Name (Print): _____

Problem 1 (1 points). What are the three Elementary Row Operations?

- Replacement: Adding a multiple of a row to another row
- Interchange two rows
- Scaling: Multiply a row by a non zero constant.

Problem 2 (4 points). Consider the system of linear equations with augmented matrix

$$\begin{pmatrix} 0 & 1 & -1 & 2 \\ 2 & 5 & 1 & 0 \\ 2 & 6 & 0 & 2 \end{pmatrix}$$

1. Reduce this augmented matrix to REF using Elementary Row Operations.

2. Determine whether the system is consistent or not.

3. Describe the solution set of the system, if it is consistent.

Solutions.

i) $\begin{pmatrix} 0 & 1 & -1 & 2 \\ 2 & 5 & 1 & 0 \\ 2 & 6 & 0 & 2 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 2 & 6 & 0 & 2 \end{pmatrix} \xrightarrow{R_3 - R_1} \begin{pmatrix} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 2 \end{pmatrix}$

$\xrightarrow{R_3 - R_2} \begin{pmatrix} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} =: C$

2) This system is consistent as the augmented column is not a pivot column.

3) Row reduce C to RREF.

$\begin{pmatrix} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 - 5R_2} \begin{pmatrix} 2 & 0 & 6 & -10 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$

So $x_1 \quad\quad + 3x_3 = -5$ $x_1 = -3x_3 - 5$

$x_2 \quad - x_3 = 2$ $x_2 = x_3 + 2$

                              x_3 free.