

Name (Print): _____

Problem 1 (1 points). Let A be an $m \times n$ matrix and $\mathbf{b} \in \mathbb{R}^m$. Determine whether the following statements are True or False:

1. _____ A vector \mathbf{b} is a linear combination of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution.
2. _____ Each $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of the columns of A if and only if A has a pivot position in every row.

Problem 2 (4 points). Let $\mathbf{x} \in \mathbb{R}^3$ be a column vector and let

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 3 \\ -6 & 3 & 3 \end{pmatrix}$$

1. How many rows of A contain a pivot position?
2. Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each $\mathbf{b} \in \mathbb{R}^3$?
3. Does the columns of A span \mathbb{R}^3 ?
4. Does the homogeneous equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?