Math 330 Section 6 - Spring 2024 - Homework 09

Published: Tuesday, March 6, 2024 Running total: 36 points
Last submission: Friday, March 22, 2024

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.2.1 – 2.7, ch.3, skim ch.4 (optional), ch.5 - 9.1, ch.11 (linear algebra) through Example 11.11

B/G (Beck/Geoghegan) Textbook:
ch.2 - 6.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, March 11, 2024 4:

- **a.** Carefully read MF ch.9.2.
- **b.** Carefully read MF ch.9.3. A through understanding of convergence and continuity is absolutely necessary to understand those topics in the abstract settings of metric and topological spaces!

Reading assignment 2 - due: Wednesday, March 13, 2024 6:

- **a.** Carefully read MF ch.9.4 and 9.5.

Reading assignment 3 - due Friday, March 15, 2024 8:

- **a.** Carefully read MF ch.9.6 and 9.7.
- **b.** Carefully read MF ch.9.8 until before Proposition 9.43 and skim the optional remainder.
- **c.** Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their liminf and limsup). The stronger students are encouraged to skim the contents, in particular the last remark.

Written assignment are on the next page.

Written assignment 1: Prove Proposition 7.13: Every infinite set contains a proper subset that is countably infinite.

Written assignment 2: Prove the following part of De Morgan's Law:
Let there be a universal set Ω which contains all elements of an indexed family of sets $(A_\alpha)_{\alpha \in I}$. Then

$$\Big(\bigcap_\alpha A_\alpha\Big)^{\complement} \subseteq \bigcup_\alpha A_\alpha^{\complement}.$$

Written assignment 3: Prove formula (8.32):
If X, Y, Z be arbitrary, nonempty sets and $f : X \to Y$, $g : Y \to Z$, $U \subseteq X$, and $W \subseteq Z$, then

$$(g \circ f)(U) \subseteq g\big(f(U)\big) \text{ for all } U \subseteq X.$$