Math 330 Section 7 - Fall 2026 - Homework 14

Published: Thursday, November 11, 2026 Running total: 49 points
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Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1 – 13.2.2 through Remark 13.14; skim/skip ch.4, 9.9, 10.3, 11.2.3 and 12.6

B/G (Beck/Geoghegan) Textbook:

ch.1 – 7.1, ch.8 – 13.4

Other material (optional):

B/K lecture notes: ch.1.1 (Introduction to sets) (optional) and ch.1.2 (Introduction to Functions)

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

Reading assignment 1 - due: Monday, November 16:

a. Carefully read the remainder of MF ch.13 (i.e., of MF ch.13.2.2). Look closely at the Riemann Rearrangement Theorem and its corollaries. It is not important that you understand the proofs, but you must understand what they assert. If there is time to prove Riemann's Rearrangement Theorem in lecture, I'll give a "proof" by picture. Unless you are really motivated to do math, you should definitely skip the proof you find in my lecture notes.

b. Read carefully the beginning of MF ch.14 through Definition 14.1. Do not skip the two introductions!

Reading assignment 2 - due: Wednesday, November 18:

a. Read carefully the remainder of MF ch.14.1. Draw plenty of pictures to understand the assertions of Proposition 14.1. It is marked as optional, so you need not remember it. Nevertheless, you are strongly advised to draw a picture for the twodimensional case.

Reading assignment 3 - due Friday, November 20:

a. Read carefully MF ch.14.2 and 14.3. Less than six pages, but it's a tough read! Understand the 3 equivalent ways to refer to compactness in metric spaces plus #4 in $\mathbb{R}^n$ Note that the "extract finite open subcovering" property needs extra careful study!

Written assignments are on the next two pages.

Written assignments:

Written assignment 1: (3 points!) Prove MF prop.11.13 (Properties of the sup norm): $h \mapsto \|h\|_\infty = \sup\{|h(x)| : x \in X\}$ defines a norm on $\mathscr{B}(X, \mathbb{R})$

This assignment is worth three points: **One point each** for pos.definite, absolutely homogeneous, triangle inequality!

Hint: Go for a treasure hunt in ch.9.2 (Minima, Maxima, Infima and Suprema), and look at the properties of $\sup(A)(A \subseteq \mathbb{R})$ to prove absolute homogeneity and the triangle inequality.

This assignment is worth three points, and you will have to earn them! The following exemplifies the level of detail I expect you to provide.

To prove that $\|\cdot\|_\infty$ satisfies the triangle inequality (11.29c) of a norm you will have to write something along the following lines:

c. Triangle inequality.
NTS: $\|f + g\|_\infty \leqq \|f\|_\infty + \|g\|_\infty$ for all $f, g \in \mathscr{B}(X, \mathbb{R})$.

Proof:

$\|f + g\|_\infty = \sup\{|f(x)| + |g(x)| : x \in X\}$ (definition of $\|\cdot\|_\infty$)
$= \ldots$ (.....)
$\leqq \ldots$ (.....)
$= \|f\|_\infty + \|g\|_\infty$ (.....)

(There may be fewer or more steps in your proof. The above only serves as an illustration!)

No need to justify properties of the absolute value $|\alpha|$ of a real number α, but you will need to justify why

$$\sup\{|\alpha f(x)| : x \in X\} = |\alpha| \sup\{|f(x)| : x \in X\},$$

and why

$$\sup\{|f(x) + g(x)| : x \in X\} \leqq \sup\{|f(x)| : x \in X\} + \sup\{|g(x)| : x \in X\}.$$

An aside: **DO NOT** write $\|f(x)\|_\infty$ when you deal with the real number $f(x)$ (and **you probably mean** the absolute value $|f(x)|$).

$\|\cdot\|_\infty$ is defined for functions f, NOT for numbers $f(x)$!

Written assignment 2 (3 points):
Prove MF thm.12.1 (Norms define metric spaces): Let $(V, \|\cdot\|)$ be a normed vector space. Then the function

$$d_{\|\cdot\|}(\cdot,\cdot) : V \times V \to \mathbb{R}_{\geqq 0}; \qquad (x,y) \mapsto d_{\|\cdot\|}(x,y) := \|y - x\|$$

defines a metric space $(V, d_{\|\cdot\|})$.

This assignment is worth three points: **One point each** for pos.definite, symmetry, triangle inequality!

Hint: You will have to show for each one of (12.1a), (12.1b), (12.1c) how it follows from def. 11.17: Which one of (11.31a), (11.31b), (11.31c) do you use at which spot?

Careful with symmetry: What is the reason that $\|a - b\| = \|b - a\|$?

This assignment is worth three points, and you will have to earn them! The following exemplifies the level of detail I expect you to provide.

To prove, e.g., that $d_{\|\cdot\|}(\cdot,\cdot)$ satisfies the triangle inequality (12.1c) of a metric you will have to write something along the following lines:

c. Triangle inequality.
NTS: $d_{\|\cdot\|}(x,z) \leqq d_{\|\cdot\|}(x,y) + d_{\|\cdot\|}(y,z)$ for all $x, y, z \in X$.

Proof:

$d_{\|\cdot\|}(x,z) = \|z - x\|$ (definition of the metric $d_{\|\cdot\|}$)
$= \ldots$ (.....)
$\leqq \ldots$ (.....)
$= d_{\|\cdot\|}(x,y) + d_{\|\cdot\|}(y,z)$ (.....)

(There may be fewer or more steps in your proof. The above only serves as an illustration!)