

Math 330 Section 1 - Spring 2026 - Homework 15

Published: Tuesday, April 14, 2026
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Running total: 54 points

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1 – ch.3, skim ch.4, ch.5.1 – ch.13.2.2 through Remark 13.14.

B/G (Beck/Geoghegan) Textbook:

ch.1 – 7.1, ch.8 – 13.4

Other material (optional):

B/K lecture notes ch.1.1 and ch.1.2, except ch.1.2.4

Stewart Calculus 7ed - ch.1.7: “The Precise Definition of a Limit”

Reading assignment 1 - due Monday, April 20.

- a. Carefully read the remainder of MF ch.13 (i.e., of MF ch.13.2.2). Look closely at the Riemann Rearrangement Theorem and its corollaries. It is not important that you understand the proofs, but you must understand what they assert. If there is time to prove Riemann’s Rearrangement Theorem in lecture, I’ll give a “proof” by picture. Unless you are really motivated to do math, you should definitely skip the proof you find in my lecture notes.
- b. Read carefully the beginning of MF ch.14 through Definition 14.1. Do not skip the two introductions!

Reading assignment 2 - due Wednesday, April 22:

- a. Read carefully the remainder of MF ch.14.1. Draw plenty of pictures to understand the assertions of Proposition 14.1. It is marked as optional, so you need not remember it. Nevertheless, you are strongly advised to draw a picture for the twodimensional case.

Reading assignment 3 - due Friday, April 24:

- a. Read carefully MF ch.14.2 and 14.3. Less than six pages, but it’s a tough read! Understand the 3 equivalent ways to refer to compactness in metric spaces plus #4 in $\mathbb{R}^n$. Note that the “extract finite open subcovering” property needs extra careful study!

Written assignments are on the next page.

Written assignments:

Written assignment 1: (3 points!) Prove Proposition 12.15:

Let $(X, \mathfrak{U})$ be a topological space. If $A \subseteq B \subseteq X$, then $A^\circ \subseteq B^\circ$.

Written assignment 2: Let (X, d) be a metric space and $A \subseteq X$, $A \neq \emptyset$. Let

$$\gamma := \gamma(A) := \inf\{d(x, y) : x, y \in A \text{ and } x \neq y\}.$$

(a) Prove that if $\gamma > 0$ and $(x_n)_n$ is Cauchy in A , then $(x_n)_n$ is constant, eventually.

(b) Use (a) to prove that if $\gamma > 0$, then A is complete.

One point each for (a) and (b)!

Written assignment 3: Let $X := \mathbb{R}$ equipped with the standard Euclidean metric $d(x, x') = |x - x'|$. Let $f_n : \mathbb{R} \rightarrow \mathbb{R}$ be the following sequence of functions:

$$f_n(x) := \begin{cases} 0 & \text{if } |x| > \frac{1}{n}, \\ nx + 1 & \text{if } -\frac{1}{n} \leq x \leq 0, \\ -nx + 1 & \text{if } 0 \leq x \leq \frac{1}{n}, \end{cases}$$

i.e., the point $(x, f_n(x))$ is on the straight line between $(-\frac{1}{n}, 0)$ and $(0, 1)$ for $-\frac{1}{n} \leq x \leq 0$, it is on the straight line between $(0, 1)$ and $(\frac{1}{n}, 0)$ for $0 \leq x \leq \frac{1}{n}$, and it is on the x -axis for all other x . Draw a picture! Let $f(x) := 0$ for $x \neq 0$ and $f(0) := 1$.

a. Prove that f_n converges pointwise to f on $\mathbb{R}$. In other words, prove that $\lim_{n \rightarrow \infty} f_n(x) = f(x)$ for all $x \in \mathbb{R}$.

b. Prove that f_n does not converge uniformly to f on $\mathbb{R}$. You may use without proof that each of the functions f_n is continuous on $\mathbb{R}$.

One point each for (a) and (b)!

Hint for part (a): For fixed $x \neq 0$, what happens eventually, i.e., for big enough n ? Transform the inequalities $\dots \leq x \leq \dots$ into inequalities for n and you should see what happens.

Example (NOT legit as a proof): If $x = 0.01$, what happens if $n > 1000$? Thus $\lim_{n \rightarrow \infty} f_n(0.01) =$ WHAT?

No need to submit those hints as part of your HW. Just use them!

Hint for part (b): Look at the (very few) propositions and theorems of Ch.13.2.1).