

## Math 330 Section 1 - Spring 2026 - Homework 10

Published: Tuesday, March 10, 2026  
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Running total: 37 points

### Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1 – ch.3, skim ch.4, ch.5.1 – ch.9.5, ch.11 through Example 11.11

B/G (Beck/Geoghegan) Textbook:

ch.1 – 5, ch.8 – 9

Other material (optional):

B/K lecture notes ch.1.1 and ch.1.2, except ch.1.2.4

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

### New reading assignments:

#### Reading assignment 1 - due Monday, March 16:

- a. Carefully read B/G ch.6 and ch.7.1. This corresponds to material in MF ch.6.
- b. Carefully read B/G ch.10. This corresponds to material in MF ch.9.1 – 9.3.

#### Reading assignment 2 - due Wednesday, March 18:

- a. Carefully read MF ch.9.6 and 9.7.
- b. Carefully read B/G ch.11. This corresponds to material in MF ch.9.4.

#### Reading assignment 3 - due Friday, March 20:

- a. Carefully read MF ch.9.8 until before Proposition 9.44. and skim the optional remainder.
- c. Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their  $\liminf$  and  $\limsup$ ). The stronger students and those who consider getting a master's degree in probability/statistics/actuarial science are encouraged to study the material, in particular the last remark.

**General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

**Written assignments:** See the next page!

### Written assignments:

**Written assignment 1:** Prove Proposition 7.13: Every infinite set  $X$  contains a proper subset  $A$  that is countably infinite. **Hint:** • Define  $X_0 := X \setminus \{a_0\}$  and then recursively,  $X_n := X_{n-1} \setminus \{a_n\}$ .

• Use an induction argument to prove that each set  $X_n$  is not empty (Thm.7.2 might come in handy). What does that imply for the number of elements of the set  $\{a_0, a_1, \dots\}$ ? • How do you use those  $a_j$  to define the set  $A$  you want? • What is the bijection  $A \xrightarrow{\sim} \mathbb{N}$  which proves that  $A$  is indeed countably infinite?

**Written assignment 2:** Prove the following part of De Morgan's Law:

Let there be a universal set  $\Omega$  which contains all elements of an indexed family of sets  $(A_\alpha)_{\alpha \in I}$ . Then

$$\left(\bigcap_{\alpha} A_{\alpha}\right)^c \subseteq \bigcup_{\alpha} A_{\alpha}^c.$$

**Written assignment 3:** Prove formula (8.33):

If  $X, Y, Z$  be arbitrary, nonempty sets and  $f : X \rightarrow Y$ ,  $g : Y \rightarrow Z$ , and  $U \subseteq X$ , then

$$(g \circ f)(U) \subseteq g(f(U)) \text{ for all } U \subseteq X.$$