irrational numbers taken together. [1.1]

real part of a complex number For the complex number $a + bi$, a is the real part. [7.4]

reciprocal The reciprocal of a nonzero real number a is $\frac{1}{a}$; also called the multiplicative inverse. [1.2]

reciprocal of a rational expression The rational expression with the numerator and denominator interchanged. [6.1]

rectangular coordinate system A coordinate system formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. [3.1]

relation A set of ordered pairs. [3.2]

Remainder Theorem If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$. [5.4]

repeating decimal A decimal formed when division of the numerator of its fractional counterpart by the denominator results in a decimal part wherein one or more digits repeat infinitely. [1.1]

right triangle A triangle that contains one 90° angle. [3.1]

root(s) of an equation The replacement value(s) of the variable that will make the equation true; also called the solution(s) of the equation. [2.1]

roster method A method of designating a set by enclosing a list of its elements in braces. [1.1]

scatter diagram A graph of ordered-pair data. [3.1]

scientific notation Notation in which a number is expressed as the product of a number between 1 and 10 and a power of 10. [5.1]

second coordinate of an ordered pair

The second number of the ordered pair; it measures a vertical distance and is also called the ordinate. [3.1]

second-degree equation An equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$; also called a quadratic equation. [8.1]

sequence An ordered list of numbers. [12.1]

series The indicated sum of the terms of a sequence. [12.1]

set A collection of objects. [1.1]

set-builder notation A method of designating a set that makes use of a variable and a certain property that only elements of that set possess. [1.1]

sigma notation Notation used to represent a series in a compact form; also called summation notation. [12.1]

simplest form of a rational expression A rational expression is in simplest form when the numerator and denominator have no common factors. [6.1]

slope A measure of the slant, or tilt, of a line. The symbol for slope is m . [3.4]

slope-intercept form of a straight line The equation $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept. [3.4]

solution of a system of equations in three variables An ordered triple that is a solution of each equation of the system. [4.2]

solution of a system of equations in two variables An ordered pair that is a solution of each equation of the system. [4.1]

solution of an equation in three variables An ordered triple (x, y, z) whose coordinates make the equation a true statement. [4.2]

solution of an equation in two variables An ordered pair whose coordinates make the equation a true statement. [3.1]

solution set of a system of inequalities The intersection of the solution sets of the individual inequalities. [4.5]

solution set of an inequality A set of numbers, each element of which, when substituted for the variable, results in a true inequality. [2.4]

solution(s) of an equation The replacement value(s) of the variable that will make the equation true; also called the root(s) of the equation. [2.1]

solving an equation Finding a root, or solution, of the equation. [2.1]

square function A function that pairs a number with its square. [3.2]

square matrix A matrix that has the same number of rows as columns. [4.3]

square of a binomial A polynomial that can be expressed in the form $(a + b)^2$. [5.3]

square root of a perfect square One of the two equal factors of the perfect square. [5.6]

standard form of a quadratic equation A quadratic equation is in standard form when the polynomial is in descending order and equal to zero. [8.1]

substitution method An algebraic method of finding an exact solution of a system of linear equations. [4.1]

summation notation Notation used to represent a series in a compact form; also called sigma notation. [12.1]

sum of two perfect cubes A polynomial that can be written in the form $a^3 + b^3$. [5.6]

sum of two perfect squares A binomial in the form $a^2 + b^2$, which is nonfactorable over the integers. [5.6]

synthetic division A shorter method of dividing a polynomial by a binomial of the form $x - a$. This method uses only the coefficients of the variable terms. [5.4]

system of equations Two or more equations considered together. [4.1]

system of inequalities Two or more inequalities considered together. [4.5]

term of a sequence A number in a sequence. [2.1]

terminating decimal A decimal formed when division of the numerator of its fractional counterpart by the denominator results in a remainder of zero. [1.1]

terms of a variable expression The addends of the expression. [1.3]

tolerance of a component The amount by which it is acceptable for the component to vary from a given measurement. [2.5]

trinomial A polynomial of three terms. [5.2]

undefined slope The slope of a vertical line is undefined. [3.4]

uniform motion The motion of an object whose speed and direction do not change. [2.3, 6.5]

uniform motion problem A problem that involves the motion of an object whose speed and direction do not change. [2.3]

union of two sets The set that contains all elements that belong to either of the sets. [1.1]

