Math 330 Section 6 - Spring 2024 - Homework 07

Published: Tuesday, February 20, 2024 Running total: 31 points
Last submission: Monday, March 11, 2024

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1; ch.2.1 - 2.6, ch.3, ch.5 - 6

B/G (Beck/Geoghegan) Textbook:
ch.2 - 6.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Friday, February 23:

a. Read carefully MF ch.7.1 - 7.3.

Reading assignment 2 - due Monday, February 26:

a. Study for the midterm!

Reading assignment 3 - due: Wednesday, February 28:

a. Read carefully the remainder of MF ch.7.

b. Carefully read MF ch.8.1. **You have been warned:** I love to ask the students in the major exams to prove (parts of) De Morgan!

c. Those of you with an academic bend are encouraged to study the optional Chapter 8.2: $(2^\Omega, \triangle, \cap)$ as a CRU (Remark 8.1)**(5)**) (but not an integral domain: If A, B are disjoint and nonempty, then $A \cap B = \emptyset$).

d. Carefully read MF ch.8.3. Be sure to understand formula (8.7):
$Y^X = \{f : f \text{ is a function with domain } X \text{ and codomain } Y\}$.

Reading assignment 4 - due Friday, March 1:

a. Carefully read MF ch.8.4 through Proposition 8.10. Skim or skip the remainder.

b. Carefully read MF ch.9.1.

Written assignments: See next page.

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1:
Prove B/G Prop. 4.7(i) by induction: Let $k \in \mathbb{N}$. Then there exists $j \in \mathbb{Z}$ such that $5^{2k} - 1 = 24j$. In other words, $24 \mid (5^{2k} - 1)$.

Written assignment 2:
Let $x_0 = 8,\ x_1 = 16,\ x_{n+1} = 6x_{n-1} - x_n$ for $n \in \mathbb{N}$.

Prove that $x_n = 2^{n+3}$ for every integer $n \geq 0$.

Hint: Use strong induction.

Written assignment 3:
Prove MF Prop. 6.7**(a)** by induction on p: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in an ordered integral domain $R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^{p} x_j \ = \ \sum_{j=m}^{n} x_j \ \oplus \ \sum_{j=n+1}^{p} x_j.$$

Hints: Think carefully about the base case: If $m = 5$ and $n = 8$, how would you choose p? If $m = -4$ and $n = 8$, how would you choose p? For general $m \leqq n$, how would you choose p?