Math 330 Section 6 - Spring 2024 - Homework 04

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A **orrection** was made to Part **c** of the Wednesday, February 7 reading assignment:

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete before the first one of this HW.

MF lecture notes:
ch.1; ch.2.1 - 2.6, ch.3, ch.5.1 - 5.2.5

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 2.2, ch.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, February 5:

- **a.** Read carefully MF ch.5.2.6 - 5.2.8 (the remainder of ch.5.2).
- **b.** Read carefully B/G ch.5 on sets and functions and B/G ch.9.1. You already have encountered the material in MF ch.2 and ch.5.1 - 5.2.

Reading assignment 2 - due: Wednesday, February 7:

- **a.** The better students are encouraged to take a look at the optional chapter 5.3.
- **b.** Read **extra carefully** MF ch.6.1. Proofs by induction will appear on all major exams!
- **c.** 8Read carefully MF ch.6.2 and 6.3. They are very brief.
- **d.** Read carefully B/G ch.2.3 on induction. Work through the proofs by induction given there.

Reading assignment 3 - due Friday, February 9:

- **a.** Carefully read MF ch.6.4 - 6.6.
- **b.** You are encouraged to look at the optional chapter 6.7 on Bernstein Polynomials. The proofs are very good examples of proofs by induction.

Written assignments: See next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1: Use anything before Proposition 3.56 to prove the following part of Proposition 3.56:

Let $(R, \oplus, \odot, P)$ be an ordered integral domain. Let $A \subseteq R$. If A has a minimum, then it also has an infimum and $\min(A) = \inf(A)$.

Hint: You must work with the definitions of the max, of the inf as the max of a certain set, of upper and lower bounds. Do not expect to et this one right at the first or even second submission!

Written assignment 2: Use the rules of working with quantifiers to negate the following statement (see B/G ch.3.3). No need at all to understand the meaning of this statement. [1]

$$\exists\, \rho > 0 \text{ such that } \forall\, x \in X\ \exists\, i \in I \text{ such that } N_\rho(x) \subseteq U_i.$$

[1] You will encounter this fragment very late in this course as part of Proposition 14.6