upper limit In a tolerance, the greatest acceptable value. [2.5]

value mixture problem A problem that involves combining two ingredients that have different prices into a single blend. [2.3]

value of a function The value of the dependent variable for a given value of the independent variable. [3.2]

variable A letter used to stand for a quantity that is unknown or that can change. [1.1]

variable expression An expression that contains one or more variables. [1.3]

variable part of a variable term The variables, taken together, that occur in a variable term. [1.3]

variable term A term composed of a numerical coefficient and a variable part. When the numerical coefficient is 1 or -1 , the 1 is usually not written. [1.3]

vertex of a parabola The point on the parabola with the smallest y-coordinate or the largest y-coordinate. [9.1]

vertical-line test A graph defines a function if any vertical line intersects the graph at no more than one point. [9.2]

whole numbers The numbers 0, 1, 2, 3, 4, 5, ... [1.1]

x-coordinate The abscissa in an xy-coordinate system. [3.1]

x-intercept The point at which a graph crosses the x-axis. [3.3]

xy-coordinate system A rectangular coordinate system in which the horizontal axis is labeled x and the vertical axis is labeled y. [3.1]

xyz-coordinate system A three-dimensional coordinate system formed when a third coordinate axis (the z-axis) is located perpendicular to the xy-plane. [4.2]

y-coordinate The ordinate in an xy-coordinate system. [3.1]

y-intercept The point at which a graph crosses the y-axis. [3.3]

zero of a function A value of x for which $f(x) = 0$. [9.1]

zero slope The slope of a horizontal line. [3.4]

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TI-30X IIS

6 $\frac{2}{3}$ 2 $\frac{3}{4}$ 3 + 3 $\frac{5}{12}$ 4 ENTER =

Operations on fractions
 $6\frac{2}{3} + \frac{3}{4} = 7\frac{5}{12}$

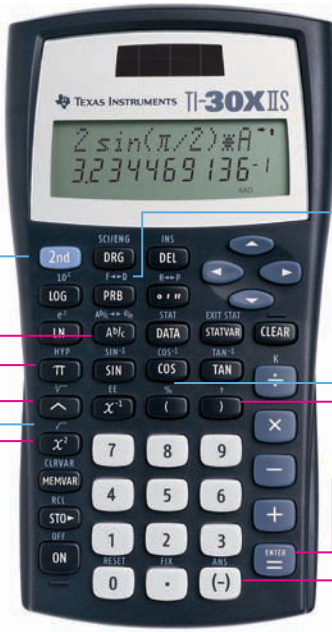
The value of π

Power of a number
(See Note 1 below.)

Square root of a number

Square a number

Access operations in blue



.4 2nd F<D> ENTER =
Change decimal to fraction or fraction to decimal

3 + 2 (10 - 6) ENTER =
Operations with parentheses

11 x 25 2nd % ENTER =
Operations with percent

Used to complete an operation

(-) 12 ÷ 6 ENTER =
Enter a negative number
(See Note 2 below.)

Photo courtesy of Texas Instruments Incorporated

fx-300MS

$\sqrt{\quad}$ 36 =

Square root of a number

6 $\frac{2}{3}$ 2 $\frac{3}{4}$ 3 + 3 $\frac{5}{12}$ 4 =

Operations on fractions
 $6\frac{2}{3} + \frac{3}{4} = 7\frac{5}{12}$

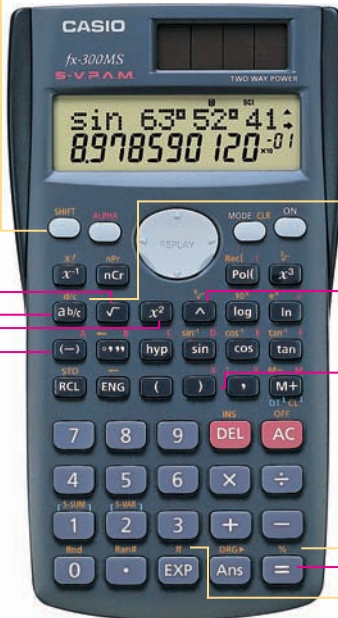
7 x^2 =

Square a number

(-) 12 ÷ 6 =

Enter a negative number
(See Note 2 below.)

Access operations in gold



.4 = d/c
Change decimal to fraction

13 ^ 4 =
Power of a number
(See Note 1 below.)

