

Math 330 Section 6 - Spring 2024 - Homework 05

Published: Tuesday, February 6, 2024

Running total: 24 points

Last submission: Wednesday, February 21, 2024

Correction to HW4: Part c of the Wednesday, February 7 reading assignment should be: Read carefully MF ch.6.2 and 6.3. They are very brief.

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1; ch.2.1 - 2.6, ch.3, ch.5 - 6.7

B/G (Beck/Geoghegan) Textbook:

ch.2.1 – 2.3, ch.3, ch.5, ch.9.1

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments: None!

Written assignments are on the next page.

Written assignments:

These written assignments are graded only once, and partial credit is given. The entire set is worth 6 points.

Written assignment 1:

Injectivity and Surjectivity

- Let $f : \mathbb{R} \rightarrow [0, \infty[; \quad x \mapsto x^2$.
- Let $g : [0, \infty[\rightarrow [0, \infty[; \quad x \mapsto x^2$.
In other words, g is same function as f as far as assigning function values is concerned, but its domain was downsized to $[0, \infty[$.

Answer the following with **true** or **false**.

- a. f is surjective c. g is surjective
- b. f is injective d. g is injective

If your answer is **false** then give a specific counterexample.

Written assignment 2: Let $X := \{-2, 2\}$, $Y := \{4\}$, and $f : X \rightarrow Y$ defined as $f(x) = x^2$. Find $A \subseteq X$ such that $f(A^c) \neq f(A)^c$.
Compute both $f(A^c)$ and $f(A)^c$.

Written assignment 3: Let $f :]-10, 10[\rightarrow \mathbb{R}; \quad x \mapsto x^2$.

- a. what is the range of f ? b. Is f injective? c. Is f surjective?
- d. $f(\{1\} \cup [4, 6]) = ?$ e. $f([2, 5]) \cap f([4, 7]) = ?$ f. $f^{-1}([4, 25]) \cap f^{-1}([16, 49]) = ?$

Hint: For d, e, f, review examples 5.24–5.27.

Written assignment 4:

You have learned in MF ch.5 that

injective \circ injective = injective,
surjective \circ surjective = surjective.

The following illustrates that the reverse is not necessarily true.

Let $A := \{a\}$ and $B := \{b_1, b_2\}$. Assume that $b_1 \neq b_2$. Find functions $f : A \rightarrow B$ and $g : B \rightarrow A$ which satisfy the following:

- The composition $h := g \circ f : A \rightarrow A$ is bijective.
- It is **not true** that both f, g are injective.
- It is **not true** that both f, g are surjective.

Hint: There are not a whole lot of possibilities. Draw all possible candidates for f and g in arrow notation as you see in MF example 5.19. There are only very few choices!