Math 330 Section 7 - Fall 2026 - Homework 15

Published: Tuesday, November 17, 2026 Running total: 52 points
Last submission: Monday, December 7, 2026

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1 – 14.3; skim/skip ch.4, 9.9, 10.3, 11.2.3 and 12.6

B/G (Beck/Geoghegan) Textbook:

ch.1 – 7.1, ch.8 – 13.4

Other material (optional):

B/K lecture notes: ch.1.1 (Introduction to sets) (optional) and ch.1.2 (Introduction to Functions)

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

Reading assignment 1 - due: Monday, November 23:

- No new material: Catch your breath and review partial orderings in Ch.5.1 and limits + continuity in ch.13.

Reading assignment 2 - due: Tuesday(!), November 24:

a. Carefully read the remainder of MF ch.14.
b. Carefully read MF ch.15.1 and 15.3.

Reading assignment 3 - due: Monday, November 30:

a. Review the end of MF ch.11.2.1, starting at Definition 11.10 (Linear dependence and independence).
b. Carefully read MF ch.15.2. and MF ch.15.4.
c. Carefully read MF ch.15.5.1. Very brief, but also abstract.

Reading assignment 4 - due: Wednesday, December 2:

a. Read carefully MF ch.15.5.2. It's OK to skip the proofs of Lemma 15.3 and Lemma 15.4. They are very technical in nature.
b. Start reviewing for the final exam.

Reading assignment 5 - due Friday, December 4:

- Keep reviewing for the final exam.

Written assignments are on the next page.

Written assignments:

Written assignment 1: (3 points!) Prove Proposition 12.15:
Let $(X, \mathfrak{U})$ be a topological space If $A \subseteq B \subseteq X$, then $A^o \subseteq B^o$.

Written assignment 2: Let (X, d) be a metric space and $A \subseteq X$, $A \neq \emptyset$. Let

$$\gamma := \gamma(A) := \inf\{d(x, y) : x, y \in A \text{ and } x \neq y\}.$$

Prove that if $\gamma > 0$ and $(x_n)_n$ is Cauchy in A, then $(x_n)_n$ is constant, eventually.

Written assignment 3: Let $X := \mathbb{R}$ equipped with the standard Euclidean metric $d(x, x') = |x - x'|$. Let $f_n : \mathbb{R} \to \mathbb{R}$ be the following sequence of functions:

$$f_n(x) := \begin{cases} 0 & \text{if } |x| > \frac{1}{n}, \\ nx + 1 & \text{if } \frac{-1}{n} \leqq x \leqq 0, \\ -nx + 1 & \text{if } 0 \leqq x \leqq \frac{1}{n}, \end{cases}$$

i.e., the point $\big(x, f_n(x)\big)$ is on the straight line between $(-\frac{1}{n}, 0)$ and $(0, 1)$ for $\frac{-1}{n} \leqq x \leqq 0$, it is on the straight line between $(0, 1)$ and $(\frac{1}{n}, 0)$ for $0 \leqq x \leqq \frac{-1}{n}$, and it is on the x–axis for all other x.

Let $f(x) := 0$ for $x \neq 0$ and $f(0) := 1$.
Draw a picture OF F and, for $n = 2$ and $n = 8$, of f_n! It will show you the following:

- Each f_n is continuous (on all of $\mathbb{R}$), but f is not. DO NOT PROVE either of this.
- PROVE that f_n does not converge uniformly to f on $\mathbb{R}$.

Hint: Look at the propositions and theorems of the very short section 13.1.2 (Uniform Continuity) and see whether one of them will make your life easy!