

Math 330 Section 7 - Fall 2026 - Homework 07

Published: Sunday, September 20, 2026
Last submission: Friday, October 9, 2026

Running total: 28 + __ points

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:

ch.1 – 6.11; skim/skip ch.4 on logic

B/G (Beck/Geoghegan) Textbook:

ch.2 – 5, ch.9

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Since there are no classes on Monday, consider reading assignments 1 and 2 as a combined assignment due Wednesday.

Reading assignment 1 - due: Monday, September 21:

- a. Read carefully the remainder of MF ch.6.
- b. Do a quick review of B/G ch.1. You already have encountered the material in MF ch.3 and ch.6.

Reading assignment 2 - due: Wednesday, September 23:

- a. Read carefully MF ch.7.1 -7.3.

Reading assignment 3 - due Friday, September 25:

- a. Read carefully the remainder of MF ch.7.

Written assignments are on the next page.

Written assignments:

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1:

Prove B/G Prop. 4.7(i) by induction: Let $k \in \mathbb{N}$. Then there exists $j \in \mathbb{Z}$ such that $5^{2k} - 1 = 24j$. In other words, $24 \mid (5^{2k} - 1)$.

Written assignment 2:

Prove MF Prop. 6.7(a) by induction on p : Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in an ordered integral domain $R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^p x_j = \sum_{j=m}^n x_j \oplus \sum_{j=n+1}^p x_j.$$

Hints: Think carefully about the base case: If $m = 5$ and $n = 8$, how would you choose p ? If $m = -4$ and $n = 8$, how would you choose p ? For general $m \leq n$, how would you choose p ?