

Math 330 Section 7 - Fall 2026 - Homework 08

Published: Tuesday, September 22, 2026
Last submission: Friday, October 23, 2026

Running total: 32 points

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:

ch.1 – 7; skim/skip ch.4 on logic

B/G (Beck/Geoghegan) Textbook:

ch.1 – 5, ch.9

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Since there are no classes on Monday, consider reading assignments 1 and 2 as a combined assignment due Wednesday.

Reading assignment 1 - due: Monday, September 28:

- a. Carefully read MF ch.8.1. **You have been warned:** I love to ask the students in the major exams to prove (parts of) De Morgan!
- b. Those of you with an academic bend are encouraged to study the optional Chapter 8.2: $(2^\Omega, \Delta, \cap)$ as a CRU (Remark 8.1)(5)) (But 2^Ω is not an integral domain: If A, B are disjoint and nonempty, then $A \cap B = \emptyset$. Thus, A and B are zero divisors.)
- c. Carefully read MF ch.8.3. Be sure to understand formula (8.10):
 $Y^X = \{f : f \text{ is a function with domain } X \text{ and codomain } Y\}.$

Reading assignment 2 - due: Wednesday, September 30:

- Prepare for the midterm!

Reading assignment 3 - due Friday, October 2:

- a. Carefully read MF ch.8.4 through Proposition 8.10. Skim or skip the remainder.
- b. Read MF ch.8.5. Indicator functions are very important for probability theory..

Written assignments are on the next page.

Written assignments:

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1:

Prove uniqueness of the “decomposition” $m = qn + r$ such that $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by equating $qn + r = m = \tilde{q}n + \tilde{r}$ and see what can be said about $|r - \tilde{r}|$ if $q \neq \tilde{q}$. More hints further down!

Written assignment 2:

Much harder than #1: Prove the existence of q and r .

Hints for #2: Review the Extended Well-Ordering principle MF thm.6.10. Its use will give the easiest way to prove this assignment: Let

$$A := A(m, n) := \{r' \in [0, \infty[_{\mathbb{Z}} : \exists q' \in \mathbb{Z} \text{ such that } r' = m - q'n\}.$$

Show that $A \neq \emptyset$ by separately examining the cases

- $m \geq 0$ (easy)
- $m < 0$ (probably the hardest part of the proof!)

Now you can apply the Extended Well-Ordering principle to the set A . How is $\min(A)$ related to your problem? (It is!) To better see what is going on, this may help:

- What is $m = nq + r$ for $n = 10$ and $m = 43$, $m = -43$, $m = -3$?
- What is $\min(A(m, n))$ in those three cases? Draw a picture!

Hint for both #1 and #2: MF Prop.3.60 and Cor.3.5 at the end of ch.3.5 will come in handy in connection with using or proving $0 \leq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following.

If $a, b \in [0, n[_{\mathbb{Z}}$, then $|a - b| \leq \max(a, b)$, i.e., $-n < a - b < n$.