

Math 330 Section 7 - Fall 2026 - Homework 03

Published: Tuesday, September 1, 2026

Running total: 16 points

Last submission: Friday, September 18, 2026

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

Status - Reading Assignments:

The reading assignments from HW 1 were:

MF lecture notes:

ch.1; ch.2.1 - 2.6, 3.1 - 3.4 until Def.3.13

B/G (Beck/Geoghegan) Textbook:

ch.2.1 – 2.2

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due: Wednesday, September 2 (FIRST LECTURE):

- a. Read carefully the remainder of MF ch.3.4.
- b. Read carefully MF ch.3.5 until Proposition 3.58.
Understand why $\inf(A)$ is also called the greatest lower bound of A and why it is a generalization of $\min(A)$. In ch.5 we will consider partial orderings. They are more general than the “linear” orderings of MF ch.4.

Reading assignment 2 - due Friday, September 4:

- a. Read carefully the remainder of MF ch.3.5 Draw plenty of pictures for Props.3.59 and 3.60 and Cor.3.4 (A = interval of real #s)
- b. Read carefully B/G ch.3 on logic. It is very brief, but all of it is extremely important.
- c. Skim MF ch.4 on logic, just so you have an idea what’s in there. Look a little bit more closely at ch.4.5.4 (Quantifiers and Negation). Note that I have marked all of ch.4 as optional, but you will be tested on B/G ch.3!

Written assignments are on the next page.

Written assignments:

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1:

(a) Let $(R, \oplus, \odot)$ be an integral domain. Use anything up-to and including MF prop. 3.27 to prove MF prop.3.28: Let $x \in R$. If $x \odot x = x$ then $x = 0$ or $x = 1$.

Hint:

- a. Show that both $x = 0$ and $x = 1$ satisfy $x \odot x = x$.
- b. Assuming that $x \neq 0$, show that $x = 1$. To do so, transform $x \odot x = x$ into a statement of the form $A \odot B = 0$ and do something that you can always do in an integral domain, but not necessarily in a CRU!

Written assignment 2: TOUGH! Get started EARLY!

Let $(R, \oplus, \odot, P)$ be an ordered integral domain. Use anything up-to and including MF prop. 3.35 to prove MF prop.3.36: The multiplicative unit 1 of R belongs to P .

Hint: This is an **indirect proof**! Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

You are **strongly advised** to study the proof of Proposition 3.33 very thoroughly before working on this problem.