3 + 2 (10 - 6) =
Operations with parentheses

11 x 25 % =
Operations with percent

Used to complete an operation

SHIFT π =
The value of π

Photo courtesy of Casio, Inc.

NOTE 1: Some calculators use the y^x key to calculate a power.
For those calculators, enter 13 y^x 4 = to evaluate 13^4 .

NOTE 2: Some calculators use the +/- key to enter a negative number.
For those calculators, enter 12 +/- ÷ 6 = to calculate $-12 \div 6$.

WINDOW
Xmin = -10
Xmax = 10
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1
Xres = 1

2ND TBLSET

TABLE SETUP
TblStart=0
ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask

2ND CALC

CALCULATE
1: value
2: zero
3: minimum
4: maximum
5: intersect
6: dy/dx
7: ∫f(x)dx

2ND TABLE

X	Y1
0	-3
1	-1
2	1
3	3
4	5
5	7
6	9

X=0

FUNCTION

1: Y1
2: Y2
3: Y3
4: Y4
5: Y5
6: Y6
7: Y7

2ND STAT PLOT

STAT PLOTS
1: Plot1...On
2: Plot2...Off
3: Plot3...Off
4: PlotsOff

Plot1 Plot2 Plot3
Y1 =
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =

2ND MATRX

NAMES MATH EDIT
1: [A] 3x4
2: [B] 3x4
3: [C] 3x4
4: [D]
5: [E]
6: [F]
7: [G]

ZOOM MEMORY
1: ZBox
2: Zoom In
3: Zoom Out
4: ZDecimal
5: ZSquare
6: ZStandard
7: ZTrig

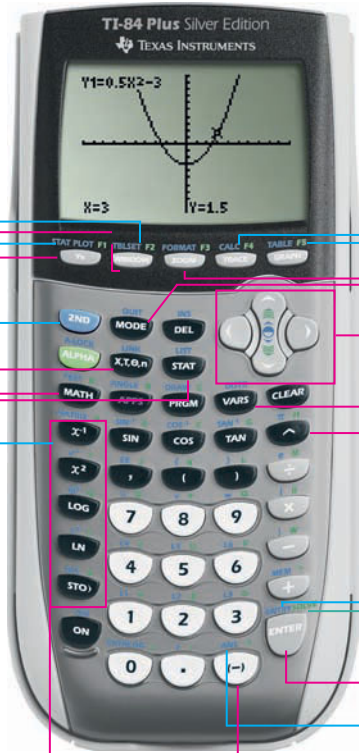
VARX Y-VARS

1: Function...
2: Parametric...
3: Polar...
4: On/Off...

VARX Y-VARS

1: Window...
2: Zoom...
3: GDB...
4: Picture...
5: Statistics...
6: Table...
7: String...

Normal Sci Eng
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T



Access functions and menus written in blue
Enter the variable x

Move the cursor on the graphics screen or scroll through a menu

Enter an exponent

2ND ENTRY - Recall last calculation

ALPHA SOLVE - Used to solve some equations and to solve for a variable in financial calculations

Used to complete an operation

2ND ANS - Recall last answer

Used to enter a negative number

MATH NUM CPX PRB
1: abs()
2: round()
3: iPart()
4: fPart()
5: int()
6: min()
7: max()

MATH NUM CPX PRB
1: conj()
2: real()
3: imag()
4: angle()
5: abs()
6: Rect
7: Polar

MATH NUM CPX PRB
1: Frac
2: Dec
3: 3
4: 3√
5: x√
6: fMin()
7: fMax()

EDIT CALC TESTS
1: 1-Var Stats
2: 2-Var Stats
3: Med-Med
4: LinReg(ax+b)
5: QuadReg
6: CubicReg
7: QuartReg

EDIT CALC TESTS
1: Edit...
2: SortA()
3: SortD()
4: CnList
5: SetUpEditor

x^{-1} - Reciprocal of last entry

x^2 - Square last entry; 2ND $\sqrt{}$: Square root of next entry

LOG - Common logarithm (base 10); 2ND 10^x : 10 to the x power, antilogarithm of x

LN - Natural logarithm (base e); 2ND e^x : Calculate a power of e

STO → - Store a number; 2ND RCL : Recall a stored variable

Photo courtesy of Texas Instruments Incorporated