Math 330 Section 6 - Spring 2024 - Homework 06

Published: Tuesday, February 6, 2024 Running total: 28 points
Last submission: Friday, February 23, 2024

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1; ch.2.1 - 2.6, ch.3, ch.5 - 6.7

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 2.3, ch.3, ch.5, ch.9.1

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, February 12:

a. Read carefully MF ch.6.8 - 6.10.
b. Read carefully B/G ch.2.4, B/G ch.4.1 - 4.3 and B/G ch.9.2. You already have encountered the material in MF ch.6.

Reading assignment 2 - due: Wednesday, February 14:

a. Read carefully the remainder of B/G ch.4 and B/G ch.5. You already have encountered the material in MF ch.5 and ch.6. Work through the proofs by induction given in B/G ch.4.
b. Read carefully MF ch.6.11.

Reading assignment 3 - due Friday, February 16:

a. Carefully read the remainder of MF ch.6.
b. Read carefully B/G ch.6.1 - 6.3. You already have encountered the material in MF ch.6.

Written assignments: See next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1:
Let $X, Y \neq \emptyset$ and $f : X \to Y$.

(a) Prove that $R := \{(x, x') \in X \times X : f(x) = f(x')\}$ is an equivalence relation on X.

(b) For the special case $f : \mathbb{R} \to \mathbb{R}; \quad x \to x^2$ compute the equivalence classes $[2], [0], [-2]$ for this equivalence relation.

One point each for **(a)** and **(b)**!!

Written assignment 2:
Prove formulas (5.15) and (5.16) of Proposition 5.3: Let $f : X \to Y$. Then

(a) (5.15) $\quad A_1 \subseteq A_2 \subseteq X \Rightarrow f(A_1) \subseteq f(A_2)$
(b) (5.16) $\quad B_1 \subseteq B_2 \subseteq Y \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

One point each for **(a)** and **(b)**!!

Hint: Start **(a)** as follows: Let $y \in f(A_1)$. ..... Start **(b)** as follows: Let $x \in f^{-1}(B_1)$. .....