Math 330 Section 6 - Spring 2024 - Homework 12

Published: Tuesday, March 26, 2024 Running total: 45 points
Last submission: Friday, April 12, 2024

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.2.1 – 2.7, ch.3, skim ch.4 (optional), ch.5 - 12.3

B/G (Beck/Geoghegan) Textbook:
ch.2 - 12

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 1, 2024:

- **a.** Carefully read MF ch.12.4 and 12.5.
- **b.** The better students are encouraged to at least skim ch.12.6

Reading assignment 2 - due: Wednesday, April 3, 2024:

- Carefully read MF ch.12.7 and 12.8. You might have problems with subspaces, since this is a very subtle concept!

Reading assignment 3 - due Friday, April 5, 2024:

- **a.** Carefully read MF ch.12.9 and 12.10 until Theorem 12.10.

Be sure to read pages 2 and 3!

Supplementary instructions for reading MF ch.12:

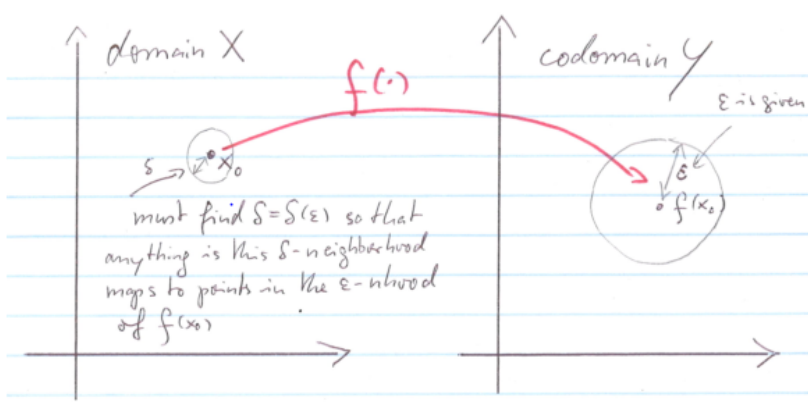
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for

- open sets and neighborhoods (ch.12.1.3)
- convergence, expressed with nhoods (the end of def.12.10 in ch.12.1.4)
- metric and topological subspaces (ch.12.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^\complement$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^\complement$ and A_1. What is $N_\varepsilon^A(x_j)$?
- Contact points, closed sets and closures (ch.12.1.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^\complement$? Use those pictures to visualize the definitions in this chapter and thm 12.6 and thm.12.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.12.2 (Continuity): Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 0.1: ε-δ continuity



Written assignments on page 3

Written assignment 1:

$$\text{Let} \qquad A_n \ := \ \begin{cases} \{\,-1, 0\,\} & \text{if } n \text{ is odd,} \\ \{\,0, 1\,\} & \text{if } n \text{ is even,} \end{cases} \qquad (n \in \mathbb{N}).$$

(Those are sequences of SETS! See Chapter 9.9: (Sequences of Sets and Indicator functions and their liminf and limsup)

(a) Compute $\liminf_{n\to\infty} A_n$

(b) Compute $\limsup_{n\to\infty} A_n$.

Provide proofs of your results! They might be more verbose than usual. You can cite any result from that chapter.

One point each for (a) and (b)!

Written assignment 2: Prove Exercise : Let Ω be a set and let $\varphi : 2^\Omega \to 2^\Omega$ satisfy $A, B \subseteq \Omega$ and $A \subseteq B \ \Rightarrow \ \varphi(A) \subseteq \varphi(B)$.

$$\text{Let} \quad \mathfrak{F} \ := \ \{A \in 2^\Omega : A \subseteq \varphi(A)\}\,, \quad A_0 := \bigcup [A : A \in \mathfrak{F}]\,.$$

The proof of Tarski's fixed point theorem (Theorem **??** on p.**??**) shows that A_0 is a fixed point for φ, i.e., $\varphi(A_0) = A_0$. Modify this proof to show the following:

$$\text{Let} \quad \mathscr{E} \ := \ \{B \in 2^\Omega : \varphi(B) \subseteq B\}\,, \quad B_0 := \bigcap [B : B \in \mathscr{E}]\,.$$

Then B_0 also is a fixed point for φ.